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ENVIRONMENTAL POLICY AND GROWTH WHEN ENVIRONMENTAL AWARENESS IS ENDOGENOUS

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This paper examines the relationship between environmental policy and growth when green preferences are endogenously determined by education and pollution. We consider an environmental policy in which the government implements a tax on pollution and recycles the revenue to fund pollution abatement activities and/or an education subsidy (influencing green behaviors). When the sensitivity of agents' environmental preferences to pollution and human capital is high, the economy can converge to a balanced growth path equilibrium with damped oscillations. We show that this environmental policy can both remove the oscillations, associated with intergenerational inequalities, and enhance the long-term growth rate. However, this solution requires that the revenue from the tax rate must be allocated to education and direct environmental protection simultaneously. We demonstrate that this type of mixed-instrument environment policy is an effective way to address environmental and economic issues in both the short and the long run.

Keywords: Environmental Policy, Endogenous Growth, Environmental Awareness, Education

1. INTRODUCTION

One of the main questions addressed by the literature on the relationship between economic growth and the environment is how environmental policy can be used to attain sustainable development, in which economic growth is compatible with environmental conservation.[1] Policy makers have at their disposal a number of economic policy instruments developed to achieve this outcome. The most obvious are pollution taxation and public pollution abatement activities (e.g., water treatment, waste management, investment in renewable energy, or

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conservation of forests), both of which have been designed in order to reduce the impact of economic activities on the environment. Yet, governments may also invest in another type of policy tool that aims to improve environmental outcomes only by modifying household behavior. Specifically, investing in education can be employed as an indirect intervention to protect the environment, as educational attainment raises environmental conscientiousness. This idea is largely supported by international organizations. For example, the United Nations declared 2005–2014 to be the "UN Decade of Education for Sustainable Development",[2] while the OECD (2008) refers to education as "one of the most powerful tools for providing individuals with the appropriate skills and competencies to become sustainable consumers". The European Commission (2005) recognizes also that education is a prerequisite for sustainable development and underlines the importance of combining a variety of policy tools to meet this challenge. This point of view is shared by the OECD (2007, 2010), which emphasizes that policy instruments are likely to reinforce each other. Moreover, it appears that pollution taxation may not yield the expected impacts without the use of complementary policies that target household behavior. Despite the growing interest in such environmental policy schemes, especially by international organizations, no theoretical studies have yet examined this combination. Thus, the purpose of this paper is to investigate how an environmental policy that can combine pollution taxation, pollution abatement, and educational support affects economic activities and the environment.

In studying policies that target consumer behavior, it is especially relevant to consider the role of agents' preferences for the environment, which determine how agents respond to pollution. In this regard, Bednar-Friedl (2012) shows that environmental preferences, and in particular differences in these preferences between emerging and industrialized countries, are key factors in determining the specific climate policy that maximizes welfare. Moreover, the empirical literature highlights that environmental awareness evolves over time and changes according to the economic and environmental context [see, e.g., Dunlap and Scarce (1991), European Commission (2008), Scruggs and Benegal (2012)]. We therefore take into account the endogeneity of individual green preferences in order to examine the consequences of environmental policies on growth. More precisely, we consider the role of two major determinants of environmental awareness identified empirically: individual human capital and pollution level. With respect to human capital, the intuition is that the more educated an agent, the more he/she is informed about environmental issues, and the greater his/her potential concern about environmental protection [see, e.g., Blomquist and Whitehead (1998), Witzke and Urfei (2001), European Commission (2008)]. Pollution is thought to influence environmental awareness mainly due to the fact that environmental issues, such as climate change and air pollution, harm welfare. High levels of pollution thus draw cognitive attention to environmental problems, compelling households to recognize the severity of these issues and, perhaps, to react [see, e.g., Dunlap and Scarce (1991), Tjernström and Tietenberg (2008), Schumacher (2009)].

With respect to endogenous environmental awareness, our analysis is related to the recent contribution made by Prieur and Bréchet (2013), who are the first to consider the effect of human capital on green preferences. The authors argue that public education may enable an economy to escape being caught in a steady-state situation with no economic growth and instead to achieve a long-term equilibrium with sustainable growth. In another recent article, Schumacher and Zou (2015) examine the case in which a pollution threshold determines the degree to which agents value the environment. They show that the level of this threshold is a key determinant of the long-term behavior of the economy, in terms of both environmental quality and economic dynamics. Here, we extend these papers by assuming that educational choices are not exogenous but stem from paternalistic altruism and that environmental preferences are driven by both human capital and pollution levels.

Studying the economic implications of an environmental policy, our analysis also contributes to the wide literature on the link between environmental policy and growth, and in particular to the consideration of the role of human capital accumulation in this relationship. For example, Gradus and Smulders (1993) conclude that an improvement in environmental quality can affect the long-term growth of the economy only when pollution directly affects human capital accumulation. More recently, Grimaud and Tournemaine (2007) and Pautrel (2012, 2015) find that a tighter environmental tax can favor education and hence growth in the long run. This occurs because the tax makes polluting activities less attractive compared to a human capital intensive sector. We depart from these papers in three major ways.

First, we analyze the effect of a "mixed-instrument" environmental policy in line with the recommendations made by international organizations regarding growth and the environment. More precisely, we model the implementation of a government tax on pollution whose revenue can be recycled into two types of environmental interventions: public pollution abatement activities and an education subsidy. The former represents an investment in environmental protection, and the latter aims to raise environmental awareness.

Second, in order to study the effect of such an educational intervention on green preferences, we take into account agents' endogenous preferences for the environment, as mentioned above.

Third, given that the economic implications of environmental policies are generally studied in the long run, we also investigate the consequences of this mixed-instrument environmental policy in the short run. Consequently, we follow the approach of contributions such as Zhang (1999) and Ono (2003), who recognize that short-term analysis is a crucial topic of study and that preferences for the environment play an important role in this type of analysis. In particular, Zhang (1999) shows that the economy may exhibit cyclical behavior when green preferences are not sufficiently high and suggests that an environmental policy may be required in order to smooth convergence toward a sustainable equilibrium. In contrast, Ono (2003) finds that high levels of concern for the environment may cause economic fluctuations and demonstrates the importance of examining both short- and long-term policy implications by showing that an increase in a tax on pollution can mitigate these fluctuations and have a nonmonotonous impact on the long-term growth depending on the size of the tax. We contribute to this literature by examining the implications of a mixed-instrument environmental policy, that employs both environmental and educational interventions, when environmental awareness is endogenous.

We develop an overlapping generations model that incorporates environmental quality, and in which growth is driven by human capital accumulation. In this model, production creates a pollution flow, which damages environmental quality, whereas abatement activities improve it. Using this model, we show that the economy can converge to a sustainable long-term equilibrium, in which both environmental quality and human capital grow. We also find that the endogeneity of environmental preferences can cause the economy to experience oscillations along the convergence path. More precisely, when household environmental awareness is highly sensitive to levels of pollution and human capital, green preferences fluctuate across generations, and this affects the household's trade-off between education and abatement activities. This dynamics leads, therefore, to significant variations in the levels of human capital and environmental quality across generations and produces intergenerational inequalities that present a challenge for policy makers.

Furthermore, we show that an increase in pollution tax can eliminate these intergenerational inequalities and improve the long-term growth rate.[3] This win–win scenario is achieved when the tax revenue is allocated to both public pollution abatement activities and an education subsidy, in particular when the allocation enables households to invest sufficiently in education while the provision of environmental maintenance is entirely public. When environmental protection is entirely done by public authorities, households can focus on education, which makes their behavior less dependent on the economic and environmental context so that intergenerational inequalities do not occur. This policy generates a level of environmental quality that is high enough to induce agents to stop contributing to abatement activities (despite a joy of giving for it), while at the same time providing sufficient support to education. In this way, the policy results in a higher sustainable rate of growth. Thus, we conclude that this mixed-instrument environmental policy can successfully address both short- and long-term environmental issues.

The paper is organized as follows. In Section 2, we introduce the theoretical model. Section 3 focuses on the long-term equilibrium and the transitional dynamics. In Section 4, we examine the short- and long-term implications associated with the environmental policy in question. Finally, Section 5 concludes. Technical details are provided in the appendix.

2. THE MODEL

We consider an overlapping generations economy, with discrete time indexed by $t = 0, 1, 2, \ldots, \infty$. Households live for two periods, childhood and adulthood, and make all decisions during the second period. At each date t, a new generation of N identical agents is born ($N > 1$). We assume no population growth.

2.1. Consumer Behavior

Individuals born in $t-1$ care about their levels of consumption c_t and the level of environmental quality Q_t when they are adults. Through paternalistic altruism, they value the human capital of their children h_{t+1}, such that parents finance their children's education [as, e.g., in Glomm and Ravikumar (1992)]. Finally, agents also have altruistic preferences for the environment. Given that environmental quality is a public good, we refer to the behavioral economics literature in order to identify the motives for private provision of this type of collective good. In this literature, both theoretical and empirical studies suggest that such contributions, including investment in environmental protection, arise from an "impure altruism" [e.g., Ribar and Wilhelm (2002), Menges et al. (2005), Crumpler and Grossman (2008)].[4] We thus consider two motives that agents may have for private provision of environmental protection: a "pure altruism" for the level of environmental quality bequeathed to their children (i.e., the future environmental quality Q_{t+1}) and a "joy of giving" associated with the act of contributing itself (i.e., environmental maintenance m_t). In addition to properly representing environmental preferences, the formalization of impure altruism is important for studying environmental policy. This is because the "joy-of-giving" motivational aspect means that public and private contributions are not perfect substitutes [see Andreoni (1990)].

The preferences of a representative agent, born in $t-1$, are represented by the following utility function:

$$U(c_t, m_t, h_{t+1}, Q_{t+1}) = \ln c_t + \gamma_{1t} \ln(\varepsilon_1\ m_t + \varepsilon_2\ Q_{t+1}) + \gamma_2 \ln h_{t+1} + \gamma_3\ \ln Q_t, \tag{1}$$

with γ_{1t}, γ_2, γ_3, ε_1, and $\varepsilon_2 > 0$.

The parameters γ_3 and γ_2 capture an agent's usual taste for current environmental quality and the preference for his/her child's human capital, respectively.[5]

The weight γ_{1t} captures environmental awareness. We assume that these environmental preferences are positively affected by the level of pollution and of individual human capital, as supported by the literature.[6] Pollution has an impact on environmental behaviors through its effect on welfare. The lower the level of environmental quality, the greater the opportunity for an individual to realize the severity of the situation, and therefore the greater his/her incentive to protect the environment, as Dunlap and Scarce (1991) and Schumacher (2009) note. Higher environmental quality, in contrast, may reduce an agent's willingness to improve the environment, as doing so appears less necessary under such conditions. The

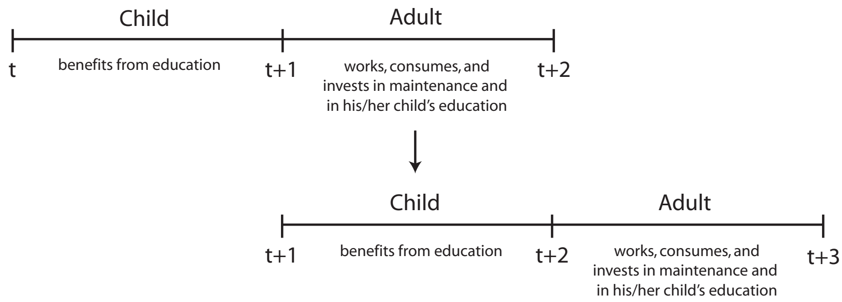

FIGURE 1. Life cycle of two generations for $t > 0$.

empirical behavioral economics literature also identifies education as a key determinant of contributions made to improving environmental quality [see Blomquist and Whitehead (1998), Witzke and Urfei (2001)]. The economic intuition behind this assertion is that the higher an agent's education level, the higher the likelihood that he/she is informed about environmental issues and their consequences, and thus the greater his/her potential environmental concern. We thus assume that $\gamma_{1t} = \gamma_1(h_t, Q_t)$, where γ_1 is increasing and concave with respect to human capital h, and decreasing and convex with respect to environmental quality Q.

For tractability reasons, we consider the following functional form, in line with the form commonly used to represent endogenous longevity, defined as the weight of future arguments in the utility function [see, e.g., Blackburn and Cipriani (2002)]:

$$\gamma_{1t} \equiv \frac{\beta h_t + \eta Q_t}{h_t + Q_t}, \tag{2}$$

with parameters β, $\eta \in [0, 1]$, and $\beta \geqslant \eta$.[7] The parameters β and η embody the weight of human capital and of environmental quality in green preferences, respectively. We note that when $\beta = \eta$, environmental awareness is constant.

As illustrated in Figure 1, during childhood, the agent does not make any decisions. He/she is reared by his/her parents and benefits from education.[8] After reaching adulthood, the agent provides an inelastic supply of one unit of labor remunerated at wage w_t according to his/her level of human capital h_t. He/she allocates this income to consumption c_t, education per child e_t, and environmental maintenance m_t.[9]

The fact that environmental awareness depends on human capital implies that public authorities can use educational support as an environmental policy tool. In this sense, we suppose that the government can subsidize education at rate $0 \leqslant \theta_t^e < 1$ using the revenue derived from a pollution tax that we will present later. This policy reduces the private cost of education, such that the budget constraint for an adult with human capital h_t is

$$c_t + m_t + e_t(1 - \theta_t^e) = w_t h_t. \tag{3}$$

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A child's level of human capital is determined by the private education expenditure and human capital level of his/her parents, so that for a child born in t, his/her level of human capital as an adult is

$$h_{t+1} = \epsilon \; {e_t}^{\mu} {h_t}^{1-\mu}, \tag{4}$$

where $\epsilon > 0$ is the efficiency of human capital accumulation. The parameter $0 < \mu < 1$ is compatible with endogenous growth and captures the elasticity of human capital with respect to private education, whereas $1 - \mu$ represents the share of human capital resulting from intergenerational transmission within the family.

The index of environmental quality, we consider in this paper, reflects an agent's perception of his/her local environment and corresponds to the amenity value derived from, for example, resource availability, the quality of air, water, and soil, or the quality of national parks. The law of motion for the environmental quality index is defined as[10]

$$Q_{t+1} = (1 - \alpha)Q_t - aY_t + b(m_t + M_t + NG_t^m), \tag{5}$$

where $\alpha > 0$ is the natural degradation of the environment, and Y_t represents the pollution flow due to production in the previous period. The parameter $a > 0$ corresponds to the emission rate of pollution, whereas $b > 0$ represents the efficiency of environmental maintenance. These pollution abatement activities are represented by a Cournot–Nash equilibrium approach. Each agent determines his/her own level of environmental maintenance activity (m_t), taking the maintenance level provided by others (M_t) as given. The government can also use the revenue of the pollution tax to directly improve environmental quality, by investing public funds in environmental maintenance activities $NG_t^m \geqslant 0$. Given our definition of the environment, public maintenance activities refer in particular to the implementation of local public projects aimed at protecting the environment.[11] This expenditure corresponds, for example, to water treatment, waste management, or the conservation of biodiversity and landscape. As a result, both public and private efforts to protect the environment co-exist and have an impact on the level of environmental quality. To avoid arbitrarily favoring one type of pollution abatement over another, we assume that the efficiency of public and private investments in environmental maintenances is equal.

The consumer program is summarized by

$$\begin{aligned} &\max_{e_t, m_t}, U(c_t, m_t, h_{t+1}, Q_{t+1}, Q_t) \\ &\quad = \ln c_t + \gamma_{1t} \ln(\varepsilon_1 \; m_t + \varepsilon_2 \; Q_{t+1}) + \gamma_2 \ln h_{t+1} + \gamma_3 \; \ln Q_t, \end{aligned} \tag{6}$$

$$s.t. \quad c_t + m_t + e_t(1 - \theta_t^e) = w_t h_t,$$

$$h_{t+1} = \epsilon \; {e_t}^{\mu} {h_t}^{1-\mu},$$

$$Q_{t+1} = (1 - \alpha)Q_t - aY_t + b(m_t + M_t + NG_t^m),$$

with $m_t \geqslant 0$.

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2.2. Production

The production of the consumption good is carried out by a single representative firm. Output of this good is produced according to a constant returns to scale technology:

$$Y_t = AH_t, \tag{7}$$

where H_t is the aggregate stock of human capital, and $A > 0$ measures a technology parameter.

Because pollution is a by-product of the production process, human capital exerts two opposite effects on the environment: a positive effect through increasing an agent's level of environmental awareness and thus his/her motivation for environmental protection, and a negative effect through an increase in production that raises the amount of waste and pollution emissions. Indeed, even if human capital is not a highly polluting input per se, it is a determining factor in the scale of production and hence of emissions.[12] In order to maintain the tractability of the analysis as well as a focus on the mechanisms linked to human capital, we assume that the only input in the production function is aggregate human capital and that the share of polluting factors in the production process is constant and represented by the parameter A.[13] Therefore, the polluting production process is composed of an index of pollution intensity A as well as the level of human capital, and together these elements determine the pollution emissions.

Defining $y_t \equiv \frac{Y_t}{N}$ as the output per worker and $h_t \equiv \frac{H_t}{N}$ as the human capital per worker, we have the following production function per capita:

$$y_t = Ah_t. \tag{8}$$

The government collects revenues through a tax rate $0 \leqslant \tau < 1$ on production, the source of pollution.[14] The firm chooses input to maximize its profit $(1-\tau)Y_t - w_t H_t$, such that

$$w_t = A(1-\tau). \tag{9}$$

2.3. The Government

The design of environmental policies represents a major challenge for governments. The OECD (2007, 2008), among others, recommends recycling pollution tax revenues as a way for governments to mitigate the externalities generated by polluting activities. This type of policy has precedent in several countries. In France, for example, the government implements a general tax on polluting activities and transfers the revenues of this tax to the French Environment and Energy Management Agency (ADEME) that funds projects to improve environmental quality.

In this model, we consider the following policy scheme. Since pollution is a by-product of the production process, the government taxes production output at rate τ and the public budget generated by the revenue from this tax is spent on public environmental maintenance NG_t^m and/or on education subsidies θ_t^e.[15] The government's budget is balanced at each period, such that

$$N(\theta_t^e e_t + G_t^m) = \tau Y_t. \tag{10}$$

To study possible policy mixes in a simple way, we define the constant share of public expenditure that is devoted to public environmental maintenance as $0 \leqslant \sigma \leqslant 1$, and the share that is devoted to an education subsidy as $(1-\sigma)$, with

$$\sigma = \frac{NG_t^m}{\tau Y_t}; \quad 1-\sigma = \frac{N\theta_t^e e_t}{\tau Y_t}, \tag{11}$$

such that the fiscal policy is summarized by two instruments: the pollution tax τ and the allocation of public revenue σ, both taken as given by consumers. With this general formalization, we examine the specific effects of each policy. We accomplish this by setting σ to 0 in order to model the case in which the government supports only education, and by setting σ to 1 in order to examine the case in which the government finances only public maintenance activities. Finally, we also model a mixed-instrument scenario in which public funds are allocated to a combination of environmental maintenance and education subsidies.

2.4. Equilibrium

The maximization of the consumer program (6) leads to optimal choices regarding education and maintenance in two regimes: an interior solution, in which both individuals and the government invest in environmental protection $m_t > 0$ (hereafter *pm*), and a corner solution in which the government is the only contributor to the public good, i.e., $m_t = 0$ (hereafter *npm*). The intertemporal Nash equilibria are given by

$$m_t = \begin{cases} \frac{\gamma_{1t} c_1 A h_t (1-\tau) - \varepsilon_2 (1+\gamma_2 \mu)[(1-\alpha) Q_t + ANh_t(b\sigma\tau - a)]}{\gamma_{1t} c_1 + c_2 + \varepsilon_2 bN(1+\gamma_2\mu)} & pm \\ 0 & npm, \end{cases} \tag{12}$$

$$e_t = \begin{cases} \frac{\gamma_2 \mu \{c_3 A(1-\tau) h_t + \varepsilon_2 [(1-\alpha) Q_t + ANh_t(b\sigma\tau - a)]\}}{\gamma_{1t} c_1 + c_2 + \varepsilon_2 bN(1+\gamma_2\mu)} + (1-\sigma)\tau A h_t & pm \\ \frac{Ah_t[\gamma_2\mu(1-\tau) + \tau(1-\sigma)(1+\gamma_2\mu)]}{1+\gamma_2\mu} & npm, \end{cases} \tag{13}$$

where c_1, c_2, and c_3 are three positive constants defined by $c_1 \equiv \varepsilon_1 + \varepsilon_2 b$, $c_2 \equiv \varepsilon_1(1+\gamma_2\mu)$, and $c_3 \equiv \varepsilon_1 + \varepsilon_2 bN$, respectively.[16]

Education spending depends positively on environmental quality. The greater the health of the environment, the lower the optimal amount of maintenance activities. As a result, education becomes more attractive and individuals are able to devote more resources to educating their children.

Moreover, public policy instruments shape education and abatement spending in different ways. An increase in the pollution tax implies a negative income effect (wage decreases) but continues to favor education spending as long as

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the share of public expenditure in education subsidies is sufficiently high (σ low). However, an increase in the pollution tax always has a negative effect on environmental maintenance activities. In addition to the negative income effect, the tax increases the amount of public pollution abatement activities, which crowds out private maintenance activities. Nevertheless, public spending on environmental maintenance is only a partial substitute for private spending due to the joy-of-giving motive in the utility function.

REMARK 1. *Below we note the implications of the model without a joy-of-giving motive for the environment (i.e., $\varepsilon_1 = 0$).*

- *If all public expenditure were devoted to pollution abatement ($\sigma = 1$), there would be a total crowding-out of private maintenance such that the environmental policy would have no effect on the environment.*
- *If the budget were allocated to both types of expenditure ($\sigma < 1$), the fall in private maintenance would outweigh the increase in public maintenance. Consequently, the overall level of abatement activity would decrease with the implementation of the environmental policy.*

This scenario contradicts the empirical and experimental literature, which demonstrates only a partial crowding-out of private contributions by government expenditure.[17] *Thus, the joy-of-giving motive (ε_1) is necessary for a meaningful policy analysis in our setting, such that the policy does not have a zero impact on the total amount of maintenance (private and public).*

To study endogenous growth, we write the environment in intensive form, by considering environmental quality per unit of human capital $\frac{Q_t}{h_t} \equiv X_t$. This ratio enables us to capture the evolution of environmental quality relative to the development of economic activities. For the remainder of the analysis, we define X_t as a green development index. Thus, environmental awareness, given by (2), can be rewritten as

$$\gamma_{1t} = \frac{\beta + \eta X_t}{1 + X_t}. \tag{14}$$

Using equations (12) and (14), we emphasize that households invest in environmental protection for

$$X_t \leqslant \frac{A\left[\frac{(\beta+\eta X_t)c_1(1-\tau)}{1+X_t} - \varepsilon_2 N(1+\gamma_2\mu)(b\sigma\tau - a)\right]}{\varepsilon_2(1+\gamma_2\mu)(1-\alpha)}. \tag{15}$$

From this inequality, we deduce that there exists a critical threshold $\Lambda(\sigma, \tau) \geqslant 0$ such that environmental protection is privately supported as long as[18]

$$X_t \leqslant \Lambda(\sigma, \tau). \tag{16}$$

When this inequality is unsatisfied [$X_t > \Lambda(\sigma, \tau)$], the level of environmental quality is sufficiently high enough, and/or the level of human capital sufficiently low enough, to reduce the private provision of abatement activities to zero. The critical threshold $\Lambda(\sigma, \tau)$ is decreasing in the tax on pollution τ and in the share of public spending devoted to environmental maintenance σ. Therefore, when the government intervenes, the economy is more likely to be characterized by no private maintenance activities since public maintenance partially substitutes for private maintenance. This result continues to hold if the recycled revenues are entirely devoted to education since agents replace some abatement activities with education as the latter becomes relatively less costly. On the other hand, the higher an agent's green preferences, the higher the level of environmental quality at which the agent decides to stop supporting environmental protection [the higher the critical threshold $\Lambda(\sigma, \tau)$].

Using the human capital accumulation (4), the environmental quality process (5), and the first-order conditions (12) and (13), we write the dynamic equation characterizing the equilibrium paths.

DEFINITION 1. *Given the initial condition $X_0 = \frac{h_0}{Q_0} > 0$, the intertemporal equilibrium is the sequence $(X_t)_{t\in\mathbb{N}}$ that satisfies, at each t, $X_{t+1} = \mathcal{F}(X_t)$, with*

$$\mathcal{F}(X_t) = \begin{cases} \{(1-\alpha)\, X_t\, [\gamma_{1t}\, c_1 + c_2] + AN\, [\gamma_{1t}\, c_1 b(1-\tau) + (\gamma_1 c_1 + c_2)(b\tau\sigma - a)]\} \\ \times \dfrac{[\gamma_2\mu\, (\varepsilon_2(1-\alpha)X_t + \varepsilon_2 AN(b\sigma\tau - a) + A\, c_3(1-\tau)) + (1-\sigma)\tau A(\gamma_{1t}\, c_1 + (1+\gamma_2\mu)c_3)]^{-\mu}}{\epsilon\, [\gamma_{1t}\, c_1 + (1+\gamma_2\mu)c_3]^{1-\mu}} & pm \\ \dfrac{(1-\alpha)X_t + AN(b\sigma\tau - a)}{\epsilon\left[\dfrac{\gamma_2\mu A(1-\tau) + (1-\sigma)\tau A(1+\gamma_2\mu)}{1+\gamma_2\mu}\right]^{\mu}} & npm \end{cases}$$

(17)

3. BALANCED GROWTH PATH AND TRANSITIONAL DYNAMICS

In this section, we examine the existence of a balanced growth path (hereafter BGP).

DEFINITION 2. *A BGP satisfies Definition 1 and has the following additional properties: The stock of human capital and environmental quality grow at the same constant rate g_i, with subscripts $i = \{pm, npm\}$ denoting, respectively, the regime with private maintenance and the regime with no private maintenance. This equilibrium path is such that the green development index X_t is constant and defined by $X_{t+1} = X_t = \bar{X}_i$.*

From Definitions 1 and 2 and equations (14) and (16), we explore the properties of the dynamic equation $\mathcal{F}$ and deduce the existence of a BGP $\bar{X}_i$ corresponding to the solutions of equation $\bar{X}_i = \mathcal{F}(\bar{X}_i)$.

PROPOSITION 1. *When $\beta > \eta$ and $\sigma_{Min}(\tau) < \sigma < \sigma_{Max}(\tau)$ for all $\tau \in$* [0, 1)*, there exists a unique positive BGP $(\bar{X}_i)$, such that, according to a critical threshold $\hat{\sigma}(\tau)$:*

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- *when $\sigma > \hat{\sigma}(\tau)$, the BGP occurs in the regime without private maintenance (npm), and*
- *when $\sigma < \hat{\sigma}(\tau)$, the BGP occurs in the regime with private maintenance (pm),*

where $\sigma_{Min}(\tau) \equiv \frac{a(\beta c_1+c_2)-b\beta c_1(1-\tau)}{b\tau(\beta c_1+c_2)}$, $\sigma_{Max}(\tau) \equiv \frac{A(\gamma_2\mu+\tau)-\epsilon^{-1/\mu}(1+\gamma_2\mu)(1-\alpha)^{1/\mu}}{A\tau(1+\gamma_2\mu)}$, and $\hat{\sigma}(\tau)$ is a decreasing function of τ, with $\lim_{\tau\to 0}\hat{\sigma}(\tau) = +\infty$ and $\lim_{\tau\to 1}\hat{\sigma}(\tau) = a/b$.

Proof. See Appendix A.2.

Such a balanced growth path corresponds to a long-term equilibrium with sustainable development, in which both human capital and environmental quality improve across generations.[19] From Proposition 1, we identify two possible long-term regimes—with or without private provision of environmental maintenance. When the share of public spending devoted to environmental protection is sufficiently large [$\sigma > \hat{\sigma}(\tau)$], households stop privately investing in abatement in the long run despite their willingness to protect the environment. This occurs due to the trade-off the agent makes between education and maintenance spending. On the one hand, low public investment in education leads individuals to increase their private efforts to finance education. On the other hand, the improvements in environmental quality made possible by the policy reduce the need for private provision of abatement activities. We note, however, that the greater the degree of environmental awareness (γ_1), the more public spending on the environment is necessary in order to facilitate the long-run equilibrium in the *no private maintenance* regime [$\hat{\sigma}(\tau)$ increases with green preferences].

Whatever the regime, the existence of a long-term sustainable development depends on several conditions, summarized in Proposition 1 by $\sigma_{\text{Min}}(\tau) < \sigma < \sigma_{\text{Max}}(\tau)$ for all $\tau \in [0, 1)$. These conditions pertain to specific economic parameters and policy instruments employed.

With respect to economic parameters, the efficiency of human capital accumulation (ϵ) and of environmental maintenance (b) must be sufficiently high. In particular, a prerequisite to obtaining sustainable development is that the environmental benefit resulting from one unit of abatement is larger than the environmental damage resulting from one unit of production ($b > a$). This seems a reasonable condition in our model, as the environmental quality that we consider is the quality that is perceived by agents. Since individuals preferentially invest in abatement activities that target the environmental issues that matter most to them and pollution is simply a side-effect of the aggregate production process, the marginal benefit of abatement can be assumed to be relatively higher than the marginal damage of production.

Second, the condition described above depends on how the government recycles pollution tax revenue. To examine this issue further, we depict $\sigma_{\text{Min}}(\tau)$ and $\sigma_{\text{Max}}(\tau)$ in Figure 2.

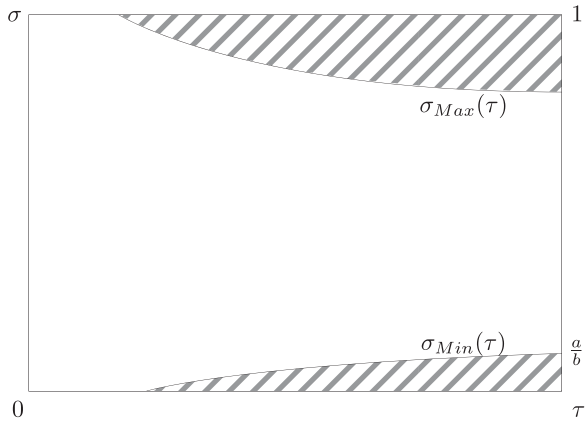

FIGURE 2. Environmental policy conditions that lead to sustainable development.

As illustrated in Figure 2, extreme allocations of public funds are excluded for high levels of the tax (shaded area). The reason for this is that when the tax rate is high, the share of an agent's income that is available for investment is low. When the share of the tax revenue used for education subsidies or environmental maintenance is relatively too great, one of the two actions that drive sustainable development is highly disadvantaged (too expensive and not sufficiently supported through public funds). Thus, in order to achieve a state in which both environmental quality and the economy improve, the government must allocate its public budget judiciously (white area).

For the remainder of the paper, we assume the condition of Proposition 1 to be true.[20]

Assumption 1. For all $\tau \in [0, 1)$, we assume that $\sigma_{\text{Min}}(\tau) < \sigma < \sigma_{\text{Max}}(\tau)$.

From the dynamic equation, we derive the stability properties of the BGP presented in Proposition 2. Note that when the BGP occurs in the *no private maintenance* regime, we obtain an explicit solution whose dynamics is easily deduced. However, in order to analyze the stability of the equilibrium in the *private maintenance* regime, we normalize $\bar{X}_{pm}$ to 1 using the scaling parameter ϵ.

PROPOSITION 2. *Under Assumption 1 and $\beta > \eta$:*

- *The BGP in the regime without private maintenance, $\bar{X}_{npm}$ [i.e., $\bar{X} > \Lambda(\sigma, \tau)$], is globally and monotonously stable.*
- *The BGP in the regime with private maintenance, $\bar{X}_{pm}$ [i.e., $\bar{X} < \Lambda(\sigma, \tau)$], is locally stable and for $N > \bar{N}(\tau, \sigma)$, there exists a $\tilde{\beta}(\tau, \sigma) \in (0, 1]$ such that*
  - *when $\beta < \tilde{\beta}(\tau, \sigma)$, the convergence is monotonous,*
  - *when $\beta \geqslant \tilde{\beta}(\tau, \sigma)$, the convergence is oscillatory,*

where $\Lambda(\sigma, \tau)$ is the critical threshold of X_t at which the economy is in the no private maintenance regime.[21]

Proof. See Appendix A.3.

Figure 3 provides an illustration of the cases identified in Proposition 2.

As underlined in Proposition 2, the economy may display damped oscillations due to endogeneity of green preferences.[22] The oscillatory dynamics emerges when an agent's environmental awareness γ_1 is highly sensitive to the green development index X ($\beta \geqslant \tilde{\beta}$) and when the population size is not too low ($N > \bar{N}$), such that individuals' behavior has a significant impact on environmental quality.[23]

The mechanism responsible for the emergence of this dynamics consists of a feedback effect between the green development index and environmental awareness, which impacts the trade-off between education and environmental maintenance spending. Indeed, in the absence of private maintenance [i.e., $\bar{X} > \Lambda(\sigma, \tau)$], this trade-off does not exist, as households focus solely on education, and the dynamics is therefore always monotonous. Reversely, when an agent invests in environmental protection [i.e., $\bar{X} < \Lambda(\sigma, \tau)$], cyclical convergence may occur and can be described as follows. An increase in environmental awareness γ_{1t} encourages private investment in the environment at the expense of investment in education. This generates a fall in human capital h_{t+1}, a rise in environmental quality Q_{t+1}, and thus a decline in green preferences γ_{1t+1} for the next generation. These changes impact private choices in several ways. Each change negatively affects private investment in environmental maintenance m_{t+1}, whereas their impact on private investment in education e_{t+1} is ambiguous. Indeed, education spending is positively affected by the fact that private provision of environmental quality becomes both less necessary (from the raise in Q_{t+1}) and less desirable (from the decrease in γ_{1t+1}), whereas it is negatively affected by an income effect due to the fall in human capital h_{t+1}. The opposite variation of human capital h_{t+1} acts therefore as a brake on oscillations and serves a stabilizing effect, such that cyclical variations are damped. As a result, the economy displays oscillations as long as the two positive impacts on education exceed the negative income effect. Note that this is possible only when environmental awareness is highly responsive to the economic and environmental context ($\beta > \tilde{\beta}$), which enables the positive impacts to dominate through γ_1.

This oscillatory dynamics implies that environmental awareness (γ_1) undergoes variations along the converging trajectory. Because human capital and environmental quality are two stock variables, there is some inertia in preferences that mitigates extreme variations across generations. Significant variations in green preferences are, however, consistent with empirical evidence. For example, Dunlap and Scarce (1991) remind us that, although public concern for the environment has increased globally since the late 1960s, it has also experienced fluctuations according to economic and environmental context. Likewise, Scruggs and Benegal (2012) show that the decline of public concern about climate change observed since 2008 in the United States is due in large part to the great recession. These studies

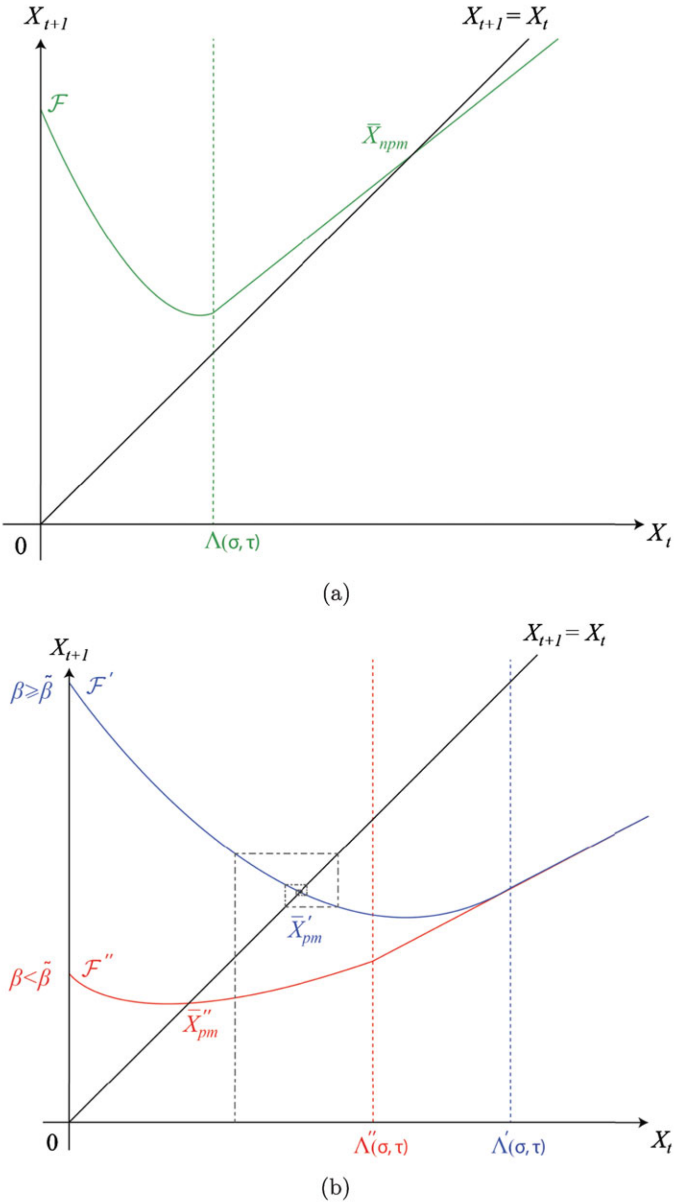

FIGURE 3. Dynamics for $N > \bar{N}$ when (a) $\sigma > \hat{\sigma}(\tau)$ and (b) $\sigma < \hat{\sigma}(\tau)$.

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demonstrate that a rise in environmental problems tends to lead to a substantial increase in public support for environmental protection, whereas an intensification of economic issues tends to have the reverse effect.

In this paper, we find that fluctuations in environmental awareness cause variations in environmental quality and human capital across generations. It follows that some generations experience higher levels and growth rates of human capital and environmental quality than others. As Seegmuller and Verchère (2004) show, cyclical convergence paths result in welfare variations across generations and are associated with intergenerational inequalities. The term inequality refers here to the fact that, when these fluctuations occur, the high levels of environmental quality that are enjoyed by some generations are obtained at the expense of other generations. As a result, this complex dynamics translates into cyclical short-run growth and hence is closely related to the concept of volatility, as Varvarigos (2011) argues. This result underscores the importance of studying the short-term effects associated with environmental policies.

4. ENVIRONMENTAL POLICY IMPLICATIONS

In this section, we analyze the consequences of the environmental policy on intergenerational inequalities in the short run and on growth in the long run.

4.1. The Short-Term Effect of the Environmental Tax

In the preceding analysis, we showed that the economy may exhibit complex dynamics when environmental awareness is endogenous. In this section, we model how a tighter environmental tax affects this short-term scenario and whether the use of a policy mix allows for a reduction in intergenerational inequalities.[24] Focusing on the BGP in the *private maintenance* regime, where damped oscillations may occur, we examine the effect of an increase in the environmental tax on transitional dynamics.[25]

PROPOSITION 3. *Under Assumption 1 and $\beta > \eta$, for a sufficient increase in the tax on pollution, $\hat{\sigma}(\tau)$ becomes lower than σ. In this way, the BGP that occurs in the regime with private maintenance moves to the regime without private maintenance, in which there are no oscillations.*

An increase in the environmental tax makes more likely that agents do not privately invest in environmental protection at the BGP [as $\hat{\sigma}(\tau)$ declines]. The main reason for this is that a tighter tax allows for an increase in the government's budget, and hence in the amount of public environmental maintenance. Moreover, as there is no longer a trade-off between private choices in this regime, transitional dynamics do not result in oscillations. Thus, the government may avoid intergenerational inequalities by fixing a sufficiently high tax on pollution and devoting a large enough share of public spending to environmental maintenance [such that

$\hat{\sigma}(\tau) < \sigma$], enabling the long-term equilibrium to move to the regime in which environmental maintenance is entirely publicly funded.

The tax needed to achieve the regime without private provision of environmental maintenance may, however, be very high. In these extreme cases, the policy appears unreasonable as it deprives agents of consumption and reduces their welfare. In this regard, it seems pertinent to study the possible effects of the policy when the economy remains in the regime with private maintenance.

REMARK 2. *Under Assumption 1 and $\beta > \eta$, an increase in τ which does not allow the BGP to reach the regime without private maintenance* [$\sigma < \hat{\sigma}(\tau)$]*, there exists a $\tilde{\sigma} \in (0, 1)$ such that*

- *for $\sigma < \tilde{\sigma}$, the cases in which oscillations occur are less frequent,*
- *for $\sigma > \tilde{\sigma}$, the cases in which oscillations occur may be more or less frequent.*

Proof. See Appendix A.4.

From Remark 2, we observe that the government intervention may either neutralize or generate oscillations and hence intergenerational inequalities. This dynamics arises from the fact that the policy shapes the trade-off between maintenance and education spending, and hence the mechanism driving oscillations. As previously mentioned, this mechanism relies on the fact that variations in spending on education stem from environmental quality Q, environmental awareness γ_1, and human capital h. Changes driven by the former two factors exacerbate oscillations, whereas changes driven by the latter attenuate oscillations.

When $\sigma < \min\{\tilde{\sigma}; \hat{\sigma}(\tau)\}$, an increase in the environmental tax reduces the frequency of cases in which oscillations arise. The reason for this is that as long as σ is low enough, public spending sufficiently supports education so that a tighter tax reinforces the impact of human capital on private education spending. Indeed, the fall in wages, caused by the pollution tax, is overcompensated by the increase in the education subsidy. Thus, spending on education is mainly driven by government action and becomes less sensitive to variations in green preferences γ_1. As a result, oscillatory trajectories are less frequent, i.e., the conditions under which we observe damped oscillations are more restricted.

When $\tilde{\sigma} < \sigma < \hat{\sigma}(\tau)$, the policy may increase the occurrence of intergenerational inequalities. The intuition is the following. Public revenue devoted to abatement activities σ is such that agents continue to invest in environmental maintenance and it is not low enough to ensure that the level of the education subsidy prevents oscillations. Indeed, the environmental tax diminishes the influence of human capital on private investment in education and oscillations may occur more frequently as a result. In this case, the conditions that are necessary for damped oscillations are more easily satisfied.

Thus, our short-term analysis of the effect of the environmental policy reveals that the government can play a role in avoiding the fluctuations in environmental preferences that are costly in equities across generations. To accomplish this, policy makers should invest sufficiently in education or in environmental protection in order to reduce the dependency of households behavior on the economic and environmental contexts. Our study is one of the few that examine the implications of environmental policy in the short run. Notable exceptions are Ono (2003) and Palivos and Varvarigos (2017) who emphasize that an environmental policy may shift the economy from a fluctuating regime to a long-term state with growth. This is the case for a sufficient increase in the environmental tax in Ono (2003) and when the policy consists in a public investment in pollution abatement in Palivos and Varvarigos (2017). In this paper, we demonstrate that the way in which the revenue from the environmental tax is used is a determining factor in producing such a change.

4.2. The Long-Term Effect of the Environmental Tax

In accordance with the concept of sustainable development and the well-known definition developed by the Brundtland Commission [WCED (1987)], environmental policy attempts to strike the right balance between economic and environmental interests in the long run. In this respect, we wish to explore what solutions the government can put in place in order to achieve higher long-term growth in both human capital and environmental quality. In this section, we therefore examine how the environmental policy affects the long-term equilibrium and the corresponding growth rate.

Using equations (4) and (13), the expression of the long-term growth rate is given by

$$1+g_H = \begin{cases} \epsilon\left[\dfrac{\gamma_2\mu\left\{A(1-\tau)c_3+\varepsilon_2\left[(1-\alpha)\bar{X}_{pm}+AN(b\sigma\tau-a)\right]\right\}+(1-\sigma)\tau A[\bar{\gamma}_1 c_1+c_3(1+\gamma_2\mu)]}{\bar{\gamma}_1 c_1+(1+\gamma_2\mu)c_3}\right]^{\mu} & pm \\ \epsilon\left[\dfrac{A[\gamma_2\mu(1-\tau)+\tau(1-\sigma)(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu} & npm. \end{cases} \quad \textbf{(18)}$$

A more strict environmental policy influences the long-term growth rate through several channels. First, it influences growth directly, by affecting the trade-off between education and maintenance activities. Second, it influences growth indirectly by modifying the green development index and environmental preferences.[26] As a result, the global impact of the policy on growth is ambiguous in the long run. In the following proposition, we show how authorities can improve the growth rate along the BGP.

PROPOSITION 4. *Under Assumption 1 and $\beta > \eta$, following an increase in τ:*

- *When the BGP remains in the pm regime [$\sigma < \hat{\sigma}(\tau)$], there exists an interval [$\underline{\sigma}(\tau), \overline{\sigma}(\tau)$] such that the growth rate increases for $\underline{\sigma}(\tau) < \sigma < \overline{\sigma}(\tau)$.*

- *When the BGP initially occurs in or moves to the npm regime [$\sigma > \hat{\sigma}(\tau)$], the growth rate improves and is greater than in the pm regime for $\sigma < \frac{1}{1+\gamma_2\mu}$.*

Proof. See Appendix A.5.

Considering a tighter tax, an economy in which agents initially invest in the private provision of environmental maintenance may switch to the other regime, in which no private investment occurs. When the BGP remains in the regime with private provision of maintenance [$\sigma < \hat{\sigma}(\tau)$], an increase in the tax favors both human capital and environmental quality if the allocation between public spending σ is at an intermediate level. This occurs because extreme allocations render one of the two private spending (maintenance and education) too expensive relative to the other, which leads agents to neglect one of the two drivers of growth. When σ is too low, the policy favors spending on education. Despite the improvement in environmental consciousness it entails, an increase in the tax makes private maintenance too expensive, leading to a deterioration of the environment and of the growth rate. Conversely, when σ is too high, tax revenues contribute mostly to public maintenance of the environment. Even if private investment in the environment diminishes in favor of educational spending, education costs remain too high, which weakens human capital accumulation. Authorities can therefore increase growth only by supporting both education and maintenance simultaneously. Note that the two thresholds $\underline{\sigma}(\tau)$ and $\overline{\sigma}(\tau)$ increase with the parameters of green preferences β and η. It follows that the higher the environmental awareness, the larger the public environmental support necessary to ensure an improvement of economic growth.

When households do not invest in private maintenance in the long run or when they stop investing in it following the government intervention [$\sigma > \hat{\sigma}(\tau)$], a tighter tax leads to the highest growth rate as long as it is accompanied by sufficient support for human capital accumulation [$\sigma < 1/(1 + \gamma_2\mu)$]. The intuition is the following. On the one hand, in the *no private maintenance* regime, maintenance is entirely publicly funded despite agents' willingness to contribute to pollution abatement activities. Environmental quality is thus sufficiently high. On the other hand, when the government also allocates a large enough portion of its budget to education, the negative effect of the tax on available income is more than offset by the positive effect of the tax that operates through the education subsidy. In this way, human capital accumulation and the environment are both enhanced. Note that when $\sigma > 1/(1 + \gamma_2\mu)$, a tighter tax is growth reducing when maintenance is initially entirely public. This result may also be observed when there is a regime switch, particularly if the increase in τ when moving the BGP to the regime without private maintenance is significant. In this case, the negative income effect exceeds the positive impact of the education subsidy and hence human capital accumulation deteriorates.

Our results contribute to the literature examining the effects of environmental policies on sustainable development [e.g., Ono (2003), Grimaud and Tournemaine

(2007), Prieur and Bréchet (2013), Pautrel (2015)]. However, we differ from the majority of this literature by calling attention to the role of an environmental policy mix that is composed of support for both environmental protection as well as education. Whatever the tax rate, we emphasize that the recycling of the fiscal revenue can lead to a win–win situation in terms of improvements in environmental quality as well as economic growth. However, this result is only ensured if the government invests in both education and environmental protection. More specifically, despite the positive effect of environmental quality on education, a policy focusing solely on public maintenance of the environment may reduce human capital and growth. In contrast, despite the fact that human capital contributes to environmental awareness, a policy providing support for education alone may not be sufficient to enhance environmental quality, and hence to favor a sustainable growth.

4.3. How Can the Government Policy Improve Both the Short- and Long-Term Scenarios?

We have previously emphasized the role that the government can play in avoiding intergenerational inequalities in the short run and in enhancing economic and environmental growth in the long run. In this section, we examine how a tighter tax can generate both short- and long-term benefits. To do so, we consider the properties of the function $\hat{\sigma}(\tau)$, that defines the regimes with and without private maintenance, in conjunction with the conditions $\sigma_{\mathrm{Min}}(\tau)$ and $\sigma_{\mathrm{Max}}(\tau)$, which determine the possible policy schemes compatible with sustainable development (see Assumption 1). We summarize the results of Propositions 3 and 4 in Figure 4, which depicts the implications of a tighter tax for a given σ.[27] Four areas are distinguished.

In areas 1 and 2, the long-term equilibrium of the economy occurs in the regime with private maintenance. Turning to the short run, a tighter tax makes the occurrence of oscillations less likely when $\sigma < \tilde{\sigma}$ (area 1), whereas the opposite holds when $\sigma > \tilde{\sigma}$ (area 2). To improve long-term growth in these areas, an increase in the pollution tax must be associated with an intermediate level of σ [$\underline{\sigma}(\tau) < \sigma < \overline{\sigma}(\tau)$]. Unfortunately, no clear conclusion emerges when comparing these values of σ with the value that corresponds to environmental and economic benefits in the short run.

As long as the tax rate is sufficiently high, the economy is able to achieve areas 3 and 4, where the BGP is in the regime without private maintenance and hence contributions to pollution abatement activities are entirely publicly funded. In this state, the short-term issue vanishes, and the economy achieves a higher long-term growth rate when the education subsidy is sufficiently high (σ low, area 4). Furthermore, as stressed in Proposition 4, for a given σ, the growth rate is higher in area 4 than in the other regime.

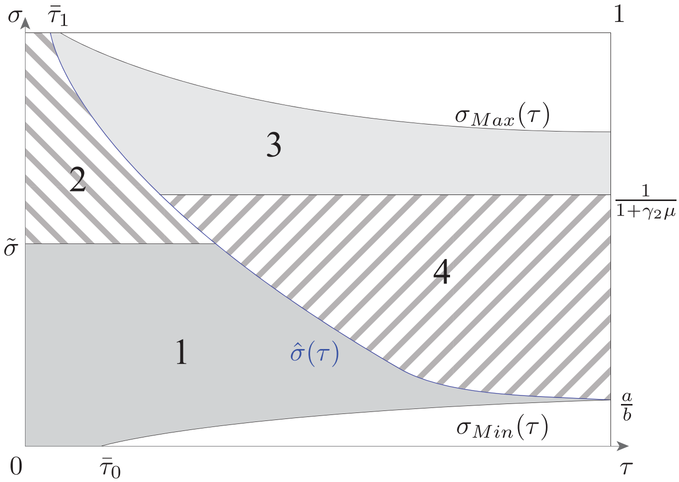

FIGURE 4. Short- and long-term implications of a tighter tax at a given σ, when $\frac{a}{b} < \frac{1}{1+\gamma_2\mu}$.

As a result, we identify the most favorable tax scheme as one in which environmental protection is funded exclusively by public investment and educational support is sufficiently high (area 4). This scheme is recommended for two reasons. First, even if the efficiency of public and private spending on environmental maintenance are the same, our findings indicate that the government should take full responsibility for environmental protection so as to avoid intergenerational inequalities. In this way, households may focus on their children's education and their spending on environmental maintenances is no longer vulnerable to potential fluctuations in green preferences. This second outcome is a particularly noteworthy result, given empirical evidence demonstrating that environmental preferences can fluctuate significantly according to the economic and environmental contexts in which agents are situated [see Dunlap and Scarce (1991), Scruggs and Benegal (2012)]. Second, in this case, the level of environmental quality is sufficiently high so that agents do not privately contribute to the environment despite the increase in welfare they might receive from doing so. Moreover, human capital accumulation is sufficiently supported to achieve both higher environmental and economic growth. Thus, we conclude that a mixed-instrument policy is a promising tool for encouraging sustainable development.

Our analysis provides additional insights through an examination of extreme allocations of the tax revenue between education and environmental maintenance. We conclude that the effects of an environmental policy that consists in investing only in education ($\sigma = 0$) is not satisfying. From Figure 4, such a policy scheme is always located in area 1. Despite the fact that this policy reduces the risk of experiencing oscillations (a source of intergenerational inequalities), it does not guarantee a higher growth rate. Indeed, for the case in which green preferences are not sufficiently sensitive to human capital, this policy leads to a low level of environmental protection, which damages economic growth. In contrast, when tax revenue is entirely devoted to maintenance ($\sigma = 1$), only areas 2 and 3 can be achieved. Thus, the policy can completely eliminate oscillatory behavior but this is not sufficient to guarantee a higher rate of growth. The negative direct effect of the policy on educational spending exceeds the positive indirect effect that operates through the improvement in environmental quality. The analysis of these two extreme cases confirms our suggestion that a policy mix is necessary to achieve the best situation in both the short and long run since it corresponds to a policy scheme in area 4 of Figure 4 that is not compatible with $\sigma = 0$ or $\sigma = 1$. Moreover, these two cases represent the most restrictive policy schemes in which the range of the pollution tax that guarantees sustainable development is limited by $\bar{\tau}_0$ when $\sigma = 0$ and $\bar{\tau}_1$ when $\sigma = 1$.[28]

Finally, note that even when we conclude that environmental maintenance activities should be entirely publicly funded, environmental awareness is important as it affects the policy to implement. When green preferences are higher, for any given tax the government should orient further its policies toward environmental protection in order to both avoid intergenerational inequalities and improve growth (area 4).

5. CONCLUSION

In this paper, we examine the implications of an environmental policy mix on sustainable development when environmental awareness is endogenously determined by both individual's human capital and pollution. We model an environmental policy that consists of implementing a tax on pollution, the revenues of which can be allocated between two categories of environmental expenditure: public pollution abatement activities and an education subsidy that aims to develop ecological consciousness. Public investment in education and environmental maintenance consequently affect agents' choices regarding private investment in these two areas. If public investment in environmental maintenance is sufficiently high, households may stop investing privately in maintenance activities despite the benefits they could derive from this investment.

Using the present model, we show that the economy can converge to a long-term equilibrium characterized by sustainable growth. However, when environmental preferences are highly sensitive to changes in human capital and pollution levels, this convergence may be oscillatory. This is due to the feedback effect of human capital and environmental quality on endogenous environmental awareness, which shapes the trade-off between private investment choices. This complex dynamics represents an important issue, as it can cause variations in welfare across generations that lead to intergenerational inequalities.

Our study reveals that an environmental policy can eliminate these inequalities in two ways: by providing sufficiently high support for environmental maintenance activities such that private contributions are no longer made, or by sufficiently subsidizing education so as to make private investment decisions less dependent

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on green preferences. Additionally, our analysis indicates that a tighter tax can also improve the long-term economic and environmental growth rate when the tax revenue is allocated between both types of public expenditures simultaneously. In this case, human capital accumulation is favored without damaging the environment. We wish to emphasize that this type of policy can result in a win–win situation, by both avoiding intergenerational inequalities as well as fostering growth of in human capital and environmental quality. More specifically, our results indicate that the most favorable policy scheme is characterized by an allocation of the tax revenue that is shared between environmental maintenance and education. This research therefore provides theoretical support for the use of mixed-instrument policies that combines public investment in environmental maintenance with other tools that aim to directly modify environmental behaviors.

As such, we have contributed a novel result concerning the effectiveness of using a combination of environmental policy instruments in order to generate both economic and environmental improvements. We note that further extensions will certainly provide additional insights. It could be worthwhile, for example, to consider the more direct effects of pollution on economic growth. In this regard, one could explore the role of the environment as an input in the production function or as a determinant of worker productivity through health effects. Including both direct and indirect impacts of the environment on production would call for a much more complex analysis but we contend that this constitutes an interesting line of future research, enabling to take into account additional effects of our policy instruments as well to analyze other types of policy mixes.

NOTES

1. See Brock and Taylor (2005) and Xepapadeas (2005) for literature reviews on this relationship.
2. See resolution 57/254 of United Nations General Assembly of 2002.
3. We do not conduct a formal welfare analysis because this is not analytically tractable in our setting, but we do provide intuitions about the economic impacts of the environmental policy under consideration.
4. This formalization is not relevant for human capital, as it is a private good.
5. Current environmental quality is not related to altruism concerns and is taken as given by agents, as agents have no impact on the current level of environmental quality.
6. As in Fodha and Seegmuller (2012), we consider pollution as the opposite of the environmental quality index Q in this paper. Thus, we will use both concepts interchangeably.
7. The assumption $\beta > \eta$ is required in order to ensure a positive effect of human capital on environmental awareness.
8. As in standard overlapping generations models, we assume that there is neither work nor explicit consumption in the first period and that a child's consumption is included in that of the parent.
9. See Kotchen and Moore (2008) for empirical evidence relating to the private provision of environmental public goods.
10. Following the seminal contribution of John and Pecchenino (1994), Q is an environmental quality index with an autonomous value of 0 in the absence of human intervention. Moreover, we consider $Q > 0$, which is a standard assumption [see, e.g., Ono (2003), Mariani et al. (2010)].
11. According to our definition of the environment, we do not address policies that tackle issues of global pollution (e.g., greenhouse gas emission limits).

12. This duality is supported by the fact that individuals in high-income economies generally have higher green preferences but also a larger ecological footprint per capita than those in low-income countries [five times higher, see WWF (2014)].

13. Introducing physical capital accumulation, for example, would render the analysis much more complex, as it adds an additional dimension to the dynamics.

14. Alternatively, with consumption as the source of pollution, the tax would be on consumption and our main results would remain valid. However, the negative income effect of the policy, which is presented in the following of the analysis, would be smaller and hence the growth benefits of the tax would be reinforced.

15. We consider a public investment in environmental maintenance rather than a subsidy for private spending because the latter is not analytically tractable. We do, however, model an education subsidy because our formalization precludes an analysis of public education spending.

16. See details in Appendix A.1.1.

17. See, e.g., Ribar and Wilhelm (2002), Menges et al. (2005), and Crumpler and Grossman (2008).

18. See technical details in Appendix A.1.2.

19. Note that, focusing on its amenity value, the long-term environmental quality is not constant. Therefore, it does not represent directly the stock of an environmental good, which may represent a limitation of the paper.

20. If this condition does not hold, either no balanced growth path exists or the growth rate of human capital is always negative, causing the economy to collapse.

21. See equation (16).

22. In the extreme case in which $\beta = \eta$, γ_1 is exogenous and the dynamics is always monotonous. The proof is available upon request.

23. The term sensitivity of preferences to X refers to the convexity and the elasticity of γ_1 to X. The second derivative is given by $\frac{2(\beta-\eta)}{(1+X)^3}$ and the elasticity by $\frac{-(\beta-\eta)X}{(1+X)(\beta+\eta X)}$. At a given X, oscillations occur for high elasticity or convexity.

24. The effect of the policy on oscillations is analyzed through local dynamics. In this case, the economy is close to the BGP but the convergence occurs across several periods, illustrating the short-run dynamics.

25. A BGP that initially occurs in the *npm* regime cannot shift to the *pm* regime, following an increase in the tax.

26. As in Prieur and Bréchet (2013), we have that the stronger the public environmental concern, the lower the growth rate. In their paper, an increase in environmental awareness always reduces physical capital accumulation. Here, γ_1 affects growth through an additional channel, as an increase in environmental quality results in increased spending on education. Nevertheless, the negative direct impact of γ_1 on education spending more than offsets its positive impact through an improvement in environmental quality. The proof is available upon request.

27. From Proposition 4, we deduce that a growth-enhancing policy exists in the regime *npm* if and only if $\frac{a}{b} < \frac{1}{1+\gamma_2\mu}$, where the ratio $\frac{a}{b}$ represents the minimum level of the threshold $\hat{\sigma}(\tau)$ above which the economy is in the regime without private maintenance.

28. $\bar{\tau}^0$ is such that $\sigma_{\mathrm{Min}}(\bar{\tau}^0) = 0$ and $\bar{\tau}^1$ is such that $\sigma_{\mathrm{Max}}(\bar{\tau}^1) = 1$.

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APPENDIX

A.1. EQUILIBRIUM

A.1.1. First-Order Conditions

The maximization of the consumer program (6) leads to the following first-order conditions on education expenditure and on environmental maintenance:

$$\frac{\partial U}{\partial e_t} = 0 \Leftrightarrow \frac{1-\theta_t^e}{c_t} = \frac{\gamma_2 \mu}{e_t}, \tag{A.1}$$

$$\frac{\partial U}{\partial m_t} \leqslant 0 \Leftrightarrow \frac{1}{c_t} \geqslant \frac{\gamma_{1t}(\varepsilon_1+\varepsilon_2 b)}{\varepsilon_1 m_t + \varepsilon_2 Q_{t+1}}. \tag{A.2}$$

From equations (3) and (5) and the first-order conditions (A.1) and (A.2), we deduce the optimal choices in terms of education and maintenance in two regimes: *pm* (where $m_t > 0$) and *npm* (where $m_t = 0$):

$$e_t = \begin{cases} \left(\frac{\gamma_2\mu}{1-\theta_t^e}\right)\left\{\frac{(\varepsilon_1+\varepsilon_2 b)w_t h_t + \varepsilon_2\left[(1-\alpha)Q_t - aY_t + b(M_t+NG_t^m)\right]}{(1+\gamma_{1t}+\gamma_2\mu)(\varepsilon_1+\varepsilon_2 b)}\right\} & pm \\ \frac{w_t h_t \gamma_2 \mu}{(1+\gamma_2\mu)(1-\theta_t^e)} & npm, \end{cases}$$

$$m_t = \begin{cases} \frac{\gamma_{1t}(\varepsilon_1+\varepsilon_2 b)w_t h_t - \varepsilon_2(1+\gamma_2\mu)\left[(1-\alpha)Q_t - aANh_t + b(M_t+NG_t^m)\right]}{(1+\gamma_{1t}+\gamma_2\mu)(\varepsilon_1+\varepsilon_2 b)} & pm \\ 0 & npm. \end{cases}$$

At the symmetric equilibrium, $M_t = m_t(N-1)$, the wage equilibrium is $w_t = A(1-\tau)$, the production function is $Y_t = ANh_t$, and the government budget constraint is given by (10). The Nash intertemporal equilibria are thus given by

$$e_t = \begin{cases} \frac{\gamma_2\mu\{c_3 A(1-\tau)h_t + \varepsilon_2[(1-\alpha)Q_t + ANh_t(b\sigma\tau - a)]\}}{\gamma_{1t}c_1 + (1+\gamma_2\mu)c_3} + (1-\sigma)\tau A h_t & pm \\ \frac{Ah_t[\gamma_2\mu(1-\tau)+\tau(1-\sigma)(1+\gamma_2\mu)]}{1+\gamma_2\mu} & npm, \end{cases}$$

$$m_t = \begin{cases} \frac{\gamma_{1t}c_1 A h_t(1-\tau) - \varepsilon_2(1+\gamma_2\mu)[(1-\alpha)Q_t + ANh_t(b\sigma\tau - a)]}{\gamma_{1t}c_1 + (1+\gamma_2\mu)c_3} & pm \\ 0 & npm. \end{cases}$$

A.1.2. Condition for the regime without private maintenance

Using equation (12), we deduce the condition such that the regime without private environmental maintenance occurs:

$$X_t \geqslant \frac{A\,[\gamma_{1t}c_1(1-\tau) - \varepsilon_2 N(1+\gamma_2\mu)(b\sigma\tau - a)]}{\varepsilon_2(1+\gamma_2\mu)(1-\alpha)}. \tag{A.3}$$

This condition can be written as

$$\begin{aligned} \mathcal{P}(X_t) \equiv\ & X_t^2\varepsilon_2(1+\gamma_2\mu)(1-\alpha) \\ & + X_t[\varepsilon_2(1+\gamma_2\mu)(1-\alpha+AN(b\sigma\tau-a)) - A\eta c_1(1-\tau)] \\ & - A[\beta c_1(1-\tau) - (b\sigma\tau - a)N\varepsilon_2(1+\gamma_2\mu)] \geqslant 0. \end{aligned} \tag{A.4}$$

Given the expression of $\mathcal{P}(X_t)$, the polynomial $\mathcal{P}(X_t) = 0$ admits at most one positive solution. We define Λ as the critical threshold of X_t over which the economy is in the *no private maintenance* regime, with $\Lambda \equiv \max\{0, \text{sol}\{\mathcal{P}(X_t) = 0\}\}$:

- When $\mathcal{P}(X_t) = 0$ does not admit a positive solution, the polynomial is positive or nul $\forall\ X_t \geqslant 0$. In this case, the economy is always in the *npm* regime (i.e., $\Lambda = 0$).
- When $\mathcal{P}(X_t) = 0$ admits a positive solution, the economy is in the *npm* regime only when X_t is sufficiently high (i.e., $\Lambda > 0$).

As a result, the economy is in the *npm* regime when $X_t \geqslant \Lambda$, with $\Lambda \geqslant 0$.

A.1.3. Growth rates

We define the endogenous growth rate of human capital g_H, and environmental quality g_Q, using equations (4), (5), (12), and (13):

$$1 + g_{Ht} = \begin{cases} \epsilon\left[\frac{\gamma_2\mu\{A(1-\tau)c_3 + \varepsilon_2[(1-\alpha)X_t + AN(b\sigma\tau - a)]\} + (1-\sigma)\tau A[\gamma_{1t}c_1 + c_3(1+\gamma_2\mu)]}{\gamma_{1t}c_1 + (1+\gamma_2\mu)c_3}\right]^{\mu} & pm \\ \epsilon\left[\frac{A[\gamma_2\mu(1-\tau)+\tau(1-\sigma)(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu} & npm. \end{cases} \tag{18}$$

In the *npm* regime, human capital growth rate is constant, as education spending does not depend on the environment. The *pm* regime is characterized by a human capital growth rate increasing in the green development index, directly and indirectly though environmental

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awareness γ_{1t}.

$$1+g_{Qt}=\begin{cases}\frac{(1-\alpha)X_t(\gamma_{1t}c_1+c_2)+AN[\gamma_{1t}\, c_1(1-\tau)b+(\gamma_{1t}c_1+c_2)(b\tau\sigma-a)]}{X_t[\gamma_{1t}c_1+c_3(1+\gamma_2\mu)]} & pm\\ 1-\alpha+\frac{AN(b\sigma\tau-a)}{X_t} & npm.\end{cases} \quad \textbf{(A.5)}$$

In the case with private maintenance, the green development index X_t has a direct negative impact on the growth rate of environmental quality and an indirect positive effect through environmental awareness γ_{1t}. In the corner solution, g_{Qt} is always negative without public abatement. However, when the government intervenes, the growth of the environmental quality at the corner may be positive for a share of policy devoted to public maintenance (σ) sufficiently high.

A.2. PROOF OF PROPOSITION 1

A.2.1. Properties of the dynamic equation

From Definitions 1 and 2 and equations (14) and (16), we emphasize the properties of the dynamic equation characterizing equilibrium paths, $\mathcal{F}(X_t)$, defined on $(0; +\infty)$:

- When $X_t \in (0\,;\ \Lambda)$ the function is given by equation (17 *pm*). We have

$$\mathcal{F}(0)=AN(\beta\, c_1(1-\tau)b+(\beta c_1+c_2)(b\tau\sigma-a))$$
$$\times\frac{[A\gamma_2\mu\,(\,c_3(1-\tau)+\varepsilon_2N(b\sigma\tau-a))+(1-\sigma)\tau A(\beta\, c_1+(1+\gamma_2\mu)c_3)]^{-\mu}}{\epsilon\,[\beta\, c_1+(1+\gamma_2\mu)c_3]^{1-\mu}}$$

  Finally, with equation (16), $\lim_{X_t\to\Lambda^-}\mathcal{F}(X_t)=\frac{(1-\alpha)\Lambda+AN(b\sigma\tau-a)}{\epsilon\left[\frac{\gamma_2\mu A(1-\tau)+(1-\sigma)\tau A(1+\gamma_2\mu)}{1+\gamma_2\mu}\right]^{\mu}}\equiv v$.
- When $X_t \in [\Lambda\,;\ +\infty)$, the function is given by equation (17 *npm*). $\mathcal{F}$ is increasing and linear in X, $\mathcal{F}(\Lambda)=v$ and $\lim_{X\to+\infty}\mathcal{F}(X)=+\infty$.

As $\lim_{X_t\to\Lambda}\mathcal{F}(X_t)=\mathcal{F}(\Lambda)$, the function is continue on $(0; +\infty)$.

A.2.2. Existence and unicity of the balanced growth path

npm solution: A BGP in the *npm* regime is characterized by $\bar{X}_{npm}=\mathcal{F}(\bar{X}_{npm})$. Using (17 *npm*), we obtain

$$\bar{X}_{npm}=\frac{AN(b\tau\sigma-a)}{\epsilon\left[\frac{A[\gamma_2\mu(1-\tau)+(1-\sigma)\tau(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu}-(1-\alpha)}. \quad \textbf{(A.6)}$$

To exist, $\bar{X}_{npm}$ has to be positive. Following (18 *npm*), if the denominator of (A.6) is negative, so does the growth rate in the *npm* regime. Therefore, the case where the denominator and the numerator of (A.6) are negative is meaningless. The existence conditions are thus $\sigma>\frac{a}{b\tau}$ and $\mathcal{A}_1>0$ with $\mathcal{A}_1\equiv\left[\frac{A[\gamma_2\mu(1-\tau)+(1-\sigma)\tau(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu}-(1-\alpha)$. Note that the condition $\mathcal{A}_1>0$ implies that $\lim_{X\to+\infty}\frac{\mathcal{F}(X)}{X}<1$.

Then, we have to check the admissibility of the steady state, i.e., if it effectively belongs to the *npm* region. To do this, we examine the sign of $\mathcal{F}(\Lambda) - \Lambda$. Under condition $\mathcal{A}_1 > 0$ and $\sigma > \frac{a}{b\tau}$, $\bar{X}_{npm}$ is admissible if $\mathcal{F}(\Lambda) \geq \Lambda$, which is equivalent to $\bar{X}_{npm} \geq \Lambda$.

pm solution: A BGP in the *pm* regime is characterized by $\bar{X}_{pm} = \mathcal{F}(\bar{X}_{pm})$. As we focus on $X > 0$, we determine the solutions $\bar{X}_{pm}$ which satisfy $\mathcal{F}(\bar{X}_{pm})/\bar{X}_{pm} = 1$. Using equations (14) and (17 *pm*), it corresponds to the following equation. For the sake of simplicity, subscripts on X are removed:

$$\begin{aligned}&\epsilon\big(\gamma_2\mu\{Ac_3(1-\tau)+\varepsilon_2[(1-\alpha)X+AN(b\tau\sigma-a)]\}\\&\quad+A\tau(1-\sigma)\left[c_1\tfrac{\beta+\eta X}{1+X}+(1+\gamma_2\mu)c_3\right]\big)^{\mu}\\&\left[\tfrac{\beta+\eta X}{1+X}c_1+c_3(1+\gamma_2\mu)\right]^{1-\mu}=(1-\alpha)\left(\tfrac{\beta+\eta X}{1+X}c_1+c_2\right)\\&\quad+\frac{AN\left[\frac{\beta+\eta X}{1+X}c_1(b(1-\tau)+b\tau\sigma-a)-c_2(a-b\tau\sigma)\right]}{X}.\end{aligned}$$

We define $\mathcal{D}_1(X)$ and $\mathcal{D}_2(X)$, respectively, as the terms on the left-hand side and on the right-hand side. Their properties are given as follows:

- $\mathcal{D}_1$ is decreasing and then increasing with X. Moreover, $\mathcal{D}_1(X) > 0$ for all X, $\lim_{X\to 0}\mathcal{D}_1(X) = \mathcal{C} > 0$, with $\mathcal{C}$ a constant and $\lim_{X\to+\infty}\mathcal{D}_1(X) = +\infty$.
- Concerning $\mathcal{D}_2$ we have that if $\mathcal{A}_2 \equiv \beta c_1[b(1-\tau)+b\tau\sigma-a]-c_2(a-b\tau\sigma) > 0$ (resp. < 0), $\lim_{X\to 0}\mathcal{D}_2(X) > 0$ (resp. < 0). Moreover, $\lim_{X\to+\infty}\mathcal{D}_2(X) = (1-\alpha)(\eta c_1+c_2) > 0$.

The condition $\mathcal{A}_2 > 0$ guarantees that the curves $\mathcal{D}_1(X)$ and $\mathcal{D}_2(X)$ cross at least once in the positive area. Thus, a positive solution exists. From equation (17 *pm*) and $\mathcal{A}_2 > 0$, we have $d\mathcal{F}(\bar{X}_{pm})/dX < 1$; hence, the positive solution $\bar{X}_{pm}$ is unique.

We check the admissibility of the steady state, i.e., if it effectively belongs to the *pm* region. Under condition $\mathcal{A}_2 > 0$, $\bar{X}_{pm}$ is admissible if $\lim_{X\to\Lambda}\mathcal{F}(X) < \Lambda$, which is equivalent to $\bar{X}_{pm} < \Lambda$. As $\bar{X}_{pm} \leqslant \bar{X}_{npm}$, the condition $\bar{X}_{npm} < \Lambda$ guarantees that $\bar{X}_{pm}$ is admissible.

The case where both $\bar{X}_{pm}$ and $\bar{X}_{npm}$ are admissible is excluded. Indeed, when $\bar{X}_{pm}$ exists (condition $\mathcal{A}_2 > 0$ verified) and is admissible, then $\mathcal{F}(\Lambda) < \Lambda$. But under condition $\mathcal{A}_1 > 0$, the slope of $\mathcal{F}$ in the *npm* regime is lower than 1, which means that $\mathcal{F}$ cannot cut the bisector in this regime. Reversely, when X_{npm} exists (conditions $\mathcal{A}_1 > 0$ and $\sigma > \frac{a}{b\tau}$ verified) and is admissible, then $\mathcal{F}(\Lambda) \geqslant \Lambda$. Under condition $\mathcal{A}_2 > 0$, this implies that $\mathcal{F}$ does not cut the bisector in the *pm* area.

To sum up, under these conditions, it is not possible to have $\bar{X}_{pm} < \Lambda \leqslant \bar{X}_{npm}$, and hence to have cases with multiple steady states: When $\bar{X}_{npm} < \Lambda$, the BGP is in the regime *pm*, whereas when $\bar{X}_{npm} \geqslant \Lambda$, the BGP is in the regime *npm*.

For the rest of the paper, we rewrite the conditions $\mathcal{A}_1, \mathcal{A}_2 > 0$, as $\sigma_{\text{Min}}(\tau) < \sigma < \sigma_{\text{Max}}(\tau)$ with

$$\sigma_{\text{Min}}(\tau) \equiv \frac{a(\beta c_1+c_2)-b\beta c_1(1-\tau)}{b\tau(\beta c_1+c_2)}$$

and

$$\sigma_{\text{Max}}(\tau) \equiv \frac{A(\gamma_2\mu+\tau)-\epsilon^{-1/\mu}(1+\gamma_2\mu)(1-\alpha)^{1/\mu}}{A\tau(1+\gamma_2\mu)}$$

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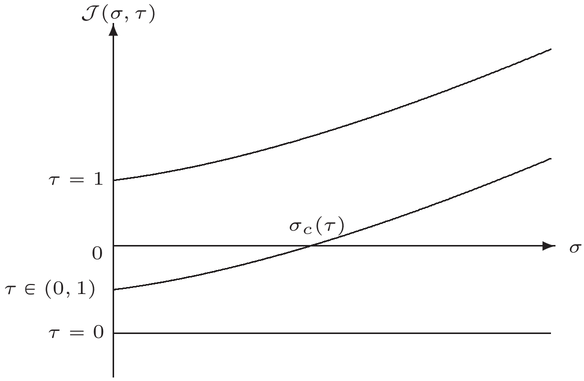

FIGURE 5. Function $\mathcal{J}$ at given τ.

For $\tau = 0$, the condition $\sigma_{\text{Min}}(\tau) < \sigma < \sigma_{\text{Max}}(\tau)$ is equivalent to

$$a(\beta c_1 + c_2) - b\beta c_1(1-\tau) < 0 < A(\tau + \gamma_2\mu) - (1 + \gamma_2\mu)\left(\frac{1-\alpha}{\epsilon}\right)^{1/\mu}$$

Given this, $\sigma_{\text{Min}}(\tau)$ is increasing in τ from $\lim_{\tau\to 0}\sigma_{\text{Min}} = -\infty$ to $\lim_{\tau\to 1}\sigma_{\text{Min}} = a/b$, whereas $\sigma_{\text{Max}}(\tau)$ is decreasing in τ from $\lim_{\tau\to 0}\sigma_{\text{Max}} = +\infty$ to $\lim_{\tau\to 1}\sigma_{\text{Max}} = 1 - \frac{1}{\epsilon^{1/\mu}A}$.

Condition for the nature of the BGP: The analysis of admissibility conditions comes down to $\bar{X}_{npm} <$ or $> \Lambda$.

From equation (A.4) in Appendix A.1.2, $\bar{X}_{npm} \geqslant \Lambda$ is equivalent to $\mathcal{P}(\bar{X}_{npm}) \geqslant 0$. We define $\mathcal{P}(\bar{X}_{npm}) \equiv \mathcal{J}(\tau, \sigma)$, where the function $\mathcal{J}$ is increasing in τ and σ. Under $\mathcal{A}_1 > 0$:

- For $\tau = 0$, $\bar{X}_{npm} < 0$, $\mathcal{J} < 0$ and does no longer depend on σ.
- For $\tau = 1$, we get $\mathcal{J} > 0 \ \forall \sigma \in [0, 1]$.

We depict a representation of $\mathcal{J}$ at given τ in Figure 5.

We deduce that there exists a $\sigma_c(\tau)$ decreasing in τ such that $\mathcal{J} = 0$, with $\lim\limits_{\tau\to 0}\sigma_c(\tau) = +\infty$ and $\sigma_c(1) < 0$. Thus, a minimum level of the tax is required to make the *npm* regime possible. When $\sigma < \sigma_c(\tau)$, we get $\mathcal{P}(\bar{X}_{npm}) < 0$, meaning that the equilibrium is in the *pm* regime. Respectively, when $\sigma \geqslant \sigma_c(\tau)$, we get $\mathcal{P}(\bar{X}_{npm}) \geqslant 0$, and from equations (A.4) and (A.6), we have $\bar{X}_{npm} \geqslant \Lambda$ if and only if $\sigma > \frac{a}{b\tau}$. Thus, the BGP is in the *npm* regime when $\sigma > \text{Max}\{\sigma_c(\tau); \frac{a}{b\tau}\} \equiv \hat{\sigma}(\tau)$ and in the *pm* regime when $\sigma < \hat{\sigma}(\tau)$. When $\sigma = \hat{\sigma}(\tau)$, the BGP is in the *npm* regime if $\hat{\sigma}(\tau) = \sigma_c(\tau)$ and in the *pm* regime if $\hat{\sigma}(\tau) = \frac{a}{b\tau}$. Note that $\hat{\sigma}(\tau) > \sigma_{\text{Min}}(\tau)$ and $\lim_{\tau\to 1}\hat{\sigma}(\tau) < \lim_{\tau\to 1}\sigma_{\text{Max}}(\tau)$.

A.3. PROOF OF PROPOSITION 2

pm solution: We use the scaling parameter ϵ in order to normalize the steady state $\bar{X}_{pm}$ to 1. There is a unique solution ϵ^* such that $\bar{X}_{pm} = 1$ and from equation (17 *pm*), the expression of the normalization constant is given by

$$\epsilon^*(\bar{X}_{pm}) \equiv \left\{(1-\alpha)\ \bar{X}_{pm}\ (\bar{\gamma}_1 c_1 + c_2) + AN[\bar{\gamma}_1 c_1 b(1-\tau) + (\bar{\gamma}_1 c_1 + c_2)(b\tau\sigma - a)]\right\}$$
$$\times \{\gamma_2\mu A\ c_3(1-\tau) + \gamma_2\mu\varepsilon_2\left[(1-\alpha)\bar{X}_{pm} + AN(b\sigma\tau - a)\right] + (1-\sigma)\tau A[\bar{\gamma}_1\ c_1$$
$$+ (1+\gamma_2\mu)c_3]\}^{-\mu}\,[\bar{\gamma}_1\ c_1 + (1+\gamma_2\mu)c_3]^{\mu-1}.$$

Then, by differentiating equation (17 *pm*) and analyzing it around the steady state $\bar{X}_{pm} = 1$ and $\epsilon \equiv \epsilon^*(\bar{X}_{pm})$, we obtain

$$\frac{dX_{t+1}}{dX_t} = \frac{(1-\alpha)(\bar{\gamma}_1 c_1 + c_2 + \bar{\gamma}_1{}' c_1) + AN\bar{\gamma}_1{}' c_1[b(1-\tau) + b\tau\sigma - a]}{\mathcal{B}_3} - \frac{\mu\mathcal{B}_2(\gamma_2\mu\varepsilon_2(1-\alpha) + (1-\sigma)A\tau\bar{\gamma}_1{}' c_1) + (1-\mu)\gamma_1' c_1\mathcal{B}_1}{\mathcal{B}_1\mathcal{B}_2} \qquad \textbf{(A.7)}$$

with $\bar{\gamma}_1 = \frac{\beta+\eta}{2}, \bar{\gamma}_1{}' = (\eta-\beta)/4$, $\mathcal{B}_1 = \gamma_2\mu A\ \ c_3(1-\tau) + \gamma_2\mu\varepsilon_2[(1-\alpha) + AN(b\sigma\tau - a)] + (1-\sigma)\tau A[\bar{\gamma}_1\ c_1 + (1+\gamma_2\mu)c_3]$, $\mathcal{B}_2 = \bar{\gamma}_1\ c_1 + c_3(1+\gamma_2\mu)$ and $\mathcal{B}_3 = (1-\alpha)(\bar{\gamma}_1\ c_1 + c_2) + AN[\bar{\gamma}_1 c_1 b(1-\tau) + (\bar{\gamma}_1 c_1 + c_2)(b\tau\sigma - a)]$.

From (A.7), we get $dX_{t+1}/dX_t < 1$. Thus, when $dX_{t+1}/dX_t > 0$, transitional dynamics is monotonous and the BGP equilibrium is locally stable. Using equation (A.7), we have $dX_{t+1}/dX_t > 0$ if and only if

$$\mathcal{B}_2(1-\alpha)\{(1-\sigma)\tau A\mathcal{B}_2 + \gamma_2\mu[A\varepsilon_1(1-\tau) + \varepsilon_2(1-\mu)]\}(\gamma_1 c_1 + c_2)$$
$$+ \mathcal{B}_2(1-\alpha)A\varepsilon_2 bN(1-\tau)[\gamma_1 c_1(1-\mu) + c_2]$$
$$+ \bar{\gamma}_1{}' c_1\mathcal{B}_5\{\gamma_2\mu[1-\alpha + AN(b-a) + bN\tau(\sigma-1)][bN(1+\gamma_2\mu)$$
$$\times\varepsilon_2(1-\mu) + \mu\mathcal{B}_2]\}$$
$$+ \bar{\gamma}_1{}' c_1\mathcal{B}_5 ANb(1+\gamma_2\mu)[\gamma_2\mu\varepsilon_1(1-\mu) + (1-\sigma)\tau\mathcal{B}_2] > 0,$$

with $\mathcal{B}_4 \equiv 1-\alpha+AN[b(1-\tau)+b\tau\sigma-a]$ and $\mathcal{B}_5 \equiv Ac_3(1-\tau)+\varepsilon_2[1-\alpha+AN(b\tau\sigma-a)]$. Rewriting this expression, we have $dX_{t+1}/dX_t > 0$ if and only if the following polynomial is positive:

$$\mathcal{R}(\beta) \equiv a_1\beta^3 + a_2\beta^2 + a_3\beta + a_4,$$

with $a_4 > 0$ and expressions for a_1, a_2, and a_3 given by

$$a_1 = \frac{c_1^3}{8}(1-\sigma)(1-\alpha)\tau A > 0,$$

$$a_2 = \frac{\frac{c_1^2(1-\alpha)}{2}}{4}\left\{\gamma_2\mu(1-\mu)\mathcal{B}_5 + (1-\sigma)\tau A[2c_3(1+\gamma_2\mu) + c_2] + \gamma_2\mu^2\varepsilon_1 A(1-\tau)\right\}$$
$$+ \frac{c_1^2}{8}\left\{3\eta c_1(1-\alpha)(1-\sigma)\tau A - \mathcal{B}_5\left[\gamma_2\mu^2\mathcal{B}_4 + bAN(1+\gamma_2\mu)\tau(1-\sigma)\right]\right\},$$

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$$
\begin{aligned}
a_3 = {} & \frac{c_1^3\eta^2(1-\sigma)(1-\alpha)\tau A}{8} + \frac{2\eta c_1^2}{4}(1-\alpha)\left\{\gamma_2\mu(1-\mu)\mathcal{B}_5\right. \\
& \left. +(1-\sigma)\tau A[2c_3(1+\gamma_2\mu)+c_2]+\gamma_2\mu^2\varepsilon_1 A(1-\tau)\right\} \\
& + \frac{c_1(1-\alpha)}{2}\{3(1-\sigma)\tau A c_2 c_3(1+\gamma_2\mu)+(1-\mu)\gamma_2\mu\mathcal{B}_5[c_3(1+\gamma_2\mu)+c_2] \\
& \quad \left. +2\gamma_2\mu^2 A(1-\tau)\varepsilon_1 c_3(1+\gamma_2\mu)\right\} \\
& \quad -\frac{c_1\mathcal{B}_5}{4}\left\{\gamma_2\mu^2 c_3(1+\gamma_2\mu)\mathcal{B}_4+bN(1+\gamma_2\mu)[\gamma_2\mu(1-\mu)\mathcal{B}_5\right. \\
& \quad +(1-\sigma)\tau A c_3(1+\gamma_2\mu)]\}\,.
\end{aligned}
$$

We have $\mathcal{R}(1)$ which is a polynomial of degree 3 in N. When $N=0$, we have $\mathcal{R}(1)>0$, whereas when N tends to ∞, $\mathcal{R}(1)<0$. As N^3 intervenes positively in $\mathcal{R}(1)$, there exists a critical threshold $\bar{N}$ over which the dynamics is oscillatory for $\beta=1$. Given that $a_1>0$ and $a_4>0$, we can conclude that for $N>\bar{N}$ there exists a $\tilde{\beta}\in(0,1]$ over which the dynamics is oscillatory.

We examine the stability of the equilibrium when the dynamics is oscillatory. From equation (A.7), we have $dX_{t+1}/dX_t>-1$ if and only if

$$
\begin{aligned}
& \{(1-\alpha)(\bar{\gamma}_1 c_1+c_2+\bar{\gamma}_1{}' c_1)+AN\bar{\gamma}_1{}'c_1[b(1-\tau)+b\tau\sigma-a]\}\mathcal{B}_1\mathcal{B}_2+\mathcal{B}_1\mathcal{B}_2\mathcal{B}_3 \\
& \quad -\left\{(1-\mu)c_1\bar{\gamma}_1{}'\mathcal{B}_1+\mathcal{B}_2\mu\left[\gamma_2\mu\varepsilon_2(1-\alpha)+(1-\sigma)A\tau\bar{\gamma}_1{}'c_1\right]\right\}\mathcal{B}_3>0.
\end{aligned}
$$

Replacing expressions $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{B}_3$, we finally obtain

$$
\begin{aligned}
& c_3(1+\gamma_2\mu)c_2(1-\alpha)(\mathcal{B}_6+\varepsilon_2 bN\mu)+\gamma_2\mu(\bar{\gamma}_1 c_1)^2[\mathcal{B}_6(1-\alpha)+\mathcal{B}_5\mathcal{B}_4] \\
& \quad +\bar{\gamma}_1 c_1(1+\gamma_2\mu)\{(1-\alpha)\varepsilon_1(\mathcal{B}_6+\varepsilon_2 bN\mu) \\
& \quad +\gamma_2\mu\varepsilon_1[1-\alpha+AN(b\tau\sigma-a)]+c_3(\mathcal{B}_4+\mathcal{B}_6)\} \\
& \quad +\gamma_2\mu\bar{\gamma}_1{}'c_1(1+\gamma_2\mu)\{\mathcal{B}_4 c_3-(1-\mu)\varepsilon_1[1-\alpha+AN(b\tau\sigma-a)]\} \\
& \quad +\gamma_2\mu^2\bar{\gamma}_1{}'\bar{\gamma}_1 c_1^2\mathcal{B}_5\mathcal{B}_4+\mathcal{B}_5 c_3(1+\gamma_2\mu)c_2[1-\alpha+AN(b\tau\sigma-a)] \\
& \quad +(1-\sigma)\tau A[\bar{\gamma}_1 c_1+c_3(1+\gamma_2\mu)]\gamma_1' c_1 bN(1+\gamma_2\mu)\mathcal{B}_5 \\
& \quad +(1-\sigma)\tau A[\bar{\gamma}_1 c_1+c_3(1+\gamma_2\mu)]^2\{\bar{\gamma}_1 c_1\mathcal{B}_4+c_2[1-\alpha+AN(b\tau\sigma-a)] \\
& \quad +(1-\alpha)(\bar{\gamma}_1 c_1+c_2)\},
\end{aligned}
$$

with $\mathcal{B}_6=c_2+\varepsilon_2 bN(1+\gamma_2\mu)>0$.

As $-\bar{\gamma}_1{}'<\bar{\gamma}_1$ and $c_3\mathcal{B}_4>bN\mathcal{B}_5$, we easily see that this term is always positive. The BGP equilibrium is always locally stable.

npm solution: The *npm* BGP is obtain from (17 *npm*) and given in Appendix A.2. We differentiate equation (17 *npm*) and obtain

$$
\frac{d\mathcal{F}(X_t)}{dX_t}=\frac{(1-\alpha)}{\epsilon\left[\frac{A[\gamma_2\mu(1-\tau)+(1-\sigma)\tau(1+\gamma_2\mu)]}{1+\gamma_2\mu}\right]^{\mu}}.
$$

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Under Assumption 1, the slope of $\mathcal{F}(X_t)$ in the *npm* regime is always positive and lower than 1, the *npm* BGP is thus monotonously stable.

A.4. PROOF OF REMARK 2

The condition to observe oscillatory cases given in Appendix A.3 (i.e., $N > \bar{N}$) depends on policy instruments. Moreover, by examining the terms a_2 and a_3 given in Appendix A.3, we conclude that if a_2 is positive, a_3 is positive as well. A necessary condition is thus required to have $N > \bar{N}$: The term a_2 has to be negative. An analysis of the impact of the policy instruments on the polynomial $\mathcal{R}(\beta)$ is not analytically tractable, but the analysis of Sign $\left\{\frac{\partial a_2}{\partial \tau}\right\}$ gives us interesting intuitions. Using the expression of a_2 given in Appendix A.3, we obtain

$$\begin{aligned}\text{Sign}\left\{\frac{\partial a_2}{\partial \tau}\right\} = &-(1-\sigma)bAN[1+\gamma_2\mu(1-\mu)][\mathcal{S}_2 - \tau A\varepsilon_2 bN(1-\sigma)][\tau(1-\sigma)\mathcal{S}_3 \\ &+(1-\tau)\gamma_2\mu A\varepsilon_1] \\ &+(1-\sigma)\{b\tau(1-\sigma)AN[1+\gamma_2\mu(1-\mu)]+\mathcal{S}_1\}(\varepsilon_2 ANb\mathcal{S}_4+\mathcal{S}_3\mathcal{S}_2) \\ &+3(1-\sigma)\eta c_1 A\mathcal{S}_4 \\ &-A\varepsilon_1\mu^3[1-\alpha+AN(b-a)]\{\mathcal{S}_1+b\tau(1-\sigma)AN[1+\gamma_2\mu(1-\mu)]\},\end{aligned}$$

with $\mathcal{S}_1 = \mu^2\gamma_2[1-\alpha+AN(b-a)]$, $\mathcal{S}_2 = A(1-\tau)\varepsilon_1 + \varepsilon_2[AN(b-a)+1-\alpha]$, $\mathcal{S}_3 = A[3\varepsilon_1(1+\gamma_2\mu)+2\varepsilon_2 bN+\gamma_2\mu\varepsilon_2 bN(1+\mu)]$ and $\mathcal{S}_4 = \gamma_2\mu\{\varepsilon_2(1-\mu)[1-\alpha+AN(b-a)]+A\varepsilon_1\}$.

We can define Sign $\left\{\frac{\partial a_2}{\partial \tau}\right\}$ as a polynomial of degree 3 in σ, with $\frac{\partial a_2}{\partial \tau} < 0$ when $\sigma = 1$ and $\frac{\partial a_2}{\partial \tau} > 0$ when $\sigma = 0$. Since Sign $\left\{\frac{\partial a_2}{\partial \tau}\right\}$ is decreasing in σ for $\sigma \in [0, 1]$, there exists a critical value $\tilde{\sigma} \in (0, 1)$ such that for $0 < \sigma < \tilde{\sigma}$, $\frac{\partial a_2}{\partial \tau} > 0$ and for $\tilde{\sigma} < \sigma < 1$, $\frac{\partial a_2}{\partial \tau} < 0$. Moreover, from Assumption 1, $\lim_{\tau\to 0}\sigma_{\text{Min}}(\tau) < \tilde{\sigma} < \lim_{\tau\to 1}\sigma_{\text{Max}}(\tau)$. When σ is sufficiently low, a tighter tax tightens the condition to observe oscillatory cases, whereas when σ is high enough the condition to observe oscillatory dynamics may be relaxed.

A.5. PROOF OF PROPOSITION 4

We examine the impact of taxation on the growth rate along the BGP.

pm solution: Using equation (18 *pm*) with $X_t = \bar{X}_{pm}$, we have

$$\begin{aligned}\text{Sign}\left(\frac{\partial g_{pm}}{\partial \tau}\right) = &\mathcal{V}_2\left[(1-\sigma)(\bar{\gamma}_1 c_1 + c_3) - \sigma\gamma_2\mu\varepsilon_1 + \gamma_2\mu\varepsilon_2(1-\alpha)\frac{\partial \bar{X}_{pm}}{\partial \tau}\right] \\ &+\frac{c_1(\beta-\eta)}{(1+\bar{X}_{pm})^2}\frac{\partial \bar{X}_{pm}}{\partial \tau}\gamma_2\mu\left[\mathcal{V}_1 - \tau(1-\sigma)A\mathcal{V}_2\right],\end{aligned}$$

with $\mathcal{V}_1 = \gamma_2\mu Ac_3(1-\tau) + \gamma_2\mu\varepsilon_2\left[(1-\alpha)\bar{X}_{pm} + AN(b\sigma\tau - a)\right] + (1-\sigma)\tau A[\bar{\gamma}_1 c_1 + (1+\gamma_2\mu)c_3]$ and $\mathcal{V}_2 = \bar{\gamma}_1 c_1 + (1+\gamma_2\mu)c_3$. From the implicit function theorem and equation

(17), we have

$$\frac{\partial \bar{X}_{pm}}{\partial \tau} = \Big(\mathcal{V}_2 \bar{X}_{pm} A \{[\bar{\gamma}_1 c_1(\sigma-1)+\sigma c_2]\mathcal{V}_1 N - \mu \mathcal{V}_3[-\sigma\varepsilon_1\gamma_2\mu + (1-\sigma)(\bar{\gamma}_1 c_1 + c_3)]\}\Big)$$
$$\times \Big[\frac{c_1 \bar{X}_{pm}(\beta-\eta)}{(1+\bar{X}_{pm})^2}\big(\mathcal{V}_1\mathcal{V}_2\{\bar{X}_{pm}(1-\alpha)+AN[b(1-\tau)+b\tau\sigma-a]\}$$
$$-\mu\mathcal{V}_2\mathcal{V}_3(1-\sigma)\tau A + (1-\mu)\mathcal{V}_1\mathcal{V}_3\big) + \mu\mathcal{V}_2\mathcal{V}_3\bar{X}_{pm}\gamma_2\mu\varepsilon_2(1-\alpha)$$
$$+AN\mathcal{V}_1\mathcal{V}_2\mathcal{V}_4\Big]^{-1},$$

with $\mathcal{V}_3 = (1-\alpha)\bar{X}_{pm}(\bar{\gamma}_1 c_1 + c_2) + AN[\bar{\gamma}_1 c_1 b(1-\tau) + (\bar{\gamma}_1 c_1 + c_2)(b\tau\sigma - a)]$ and $\mathcal{V}_4 = c_2(b\tau\sigma - a) + \bar{\gamma}_1 c_1 b[1-\tau(1-\sigma)]$.

Thus, substituting $\frac{\partial \bar{X}_{pm}}{\partial \tau}$ in Sign $\left(\frac{\partial g_{pm}}{\partial \tau}\right)$, we finally obtain

$$\text{Sign}\left(\frac{\partial g_{pm}}{\partial \tau}\right) = \left(\gamma_2\mu\varepsilon_2(1-\alpha)\mathcal{V}_2\bar{X}_{pm}AN + \frac{c_1\bar{X}_{pm}AN(\beta-\eta)}{(1+\bar{X}_{pm})^2}[\mathcal{V}_1 - \tau(1-\sigma)A\mathcal{V}_2]\right)$$
$$\times[\bar{\gamma}_1 c_1(\sigma-1)+\sigma c_2] + \left(\frac{c_1\bar{X}_{pm}(\beta-\eta)}{(1+\bar{X}_{pm})^2}\big[\mathcal{V}_2\{\bar{X}_{pm}(1-\alpha)\right.$$
$$\left. + AN[b(1-\tau)+b\tau\sigma-a]\} + \mathcal{V}_3\big] + \mathcal{V}_2\mathcal{V}_4 AN\right)$$
$$\times[-\sigma\varepsilon_1\gamma_2\mu + (1-\sigma)(\bar{\gamma}_1 c_1 + c_3)]\mathcal{V}_1.$$

Under Assumption 1, policy improves the BGP growth rate when the following sufficient condition is satisfied:

$$f_1(\sigma) < \sigma < f_2(\sigma),$$

with $f_1(\sigma) \equiv \frac{\bar{\gamma}_1 c_1}{\bar{\gamma}_1 c_1 + c_2} < 1$ and $f_2(\sigma) \equiv \frac{\bar{\gamma}_1 c_1 + c_3}{\bar{\gamma}_1 c_1 + c_2 + \gamma_2\mu\varepsilon_1} < 1$. These two functions are increasing in $\bar{\gamma}_1$, and as $\frac{\partial \bar{\gamma}_1}{\partial \bar{X}_{pm}} < 0$ and $\frac{\partial \bar{X}_{pm}}{\partial \sigma} > 0$, they are decreasing in σ. As a result, there exists a unique range of value $[\underline{\sigma}(\tau); \overline{\sigma}(\tau)]$ that satisfies this condition. Moreover, under Assumption 1, $\lim_{\tau\to 0}\sigma_{\text{Min}}(\tau) < \underline{\sigma}(\tau) < \overline{\sigma}(\tau) < \lim_{\tau\to 1}\sigma_{\text{Max}}(\tau)$.

npm solution: We use equation (18 *npm*) with $X_t = \bar{X}_{npm}$ and deduce

$$\text{Sign}\left(\frac{\partial g_c}{\partial \tau}\right) = 1 - \sigma(1+\gamma_2\mu).$$

A tighter tax is growth promoting as long as $\hat{\sigma}(\tau) < \sigma < 1/(1+\gamma_2\mu)$.

Regime switch: We consider the case in which an increase in τ leads the economy from a *pm* regime to an *npm* regime. The opposite switch cannot be observed as $\hat{\sigma}(\tau)$ is decreasing in τ. For a given σ, we compare equations (18 *npm*) and (18 *pm*), by considering a higher tax rate in the *npm* regime (τ_N) than in the *pm* one (τ_P). The growth rate in the *pm*

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regime is higher than in the *npm* if and only if

$$\gamma_2\mu(1+\gamma_2\mu)\left\{c_3A(\tau_N-\tau_P)+\varepsilon_2[(1-\alpha)\bar{X}_{pm}+AN(\sigma b\tau_P-a)]\right\}$$
$$-(1-\sigma)A[\gamma_1c_1+(1+\gamma_2\mu)c_3](\tau_N-\tau_P)-A\gamma_2\mu(1-\tau_N)\gamma_1c_1>0.$$

This expression is increasing in σ and from (16) is never satisfied when $\sigma = 1/(1+\gamma_2\mu)$. And according to Appendix A.2, the *npm* regime exists only if $\frac{a}{b} < \sigma$. Thus, for a given $\sigma \in (a/b\,;1/(1+\gamma_2\mu))$, the growth rate in the *npm* regime is higher than in the *pm* one. Moreover, under Assumption 1, $\sigma_{\text{Min}}(\tau) < 1/(1+\gamma_2\mu) < \sigma_{\text{Max}}(\tau)$.

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