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Robust and adaptive control of three dimensional revolute-jointed cooperative manipulators for handling automation

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SUMMARY

This paper presents a study of the application of adaptive and robust control methods to a cooperative manipulation system which is developed for handling an object by three dimensional revolute-jointed manipulators. The adaptive control algorithm supports the parameter adaptive law that provides guaranteed stability for uncertain systems. In designing the robust control structure, contact and friction constraints for grasp and bearing conditions, structural flexibility or such similar factors as various unmodeled dynamics are considered as uncertainties that determine available values of control parameters. The novelty of results in the present paper is to define new control inputs using parametric uncertainties and the Lyapunov based theory of guaranteed stability of uncertain systems for handling objects in a spatial workspace.

KEYWORDS: Spatial cooperative manipulation; Robust control; Adaptive control; Parameter uncertainty; System stability.

I. INTRODUCTION

Cooperative manipulations comprising multiple robots have attracted attention in such various applications as material handling, assembling, etc. In the case of this type of operations, it is important to develop a proper dynamic model of real systems so as to determine appropriate control parameters in the trajectory or force control in associated with the work accomplished by cooperative manipulation. Recent advances focus on determining suitable control strategies and so available control parameters, which minimise the deviation of control variables from its desired values due to such effects as parametric uncertainties in the dynamic model of cooperative manipulation.

Huang and Chen developed a dynamic model of cooperative manipulation system to handle objects by two different robots. In this system, an adaptive controller is used to eliminate uncertainties due to the dynamics of the object and robots.[1]

Yoshikawa and Zheng present also another study on hybrid position/force control application to a cooperative manipulation, considering the dynamics concerning handling of an object by cooperating manipulators. Thus, the required values of the position/force control parameters have been investigated for the control process of the model system.[2]

In order to elaborate the dynamic model of a cooperative manipulation, it is necessary to determine the kinematic constraints for the cooperative motion of the object being handled and the robot manipulators. Supposing that the kinematic constraints are known analytically, Kleinger and Khalil developed an algorithm for the dynamic modeling of robot manipulators containing closed kinematic chains.[3]

Vukobratovic and Potkonjak deal with dynamic modeling of a bilateral manipulation system in which cooperative manipulators are directly connected by their grippers. Appropriate actuator torques are defined considering inertial parameters and necessary forces.[4]

In solving the control problem of robotic system, several approaches have been suggested. One of them is the master-slave approaches.[5–7] A new type of master-helper control strategy solves the problem of task distribution among the robots in a simple and cooperative way.

Li, Hsu & Sastry treated two fundamental problems of grasping and coordinated manipulation by a multi-fingered robot hand.[8] First, dual notions of grasp stability and grasp manipulability are improved, and given a task, a new model is suggested. Secondly, by considering the contact model above mentioned, a computed torque based control algorithm is developed for designing controllers in controlling the trajectory and grasping force.

Some researches focused on position and force control in cooperative manipulation are as follows: Perdereau and Drouin suggested different control schemes for the hybrid position/force control in the coordinated motion of cooperative manipulators.[9] Lian and Lin also carried out an investigation concerning the application of a sliding-mode motion/force control of constrained robots.[10] In another study, an adaptive control scheme is proposed for similar systems.[11]

Su and Stepanenko proposed an adaptive sliding mode control algorithm for coordinating multiple robot arms so that they grasp a constrained object and handle it along a rigid surface. The stability analysis of the system controlled using the mentioned rule shows that the proposed algorithm can achieve satisfactory tracking performance.[12]

Luecke and Lai studied a joint error-feedback control approach to internal force regulation in cooperative manipulator systems.[13] Liu and Arimato developed decentralized

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adaptive and non-adaptive position/force controllers for redundant manipulators in cooperation.[14]

In a study on adaptive control with impedance of cooperative multi-robot system, the manipulation of a smooth object is effected by an adaptive control scheme, and the asymptotic stability is provided using the Lyapunov function.[15]

Itoh, Murakami and Ohnisni suggest a novel approach to control cooperating robots and to manipulate an object being grasped. In this method, at first, the grasp and acceleration forces of end-effector are calculated, and then by using backward iteration, the required joint torques of manipulators are found. Finally, a workspace observer is adopted so that tip deflections are corrected. Thus, the control structure is simplified, and it may be possible to utilize decentralized control technique for the cooperative manipulators to control.[16]

Liu and Abdel-Malek developed a robust control method for a planar dual-arm manipulator system. Considering the contact and friction constraints for the grasp conditions, a robust controller is proposed using a switching-sliding algorithm for modeling imprecision and disturbances.[17]

Zribi, Karkoub and Huang addressed the problem of modeling and controlling two manipulators handling a constrained object. A reduced order dynamics of system is derived, and using this model, an adaptive control scheme that guarentees the asymptotic convergence of the object's position and the force acting on the object to their desired values is achieved.[18]

In order to assign a manipulation task for a cooperative system, a suitable definition of the variables describing the motion of system has to be adopted. A cooperative task space formulation developed by Chiacchio, Chiaverini and Siciliano, which fully characterizes the coordinated motion of system, at the same time, allows the user to specify the task in terms of meaningful variables.[19] Based on this formulation, a kinematic control strategy, a task space control law and a task space regulation scheme for a system of two cooperative manipulators tightly grasping a rigid object have been proposed.[20,21]

Takubo et al. presented a robotic assistance system which enables a human to handle unwieldy long objects by providing only one point of support.[22] The robot grasps one end of the object and helps the human operator carry it at the other end. The proposed control method uses a virtual nonholonomic constraint. In spite of the constraint, the object can reach any position and orientation due to the nonholonomy. Experimental results show that an operator can easily handle a long object when aided by the robot.

That paper also presents a study of the application of adaptive and robust control methods to a cooperative manipulation system which is developed for handling an object by three dimensional revolute-jointed manipulators. It is here assumed that there is no relative motion among the grippers of manipulators and the object. The dynamics of the cooperative manipulation first will be introduced. The adaptive and robust control schemes corresponding to the cooperative manipulation system are described in successive sections. The adaptive control algorithm ensures a parameter adaptation law that guarantees the stability of uncertain systems. In designing the robust control structure, contact and friction constraints for grasp and bearing conditions, structural flexibility or such similar factors as various unmodeled dynamics are considered as the uncertainties that determine the available values of control parameters. The robust and adaptive control algorithms are applied separately to the developed dynamic model for controlling the trajectory. The novelty of results in the present paper is to define new control inputs using the parametric uncertainties and the Lyapunov based theory of guaranteed stability of uncertain systems. In addition, proposed control schemes have been developed for handling tasks a in spatial workspace.

II. PARAMETERS OF THE COOPERATIVE HANDLING

Besides the parameters of manipulators involved in cooperative manipulation, it is necessary to define the parameters that characterize the object handled by a multi-robot system, its relative position to the grippers of manipulators, and the relative position of the manipulators themselves. For that purpose, local reference systems, T_i, each bound to the last segment of the corresponding manipulator Z_i are introduced. However, in defining the object's motion, the local coordinate frame T_0, which is bound to the object, with the origin in the center of mass of the object is also considered.[23] The model of cooperative manipulation is composed of an object and three dimensional revolute-jointed manipulators such that the object is handled by these manipulators along a given path in a spatial workspace. The model and its parameters are shown in Fig. 1.

The model parameters are;

Z_i : ith manipulator,
r_i : Relative position of T_o with respect to T_i, bounds to the last segment of Z_i
R_i^o : Transformation matrix that maps vectors from system T_o to vectors in T_i.
A_1^i : Homogenous transformation matrix that maps vectors from the base coordinate system of Z_i to vectors in the base coordinate system of Z_1.
m_o : mass of the object.
I_o : Inertia of the object in the system T_o

III. MOTION EQUATION OF OBJECT

By assuming that there is no relative motion among grippers of manipulators and the object, the motion of the object under the influence of forces and torques applied by manipulators can be treated. The notation to be used is;[23]

F_i , M_i : Force and torque applied upon the object by manipulator Z_i in system T_i,
a_o, α_o, ω_o : Linear and angular acceleration, and angular velocity of the object in system T_o, respectively.
R_B^3 : Transformation matrix between T_3 and the absolute system bound to the base of Z_1.
g : Gravitational acceleration,

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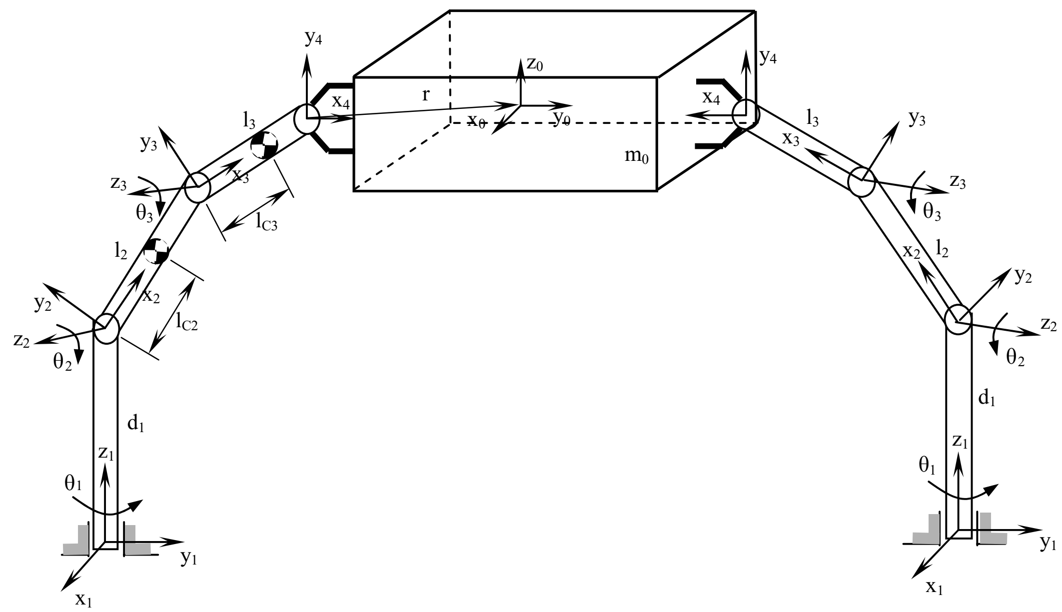

Fig. 1. Dynamic model of the cooperative manipulation.

The equation of the force balance acting upon the object, in the system T_3 is

$$\sum_{i=1}^{n} R_3^0 R_i^{0^T} \vec{\mathbf{F}}_i = m_o R_3^0 \vec{\mathbf{a}}_0 - m_0 R_B^{3^T} \vec{\mathbf{g}} \qquad (1)$$

where

$$R_3^0 \vec{a}_0 = \vec{a}_3 + \vec{\alpha}_3 \otimes \vec{r}_3 + \vec{\omega}_3 \otimes (\vec{\omega}_3 \otimes \vec{r}_1) \qquad (2)$$

If α_o and ω_o are mapped in T_3, the equation of torque balance with respect to the origin of the frame T_o, in the system T_3, is

$$\sum_{i=1}^{n} R_3^0 R_i^{0^T} (\vec{M}_i - \vec{r}_i \otimes \vec{F}_i) = R_3^0 I_0 R_3^{0^T} \vec{\alpha}_3 - \left(R_3^0 I_0 R_3^{0^T} \vec{\omega}_3\right) \otimes \left(R_3^{0^T} \vec{\omega}_3\right) \qquad (3)$$

By making various arrangements, the following notations are used to describe the object motion in matrix form.

B_i : Transformation matrix
f_i : Force and torque vectors applied upon the object
$C\ddot{q}_1$: Equivalent inertia force in the joint coordinates of the first link
D : Centripetal/coriolis and gravitational force of the object

From above notation, the equation of object motion can be unified in the matrix form,

$$\sum_{i=1}^{n} B_i f_i = C\ddot{q}_1 + D \qquad (4)$$

where q shows the joint variables of manipulators, J(q) also corresponds to the Jacobian matrix.

IV. COOPERATIVE MANIPULATION DYNAMICS

In order to obtain a dynamic model of cooperative manipulation, it is necessary to unify dynamic models of the manipulators and the model of interaction, represented by Eq. (4). This can be done by eliminating the interaction forces f_i from the above model. The dynamic model of the manipulator Z_i is

$$H_i(q_i)\ddot{q}_i + h_i(q_i, \dot{q}_i) = P_i - J_i^T(q_i) f_i \qquad (5)$$

where P_i denote joint torques of the considered robot, J(q) Jacobian and negative sign implies that the second term corresponds to the external force/torque vector.

If the value of f_i obtained from Eq. 4 is substituted into Eq. 5 and the dynamic model is arranged for the manipulator Z_1, the model becomes

$$\left(H_1 + J_1^T B_1^{-1} C\right)\ddot{q}_1 + h_1 + J_1^T B_1^{-1} D = P_1 + J_1^T B_1^{-1} \sum_{i=2}^{n} B_i \left(J_i^T\right)^{-1} P_i \qquad (6)$$

Consequently, the dynamic model for the system of n cooperating robot to handle an object can be written in a matrix form.

$$X\ddot{q}_1 + x = \sum_{i=1}^{n} \emptyset_i P_i \qquad (7)$$

where;

$$\begin{aligned} \emptyset_i &= J_1^T B_1^{-1} B_i \left(J_i^T\right)^{-1} \\ X &= H_1 + J_1^T B_1^{-1} C \\ x &= h_1 + J_1^T B_1^{-1} D \end{aligned} \qquad (8)$$

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where $X\ddot{q}_1$ is inertia forces of cooperative manipulation, x the sum Coriolis, centrifugal and gravitational forces and $\sum_{i=1}^{n} \O_i P_i$ the torque vector applied by the cooperative manipulators.

The dynamics of the cooperative manipulation system including two robots can be modified into

$$H(q)\ddot{q} + h(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\pi \quad (9)$$

where π is a 12-dimensional vector of constant robot parameters and Y is a 3×12 matrix which is a function of joint positions, velocities and accelerations of the spatial manipulators. Details dealing with such parameters as X, h and Y are given in the appendix.

V. ADAPTIVE CONTROLLER

The possibility of finding adaptive control laws is ensured by the property of linearity in the parameters of the dynamic model of a manipulator. In fact, it is always possible to express the nonlinear equations of motion in a linear form with respect to a suitable set of constant dynamic parameters as in Eq. (9). For the purpose of control, Eq. (9) can be modified into

$$H(q)\ddot{q} + h(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\pi = \tau \quad (10)$$

where τ is a 3-dimensional vector of control input employed for the system to control. For any specific trajectory, consider known the desired position, velocity and acceleration vectors q_d, $\dot{q}_d$ and $\ddot{q}_d$ and measured the actual position and velocity, and then the position and velocity errors $\tilde{q} = q_d - q$, and $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$. Using the above information a corrected desired velocity and acceleration vectors for nonlinearities and decoupling effects are proposed as

$$v_0 = \dot{q}_d + \Lambda\tilde{q} \qquad a_0 = \ddot{q}_d + \Lambda\dot{\tilde{q}} \quad (11)$$

where Λ is a positive definite matrix. Then the following control law is considered.

$$\tau = H(q)a_0 + h(q,\dot{q})v_0 + G(q) + K_D\sigma \quad (12)$$

where $\sigma = v_0 - \dot{q} = \dot{\tilde{q}} + \Lambda\tilde{q}$ is a corrected velocity error and $K_D\sigma$ is the vector of PD action Suppose that the computational model has the same structure as that of the manipulator dynamic model, but its parameters are not known exactly. Hence the control vector can be defined in the following form.

$$\begin{aligned} \tau &= \hat{H}(q)a_0 + \hat{h}(q,\dot{q})v_0 + \hat{G} + K_D\sigma \\ &= Y(q,\dot{q},v_0,a_0)\hat{\pi} + K_D\sigma \end{aligned} \quad (13)$$

where $\hat{\pi}$ represents the available estimate on the parameters, $\hat{H}$, $\hat{h}$ and $\hat{G}$ denote the estimated terms in the dynamic model. Substituting (13) into (10) gives

$$\begin{aligned} H(q)\dot{\sigma} + h(q,\dot{q})\sigma + K_D\sigma &= -\tilde{H}(q)a_0 - \tilde{h}(q,\dot{q})v_0 - \tilde{G}(q) \\ &= -Y(q,\dot{q},v_0,a_0)\tilde{\pi} \end{aligned} \quad (14)$$

where the parameter error vector is

$$\tilde{\pi} = \hat{\pi} - \pi \quad (15)$$

System modelling errors can be expressed as:

$$\tilde{H} = \hat{H} - H, \quad \tilde{h} = \hat{h} - h, \quad \tilde{G} = \hat{G} - G \quad (16)$$

Consider the Lyapunov function candidate as

$$V(\sigma,\tilde{q},\tilde{\pi}) = \frac{1}{2}\sigma^T H(q)\sigma + \frac{1}{2}\tilde{q}B\tilde{q} + \frac{1}{2}\tilde{\pi}^T K_\pi\tilde{\pi} > 0 \quad (17)$$

The time derivative of V along the trajectories of the system (14) is

$$\dot{V} = \sigma^T H(q)\dot{\sigma} + \frac{1}{2}\sigma^T\dot{H}(q)\sigma + \tilde{q}^T B\dot{\tilde{q}} + \tilde{\pi}^T K_\pi\dot{\tilde{\pi}} \quad (18)$$

Substituting Eq. (14) into Eq. (18) yields

$$\begin{aligned} \dot{V} &= \sigma^T\left[\frac{1}{2}\sigma^T\dot{H}(q) - h(q,\dot{q})\right]\sigma - K_D\sigma + \tilde{q}^T B\dot{\tilde{q}} \\ &\quad -Y(q,\dot{q},v_0,a_0)\tilde{\pi} + \tilde{\pi}^T K_\pi\dot{\tilde{\pi}} \end{aligned} \quad (19)$$

where $\frac{1}{2}\dot{H}(q) - h(q,\dot{q})$ is the skew-symmetry matrix and equals to zero. In the view expression of σ, with diagonal Λ and K_D, it is convenient to choose $B = 2\Lambda K_D$; this leads to

$$\dot{V} = -\sigma^T K_D\sigma + \tilde{q}^T B\tilde{q} - Y(q,\dot{q},v_0,a_0)\tilde{\pi} + \tilde{\pi}^T K_\pi\dot{\tilde{\pi}}$$

Then

$$\dot{V} = -\dot{\tilde{q}}^T K_D\dot{\tilde{q}} - \tilde{q}^T\Lambda K_D\Lambda\tilde{q} + \tilde{\pi}^T[K_\pi\dot{\tilde{\pi}} - Y^T(q,\dot{q},v_0,a_0)] \quad (20)$$

If the estimate of the parameter vector is updated as in the adaptive law

$$\dot{\hat{\pi}} = K_\pi^{-1}Y^T(q,\dot{q},v_0,a_0)\sigma \quad (21)$$

thus the last term in Eq. (20) becomes zero, and consequently the following expression is obtained.

$$\dot{V} = -\dot{\tilde{q}}K_D\dot{\tilde{q}} - \tilde{q}^T\Lambda K_D\Lambda\tilde{q} \le 0 \quad (22)$$

So, $\dot{V}$ is negative semidefinite and the system defined by Eq.14 is stable. It should be noted that $\dot{\hat{\pi}} = \dot{\tilde{\pi}}$ (π is constant).[24]

VI. ROBUST CONTROL LAW

Consider the nominal control vector for the model system described by Eqs. (9) and (10).

$$\begin{aligned} \tau_0 &= H_0(q)a_0 + h_0(q,\dot{q})v_0 + G_0(q) - K_D\sigma \\ &= Y(q,\dot{q},v_0,a_0)\pi_0 - K_D\sigma \end{aligned} \quad (23)$$

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The nominal control vector τ_0 in Eq. (23) is defined in terms of fixed parameters given by π_0.The other quantities as distinct of from adaptive law are given by

$$\tilde{q} = q - q_d \quad \dot{\tilde{q}} = \dot{q} - \dot{q}_d \quad a_0 = \ddot{q}_d - \Lambda\dot{\tilde{q}}$$
$$\upsilon_0 = \dot{q}_d - \Lambda\tilde{q} \quad \sigma = \dot{\tilde{q}}\Lambda + \dot{\tilde{q}} \tag{24}$$

It is supposed that the parameter vector π is uncertain and both π_0 and ρ are known. The parametric uncertainty $\tilde{\pi}$ is,

$$\|\tilde{\pi}\| = \|\pi - \pi_0\| \leq \rho \tag{25}$$

The control input τ can be defined in terms of the nominal control vector τ_0

$$\tau = \tau_0 + \mathrm{Y}(q, \dot{q}, \upsilon_0, a_0)\mathrm{u} = \mathrm{Y}(q, \dot{q}, \upsilon_0, a_0)(\pi_0 + \mathrm{u}) - \mathrm{K}_D\sigma \tag{26}$$

Let $\varepsilon > 0$ and the additional control vector is defined by Spong as[25]

$$\mathrm{u} = \begin{cases} -\rho\dfrac{\mathrm{Y}^T\sigma}{\|\mathrm{Y}^T\sigma\|} & if\ \|\mathrm{Y}^T\sigma\| > \varepsilon \\ -\rho\dfrac{\mathrm{Y}^T\sigma}{\varepsilon} & if\ \|\mathrm{Y}^T\sigma\| \leq \varepsilon \end{cases} \tag{27}$$

As a measure of parameter uncertainty on which the additional control input is based, ρ can be defined as

$$\rho = \left(\sum_{i=1}^{p} \rho_i^2\right)^{1/2} \tag{28}$$

Having a single number ρ to measure the parameter uncertainty may lead to overly conservative design, higher than necessary gains, etc. For this purpose, different gains to the components of u may be assigned. Hence, the uncertainty for each parameter $\tilde{\pi}_i$ can be defined separately using the parameter measure given by Eq. (28)

$$\lfloor\tilde{\pi}_i\rfloor \leq \rho_i \qquad i = 1, 2, \ldots, p \tag{29}$$

Let υ_i denote the ith component of the vector $\mathrm{Y}^T\sigma$, ε_i represents the ith component of ε, and define the ith component of the control input u_i as[25]

$$\mathrm{u}_i = \begin{Bmatrix} -\rho_i\upsilon_i/|\upsilon_i| & if\ \upsilon_i > \varepsilon_i \\ -(\rho_i/\varepsilon_i)\upsilon_i & if\ \upsilon_i \leq \varepsilon_i \end{Bmatrix} \tag{30}$$

VII. UNCERTAINTY BOUNDS IN PRACTICE

Although the computational model of the robot inverse dynamics has the same structure as that of the true manipulator dynamic model, parameter estimate uncertainty does exist. Therefore, in such control applications, as trajectory control, the error arising from parameter uncertainties can be compensated using the uncertainty bounds. Available values of these bounds are determined by considering such parameters as modelling imprecision and several disturbances, etc. In the case of implementation of robust control techniques, uncertainty bounds are particularly defined to derive the control law.

Contact and friction constraints for grasp and bearing conditions, unmodelled dynamics such as structural flexibility are considered as uncertainties that determine appropriate control law.

These constraints can be represented by friction effect at the contact surfaces among the grippers and object. If the friction coefficient at contact surfaces is supposed to be 0.15,[26] in order to compensate for that kind of losses, an additional force, which corresponds to 7.5 percent of the object's weight, has to be acted upon the object by cooperating manipulators.

Link's flexibility also causes some uncertainties. Assuming manipulator links to be a uniform thin rod with one end fixed and a mass attached at the other end, and the tip deflections of the links have to be smaller than 0.5 cm, an extra force or an additional weight is taken into account for determining the parameter uncertainty bounds. The value of this extra load can be supposed to be equivalent to 40–50 percent of the link mass. The tip deflection limits used for manipulator links are obtained as a result of experimental analysis.[27] Furthermore, if basic mechanics is considered, it can be assumed that additional force or mass corresponds to the loss due to tip deflection of links.

Besides the factors above mentioned, the losses at the joint bearings have to be considered in estimating true links' parameters. Therefore, in consequence of friction at link's joints it is supposed that there is a average loss of 3% at each joint.

Consequently, as a result of unmodelled dynamic effects above explained, the object and links' parameters m_o, m_1 and m_2 are changed in the following intervals so that the proposed control law satisfies required robustness.

Hence the changing interval of the mass handled by means of cooperative manipulation is bounded as

$$0 \leq \Delta m_o \leq 0.6 \tag{31}$$

and then the parameter bounds for the second and first link's masses are chosen, respectively.

$$0 \leq \Delta m_2 \leq 3.6$$
$$0 \leq \Delta m_3 \leq 1.6 \tag{32}$$

Adaptive and robust control algorithms have been applied to a cooperative manipulation model as shown in Fig.1, and for that purpose the system parameters are

$$\pi_1 = 2I_d; \quad \pi_2 = 2m_2 l_{c2}^2 + 2I_2; \quad \pi_3 = 2m_3 l_2^2;$$
$$\pi_4 = 2m_3 l_1 l_{c3}^2; \ \pi_5 = 2m_3 l_{c3}^2 + 2I_3; \quad \pi_6 = 2m_2 l_{c2};$$
$$\pi_7 = 2m_3 l_2; \ \pi_8 = 2m_3 l_{c3}; \quad \pi_9 = I_z; \ \pi_{10} = I_y;$$
$$\pi_{11} = I_x; \quad \pi_{12} = m_o \tag{33}$$

For illustrative purposes, the parameters of the unloaded manipulator are assumed to be known and are given by Table I. Using these values, the components of the π, obtained from Eq.(33), are shown in Table II, and the defined

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Table I. Parameters of the arms and the carried mass.

m_1 (kg)	m_2 (kg)	m_3 (kg)	d_1 (m)	l_2 (m)	l_3 (m)	l_{c2} (m)	l_{c3} (m)	I_d (kgm^2)	I_x (kgm^2)	I_y (kgm^2)	I_z (kgm^2)	m_0 (kg)
5	3	1	0.5	0.4	0.3	0.2	0.15	0.03	0.01	0.0017	0.12	0.5

Table II. π_i for the unloaded arm and the carried mass.

π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}
0.03	0.16	0.16	0.06	0.03	0.6	0.4	0.15	0.012	0.0177	0.0105	0.5

values also show the lower bounds of parameters for the adaptive control rule. π_0 is chosen as a vector of nominal parameters in robust control law, and appropriate parameters are determined considering the uncertainty range in Eqs. (31) and (32). The obtained values also show the uncertainty upper bounds in adaptive control law, and the components of π_0 are given in Table III. The difference between the values given in of Tables II and III denotes the uncertainty bounds ρ_i in robust control law (30), and the relevant values are given in Table IV.

VIII. DISCUSSION AND CONCLUSION

In the presence of the uncertainty, adaptive or robust control laws are used. In adaptive control strategy, parameters are estimated with estimation law to control the system properly and the parameters are assumed to be unknown and bounded. Updating algorithm stops when it reaches its known bound and resumes updating as soon as corresponding adaptation algorithm changes signs.[28] In robust control, parameters are not estimated like in adaptive control, and control input is defined as a function of fixed parameters and uncertainty bound.

In this paper, an adaptive and robust control laws that ensure limited tracking error have been derived for a cooperative manipulation system using the parameter estimation and uncertainty bounds, which guarantee the stability of an uncertain system, respectively.

In the adaptive control law, the parameter adaptive law, Eq. (21), does not ensures that $\hat{\pi}$ tends to π; indeed, convergence of parameters to their true values depends on the structure of the matrix $Y(q, \dot{q}, v_0, a_0)$ and then on the desired and actual trajectories. The term $Y\hat{\pi}$ ensures an approximate compensation of nonlinear effects, $K_D\sigma$ introduces a stabilizing linear control action and the matrix K_π determines the convergence rate of parameters to their asymptotic values.

As it is known, a robust control law is preferred in the presence of external disturbances and unmodeled dynamics, such as structural flexibility, unless the algorithm is modified. For that reason, as expressed in the previous section, the parameter uncertainty bounds corresponding to the unmodelled dynamics have been obtained as the values guaranteed the system stability using a Lypunov function candidate.

Handling an object on the desired trajectory is adopted as a task achieved by the cooperating manipulators given in Fig. 1. To illustrate the tracking performances of two different controllers, each control scheme under uncertainty are analyzed. For that purpose, the control parameters are chosen as high as possible, and the parameters are changed until the best performance is obtained for each case. Appropriate values of the control parameters making tracking error minimum for given trajectory are defined according to different control laws.

Figs. 2–4 show the tracking performances obtained for the straight-line trajectory for adaptive and robust control techniques. As seen from the relevant figures, the tracking performance of an adaptive controller is better than by a robust controller.

In order to investigate the performances of the controllers for different trajectories, in conformity with practice, for the case of part feeding to machine tool, a second trajectory is considered. The object is carried from position 1 to position 2 as defined for the previous trajectory, and the reference path that corresponds to the second trajectory is a semi-circle in space. The tracking responses of the adaptive and robust controllers are given in Figs. 5–7. As seen from these figures, a certain amount of difference in tracking error cannot be found for different paths.

The adaptive control algorithm incorporates some sort of on-line parameter estimation, while a robust control is usually a fixed controller design to satisfy performance specification over a given range of uncertainty. An adaptive controller can learn from experience in the sense that parameters are changed on-line, whereas a robust controller does not usually learn from earlier performance. Adaptive control

Table III. Nominal parameter vector π_0.

π_{01}	π_{02}	π_{03}	π_{04}	π_{05}	π_{06}	π_{07}	π_{08}	π_{09}	π_{010}	π_{011}	π_{012}
0.03	0.352	0.4160	0.1560	0.078	1.3200	1.04	0.39	0.0264	0.0389	0.0231	1.1

Table IV. Uncertainty bound.

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	ρ_{12}
0.01	0.1920	0.2560	0.0960	0.0480	0.7200	0.6400	0.240	0.0144	0.0212	0.0126	0.6

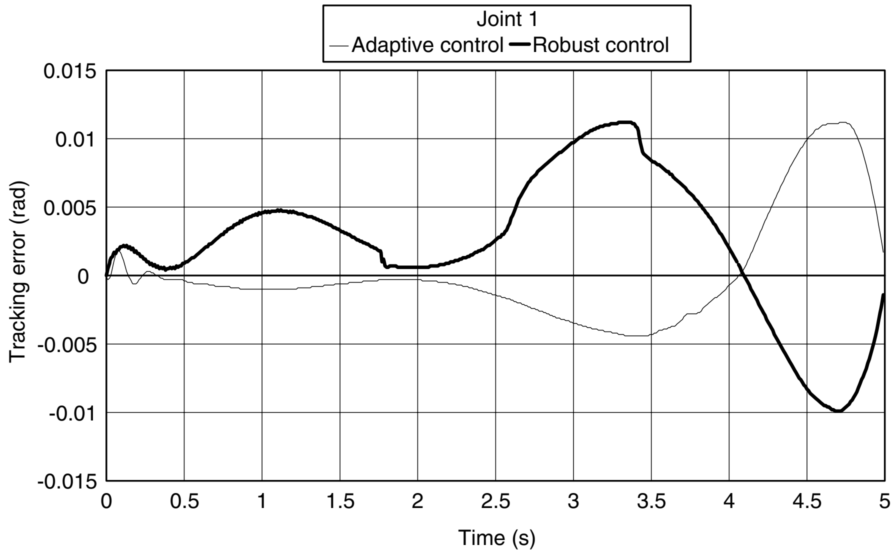

Fig. 2. Tracking error of the joint 1 for straight line trajectory for the adaptive control: $K_\pi = \mathrm{diag}([0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 2\ 2\ 8\ 8\ 8\ 8])$; $\Lambda = \mathrm{diag}(50\ 50\ 50)$; $K_D = \mathrm{diag}(15\ 30\ 10)$; for the robust control: $\Lambda = \mathrm{diag}(30\ 30\ 30)$, $K_D = \mathrm{diag}(25\ 22\ 5)$; $\varepsilon = 1$.

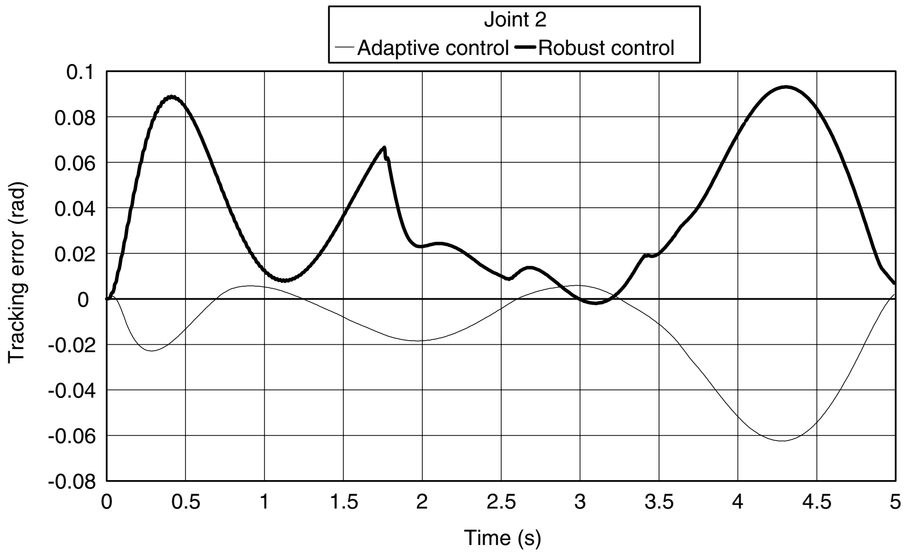

Fig. 3. Tracking errors of the joint 2 for straight line trajectory for the adaptive control: $K_\pi = \mathrm{diag}([0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 2\ 2\ 8\ 8\ 8\ 8])$; $\Lambda = \mathrm{diag}(50\ 50\ 50)$; $K_D = \mathrm{diag}(15\ 30\ 10)$; for the robust control: $\Lambda = \mathrm{diag}(30\ 30\ 30)$, $K_D = \mathrm{diag}(25\ 22\ 5)$; $\varepsilon = 1$.

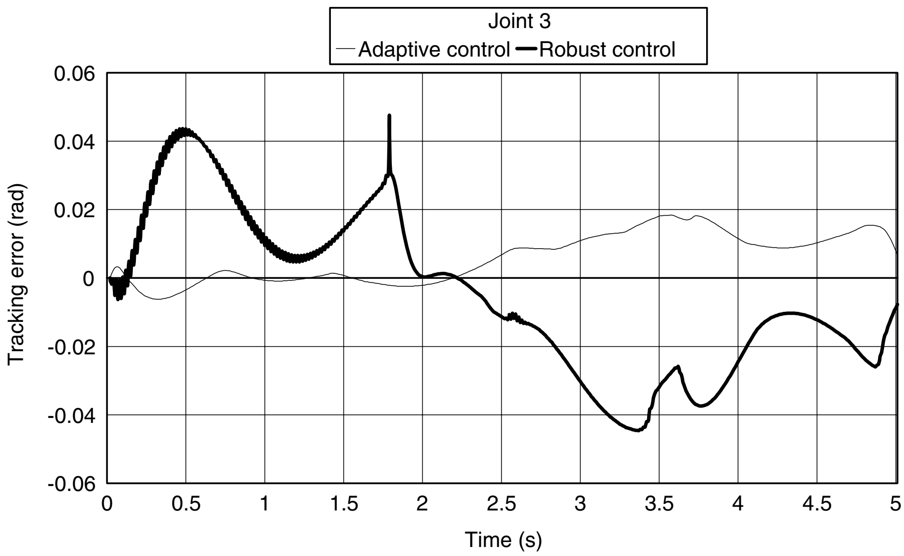

Fig. 4. Tracking error of the joint 3 for straight line trajectory for the adaptive control: $K_\pi = \mathrm{diag}([0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 2\ 2\ 8\ 8\ 8\ 8])$; $\Lambda = \mathrm{diag}(50\ 50\ 50)$; $K_D = \mathrm{diag}(15\ 30\ 10)$; for the robust control: $\Lambda = \mathrm{diag}(30\ 30\ 30)$, $K_D = \mathrm{diag}(25\ 22\ 5)$; $\varepsilon = 1$.

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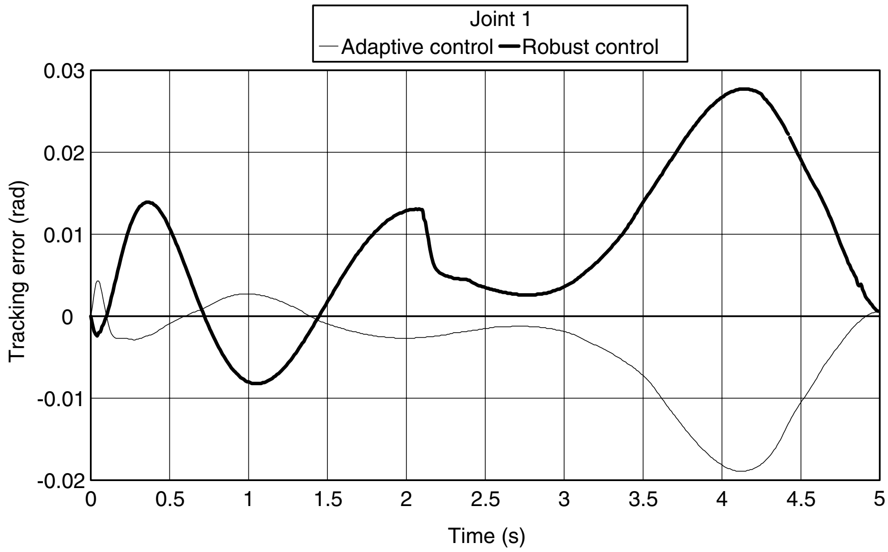

Fig. 5. Tracking error of the Joint 1 for second trajectory for the adaptive control: $K_\pi = \text{diag}([0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 2\ 2\ 8\ 8\ 8\ 8])$, $\Lambda = \text{diag}[55\ 55\ 55]$, $K_D = \text{diag}[30\ 35\ 8]$; for the robust control: $\Lambda = \text{diag}[30\ 30\ 30]$ $K_D = \text{diag}[25\ 15\ 5]$, $\varepsilon = 1$.

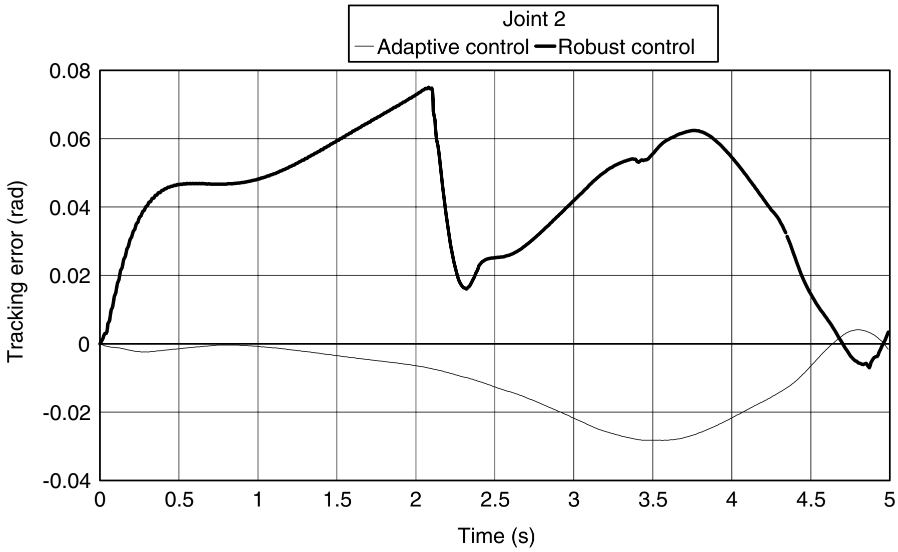

Fig. 6. Tracking error of the Joint 2 for second trajectory for the adaptive control: $K_\pi = \text{diag}([0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 2\ 2\ 8\ 8\ 8\ 8])$, $\Lambda = \text{diag}[55\ 55\ 55]$, $K_D = \text{diag}[30\ 35\ 8]$; for the robust control: $\Lambda = \text{diag}[30\ 30\ 30]$ $K_D = \text{diag}[25\ 15\ 5]$, $\varepsilon = 1$.

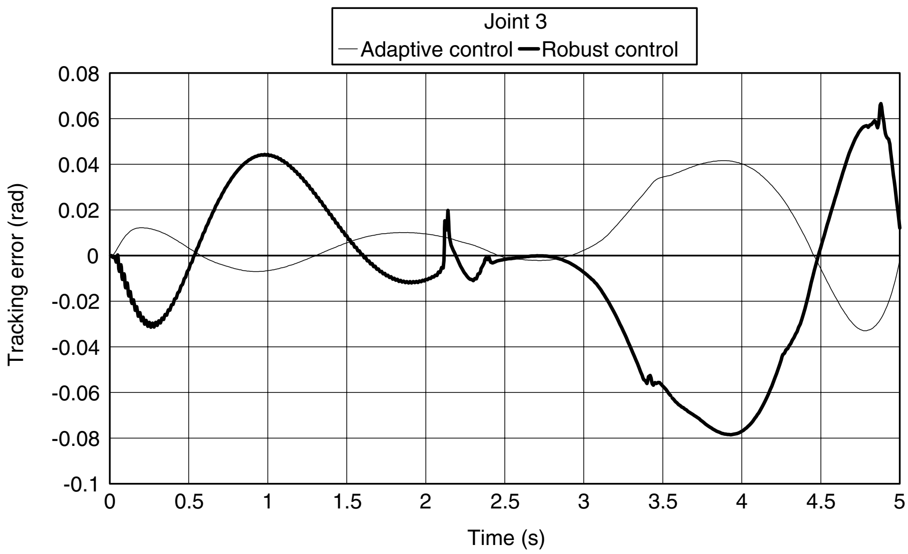

Fig. 7. Tracking error of the Joint 3 for second trajectory for the adaptive control: $K_\pi = \text{diag}([0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 2\ 2\ 8\ 8\ 8\ 8])$, $\Lambda = \text{diag}[55\ 55\ 55]$, $K_D = \text{diag}[30\ 35\ 8]$; for the robust control: $\Lambda = \text{diag}[30\ 30\ 30]$ $K_D = \text{diag}[25\ 15\ 5]$, $\varepsilon = 1$.

works better if the uncertainty is large but its performance is poor in the presence of external disturbances and unmodelled dynamics. As shown Figs. 2–7, the tracking error is small for both controllers, but the adaptive control approach has the best tracking performance and smooth time behavior. The question of whether to use robust or adaptive control does not have an obvious answer. If the uncertainty is large and there is a computational model, adaptive control is better. In the presence of external disturbances and unmodelled dynamics such as structural flexibility unless the algorithm is modified.[29] the performance of an adaptive controller is poor.[25] The robust control algorithm is also simple but suffers from uncertainty if it is large and chattering happens. However, the robust control law is simpler in design than the previous robust algorithms, and if the uncertainty is not large, it may be an alternative to the adaptive control.[25] As shown in the relevant figures, the adaptive controller has the best final tracking accuracy and the best tracking performance, and does not suffer from uncertainty as much as robust control law. Adaptive control also guarantees asymptotic compensation in tracking without chattering. Consequently, since the dynamics of the spatial cooperative manipulation is complex with respect to a simple manipulation with a single robot manipulator, the uncertainties dealing with the system parameters cause the various difficulties in controlling by means of conventional control laws. Therefore adaptive and robust control laws eliminate disturbances arising from uncertainties in parameters and unmodelled dynamics.

Besides, if the results obtained for spatial cooperative manipulation are compared to that of planar cooperative manipulation.[30] it is obvious that the performances of the applied control laws are the same for both models, that is, there is no difference in obtained tracking errors.

APPENDIX

$$X(q) = \begin{bmatrix} \pi_1 + \pi_2 c_2^2 + \pi_3 c_2 + 2\pi_4 c_{23} c_2 + \pi_5 c_{23} & 0 & \pi_9 \\ 0 & \pi_2 + \pi_3 + 2\pi_4 c_3 + \pi_5 + \pi_{10} & \pi_4 c_3 + \pi_5 \\ -\pi_{11} & \pi_4 c_3 + \pi_5 & \pi_5 \end{bmatrix}$$

$$h(q, \dot{q}) = \begin{bmatrix} 2\pi_5 s_{23} c_{23} (\dot{q}_2 + \dot{q}_3) & (2\pi_2 s_2 c_2 + 2\pi_3 s_2 c_2 + 2\pi_4 s_2 c_{23})\dot{q}_1 & (2\pi_4 c_2 c_{23})\dot{q}_1 \\ -(\pi_2 s_2 c_2 + \pi_3 s_2 c_2 + \pi_4 s_2 c_{23} + \pi_5 s_{23} c_{23})\dot{q}_1 & 2\pi_4 s_3 \dot{q}_3 & \pi_4 s_3 \dot{q}_3 \\ -(\pi_4 c_2 s_{23} + \pi_5 s_{23} c_{23})\dot{q}_1 & -\pi_4 s_3 \dot{q}_2 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} g\pi_{12} \\ g\pi_6 c_2 + g\pi_7 c_2 + g\pi_8 c_{23} \\ g\pi_8 c_{23} \end{bmatrix}$$

$$y(1,1) = \dot{q}_1$$
$$y(1,2) = \cos(q_2)^2 \dot{q}_1 + 2\sin(q_2)\cos(q_2)\dot{q}_1\dot{q}_2$$
$$y(1,3) = \cos(q_2)\dot{q}_1 + 2\sin(q_2)\cos(q_2)\dot{q}_1\dot{q}_2$$
$$y(1,4) = 2\cos(q_2+q_3)\cos(q_2)\dot{q}_1 + 2\cos(q_2)\sin(q_2+q_3) \times \dot{q}_1\dot{q}_3 + 2\sin(q_2)\cos(q_2+q_3)\dot{q}_1\dot{q}_2$$
$$y(1,5) = \cos(q_2+q_3)\dot{q}_1 + 2\sin(q_2+q_3) \times \cos(q_2+q_3)(\dot{q}_1\dot{q}_2 + \dot{q}_1\dot{q}_3)$$
$$y(1,6) = 0; \quad y(1,7) = 0; \quad y(1,8) = 0;$$
$$y(1,9) = \ddot{q}_3 y(1,10) = 0; \quad y(1,11) = 0; \quad y(1,12) = g;$$
$$y(2,1) = 0;$$
$$y(2,2) = \ddot{q}_2 - \sin(q_2)\cos(q_2)\dot{q}_1^2$$
$$y(2,3) = \ddot{q}_2 - \sin(q_2)\cos(q_2)\dot{q}_1^2$$
$$y(2,4) = \ddot{q}_3\cos(q_3) + 2\cos(q_3)\ddot{q}_2 - \sin(q_2)\cos(q_2+q_3)\dot{q}_1^2 + \sin(q_3)\dot{q}_3^2 + 2\sin(q_3)\dot{q}_2\dot{q}_3$$
$$y(2,5) = \ddot{q}_2 + \ddot{q}_3 - \sin(q_2+q_3)\cos(q_2+q_3)\dot{q}_1^2$$
$$y(2,6) = g\cos(q_2) \quad y(2,7) = g\cos(q_2)$$
$$y(2,8) = g\cos(q_2+q_3); \quad y(2,9) = 0; \quad y(2,10) = \ddot{q}_2;$$
$$y(2,11) = 0; \quad y(2,12) = 0;$$
$$y(3,1) = 0; \quad y(3,2) = 0; \quad y(3,3) = 0;$$
$$y(3,4) = \cos(q_3)\ddot{q}_2 - \cos(q_2)\sin(q_2+q_3)\dot{q}_1^2 - \sin(q_3)\dot{q}_2^2$$
$$y(3,5) = \ddot{q}_3 + \ddot{q}_2 - \sin(q_2+q_3)\cos(q_2+q_3)\dot{q}_1^2$$
$$y(3,6) = 0; \quad y(3,7) = 0; \quad y(3,8) = g\cos(q_2+q_3);$$
$$y(3,9) = 0; \quad y(3,10) = 0; \quad y(3,11) = -\ddot{q}_1;$$
$$y(3,12) = 0;$$

$Y0(q, \dot{q}, v_0, a_0)$ has the following components

$$y0(1,1) = a_{01}$$
$$y0(1,2) = \cos(q_2)^2 a_{o1} + 2\sin(q_2)\cos(q_2)\dot{q}_1 v_{o2}$$
$$y0(1,3) = \cos(q_2)a_{o1} + 2\sin(q_2)\cos(q_2)\dot{q}_1 v_{o2}$$
$$y0(1,4) = 2\cos(q_2+q_3)\cos(q_2)a_{01} + 2\cos(q_2) \times \sin(q_2+q_3)\dot{q}_1 v_{03} + 2\sin(q_2)\cos(q_2+q_3)\dot{q}_1 v_{02}$$
$$y0(1,5) = \cos(q_2+q_3)a_{01} + 2\sin(q_2+q_3)\cos(q_2+q_3) \times (\dot{q}_2 v_{01} + v_{01}\dot{q}_3)$$
$$y0(1,6) = 0; \quad y0(1,7) = 0; \quad y0(1,8) = 0;$$
$$y0(1,9) = a_{03}; \quad y0(1,10) = 0;$$
$$y0(1,11) = 0; \quad y0(1,12) = g; \quad y0(2,1) = 0;$$

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$$y0(2,2) = a_{02} - \sin(q_2)\cos(q_2)\dot{q}_1 v_{01}$$

$$y0(2,3) = a_{02} - \sin(q_2)\cos(q_2)\dot{q}_1 v_{01}$$

$$y0(2,4) = a_{03}\cos(q_3) + 2\cos(q_3)a_{02} - \sin(q_2) \times \cos(q_2+q_3)\dot{q}_1 v_{01} + \sin(q_3)\dot{q}_3 v_{03} + 2\sin(q_3)\dot{q}_3 v_{02}$$

$$y0(2,5) = a_{02} + a_{03} - \sin(q_2+q_3)\cos(q_2+q_3)\dot{q}_1 v_{01}$$

$$y0(2,6) = g\cos(q_2); \quad y0(2,7) = g\cos(q_2);$$

$$y0(2,8) = g\cos(q_2+q_3); \quad y0(2,9) = 0; \quad y0(2,10) = a_{02};$$

$$y0(2,11) = 0; \quad y0(2,12) = 0; \quad y0(3,1) = 0;$$

$$y0(3,2) = 0; \quad y0(3,3) = 0;$$

$$y0(3,4) = \cos(q_3)a_{02} - \cos(q_2)\sin(q_2+q_3)\dot{q}_1 v_{01} - \sin(q_3)\dot{q}_2 v_{02}$$

$$y0(3,5) = a_{03} + a_{02} - \sin(q_2+q_3)\cos(q_2+q_3)\dot{q}_1 v_{01}$$

$$y0(3,6) = 0; \quad y0(3,7) = 0; \quad y0(3,8) = g\cos(q_2+q_3);$$

$$y0(3,9) = 0; \quad y0(3,10) = 0;$$

$$y0(3,11) = -a_0; \quad y0(3,12) = 0;$$

References

1. H. P. Huang and R. S. Chen, "Modeling and adaptive coordination control of two-robot system," *Journal of Robotic Systems* **9**(1), 65–92 (1992).
2. T. Yoshikawa and X.-Z. Zheng, "Coordinated dynamic hybrid position/force control of for multiple-robot manipulators handling one constrained object," *Int. J. Robotics Research*, **1**(3), 219–230 (1993).
3. J. F. Kleinfinger and W. Khalil, "Developed an algorithm for dynamic modeling of robot containing closed kinematic chains," *Proc.16th ISIR*, Brussels, Belgium (1986) pp. 45–56.
4. M. Vukobratovic and V. Potkonjak, *Applied Dynamics and CAD of Manipulation Robots* (Springer, Berlin, 1985).
5. C. O. Alford and S. M. Belyeau, "Coordinated control of two robot arms," *Proc. IEEE Int. Conf. on Robotics* (1984) pp. 468–473.
6. S. Arimato, F. Miyazaki and S. Kavamura, "Cooperative motion control of multiple-robot arms or fingers," *IEEE Int. Conf. on Robotics and Automation* (1987) pp. 1407–1412.
7. G. Weibing and C. Mian, "A new control strategy for robotic system in tracking task," *Mechatronics* **1**, No. 3, 353–366 (1991).
8. Z. X. Li, P. Hsu and S. Sastry, "Grasping and coordinated manipulation by a multi-fingered robot hand," *Int. J. Robotics Research* **8**(4), 33–50 (1989).
9. V. Perdereau and M. Drouin, "Hybrid external control for two-robot coordinated motion," *Robotica,* **14** Part 2, 141–153 (1996).
10. K. Y. Lian and C. R. Lin, "Sliding-mode motion/force control of constrained robots," *IEEE Transaction Automatic Control* **43**, No. 8, 1101–1107 (1998).
11. J. H. Jean and L. C. Fu, "An adaptive control scheme for coordinated multi-manipulator system," *IEEE Transaction on Robotics and Automation* **9**, No. 12, 226–231 (1993).
12. C. Y. Su and Y. Stepanenko, "Adaptive sliding mode control of multiple-robot arms attached to a constrained object," *IEEE Transactions on Systems Man and Cybernetics* **25**, No. 5, 871–878 (1995).
13. G. R. Luecke and K. W. Lai, "A joint error-feedback control approach to internal force regulation in cooperation manipulator systems," *Journal of Robotics Systems* **14**, 631–648 (1997).
14. Y. H. Liu and S. Arimato, "Decentralized adaptive and non-adaptive position/force controllers for redundant manipulators in cooperation," *Int. J. Robotics Research* **17**, 232–247 (1998).
15. A. Rodriguez-Angeles and V. Parra-Vega, "Adaptive control with impedance of cooperative multi-robot system," *Proc. 1998 IEEE International Conference on Robotics & Automation* Leuven, Belgium (1998) pp.1522–1527.
16. M. Itoh, T. Murakami and K. Ohnisni, "Decentralized control of cooperative manipulators based on virtual force transmission algorithm," *Proc. 1999 IEEE Conference*, Kohala Coast-Island of Havai, USA (1999) pp. 874–889.
17. J. F. Liu and K. Abdel-Malek, "Robust control of planar dual-arm cooperative manipulators," *Robotics and Computer Integrated Manufacturing* **16**, 109–119 (2000).
18. M. Zribi, M. Karkoub and L. Huang, "Modelling and control of two robotic manipulators handling a constrained object," *Applied Mathematical Modeling* **24**, 881–898 (2000).
19. P. Chiacchio, S. Chiaverini and B. Siciliano, "Direct and inverse kinematics for coordinated motion task of a two-manipulator system," *Transactions of the ASME Journal of Dynamics Systems, Measurement and Control* **118**, 691–697 (1996).
20. F. Caccavale, P. Chiacchio and S. Chiaverini, "Stability analysis of a joint space control law for a two-manipulator systems," *IEEE Transactions on Automatic Control* **44**, 85–88 (1999).
21. F. Caccavale, P. Chiacchio and S. Chiaverini, "Task-space regulation of cooperative manipulators," *Automatica* **36**, 879–887 (2000).
22. T. Takubo, H. Arai, Y. Hayashibara and K. Tanie, "Human-Robot Cooperative Manipulation Using a Virtual Nonholonomic Constraint," *Int. J. Robotics Research* **21** Issue 5/6, 541–553 (2002).
23. M. D. Djurovic and M. K. Vukobratovic, "A contribution to dynamic modeling of cooperative manipulation," *Mech. Mach. Theory* **25**(4), 407–415 (1990).
24. L. Scıavıcco and B. Siciliano, *Modeling and Control of Robot Manipulators* (The McGraw-Hill Companies, New York, 1996).
25. M. W. Spong, "On the robust control of robot manipulators," *IEEE Trans. Automatic Cont.* **37**, 1782–1786 (1992).
26. D. M. Considine and G. D. Considine, *Standart Handbook of Industrial Automation* (Chapman and Hall., New York, 1986).
27. H. Ertas, "Simulation of tip deviations for flexible manipulator link," *PhD Thesis* (Erciyes Univ., Inst. of Science, Turkey, 1995).
28. J. J. Slotine and W. Lee, "On the adaptive control of robotic manipulators," *Int. J. Robotics Research* **6**, No. 3, 49–59 (1987).
29. F. Ghorbel, A. Fitzmorris and M. W. Spong, "Robustness of Adaptive Control of Robots: Theory and Experiment," *Workshop Nonlinear Adaptive Control Issues Robot*, Grenoble, France, Nov. 21–23 (1990).
30. Uzmay, R. Burkan and H. Sarıkaya, "Application of robust and adaptive control techniques to cooperative manipulation," *Control Engineering Practice* **12**, 139–148 (2004).

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