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Does Frege Have a Metalinguistic Truth-Predicate in *Begriffsschrift*?

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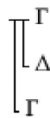
Abstract

In the explanations of logical laws and inference rules of the mature version of *Begriffsschrift* in *Grundgesetze*, Frege uses the predicate "... is the True." Scholars like Greimann maintain that this predicate is a metalinguistic truth-predicate for Frege. This paper examines an argument for this claim that is based on the "nominal reading" of Frege's conception of sentences—the claim that for Frege a sentence "*p*" is equivalent to a nonsentential phrase like "the truth-value of the thought that *p*." In particular, this paper attempts to establish two points concerning this argument based on the nominal reading. First, the argument implies a claim about the nature of assertion which Frege repeatedly denies in his mature works. Secondly, the nominal reading on which the argument depends is false. A sentence "*p*" is not equivalent to a nonsentential phrase like "the truth-value of the thought that *p*" for Frege. Our discussion will lead to an important lesson about Frege's conception of sentences and of assertion.

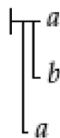
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Does Gottlob Frege have a metalinguistic truth-predicate in his logic, *Begriffsschrift*? Many commentators say "No"; they claim that Frege's conception of logic differs from the contemporary semantic conception of logic that demands a metalinguistic truth-predicate.¹ Nonetheless, scholars like Heck (2012) and Greimann (2008) argue that Frege's conception of logic substantively depends on semantics. Some of them such as Greimann (2007, 2008) even contend that we can find a metalinguistic truth-predicate in Frege's mature works. Look at, e.g., the following passage of *The Basic Laws of Arithmetic* (Frege 1893; hereafter, *Grundgesetze*):

According to section 12,



would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible; accordingly [Law I]



(Frege 1893, sec. 18)²

¹For this point, see, e.g., Burge (1986), Ricketts (1996, 2003), Goldfarb (2001), etc.

²For the translation of *Grundgesetze*, I depend on Ebert and Rossberg's (Frege 2013). For the translation of Frege's published or unpublished articles, I depend on Frege 1970, 1979, 1997.

Frege is explaining Law I of *Begriffsschrift*. Note that Frege's explanation looks like a metalinguistic justification of an axiom of a logical system. Also, note that the predicate "... is the True" looks like a truth-predicate in such a metalinguistic justification. We can find similar passages for other laws and inference rules of *Begriffsschrift*, and can also find the same predicate used in those passages. The claim of scholars like Greimann is that this predicate is a metalinguistic truth-predicate for Frege.³ I will call this claim "the TP (truth predicate) thesis."

There is an initial objection to the TP thesis. The predicate "... is the True" is related to Frege's unique conception of sentences. For Frege in his mature career, everything is either an object or a function.⁴ Now, a function is "unsaturated," i.e., has empty places for arguments. A function is thus referred to by an "unsaturated" expression such as " x is prime" or "the value-range of x ." On the other hand, an object is "saturated," i.e., has no empty place for arguments. An object is thus referred to by saturated expression such as "Gottlob Frege" or "the capital of England." Sentences are "saturated." Hence, what they refer to are objects. The reference of a sentence is one of the truth-values, the True or the False. Therefore, truth-values are objects and sentences are singular referential terms like proper names (cf. Frege 1891, 1892, 1893, 1906a, 1914).⁵ This means that sentences can be put in the position of " x " or " y " in the identity sign " $x = y$." For instance, informally,

$$(\text{the sum of 2 and 2 is 4}) = (\text{the sum of 2 and 3 is 5})$$

can be taken as a sentence that refers to the True, because both the left and the right refer to the True. Now, the predicate "... is the True" can be regarded as the identity predicate " x is identical with the True" such that we acquire a sentence that refers to the True by putting a name of the True in " x ." Given this explanation of "... is the True," it rather looks as if the predicate is not a metalinguistic truth-predicate, but just an identity predicate like " $x = 2$ " in Frege's formal language.

However, there is an argument that the predicate "... is the True" *as such* still can, and does, function as a truth-predicate in the metalinguistic sense. This argument, constructed by Greimann (2008), is based on a reading of Frege's conception of sentences, which is shared by commentators such as Klement (2001), Landini (1996), and Pedriali (2017). This interpretation of Frege's conception of sentences argues that for him the sentence " p " is equivalent to a nonsentential phrase like "the truth-value of the thought that p ." I call this interpretation "the nominal reading (of Frege's conception of sentences)." The aim of this paper is to critically examine the argument for the TP thesis based on the nominal reading.

Joan Weiner (2008) provides an extensive discussion of this argument. However, this paper's approach to the argument differs from Weiner's. Specifically, this paper establishes two points against the argument. First, the argument implies a claim about assertion which Frege repeatedly denies in his mature works. Secondly, a sentence " p " is not equivalent to a nonsentential phrase such as "the truth-value of the thought that p " for Frege. Our discussion leads to a crucial lesson about how we can understand the act of assertion under Frege's unique conception of sentence.

Section 1 introduces the target argument for the TP thesis by following Greimann's discussion. Section 2 criticizes the argument. Section 3 draws the lesson about Frege's conception of assertion from our discussion of the target argument.

³Künne (2008) and Taschek (2008) also identify this predicate as a truth-predicate for Frege.

⁴Greimann (2007) claims that truth is neither an object nor a function. However, Kim (2019) shows that his claim is grounded in a mistaken interpretation of Frege's remarks.

⁵The argument that truth-values are objects here follows Frege's own argument in "Function and Concept" (1891).

1. The argument for the TP thesis based on the nominal reading

Greimann (2008, 409) argues that if we can formulate Tarskian T-sentences with "... is the True," we can take the predicate as a metalinguistic truth-predicate. Greimann claims that it is possible to construct T-sentences with "... is the True": "the truth-value of 'Grass is green' is the True iff grass is green." For Frege, however, sentences are true or false only derivatively, i.e., only in the sense that the thoughts expressed by them are true or false. So the relevant kind of (quasi) T-sentences would be something like this:

The thought that grass is green is true iff grass is green.

Greimann (2008, 410) claims that we can construct a schema for the relevant kind of T-sentences out of "... is the True":

(FSch) The truth-value of the thought that p is the True iff p .

Hence, there is no problem with taking "... is the True" as a truth-predicate in the relevant sense. Or so argues Greimann.

An immediate objection, however, would be that (FSch) uses the operator "the truth-value of." Frege never uses such an operator in his explanations of logical laws. It rather seems as if Frege is using "... is the True" simply as the identity predicate " $x = \text{the True}$," so to speak, as in

(S) $2 + 2 = 4$ is the True.

If so, we cannot confirm that Frege is using "... is the True" like a truth-predicate in the relevant sense (Greimann 2008, 410).

As a response to this objection, Greimann appeals to the nominal reading of Frege's conception of sentence (Greimann 2007, 2008; Klement 2001; Landini 1996; Pedriali 2017), according to which the sense of any sentence " p " is identical with the sense of the phrase "the truth-value of: (the thought) that p " in Frege. If so, then even sentences such as (S) are in fact instances of (FSch). But why do we have to accept the nominal reading?⁶

In *Grundgesetze*, Frege writes:

" $0^2 = 4$," " $1^2 = 4$," " $2^2 = 4$," " $3^2 = 4$ "

are expressions of thoughts, some true, some false. I express it like this: the value of the function $\zeta^2 = 4$ is the *truth-value* of the true, or that of the false. It is already clear from this that I do not want to assert anything yet when I simply write down an equation, but that I merely *designate* a truth-value; just as I assert nothing when I simply write down " 2^2 ," but merely *designate* a number. (1893, sec. 2)

What Frege says here is related to the point that a sentence refers to a truth-value. In a sentence, or an equation in this case, we have a predicate: a word that refers to a concept, i.e., a function whose value is always a truth-value. For instance, in the above equations, we find the predicate " $\zeta^2 = 4$." Now, e.g., " $2^2 = 4$ " refers to the value of the concept $x^2 = 4$ saturated with the argument 2. Thus, it refers to the True. Because the value of a function is an object, truth-values are objects, and so the reference of a sentence, or an equation, is an object. Now, equations as such express thoughts. In Frege's *Begriffsschrift*, however, writing an equation down is not asserting that the thought it expresses is true. By writing it down, we only *designate* a truth-value. For asserting, a special sign is necessary:

⁶I do not dispute Greimann's claim that (FSch) gives us quasi T-sentences. Further, I do not dispute the claim that Frege provides a sort of metalinguistic justifications of logical laws. The aim of this paper is to critically examine this particular argument for those claims, which will be explained in the following.

Above it is already stated that within a mere equation no assertion is yet to be found; with “ $2 + 3 = 5$ ” only a truth-value is designated, without its being said which one of the two it is. Moreover, if I wrote “ $(2 + 3 = 5) = (2 = 2)$ ” and presupposed that one knows that $2 = 2$ is the True, even then I would not thereby have asserted that the sum of 2 and 3 is 5; rather I would only have designated the truth-value of: that “ $2 + 3 = 5$ ” refers to the same as “ $2 = 2$.” We are therefore in need of another special sign in order to be able to assert something as true. To this end, I let the sign “ \vdash ” precede the name of the truth-value, in such a way that, e.g., in

$$(B) \quad \vdash 2^2 = 4$$

it is asserted that the square of 2 is 4. (Frege 1893, sec. 5)

Expressions like (B) are well-formed *sentences* of *Begriffsschrift* (*Begriffsschriftsatz*).

If (B) is a sentence, then a question is raised. The reference of a sentence is always a truth-value. Hence, (B) is also the name of a truth-value. Now, say $\vdash \zeta^2 = 4$ is a concept such that it gives the True as its value if 2 is given as an argument and the False otherwise. Then, it follows that (B) also designates a truth-value as the equations in the earlier passage do. However, Frege clearly denies it:

The assertion sign cannot be used to construct a functional expression; for it does not serve in conjunction with other signs, to designate an object. “ $\vdash 2 + 3 = 5$ ” does not designate anything; it asserts something. (1891, 34n)

This raises the question of how (B) can be a name of a truth-value while $\vdash \zeta^2 = 4$ is not a concept.

Greimann maintains that Frege distinguishes sentences from (mere) singular terms for truth-values. Only expressions such as (B) are sentences. Equations such as “ $2^2 = 4$ ” are not sentences but singular terms for truth values. Both sentences and singular terms for truth-values are of course proper names of truth-values; they “denote (*bedeuten*)” (2008, 414) truth-values. However, singular terms for truth-values differ from sentences in that only the former can be used for the speech-act of reference qua a proper part of the speech act of assertion; only singular terms for truth-values “designate (*bezeichnen*)” (2008, 414) in this sense.

Then, the sense of an equation such as “ $2^2 = 4$,” which would be taken as a sentence, must be that of an expression that can be used as a part of a sentence like (B) for the sake of the speech-act of reference qua a proper part of assertion. Indeed, Greimann argues, Frege is *paraphrasing* the sentence in the ordinary sense “ $(2 + 3 = 5) = (2 = 2)$ ” into the nonsentential phrase: “the truth-value of: that “ $2 + 3 = 5$ ” refers to the same as “ $2 = 2$ ”.” Greimann insists that, in general, the sense of a singular truth-value term “ p ” is identical with that of “the truth-value of: that p .” Therefore, even “ $2 + 2 = 4$ is the True” is indeed equivalent to the (quasi) T-sentence “The truth-value of: that $2 + 2 = 4$ is the True.”⁷ Greimann concludes: “... is the True” is a truth-predicate in the relevant sense.

2. Criticism: The nominal reading can't be justified

The argument for the TP thesis heavily depends on the nominal reading according to which in Frege “ $2 + 2 = 4$ ” can be paraphrased into “the truth-value of: that $2 + 2$ is 4.” Consider the following objection to this interpretation. In a number of places, Frege says that the sense of a sentence in the usual sense is a thought. Since “ $2 + 2 = 4$ ” is a sentence in the usual sense, its sense must be a thought, i.e., the thought that $2 + 2$ is 4 or $\langle 2 + 2 \text{ is } 4 \rangle$. On the contrary, “the truth-value of: that $2 + 2$ is 4” is not

⁷See Greimann (2007, 136). Here, Greimann argues that because the thought of a sentence is a condition when the sentence refers to the True, “ $2 + 3 = 5$ ” indeed has the same sense as “the truth-value of: that $2 + 3$ equals 5.” He does not repeat the same argument in his 2008 paper. This argument, however, does not work if our discussion is along the right lines. For the critical discussion of Greimann (2007), see Kim (2019).

a sentence in the usual sense. So its sense is not a thought. Thoughts differ from senses of nonsentential singular terms. For an arbitrary thought $\langle p \rangle$, it is always identical with $\langle \langle p \rangle$ is true \rangle (Frege 1892, 1897, 1906a, 1914, 1915, etc.). But the same point is not applied to a nonsentential singular term. A case in point: “the truth-value of: that $2 + 2$ is 4.” Its sense differs from \langle The truth-value of: that $2 + 2$ is 4 is true \rangle .⁸ If “the truth-value of: that $2 + 2$ is 4” and “ $2 + 2$ is 4” have different senses, then one cannot be the paraphrase of the other.

Of course, as we have seen, Frege takes expressions such as “ $(2 + 3 = 5) = (2 = 2)$ ” to be legitimate, and thus he considers “ $2 + 3 = 5$ ” to be a proper name such as “the truth-value of: that $2 + 3$ is 5” in the *syntactical* level. One might object that this syntactic assimilation of a sentence to a proper name strongly suggests that he also *semantically* assimilates a sentence to a proper name like “the truth-value of: that ...” However, there is no textual evidence for this claim. Moreover, Frege’s text rather seems to show that this syntactic assimilation does not lead to the suggested semantic assimilation. In “On Sense and Reference,” immediately after Frege classifies sentences as proper names (1892, 63), he says that $\langle \langle 2 + 3 = 5 \rangle$ is true \rangle is identical with $\langle 2 + 3 = 5 \rangle$ (64). As we have just said, the sense of “the truth-value of that $2 + 3$ is 5” is not identical with \langle The truth-value of: that $2 + 3$ is 5 is true \rangle . We will come back to this issue at the end of this section.

What is important is that Greimann has a different response to the above objection against the nominal reading, which does not depend on the controversial claim that Frege’s syntactic assimilation of sentences to proper names implies that he semantically assimilates sentences to the phrases like “the truth-value of: that p .” According to Greimann, the nominal reading does not reject that a thought as the sense of a sentence differs from the sense of a singular term like “the truth-value of: that $2 + 2$ is 4.” When the nominal reading says that “the truth-value of: that $2 + 2$ is 4” is a paraphrase of “ $2 + 2 = 4$,” Greimann says, it is arguing that the latter as an expression of *Begriffsschrift* is “not to be considered as a sentence but as a complex singular term” (2008, 411). Let me explain further. According to Greimann, it is not the case that Frege “wants to defend the absurd view that in natural languages an expression like ‘ $2^2 = 4$ ’ is used as a singular [truth-value] term [like ‘the truth-value of: that 2^2 equals 4’]” (412). In natural language, it certainly expresses $\langle 2^2$ equals 4 \rangle and, as such, “can be used to assert something, but not to designate something” (412).⁹ The point of Greimann’s defense of the nominal reading is thus that Frege turns what we regard as sentences in natural languages into singular truth-value terms in *Begriffsschrift* because he makes a clear distinction between a mere entertainment of a thought, i.e., what he calls “predication,” and assertion. With ordinary language sentences like “ $2 + 2$ is equal to 4,” however, we cannot clearly mark the distinction. For we can use ordinary language sentences to perform both predication—entertainment of a thought—and assertion (412). The nominal reading claims that in order to make the distinction clearly shown, Frege regards “ $2 + 2 = 4$ ” qua an expression of his formal language to mean “the truth-value of: that $2 + 2$ equals 4,” which alone cannot be used for the act of assertion because it is not a sentence. A sentence is constructed and assertion is performed only if we write “ \vdash ,” a special truth-predicate or truth-operator that can be translated into “... is the True” (Greimann 2004, 2007), in front of such a term.

Several clarifications are necessary here. First, Greimann claims that while “... is the True” in “The truth-value of: that grass is green is the True” is a truth-predicate, it does not function as a truth-predicate when it is “applied to a singular term that does not *express a thought*, as in ‘5 is the True’” (2008, 414). This shows that Greimann takes the phrase “the truth-value of: that grass is

⁸What only thoughts—but not nonsentential singular terms—have is not assertoric force but positive predication. Positive predication itself does not amount to assertoric force. Thoughts do not carry assertoric force with themselves, as we will see below. Frege’s thesis that $\langle p \rangle$ is identical with $\langle p \rangle$ is true \rangle is rather his argument that a thought does not contain assertoric force.

⁹Greimann (2008, 412) also claims that Frege’s functional analysis of sentences is only intended to be applied to formal sentences. But this is not really clear. For instance, when Frege (1906b, 177–78) explains the sentence “2 is prime” in terms of the saturation of a concept, does he talk about formal languages or natural languages? Or, is he talking about language in general? Greimann’s claim seems to be question-begging without further discussion.

green”—unlike “5”—to express a thought. If he does so, there is a conflict between Greimann’s terminology and mine. I use “thought” in order to refer to the sense of a sentence like “ $2 + 2$ equals 4” in *English*. I also say that a term expresses S just in case a term has S as its sense—following Frege’s own terminology (1892, 61).¹⁰ In my terminology, “the truth-value of: that $2 + 2$ equals 4” does not express a thought because it is not a sentence and so its sense is not a thought. Recall that Greimann accepts that “ $2 + 2$ is 4” and “the truth-value of: that $2 + 2$ is 4” have different senses in English and that only expressions like the former count as sentences in it. Hence, he would accept that only “ $2 + 2$ is 4” expresses a thought in my sense. Thus, when Greimann says that “the truth-value of: that p ” expresses a thought, he must be using “express a thought” in a different sense. Perhaps, he takes such a phrase to express a thought in the sense that it includes the name of a thought. What is important is that while following my terminology, we can still capture Greimann’s point safely as follows: “ $2 + 2 = 4$ ” in Frege’s formal language does not have $\langle 2 + 2 \text{ is } 4 \rangle$ —the sense of “ $2 + 2$ is 4” in English—as its sense. In fact, Greimann’s whole point is that no expression other than the ones prefixed with “ \vdash ” can have a thought—the sense of a sentence—as its sense. Given this clarification, keeping my own terminology in the following discussion with care will not distort Greimann’s reading.

Secondly, one might think that saying that Greimann considers “ \vdash ” to be a formal counterpart of “... is the True” is misleading because he says that the horizontal—the short horizontal stroke attached to the judgment-stroke—is such a formal counterpart (2008, 418). Frege indeed takes the horizontal to refer to a concept such that it yields the True if the True is given as an argument and the False otherwise (Frege 1893, sec. 5)—the concept *being identical with the True*. However, we need to note that by Greimann’s own light, “ $- 2 + 3 = 5$,” which is a legitimate expression of *Begriffsschrift*, does not correspond to the sentence “ $2 + 3 = 5$ is the True.” Because it does not have the judgment-stroke, it can only correspond to “the truth-value of: that $2 + 3 = 5$ is the True.” An expression that corresponds to the sentence “ $2 + 3 = 5$ is the True” is made only when we add the judgment-stroke at the left side of the horizontal. Greimann does not further explain how the judgment-stroke and the horizontal interact with each other to construct such a sentential expression. In any event, what is important to our purpose is that the predicate “... is the True” qua an expression that can produce a sentence of the form “ p is the True” only corresponds to “ \vdash ,” not the horizontal. That appears to be why Greimann (2008, 418) regards the sign “ \vdash ” including the judgment-stroke, but not the horizontal alone, as a truth-operator. In the remainder of this paper, “... is the True” is a predicate as a part of the sentence of the form “ p is the True.” Again, “ \vdash ” is a formal counterpart of “... is the True” as such.

Lastly, it is *not* Greimann’s point that just as *Begriffsschrift* needs “ \vdash ” to construct a sentence and make an assertion, we need “... is the True” in order to do so in natural language. According to Greimann, for Frege, we perform sentence-construction/assertion in natural language with “the syntactic mood of assertoric sentences” (2000, 219), not with such a predicate. Therefore, the *functional* counterpart of “ \vdash ” in natural language is the syntactic mood of assertoric sentences in Greimann’s reading. However, as we have seen, precisely because in natural language we need to perform sentence-construction/assertion with the syntactic mood of assertoric sentences, natural language blurs the distinction between assertion and mere predication. For that reason, Greimann says, Frege makes a clear division between mere predication and assertion by interpreting “ $\vdash p$ ” as a combination of a nonsentential expression such as “the truth-value of: that p ”—which does not express truth—and the truth-predicate “... is the True,” i.e., the judgment-stroke.

To recap the emerging explanation of *Begriffsschrift*, say—following Greimann—“ $- 2 + 2$ equals 4_E ” is an English sentence and “ $2 + 2 = 4_F$ ” is a corresponding singular term in *Begriffsschrift*. “ $2 + 2$ equals 4_E ” expresses $\langle 2 + 2 \text{ equals } 4 \rangle$, and is a sentence in the sense that it can be used to assert, but not to designate, an object. But “ $2 + 2 = 4_F$ ” does not express such a thought; it only expresses the sense of a singular term. Therefore, it is not a sentence in the same sense. We come to have a sentence in this

¹⁰Strictly speaking, a name refers to its reference and expresses its sense (Frege 1892, 61; Frege 1893, sec. 2).

sense only if we put “┆” in front of it. Now, “┆ $2 + 2 = 4$ ” does express a thought, namely, ⟨The truth-value of: that $2 + 2$ equals 4 is the True⟩. Hence, “┆” is a formal version of “... is the True” as a part of a sentence like “ $2 + 3 = 5$ is the True” and is the *only genuine* predicate—which is also a truth-predicate or -operator—of *Begriffsschrift* in the sense that only with “┆” can we have a sentence.¹¹

One problem with taking “┆” to be a truth-predicate is that “┆ 2 ” can also count as a *Begriffsschriftsatz*. Greimann ought to accept that, in such a case, “┆” is not a truth-predicate, but a mere identity predicate. Then, why don’t we say that it is merely such an identity predicate? This leads to a doubt against his reading of “... is the True” because “ 2 is the True” seems to be a perfectly legitimate. But Greimann does not seem to take this issue seriously. He simply admits that, in such a case, “... is the True” fails to work as a truth-predicate. Greimann’s idea seems to be that as long as the predicate produces all instances of (FSch), the fact that it can play a different role does not prevent us from regarding it as a truth-predicate. That idea still seems controversial.¹² However, this is not the major issue I would like to raise here.

According to the suggested interpretation of *Begriffsschrift*, we do not have a sentence that expresses a thought without “┆” in *Begriffsschrift*. To put it in a different way, the sense of “┆” contributes to the formation of the thought ⟨... is the True⟩. At the same time, “┆” stands for assertoric force. Therefore, here, “┆” (i) contributes to the formation of a thought and (ii) indicates assertoric force. In a significant sense, this explanation takes assertoric force to contribute to the formation of a thought. However, this is what Frege emphatically rejects in many places (e.g., Frege 1892, 1897, 1906a, 1914, 1915). For instance, Frege writes in “*On Schoenflies*”:

We can of course express a thought, without stating it to be true. The thought is strictly the same whether we merely express it or whether we also put it forward as true. Thus assertoric force, which is often connected with the copula or else with the grammatical predicate, does not belong to the expressions of the thoughts... (1906b, 177)¹³

In fact, the main logical defect of natural language concerning assertoric force—and assertion—is that it leads us to believe that assertoric force comes as a part of thought. In “My Basic Logical Insights,” Frege writes:

So the word “true” [in the sentences of the form “The thought that p is true”] seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought. (1915, 252)

“This attempt miscarries,” Frege says, because

the word “true” [in “The thought that p is true”] has a sense that contributes nothing to the sense of the whole sentence in which it occurs as a predicate. (1915, 252)

He is pointing to his claim that “ p ” and “The thought that p is true” have the same thought. Now, Frege writes:

Although this attempt miscarries, or rather through the very fact that it miscarries, it indicates what is characteristic of logic. (1915, 252)

¹¹Thus, Greimann denies the interpretation that takes the judgment-stroke to be a mere force indicator (2000, 215; 2008, 418). According to him, the judgment-stroke is also a truth-operator for “predicating truth” (2008, 418). Indeed, the early version of *Begriffsschrift* (Frege 1879) understands “┆” in this way. However, in the version in *Grundgesetze*, Frege clearly denies this view because the judgment stroke is not a predicate anymore (the judgment stroke is not identified as a primitive function). Frege has reasons for this change according to Heck (2012, sec. 2.3).

¹²For instance, see Weiner’s 2008 discussion.

¹³Also, Frege (1914, 233) says that to assert “we do not need a special predicate.” Greimann’s reading makes “┆” a special predicate for assertion. See section 3 for discussion of this point.

We can see what Frege is getting at only if we take into account what he says about judgment and assertion in the first half of this short manuscript:

Making a judgment (assertion) does not alter the thought that is recognized to be true. When something is judged (asserted) to be the case, we can always cull out the thought that is recognized as true; the act of judgment (assertion) forms no part of this. . . . If I assert “It is true that sea-water is salt,” then I assert the same thing as if I assert “Sea-water is salt.” This enables us to recognize that the assertion is not to be found in the word “true’, but in the assertoric force with which the sentence is asserted. (1915, 25)

The truth-predication that occurs in “The thought that p is true” gives an expression to the nature of assertion because it does not add anything to the thought of which truth is predicated just as assertion does not alter the asserted thought.¹⁴ The point here is not that the truth-predicate of a natural language is defective because it fails to make the assertoric force contribute to the formation of a thought. It is instead that to attempt to make the assertoric force be a part of a thought is defective because the assertoric force is not a part of a thought. It is hard to believe that Frege would let his logic commit the mistake he himself points out.

One might object that my argument depends on my terminology of “thought.” Specifically one might insist that “thought” in the above passages refers to the sense of “the truth-value of: that p .” However, that is not the case. In the above passages, Frege is analyzing natural language and diagnosing its problem. As Greimann himself admits, a sentence of natural language has the sense of a sentence, i.e., a thought in my terminology. Note: the thought that p in the above passages is identical with the thought that the thought that p is true. As we have already seen, the sense of “the truth-value of: that p ” cannot be said to be identical with ⟨the truth-value of: that p is true⟩. Therefore, Frege is talking about a thought in my terminology, i.e., a thought qua the sense of a natural language sentence. His point in the above passages is that the thought expressed by a sentence of natural language does not contain assertoric force in itself because assertoric force is not something expressed by a component of a sentence.

I believe that Greimann is right when he argues that (i) Frege needs a category of “expressions that are supposed to express a thought without simultaneously asserting its truth” (Greimann 2008, 412) and that (ii) “ $2 + 2 = 4_F$ ” falls under such a category. The problem is, I believe, that from those true claims, he proceeds to the conclusion that “ $2 + 2 = 4_F$ ” must correspond something like “the truth-value of: that $2 + 2$ is 4” in English.¹⁵ It is not clear at all why Frege ought to turn “ $2 + 2 = 4_F$ ” into a nonsentential term in order to deprive assertoric force of it. Why can’t he simply regard “ $2 + 2 = 4_F$ ” as corresponding to “ $2 + 2$ is 4” in English to which assertoric force is not added, e.g., the sentence uttered by an actor on a stage (cf. Frege 1892)? Is that because as soon as we form a sentence and thereby express a thought, we come to make an assertion, i.e., because sentences or thoughts contain assertoric force in themselves? But, as we have seen, that is just what Frege rejects. It does not appear that in order to detach assertoric force from “ $2 + 2 = 4_F$ ” we ought to turn it into a nonsentential term. All we need to do is to realize that assertoric force is not its semantic component. If we realize it, then we can take “ $2 + 2 = 4_F$ ” to be a sentence that expresses ⟨ $2 + 2$ is 4⟩ but is not asserted. Assertoric force is added to the thought by “┆.”

Indeed, “ $2 + 2 = 4_F$ ” or what Greimann calls “singular truth-value term” in general is a sentence that expresses a thought. After introducing eight primitive functions, Frege writes:

¹⁴For Frege, judging or asserting is not adding something to a thought, but taking a step from a thought to its truth-value. See Frege 1892.

¹⁵Klement (2001), Landini (1996), and Pedriali (2017) make exactly the same move. Thus my criticism here is also applicable to their versions of the nominal reading.

However, not only a reference but also a sense belongs to all names correctly formed from our signs. Every such name of a truth-value *expresses* a sense, a *thought*. For owing to our stipulations [for those primitive functions], it is determined under which conditions it refers to the True. The sense of this name, the *thought*, is: that *these conditions are fulfilled*. Now, a [*Begriffsschriftsatz*] consists of a judgment-stroke with a name, or a Roman marker, of a truth-value. (1893, sec. 32)

First, note that Frege takes a *Begriffsschriftsatz* like

$$(B) \quad \vdash 2^2 = 4$$

to be the combination of the judgment-stroke (the vertical stroke) and a name of a truth-value. Therefore, what Frege calls “name of a truth-value” is not a *Begriffsschriftsatz*. Rather, something like the following is an instance of such a truth-value name:

$$(N) \quad -\hat{\varepsilon}(-\varepsilon)$$

By the stipulations given for the functions involved in (N), we can specify when it refers to the True. Frege’s point entails that (N)’s sense is \langle Those conditions are met \rangle , i.e., \langle The value-range of the concept referred by the horizontal falls under the concept \rangle .¹⁶ (N) is a typical case of what Greimann takes as a singular truth-value term: it certainly refers to a truth-value though it is not a *Begriffsschriftsatz*. Frege’s remarks are explicit enough to affirm that such a singular truth-value term “*p*” in general expresses $\langle p \rangle$, not the sense of “the truth-value of: that *p*.” One might object that because “*p*” and “the truth-value of: that *p*” have the same condition for referring to the True, they should be taken to be equivalent. However, this objection is refuted by Frege’s remarks here. Though both “*p*” and “the truth-value of: that *p*” refer to the True under exactly the same condition, only $\langle p \rangle$ but not the sense of “the truth-value of: that *p*” says that the condition *is met*. Again, singular truth-value terms express thoughts. A thought is the sense of a sentence. It follows that Frege takes singular truth-value terms to be sentences. Then a singular truth-value term “*p*” cannot be translated into a nonsentential term such as “the truth-value: that *p*.” The argument for the TP thesis based on the nominal reading must be rejected because the nominal reading is incorrect.

3. The remaining problem and the lesson about *Begriffsschrift*

The following still needs to be explained: only expressions like (B) count as sentences in Frege’s *Begriffsschrift* by means of which we can assert but cannot designate a truth-value, while expressions such as “ $2 + 2 = 4$ ” only designate a truth-value. If our discussion is along the right lines, an expression cannot be used alone for asserting *even if* it expresses a thought. We need the sign “ \vdash ” in front of such an expression. Why do we need such a sign in addition to a term that expresses a thought? The answer is in one of the passages from *Grundgesetze* we’ve already seen:

Above it is already stated that within a mere equation no assertion is yet to be found; with “ $2 + 3 = 5$ ” only a truth-value is *designated, without its being said which one of the two it is*. (Frege 1893, sec. 5; italics mine)

Designating a truth-value corresponds to writing down a name of a truth-value such as “ $2 + 3 = 5$ ” *without* saying whether it refers to the True or the False. Then, to assert that $2 + 3$ is 5, i.e., to write down “ $\vdash 2 + 3 = 5$ ” is to “say” that $2 + 3 = 5$ is the True.

¹⁶The sentence in the latter angled brackets is a natural language analogue of (N).

To this end [of asserting] I make use of a vertical stroke at the left end of the horizontal, so that, e.g., by writing

$$\vdash 2 + 3 = 5$$

we assert that $2 + 3$ equals 5. Thus here we are not just writing down a truth-value, as in

$$2 + 3 = 5,$$

but also at the same time *saying that it is the True* (Frege 1891, 34; italics mine).

To assert that $2 + 3$ equals 5 is to identify $2 + 3 = 5$ with the True. We now have a neat explanation of the point in question. Assertion as such identification is distinguished from designation of a truth-value. Thus, to write down “ $\vdash 2 + 3 = 5$,” i.e., to assert that $2 + 3$ equals 5, is not to designate a truth-value. Writing down “ $2 + 3 = 5$ ” cannot be asserting that $2 + 3$ is 5 because it is not identifying the Truth with $2 + 3 = 5$.

I believe that it is in Frege’s assimilation of assertion with identification that the standard interpretation of assertion (judgment) in Frege (Burge 1986; Dummett 1993; Heck and May 2018; Heck 2012) is grounded. For instance, Heck (2012, sec. 2.3) says that we can understand asserting that p in Frege as referring the True via $\langle p \rangle$ or by the sentence “ p .” Note that when we refer to an object o by the name “N”—as we refer to the True by the sentence “ p ”—we thereby identify o with the reference of “N.” We can take Heck to be trying to make sense of Frege’s assimilation of assertion with identification by way of the notion of the speech-act of referring to an object by a name.

One might object that because an act of identifying an object O_1 as O_2 is an act of assertively predicating “ $x = O_2$ ” of “ O_1 ,” “ \vdash ” is a formal version of “... is the True” as Greimann argues. But again that cannot be correct. Return to another passage we have seen:

Moreover, if I wrote “ $(2 + 3 = 5) = (2 = 2)$ ” and presupposed that one knows that $2 = 2$ is the True, even then I would not thereby have asserted that the sum of 2 and 3 is 5; rather I would only have designated the truth-value of: that “ $2 + 3 = 5$ ” refers to the same as “ $2 = 2$.” (Frege 1891, 34)

What Frege wants to point out is that one cannot “say” that p is the True even by predicating the identity predicate “ $x = T$ ” of “ p ” where “T” is the most obvious name of the True.¹⁷ This shows that to assert that p , i.e., to “say” that p is the True, cannot be done by predicating “... is the True” of “ p .” Thus, “ \vdash ,” the device for asserting, cannot be such a predicate.

Then how do we have to understand “ \vdash ”? Recall that the horizontal is a predicate that refers to the concept *being the True*. Thus, the horizontal is indeed “... is the True” (as in “ p is the True”). Then, one can argue that the judgment-stroke marks assertoric force attached to this predicate and thus that by writing down “ $\vdash 2 + 3 = 5$ ” we assert that $2 + 3 = 5$ is the True—identifying $2 + 3 = 5$ with the True. This reading seems to fit what Frege says in *Grundgesetze*.

Of the two signs of which “ \vdash ” is composed, only the judgment-stroke contains assertion (Frege 1893, sec. 5).

Although this reading is tempting, our discussion in section 2 casts doubt on it. Recall that Frege repeatedly claims that assertoric force does not belong to any predicate. However, if the judgment-stroke really marks assertoric force, there is a significant sense in which assertoric

¹⁷It is not clear whether Frege really intends to provide a nonsentential paraphrase of “ $(2 + 3 = 5) = (2 = 2)$.” Weiner’s claim that such a phrase “may be simply one of many ways of directing his readers to which truth-value a particular truth-value name names” (2008, 442), seems to be much more convincing. In any event, Greimann’s point is not relevant to understanding this passage.

force belongs to a particular predicate, i.e., the horizontal. That is because the judgment-stroke must always be attached to the horizontal. “ $| 2 + 3 = 5$ ” is not even a legitimate expression of *Begriffsschrift* although there is a legitimate predicate—the identity sign—and the judgment-stroke as a marker of assertoric force. We can make an assertion only by “ $\vdash 2 + 3 = 5$,” i.e., only when we attach the judgment-stroke to the horizontal. If the judgment-stroke marks assertoric force, then the horizontal monopolizes assertoric force in *Begriffsschrift* and there is a substantive sense in which assertoric force belongs to it. Moreover, Frege maintains that because assertoric force is not a component of thoughts, “we do not need a special predicate” (1914, 233) in order to assert. However, if the above reading is correct, and so asserting that p is asserting that p is the True, then we need a special predicate for asserting—the identity predicate “... is the True” or the horizontal. Thus the above reading of “ \vdash ” goes against Frege’s repeated claims about assertion.¹⁸

Then, is there an alternative? One alternative is to take “ \vdash ” to be a pure performative for performing a *nonassertive* act of identification. We are not unfamiliar with the nonassertoric conception of identification. According to Millikan (1998, 2000) or Camp (2002), the act of identifying is neither an act of predicating an identity predicate nor asserting an identity. Identification is an activity of its own kind that cannot be explained in terms of predication or assertion. I believe that Frege is appealing to such a conception of identification when he elucidates that asserting that p is taking a step from $\langle p \rangle$ to its truth-value in “On Sense and Reference” (Frege 1892).¹⁹ To go into the detail of this seminal article goes beyond the scope of this paper. What is important is that, given all his remarks we have seen, Frege would rather understand “ \vdash ” as a performative for such a nonassertoric identification if he indeed takes the act of asserting that p to be that of identifying the True with p . Now, the remaining question is how we can understand the compositionality of “ \vdash ” if it is such a performative. The reading I suggest is to regard the judgment-stroke as a performative generator that produces a performative for relevant nonassertive identification when it is combined with an identity predicate. Thus, when the judgment-stroke is combined with the horizontal, we have a performative for nonassertive identification of the True. If we understand the compositionality of “ \vdash ” in this way, *Begriffsschrift* does not imply that we do not need the horizontal in order to assert. Of course, we need the horizontal to *acquire the performative*. However, that does not imply that we need the horizontal to assert. What we need for making an assertion is the act marked by “ \vdash ,” not the components of the sign. The compositionality of the sign is only for making explicit the nature of the action we perform. The horizontal marks the point that what we affirm by our action is the identity with the True, and the judgment-stroke marks the point that what we perform is not assertion of identity. Also, assertoric force does not belong to the horizontal anymore in this interpretation. For the judgment-stroke is not a marker of assertoric force. Furthermore, we can say that the act of assertion is only contained in the judgment-stroke because it is a performative generator while the horizontal is a mere predicate.

The expressions like “ $\vdash \zeta^2 = 4$ ” are not functional expressions, because “ \vdash ” is not a functor at all. What we have by putting “2” in the position of “ ζ ” is not a truth-value, but an act of asserting that 2^2 equals 4, i.e., “saying” that $2^2 = 4$ is the True. *Begriffsschriftsätze* are not names of truth-values. Thus, they do not designate truth-values.²⁰

¹⁸There are other problems with this reading. See Kim (Forthcoming, sec. 2).

¹⁹Kim (2019, sec. 4) points to this interpretation of “On Sense and Reference”; If judging is indeed such a fundamental act of identifying, we can make better sense of Frege’s claim that “judgment is something peculiar and incomparable” (Frege 1892, 65).

²⁰Note that, in section 32 of *Grundgesetze*, Frege does not classify *Begriffsschriftsätze* as names of truth-values. Also, recall that Frege says that a *Begriffsschriftsatz* asserts something. Sentences in the ordinary sense do not assert; we assert with sentences. *Begriffsschriftsätze* are not sentences in the ordinary sense. Greimann’s reading, however, compares *Begriffsschriftsätze* to sentences in the ordinary sense. That point is what distinguishes his reading from mine.

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