

# Trapping of electromagnetic radiation in self-generated and preformed cavities

SHIXIA LUAN,<sup>1</sup> WEI YU,<sup>1</sup> JINGWEI WANG,<sup>1</sup> MINGYANG YU,<sup>2</sup> SUMING WENG,<sup>3</sup>  
MASAKATSU MURAKAMI,<sup>3</sup> JINGWEI WANG,<sup>3</sup> HAN XU,<sup>4</sup> AND HONGBIN ZHUO<sup>4</sup>

<sup>1</sup>Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai, China

<sup>2</sup>Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou, China; Institute for Theoretical Physics I, Ruhr University, Bochum, Germany

<sup>3</sup>Institute of Laser Engineering, Osaka University, Osaka, Japan

<sup>4</sup>College of Science, National University of Defense Technology, Changsha, China

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## Abstract

Laser light trapping in cavities in near-critical density plasmas is studied by two-dimensional particle-in-cell simulation. The laser ponderomotive force can create in the plasma a vacuum cavity bounded by a thin overcritical-density wall. The laser light is self-consistently trapped as a half-cycle electromagnetic wave in the form of an oscillon-caviton structure until it is slowly depleted through interaction with the cavity wall. When the near-critical density plasma contains a preformed cavity, laser light can become a standing wave in the latter. The trapped light is characterized as multi-peak structure. The overdense plasma wall around the self-generated and preformed cavities induced by the laser ponderomotive force is found to be crucial for pulse trapping. Once this wall forms, the trapped pulse can hardly penetrate.

**Keywords:** Laser light trapping in cavities; Near-critical density plasmas; Oscillon-caviton structure; Overdense plasma wall; Preformed cavity

## 1. INTRODUCTION

Laser propagation in plasma depends on the laser properties as well as the plasma density. In low-density plasmas ( $n_0 \ll n_c$ , where  $n_c$  is the critical density), the laser pulse can propagate a considerable distance. In solid-density plasmas ( $n_0 \gg n_c$ ), on the other hand, it can penetrate only about a skin length and is reflected. In the recent years, the interaction of short and intense laser pulses (Perry & Mourou, 1994; Umstadter, 2003) with gaseous (Tajima & Dawson, 1979; Wang *et al.*, 2009; 2010; 2011; Sun *et al.*, 1987; Borghesi *et al.*, 1997), solid-density (Kruer & Estabrook, 1985; Luan *et al.*, 2011; 2012a; Brunel *et al.*, 1987), as well as near-critical density (Willingale *et al.*, 2011; 1997; Kuznetsov *et al.*, 2001; Mourou *et al.*, 2002; Nakamura *et al.*, 2008) plasmas has been widely investigated. However, creating near-critical density plasma with steep density gradient and optically probing the laser-plasma interaction region is experimentally challenging (Willingale *et al.*, 2011; 1997). Recently, low-density foam targets have been fabricated using supersonic gas jets

and *in situ* polymerization (Willingale *et al.*, 1997), thus making available near-critical density plasmas suitable for experiments.

Electromagnetic (EM) solitary waves (Yu *et al.*, 1978; Luan *et al.*, 2008; 2012b; Bulanov *et al.*, 1999; Esirkepov *et al.*, 1998; 2002; Sentoku *et al.*, 1999; Weber *et al.*, 2005; Farina *et al.*, 2000; Sanchez-Arriaga *et al.*, 2011a; Sanchez-Arriaga *et al.*, 2011b) can self-consistently appear when a laser pulse propagates in a near-critical density plasma (Yu *et al.*, 1978; Luan *et al.*, 2012b). On the ion time scale, they can appear as post-solitons (Yu *et al.*, 1978; Luan *et al.*, 2012b; Naumova *et al.*, 2001; Bulanov *et al.*, 2002; Borghesi *et al.*, 2002), in which the EM radiation sits inside a self-generated plasma cavity. Depending on the laser and plasma conditions, the latter can co-move with the laser pulse or be stationary (Yu *et al.*, 1978; Luan *et al.*, 2012b; Naumova *et al.*, 2001; Bulanov *et al.*, 2002; Borghesi *et al.*, 2002; Sarri *et al.*, 2010; Zhu *et al.*, 2012). Experiments with near-critical density plasmas have demonstrated the existence of well-isolated solitons and post-solitons (Borghesi *et al.*, 2002; Sarri *et al.*, 2010; Zhu *et al.*, 2012).

This paper investigates EM radiation trapping in self-generated as well as preformed cavities using two-dimensional

Address correspondence and reprint requests to: Shixia Luan, No.390, Qinghe Road, Jiading, Shanghai, 201800, China. E-mail: sxluan@siom.ac.cn

(2D) particle-in-cell (PIC) simulation. In Section 2, simulations are performed to detect laser trapping in the self-generated cavity when the laser enters near-critical density plasma. The laser pulse is partially reflected and partially transmitted into the plasma. The ponderomotive force of the light pulse modifies the local plasma density and self-consistently creates in the cavity an oscillon (Stenflo & Yu, 1989; 1996), or a half-cycle EM standing wave. Section 3 considers laser-light trapping in a preformed cavity in near-critical density plasma. In this case, a multi-peaked EM wave is formed in the cavity. A discussion is given in Section 4.

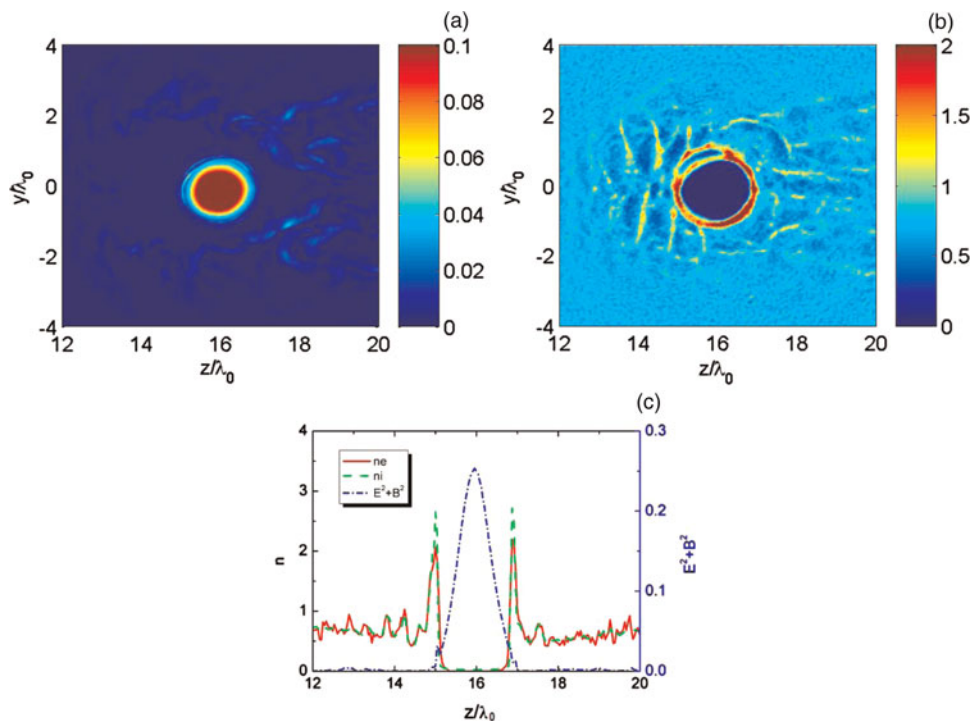
## 2. LIGHT TRAPPING IN SELF-GENERATED CAVITY

We performed two-dimensional in space and three dimensional in velocity (2D3V) PIC simulations for laser light trapping in near-critical density plasma. A Gaussian laser pulse is incident along the  $z$  axis from the left vacuum region into a homogeneous plasma located at  $10 < z/\lambda_0 < 50$ . The computation box is  $60\lambda_0 \times 30\lambda_0$  in size. The spatial mesh contains  $1500 \times 750$  cells, and each cell contains 16 ions and 16 electrons. The ion-electron mass ratio is 1836, and the initial electron density is  $n = n(0) = Zn_i(0) = 0.7$ , where  $n_i$  is the ion density. The incident laser strength, pulse duration, and spot size are  $a_L = 0.4$ ,  $\tau = 10T_0$ , and  $b = 5\lambda$ , respectively.

When the laser pulse is incident on the left vacuum-plasma boundary, it is partially reflected back to the left vacuum and partially transmitted into the near-critical density plasma. As

the transmitted light propagates, the laser ponderomotive force, proportional to the gradient of EM energy density, pushes the plasma away from the region where the EM field is strong. Figure 1 is for the EM energy density  $E^2 + B^2$ , the electron and ion densities at  $t = 77T_0$ , which is well into the ion time scale. We can see in Figures 1a and 1b that the electrons pushed away by the laser light pile up to form an overcritical-density plasma wall around a cavity, which traps the light. The maximum EM energy density occurs at the cavity center, and the EM field vanishes a short distance (of the order of the effective skin depth) into the high-density plasma wall. Figure 1c is for the densities of the EM energy, electrons, and ions along the laser axis. As expected, the electrons and ions behave almost as a quasi-neutral plasma, with only very little charge separation. The density of plasma wall is more than twice the critical density, thereby preventing the EM radiation from leaving the cavity. Clearly, the near-critical density background plasma plays a crucial role in the existence of the localized structure: the laser light is effectively trapped and stopped by the overdense cavity wall induced by its ponderomotive pressure. Otherwise, a laser can propagate through underdense plasma and be reflected by overdense plasma.

The EM pulse trapped inside the self-generated cavity oscillates in time. Figure 2 is for the normalized electric field component  $E_x$  at  $t = 102T_0$  (1) and  $t = 103T_0$  (2), about half an oscillation period ( $\sim 2T_0$ ) apart. The trapped half-cycle standing EM wave can be identified as an oscillon (Stenflo & Yu, 1996), and the self-generated cavity a caviton



**Fig. 1.** (Color online) Normalized EM energy density  $E^2 + B^2$  (a) and the electron density (b), as well as the EM energy density (blue dashed-dotted line), electron (red solid line), and ion (green dashed line) densities along the laser axis (c) at  $t = 77T_0$ , for  $a_L = 0.4$ ,  $\tau = 10T_0$ ,  $b = 5\lambda_0$ , and  $n_0 = 0.7n_c$ . One can see a standing half-cycle EM wave trapped in a self-generated plasma cavity.

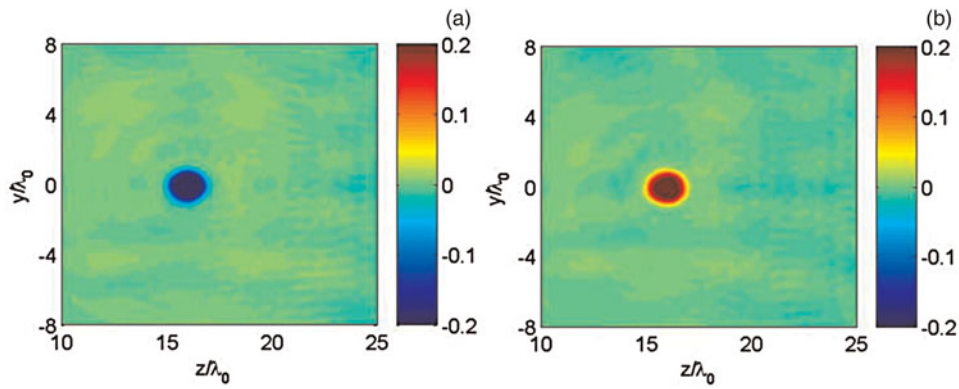


Fig. 2. (Color online) The normalized electric field  $E_x$  at  $t = 102T_0$  (a) and  $t = 103T_0$  (b), about half an oscillation cycle apart.

(Wong *et al.*, 1977; Cheung *et al.*, 1982) with overdense plasma wall.

EM radiation can be trapped in an empty cavity surrounded by an overdense plasma wall. Furthermore, the trapped EM radiation should satisfy the vacuum wave equation

$$\nabla^2 \mathbf{E} - \partial_{tt} \mathbf{E} = 0, \tag{1}$$

where time and space coordinates are normalized by  $\omega_0^{-1} k_0^{-1}$ , respectively. A simple cylindrically symmetric solution of the vacuum wave equation is

$$E(r, t) = C \cos(t) \sin(r)/r, \tag{2}$$

where  $E(r, t)$  is the radial electric field, the constant  $C = 0.2$  is determined by best-fit of this solution to that of the simulation. Figure 3 shows  $E(r, t)$  at  $t = \cos^{-1}(-1)$  and  $\cos^{-1}(1)$ , namely half an oscillation period apart. We see that the behavior of  $E(r, t)$  is consistent with that from the simulation, even though the boundary and initial conditions have not been applied.

### 3. LIGHT TRAPPING IN PREFORMED CAVITY

We now consider propagating a laser pulse in plasma containing a preformed cavity. As illustrated in Figure 4, a laser pulse enters near-critical-density plasma with a preformed vacuum cavity. The cavity radius is  $R = 2\lambda_0$ , and its center is at  $(y, z) = (0, 16\lambda_0)$ .

Laser propagation in plasma depends on the plasma density, or the dielectric coefficient  $\epsilon = 1 - n_e/n_c$ , such that  $\epsilon \sim 0$  in near-critical density plasma and  $\epsilon \sim 1$  inside the preformed vacuum cavity. Figure 5 shows the electric field component  $E_x$ , and the electron density  $n_e$  along the laser axis, at  $t = 21T_0$  and  $27T_0$ . The plasma density and the laser parameters are the same as that in the preceding case. As the laser pulse reaches the preformed cavity at  $t = 21T_0$ , its EM field is strongly modified because of the abrupt change in the plasma density: at the plasma-vacuum boundary a part of the laser light is back-scattered, enhancing its tail and exciting electron density oscillations with amplitude remaining less than  $n_c$ . The rest of the laser pulse enters the cavity and interacts with the plasma at its front. A thin enhanced electron layer of maximum density  $n_e \sim 1.2n_c$  is formed in front of the cavity by the compressed electrons, as can be seen in Figure 5d for  $t = 27T_0$ . The laser light is reflected, forming a standing EM wave in the slightly modified cavity. However, a part of the laser energy can tunnel through

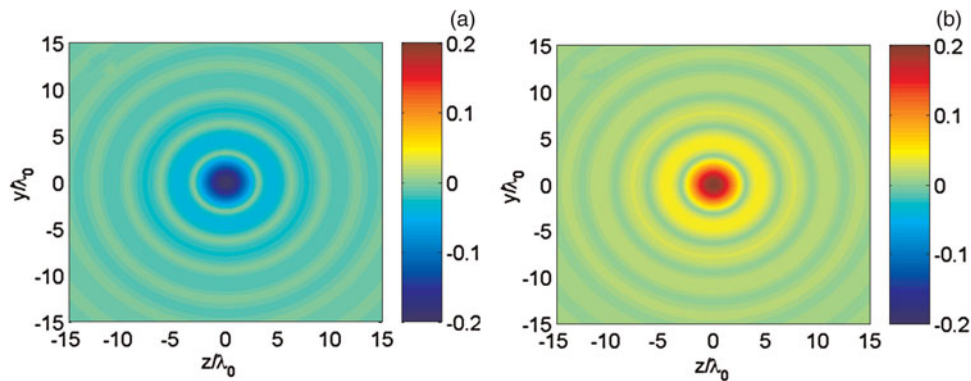
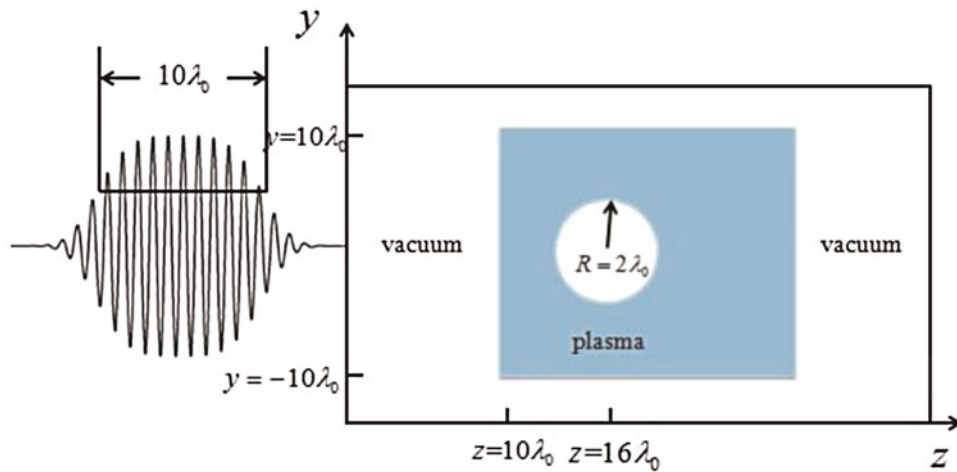


Fig. 3. (Color online) The normalized electric field  $E_x$  at  $t = \cos^{-1}(-1)$  (a) and  $\cos^{-1}(1)$  (b), from Eq. (2) for  $C = 0.2$ .

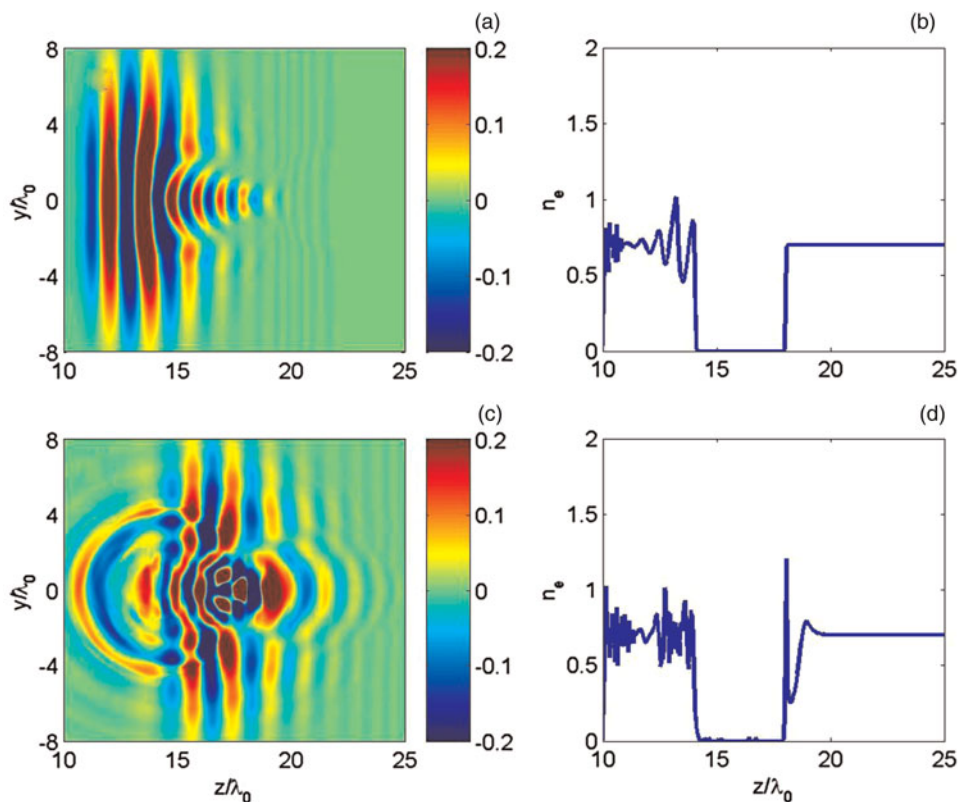


**Fig. 4.** (Color online) Schematic of laser-pulse entering a near-critical density plasma containing a preformed cavity. The plasma density and the laser parameters are the same as that in the preceding case.

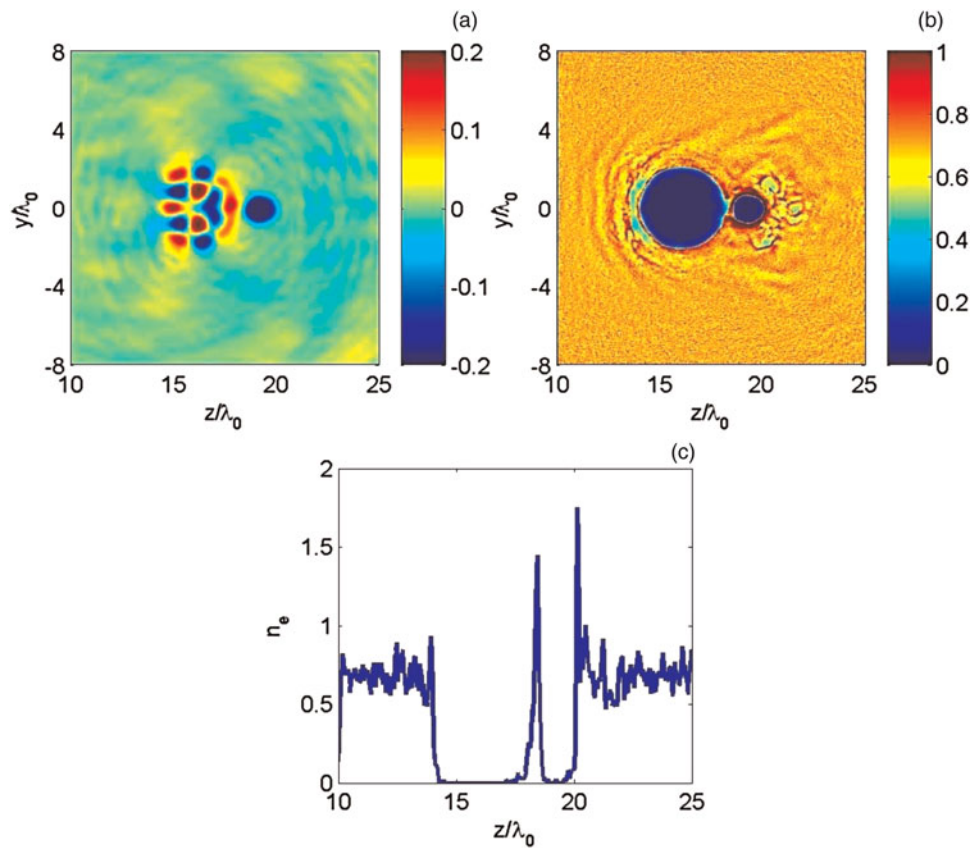
the narrow above-critical-density electron layer, forming in its front an oscillon-caviton structure, similar to that in a plasma without preformed cavity, but much smaller.

Figure 6 shows the electric field component  $E_x$  and the electron density  $n_e$  at  $t = 81T_0$ , as well as the electron density along the laser axis direction. EM fields are trapped in the two cavities bounded by ponderomotive-force driven

overdense plasma walls that prevent the light from leaving. As a result, the radiation can be trapped for rather long times. As aforementioned, the self-generated small cavity is attributed to radiation tunneling from the preformed cavity. The trapped radiation in this self-generated small cavity again undergoes single-peak oscillation. However, in the preformed cavity with radius larger than laser wavelength, EM



**Fig. 5.** (Color online) The normalized electric field component  $E_x$  at  $t = 21T_0$  (a) and  $t = 27T_0$  (c), and the electron density  $n_e$  (blue solid curve) along the laser axis at  $t = 21T_0$  (b) and  $t = 27T_0$  (d), for  $a_L = 0.4$ ,  $\tau = 10T_0$ ,  $n_0 = 0.7n_c$ , and  $b = 5\lambda_0$ . One can see that at  $t = 27T_0$  the electrons in a thin layer at the leading edge of the preformed cavity has become overdense.



**Fig. 6.** (Color online) The electric field  $E_x$  (a) and the electron density  $n_e$  (b) at  $t = 81T_0$ , and the electron density (c) along the laser axis direction. A caviton containing an EM oscillon appears in front of the preformed cavity.

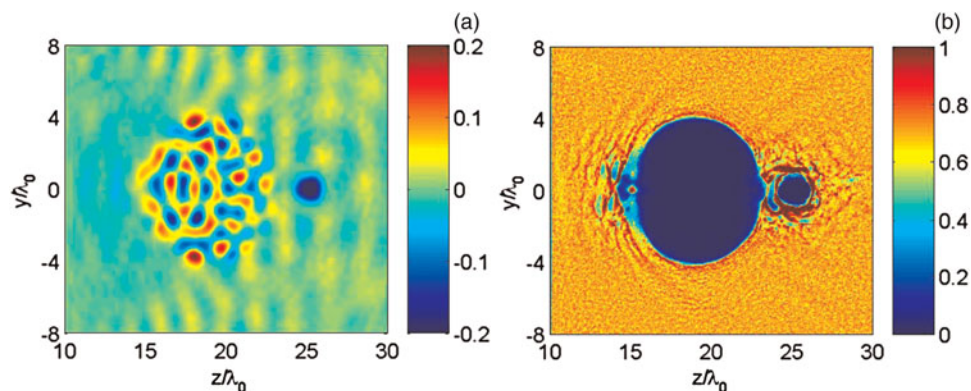
radiation becomes a multi-peak structure, as shown in Figure 7a. Our simulation shows that at  $t = 81T_0$  about 7% of the incident laser energy is still in the cavities.

Figure 7 shows the normalized laser electric field  $E_x$  (1) and electron density (2) at  $t = 151T_0$  for a larger preformed cavity of radius  $R = 4\lambda_0$ . The other parameters are the same as that in Figure 6. We can see in Figure 8 for the evolution of the EM energy in the preformed cavity that about 4% of incident laser energy is still trapped there at such long times. This is because

in the larger cavity EM energy depletion, which takes place only at the cavity boundary, occurs very slowly since there the EM wave intensity is very low. In fact, the laser light can be stably trapped in the preformed cavity for more than  $300T_0$ .

#### 4. DISCUSSION

In this paper, we have investigated light trapping in self-generated and pre-formed cavities during the propagation



**Fig. 7.** (Color online) The normalized laser electric field  $E_x$  (a) trapped in a preformed cavity of radius  $R = 4\lambda_0$ , and the electron density (b), at  $t = 151T_0$ .

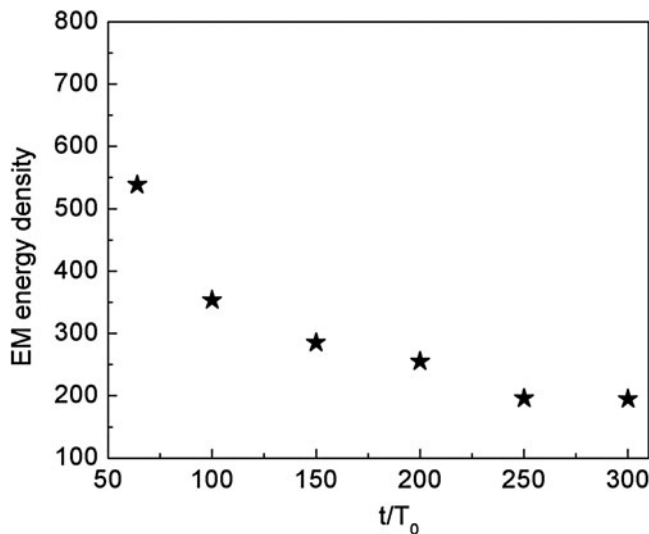


Fig. 8. Evolution of the energy of the EM wave trapped in the preformed cavity of radius  $R = 4\lambda_0$ .

of a laser pulse in near-critical density plasmas using 2D PIC simulation. A considerable part of the laser light is trapped as an EM oscillon in the self-generated caviton bounded by an overdense plasma wall. In near-critical density plasma with a preformed cavity, however, the laser ponderomotive force only slightly increases the density of the bounding plasma, but sufficient to reflect and trap the light, forming a standing wave structure. However, a part of the laser light can tunnel through the thin overdense plasma layer at the boundary, and self-consistently form an oscillon-in-caviton structure in front of the preformed cavity. Clearly, the self-consistent overdense plasma boundary layer induced by the ponderomotive force plays an important role for the long-term trapping of the EM oscillon as well as the standing EM wave.

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