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Commentary: Teach network science to teenagers

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Abstract

We discuss our outreach efforts to introduce school students to network science and explain why researchers who study networks should be involved in such outreach activities. We provide overviews of modules that we have designed for these efforts, comment on our successes and failures, and illustrate the potentially enormous impact of such outreach efforts.

Keywords: outreach, mathematics, physics, information science, sociology

1 Introduction and motivation

The World Wide Web, friendship networks at schools, and transportation networks are all complex interconnected systems that are based not only on components but also on interactions between components (Newman, 2010). To try to understand such systems, one can use network science (i.e., the science of connectivity). The notion of networks has become a mainstream part of everyday life, and scholars from sociology, physics, computer science, mathematics, and many other disciplines have developed theoretical tools and conducted empirical analyses to obtain profound insights on a diverse array of natural and designed phenomena.

In the present article, we describe our outreach efforts to teach network science to teenagers and the materials that we have developed to do this. We are mathematicians, and our perspectives on the study of networks as well as our goals in undertaking outreach activities are (of course) colored by our backgrounds. It is

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important to us to try to inspire more students to pursue the mathematical sciences in their studies at universities and, more generally, to develop a greater appreciation of mathematics and its applications. This is a major goal of our efforts and is a core reason why they were funded. Importantly, network science is a fundamentally interdisciplinary subject (which is reflected in our choice of modules), and outreach activities from people with different disciplinary backgrounds (and different goals) can—and should—reflect those backgrounds. For example, many of our modules include topics from the social and biological sciences, and they can be adjusted so that they reflect more sociological or biological perspectives. We will present this article and its associated outreach materials using our perspective as mathematicians.

Networks are fundamentally interesting to children and scientists alike. They are accessible on an intuitive level using visualizations and simple calculations, they arise in myriad areas of study (and the interdisciplinary science of networks benefits from all of them), and we believe that scholars and the lay public are interested in networks for essentially the same reasons. The ubiquity of networks offers a glimpse into the types of questions that scholars consider when studying networks, and many of these questions—and, at an intuitive level, even their solutions—can already be appreciated by children with just a primary school education. Most children have regular contact with networks (through Facebook, Google, etc.), although one would expect that almost none of them would appreciate their deep connections to science and mathematics. The habitual contact that children have with networks makes it possible to impart intuitive understanding of key concepts from network science in short outreach sessions. Importantly, because networks are omnipresent, these sessions provide an excellent way to expose students to an important side (which differs from their usual experiences) of subjects like mathematics and, ideally, to motivate them to pursue mathematics and other quantitative subjects in greater depth (or at least to improve the regard that they hold for such subjects).

There are many ways to help inspire children to study mathematics and other scientific disciplines. For example, science books written for adolescents can be extremely valuable (Preskill, 2013). Other resources, such as the "Being a Professional Mathematician" project, demonstrate that science is fundamentally a human endeavor (Mann & Good, 2013). Still others, such as the Ig Nobel Prizes, illustrate the sheer joy of scientific discovery (Abrahams, 2013). In this article, we advocate outreach activities (Saab, 2010; Dwyer, 2001a; Dwyer, 2001b; Dwyer, 2013), which provide a medium for students to interact with specialists and learn first-hand what scientists do.

We have developed a program of outreach activities to teach school students about pure and applied mathematics using network science. People from many disciplinary backgrounds have engaged in outreach efforts or developed interesting materials that can be used for outreach on network science and related topics (see, e.g., Jenkinson et al., 2013; Uzzo & Sayama, 2012; Trunfio et al., 2013; Barabási et al., 2012; Uzzo et al., 2013; Carley et al. , 2013; Meeks & Krishnan, 2013; Budd, 2013). With our outreach efforts, we hope to convey our enthusiasm for networks and mathematics (and science) more generally. We concentrated on ages 13–16, as the younger members of this age group in the United Kingdom have not yet chosen their subject specializations. We conducted our outreach activities both at local schools and at Somerville College in Oxford, and they lasted anywhere from a couple of hours to half a day. At each outreach event, we presented various aspects of network science and discussed more general issues such as studying mathematics at universities.

An outreach setting makes it possible to discuss with students what scientists actually do. Rather than showing a result after it has been cleaned and packaged, we guide the students toward their own discovery of network principles. This contrasts with the usual (and perhaps somewhat dry) material that the students are used to seeing, and we hope that it inspires them to engage in further study. We also try to illustrate that mathematics—not just networks—is a fundamental part of the students' lives by virtue of its connection to known everyday experiences like traversing a network of roads or searching for information on the Web.

Our outreach activities highlight a side of mathematics that is crucial but likely unfamiliar to the students:[1] curiosity to understand a vexing (and perhaps ambiguous) question is the main drive, and repetitive calculation (which comes later) can temporarily be pushed aside in favor of a big picture. The distinction between repetition and creative thinking about mathematics and science is common at universities, and we believe that it can—and should—be available for people of all ages. One of our main objectives with these activities is to demystify mathematics, so we present it as a discipline that is relevant to daily lives and accessible to everybody (not just to those who do it for a living). Our choices of problems for the activities were influenced both by our own research interests as applied mathematicians (of course) and by the need to present accessible, engaging, and self-contained problems that are suitable for sessions of 30–60 minutes. Networks are particularly well-suited for our objectives, although many other topics can also be used to develop fascinating outreach activities.

The rest of this article is organized as follows. We introduce our modules in Section 2 (and discuss them in detail in appendices). We present our outreach activities in Section 3, examine their successes and challenges in Section 4, and provide arguments in favor of teaching network science to teenagers in Section 5. In Section 6, we present an outlook and encourage others to participate in and develop similar activities. In Supplementary Online Material (SOM), we include lesson materials for five modules, and we encourage readers to use and adapt them for their own activities.

2 Lesson materials

Our session plans are designed for modules that last about 30–60 minutes, but they can be adapted readily for other formats. We have occasionally merged multiple modules into a single module that covers a broader set of topics. The modules are interactive, and each one allows participating students to learn about one or more areas (or applications) of network science.

In the appendices, we describe the following modules:

1. Appendix A: food webs and climate change;
2. Appendix B: small worlds and social networks;

[1] School students might have some experience with this side of some of the sciences, but we assert that this is manifestly not the case for mathematics.

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3. Appendix C: disease spread and vaccination strategies;
4. Appendix D: Google's PageRank algorithm;
5. Appendix E: coloring maps and other puzzles as an introduction to proving theorems in graph theory;
6. Appendix F: why your friends have more friends than you do;
7. Appendix G: structural balance in networks.

Modules A–E were our most successful ones, though we are sure that they can be improved further. We discuss all seven of the above modules in the appendices and go into greater depth for Modules A–E in the SOM.

As one can see from the topics above, network science is an interdisciplinary subject, and it is important to convey this when working with students. We have chosen the topics in our modules based on a combination of our interests and what we believe can be discussed successfully with teenagers. We have emphasized the role of mathematics in these topics, but the modules can be adjusted to focus on other perspectives. The study of networks benefits greatly from the contributions of scholars in sociology, biology, economics, physics, and myriad other disciplines. The choice of applications in the modules reflects this diversity, although we have focused on a mathematical approach to studying them.

3 Our outreach activities

We ran events in which students visited us at Somerville College in Oxford and others in which we visited them. The number of students varied significantly (from about 5 to about 50) from one event to another, and their ages ranged from 13 to 16. After introducing ourselves to the students and teachers, we gave a short introductory talk[2] in which we defined the concept of a "network" and some other relevant terms. We also introduced "network science" as the science of connectivity, presented several diverse examples of networks, and gave tantalizing hints on how investigating network structure can provide information about dynamics or function. We then split the students into smaller groups (e.g., 50 students can yield three groups) for breakout sessions so that they could delve deeper into a specific topic.

We have posted an example of an introductory talk on networks (Porter, 2012). This introductory talk takes 20–30 minutes. (We varied the amount of time that was spent discussing the various examples.) The presentation gives the definition of a network and shows how to represent a network as an adjacency matrix[3] using a small example. The speaker brings up different types of networks (e.g., unweighted, weighted, directed, etc.) and asks the students to think about how a matrix representation can be generalized for the different cases. The presentation then includes numerous examples, which are introduced via pretty pictures (just like in other talks for general audiences), and comments on how networks relate to the students' experiences and to various disciplines of study. Examples that we discussed include London's metropolitan transportation network ("The Tube"),

[2] For events at Somerville College, we also included a short introduction to the University of Oxford, studying mathematics and science at a university, and Somerville College before we started discussing network science.

[3] Some of the students had seen matrices before, but others had not.

Facebook friendships, food webs, networks in online role-playing games, and Web pages connected by hyperlinks. The talk also purposely introduces a *small* amount of jargon—such as "node," "edge," "small world," and "degree"—to provide some terminology to facilitate discussions in the breakout sessions. This talk covers many topics in a short period of time and it challenges the students (arguably too much on occasion), but it provides common ground and makes it possible to introduce vocabulary for the second time rather than the first during the breakout sessions. The introductory talk also includes hints of the mathematics and science that underlies networks, but it focuses predominantly on a few important ideas, which are painted using broad brush strokes.

After the introductory talk, we break out into sessions that allow students to explore some topic or set of topics in detail in smaller groups. (See Section 2 for the topics, and see the appendices and SOM for additional details and handouts.) Each of these sessions, which had 3–6 students per volunteer, was led by 1–2 people with 1–2 others helping. In many cases, the school teachers participated actively and were extremely helpful. Depending on the event, our sessions lasted 30–60 minutes, and each student participated in 1–2 such sessions.[4] On many occasions, we also asked the students to present their findings to the other groups. Naturally, our sessions had broader aims beyond discussions of specific ideas from network science: we wanted the students to think independently (and in small groups) and to investigate difficult, open-ended problems instead of problems with neat answers (to which they were more accustomed). This format gives the students a sense of how a scientist might try to tackle a problem and, at minimum, a better idea of what a university student might experience when trying to master scientific topics.

We conducted two types of outreach events: (1) ones in which students and teachers visited us at Somerville College in Oxford, and (2) ones in which we visited schools. For the events at Oxford, the logistics—including essential items like food, travel, and coffee—were arranged and run by Amy Crosweller (Somerville College's Access and Communications Officer). Amy also helped recruit Somerville undergraduates to assist us by giving tours of the College, answering questions about what it is like to be an undergraduate at University of Oxford and in Somerville College, and occasionally even acting as helpers for the breakout sessions. (When possible, the undergraduate volunteers were mathematics majors.) In one of the events in Oxford, we had a panel discussion about careers that included a volunteer from Google.

Hosting the students in Somerville College had several advantages—it made it possible to attract students from different schools to the same event, it allowed more control and knowledge of the local facilities (e.g., via the availability of flip charts, markers, etc.), and it gave us access to friendly and helpful undergraduates. However, we ultimately decided that the "traveling road show" format was more effective. This format makes it possible to work with students who are farther away geographically, as only nearby schools came to the Oxford events in practice even though we had funds for students from more distant locations to travel to Oxford and stay for a night. A traveling road show is also important for attempting to

[4] As we became more experienced, we concluded that shorter sessions tended to be a better format than longer sessions for most of the students.

work with students from schools who do not typically send students to University of Oxford (or, in some cases, who do not even consider the University of Oxford as a viable university to attend). It also makes it easier to have an event on a weekday rather than during a weekend,[5] as the students can attend this special event rather than a normal class for an hour or two.

We varied details of the event components to accommodate school schedules, road-show versus Oxford events, number of volunteers, number and type (e.g., age and quality) of students who would be working with us, and lessons that we learned regarding what seemed to work and what did not. For example, in one event that was held in Somerville College, we worked with the same set of students for 6 hours:[6] we had two 45-minute sessions (with parallel breakout modules during each session), discussions over lunch, and more. Our other events were shorter. In our visits to schools, we experienced a wide range of abilities and behavior among the students. Sometimes we had students from the same grade level, but the occasions when students from different grades were mixed together were particularly enjoyable, as it appears to be unusual for students of different ages to work with each other on equal footing. Schools varied on whether they asked us to work with their top, average, or struggling students.

Our outreach events benefited tremendously from numerous volunteers—including professors, postdoctoral scholars from University of Oxford and Imperial College London, doctoral students from several different disciplines (predominantly mathematical scientists but also several biologists), visiting researchers, undergraduates, staff members, and the students' own teachers. Our outreach efforts have been highlighted on the following Web pages:

- http://www.some.ox.ac.uk/191-5717/all/1/Somerville tutor helps students harness the science behind social_networks.aspx
- http://blogs.some.ox.ac.uk/access/2012/04/30/motivating-maths-pupils-and-reflecting-on-the-latest-sutton-trust-survey/

The University of Oxford has also produced a public-relations video to describe our outreach efforts. It is available at http://www.youtube.com/watch?v=9dcdjcyA-8E.

4 Successes and challenges

Our outreach events have been largely successful, and (unsurprisingly) more iterations have led to greater polish and greater success. In this section, we highlight a few points that we hope will be helpful for outreach events by others.

The students found the introductory talk (Porter, 2012) to be extremely challenging, but it seemed to interest them immensely[7] and its introduction to network theory, its applications, and some of its jargon helped to provide a solid foundation for the interactive modules to follow. Put another way, the initial challenge helped

[5] On one occasion, we tried having a weekday event in Oxford, but attendance was sparse.
[6] In retrospect, this was probably too long for them.
[7] We draw this conclusion based both on our perception of student excitement and on the surveys that we gave to the students for the outreach events that we held in Somerville College.

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make life easier for the rest of the event. We worked with students from very different backgrounds, but (for the most part) they were genuinely interested in working with us. We learned from our first outreach event that many students learned the key material in our lesson plans *extremely* quickly (despite the fact that we discussed topics on which there is active research). To address this, we sometimes needed to come up with impromptu activities to fill the allotted time. In Module 2 (Small Worlds), for example, we sometimes discussed material from a different module (such as Networks and Diseases, which is Module 3) that we were not running that day. When we were able to work with the same students on more than one module, students usually picked up the key ideas faster in the second module.

Our most successful modules were the most interactive ones, and we tried to increase the interactivity in the modules as we refined them. The sessions instigated many interesting and thought-provoking conversations, and sometimes the best thing we could do was to let the students discuss ideas among themselves without interrupting them. In Module 1 (Networks and the Environment), for example, students used a network-science perspective to think about the importance of grass at the bottom for survival of an entire food web. It was inspiring to see individual thinking, group discussions, and "Aha!" moments without our intervention. Much of what we write is of course true very generally and is not specific to network science, but the accessibility of networks as a subject of study helps a great deal. One of the most important outcomes of our outreach activities was that the students with whom we worked learned that many seemingly disparate problems can be solved using similar tools once they have been adequately framed (in this case, as networks). In other words, the students were able to grasp the notion of abstraction successfully, and they were able to see how the same methods can be used to achieve insights in applications from sociology, ecology, computer science, epidemiology, and other subjects. This is what many applied mathematicians, scientists, and other scholars do for a living. Additionally, examining problems from graph theory gives the students a taste of pure mathematics.

Despite the overall success of our efforts, we faced many challenges, and we have not yet figured out how to overcome all of them. We have gained much more of an appreciation for the difficulties faced by primary and secondary school teachers than we had before.[8] For example, sometimes it was difficult to get students to choose a module in the first place. (We preferred, when possible, not to choose modules for them.) Sometimes the majority of students picked the same topic, and we had to balance group sizes with the students' interests. Many students were shy and hesitant to participate, and we sometimes had to tailor the module format to encourage more group-based answers rather than individuals ones. Some students were disruptive, though teachers from most of the schools were very helpful for managing these situations and otherwise maintaining order. (This became a serious problem at only one school.) It is also worth remarking that we often noticed a difference in student comfort level when they visited us versus when we visited them.

It is also important to note that the smoothest outreach efforts are not necessarily the most important ones to undertake. For example, our outreach activities were

[8] Occasional outreach activities are wonderful, but we would not want to do this every day!

more difficult when we worked with weaker students, but this type of struggle is a *necessary* one. We welcome ideas for how to improve our outreach activities to make them more accessible to a wide variety of students and schools.

5 Why bother?

We found that some schools could not fit our outreach activities into their existing (squeezed) school schedule due to the time that it takes away from the current curriculum, whereas others worked to find an optimal time for their students either during the school day or on a weekend.[9] Whether the students came to Oxford or we went to their school, we think that learning about network science is a valuable use of time for any teenager—whether it is done via an outreach event like ours, in after-school sessions that are led by school teachers (e.g., in a science or math club), or part of the regular curriculum. Network science offers an exciting supplement to standard mathematics curricula in schools. Examining problems involving networks gives a wonderful opportunity to show that mathematics is about ideas and abstractions—and not just calculations—in a way that is both visual and closely connected to students' everyday experiences. Network science benefits from a confluence of diverse perspectives from many disciplines—including not only mathematics but also sociology, computer science, biology, physics, and myriad other areas—so examining networks in a quantitative, scholarly way allows students to experience and demonstrate interdisciplinary thinking. Moreover, working through problems like those in our modules helps students (and teachers) to see a side of mathematics that is different from most of what they have experienced. We emphasize intuition, modeling, problem-solving, and insights that draw ideas from multiple disciplines in our modules, and this provides a nice complement to the calculations to which students are accustomed. Because of the ubiquity of networks in everyday life, network science is an ideal subject for these kinds of outreach efforts. Once we explain the notion of a network to students, they—just like professional scholars—see networks everywhere they look. We hope that they will also start to see mathematics everywhere.

Based on our discussions with the students (and their teachers) and the comments in the surveys that they filled out for the events in Oxford, the students with whom we have worked certainly seem to have gained a significant appreciation and interest in networks, mathematics, and the applications that we discussed. They viewed the problems that we presented as puzzles to ponder rather than rote calculations to get out of the way. It is also a positive experience for students and teachers to have direct contact with professional mathematicians, scientists, and other scholars to get a better idea of what it is that we do (which is to try to *solve problems*, just like the students were doing). It is also good to promote a view of scientists as regular people, which contrasts sharply with the cold, detached stereotype that seems to be prevalent. Meanwhile, such outreach activities help improve the communication skills of the instructors (i.e., people like us), and that too is a significant boon.

We target our outreach activities for students of ages 13–16, as the younger students in this range have still not decided what subjects to study at university.

[9] All weekend events were held in Oxford.

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(Students specialize very early in the United Kingdom.) We hope to persuade more students to pursue mathematics and science, and we hope that those who do not pursue such disciplines will at least gain a greater appreciation of mathematics (and other quantitative disciplines) as well as its importance. Network science is an eminently accessible subject—that is why many professionals enjoy it, after all—which makes it perfect for these kinds of outreach efforts. Although our target audience was students in a particular age range, the materials can be made accessible to a broader spectrum of ages (including adults) with minimal—and mostly cosmetic—adjustments. It is desirable to increase literacy about networks among the public more generally, and the modules that we have developed will hopefully be helpful for such efforts.

One can also debate whether it is better for professionals to undertake outreach activities or for networks to be incorporated directly into school curricula. We believe that both are extremely important.

6 Conclusions and outlook

We are continuing to conduct outreach activities across England, but the only way to make a really big impact is if these activities spread far and wide. We hope that we have whet appetites for conducting outreach activities in schools, and we encourage people to use, borrow, adapt, and improve any material in this article or in the SOM.

We recognize that many people will have different opinions as to what should constitute the contents of modules, and we have already seen that the outreach activities and modules need to be adaptable from one school to another. We encourage people to create new modules and to improve our modules, so that the scientific community can develop a large repository of high-quality, networks-related school activities. Some such modules already exist (see, e.g., Jenkinson et al., 2013). We are also happy to provide advice and encouragement to anybody who wishes to contact us.

We also urge school teachers to include network science in their lessons and math/science-club projects (and, of course, to use our materials and to contact us for any desired discussions or feedback). We have written only a few modules, but we hope that they convey the hugely important role that networks have the potential to play in getting school students interested in mathematics and other scientific subjects. This is a big challenge, but it is also an exciting opportunity.

We hope that we have convinced people to engage in outreach efforts. It is a valuable use of time, and it can have a very large impact. Please do not hesitate to contact us to discuss this further.

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Supplementary materials

For supplementary material for this article, please visit http://dx.doi.org/10.1017/nws.2013.11.

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Appendix A: Networks and the environment

In this module, which was designed by Laura Keating and Puck Rombach, the students examine how climate change can affect animal species. The students construct and analyze food webs, and they investigate how the extinction of one species can affect other species.

We start the session by informing the students that they have been hired by the government to advise how climate change might affect species survival in the Arctic. Many of the students already possess some knowledge of food chains from their biology lessons, and we build on this to present food webs as networks rather than chains. To help introduce background material, the students first fill out a worksheet that includes questions about what a food web is and how one might construct a food web as a network. These initial questions also introduce relevant network properties, such as directedness of edges to describe unidirectional energy flow between species.

To help the students obtain a better grasp of representing a food web as a network, we construct a network using a small set of familiar species (e.g., grass, a mouse, an insect, a small bird, and a large bird). We then explore what happens if one or more of those species becomes endangered. We begin to discuss the potential for extinction cascades and the implications that network structure (e.g., high in-degree or high out-degree) can have for an entire ecosystem.

The final part of the module is spent on a “game” that explores food webs in Arctic ecosystems. We split the students into multiple groups and give each group a set of Arctic species that are either aquatic or terrestrial. Both sets of animals include the polar bear, which connects the aquatic and terrestrial food webs. See the Supplementary Online Material (SOM) for further details and the handouts that we give to the students. Each group of students constructs a network from their set of species, and they write down the adjacency matrix representation of it. We then ask the students what happens if one of the nodes is removed (i.e., if a species becomes extinct). Even these small networks are able to help illustrate the complicated and far-reaching effects of species extinction. At the end of the module, the students present their findings—including which species are most vulnerable, how climate change can affect food webs, and potential ways to reduce the negative effects of extinction.

Appendix B: Social networks and small worlds

This module, which was designed by Mariano Beguerisse-Díaz and Pau Erola, illustrates that social networks are everywhere and introduces some of their features. We examine the notion of a small world—the introductory talk included the jargon and a picture of a Watts-Strogatz network—using several examples that we hope are poignant for the students.

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At the beginning, we ask the students whether they have heard of the idea of "six degrees of separation." The term itself tends to be unfamiliar to most students, but many of them recognize the idea as a familiar one once we explain the jargon. We discuss Stanley Milgram's package-passing experiments and how it reached a mainstream audience through venues such as *The Oracle of Bacon* (Reynolds, 2013). We play the Kevin Bacon game with the students to try to find paths between movie actors so that they can find their own "surprisingly" short paths (e.g., a short path between an actor in a horror film and one in a children's movie), and we also think about trying to navigate networks. See the SOM for an example of a handout that we used for this activity.

We ask the students to explain what is meant by a "social network" and to give examples (and to indicate the nodes and edges). We expand on the Kevin Bacon game by examining short paths and navigation in social networks. For example, we might ask the students "How many degrees of separation are there between you and the Queen of England?" We also tried other famous people, and students on two occasions had very short paths to Nelson Mandela. (One knew him and another knew a person who knew him.) We again find short paths and try to reconcile these close connections with the fact that famous people are supposedly distant and unreachable.

We discuss who in a social network might help lead to the existence of many short paths, introduce the notion of hubs, and ask the students to consider this in the context of their Facebook friendships (Ugander et al., 2011; Backstrom et al., 2012). We stress that many short paths go through hubs and ask the students to consider how hubs can help lead to small worlds. We also discuss different "communities" of people and which types of connections might serve as "shortcuts" between different communities rather than "local" connections within a community. In our discussions, typical examples of the former arose via friendships from vacations or a summer spent abroad, and naturally the latter tended to be people—classmates, neighbors, etc.—in close geographical proximity.

As we developed this module, we added more exercises for the students, as early versions of the module were sometimes less interactive than they should have been. One exercise, which did not include a worksheet, concerns how Facebook chooses which friendships it recommends to its users. We encouraged the students to develop their own recommendation algorithms, and we discussed their ideas on a whiteboard or flip chart. In another (very successful) exercise, we returned to movies and considered the social networks of characters in movies such as *Toy Story* (see the SOM and Kaminski & Schober, 2013). We gave the students a handout with an unlabeled version of the *Toy Story* network, and we asked them to identify which nodes represent the protagonists and other characters. We also asked the students to identify some communities of toys that had things in common.[10] Obviously, the choice of *Toy Story* as an example network is geared toward a young audience, and the same exercise can use different movies that are more appropriate for other audiences.

[10] We did not use words like "homophily," though social scientists who undertake similar outreach efforts might wish to do so.

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Appendix C: Networks and diseases

This module, which was designed by Heather Harrington and Mariano Beguerisse-Díaz, illustrates how thinking about networks arises naturally when one tries to understand how diseases spread and how to develop good strategies to contain them.

We start this module by asking students to discuss the main characteristics of an infectious disease and to think about how it might spread. We encourage the students to think about a fictional disease that can only spread by shaking hands and to discuss how it might spread in their school. This quickly leads to the notion of disease spread on social networks in schools, and we sometimes combined this module with Module 2 (which is concerned with small worlds and social networks). For this discussion, it can be useful to distinguish between online and offline social networks. It is also worth discussing differences between the spread of ideas (and rumors) and the spread of diseases.

We also briefly discuss what other types of networks (e.g., transportation networks or trade networks) might be important for understanding, containing, and preventing diseases. We steer the discussion toward how different types of network topologies can affect disease spread and vaccination strategies. (One can ask similar questions in Module 2.) If it is too expensive to vaccinate everybody, then who should be vaccinated? Some students brought up node labels in this discussion—one student proposed that the youngest people should be vaccinated because they (supposedly) had the most time left to live—though our primary focus was on network topology and how it affects disease spread. In some sessions, students brought up air travel, which led to a discussion of how such travel has changed the ways that diseases spread. Students also pointed out that some diseases could be spread by insects like mosquitoes, and we used this opportunity to introduce bipartite networks and to explain how vaccination strategies differ in this situation. For example, fumigation can eliminate many mosquitoes (which reduces the number of one type of node) and slow down the spread of a disease.

The largest portion of the module is a hands-on activity in which we distribute handouts (see the SOM) with various example networks (see Figure 1) to the students and ask them to devise possible vaccination strategies in each case if they are only allowed to vaccinate three or fewer nodes. The students realized quickly that this question was much harder to answer for some network topologies than for others. This is an interesting point of discussion, as it allows the students to consider how one might develop a vaccination strategy in real networks (which are, of course, much more complicated). Moreover, given that one needs to think about the answer even if one knows network structure exactly, one can discuss how to develop strategies when some (or even a lot) of a network's structure is not known. We ask the students what would they do if one only knows that a network has a particular structure (e.g., suppose that one knows that it was generated using a Barabási–Albert mechanism) but do not know anything else. This question tends to generate a lively discussion. A useful hint for many students is to ask what would happen if we choose a random person (i.e., node) in a network and then ask him/her to choose a friend to vaccinate (rather than vaccinating the original node). We also sometimes discuss the time-ordering of contacts in social networks and how that can influence disease spread.

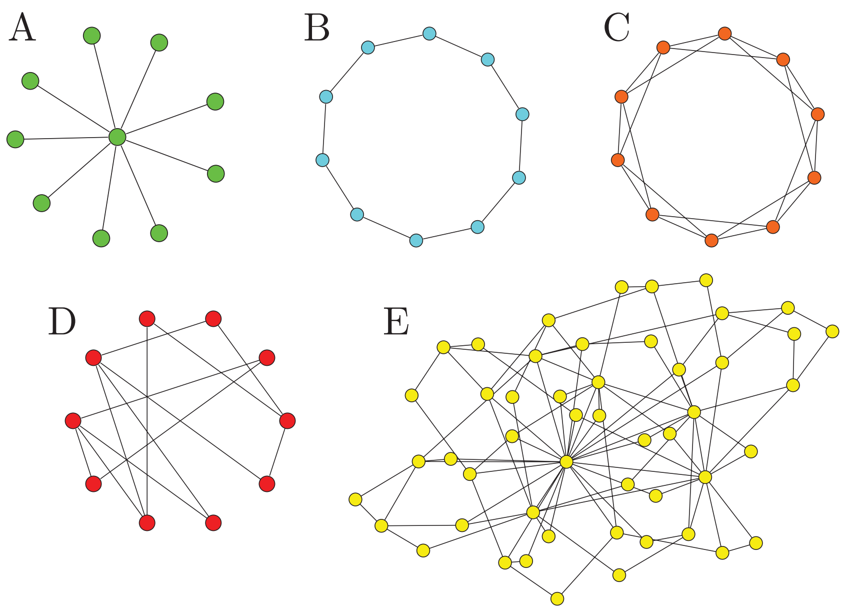

Fig. 1. Small example networks for which one can apply different vaccination strategies (see the SOM). (**A**) A star network, (**B**) a circular lattice, (**C**) a regular (lattice-like) network in which each node has four neighbors, (**D**) an Erdős–Rényi random graph, and (**E**) a Barabási–Albert network. (color online)

In addition to discussing diseases specifically, it can be useful to encourage the students to think about other contexts in which "vaccination" strategies might be employed. One key issue is distinguishing between the spread of an idea and the spread of a disease, and one might also wish to discuss other dynamical processes on networks.

This module is particularly nice for illustrating that mathematics shows up in many situations that the students (and their teachers) did not previously consider to be mathematical. This occasionally came up in discussions of viable careers for people who study mathematics at the university level, and we highlighted that nowadays mathematicians work closely alongside health professionals.

Appendix D: How Google works

This module—which was designed by Mariano Beguerisse-Díaz, Sang Hoon Lee, and Lucas Jeub—aims to introduce Google's PageRank algorithm for ranking pages on the World Wide Web (Brin & Page, 1998; Newman, 2010). (Daniel Kim also assisted in developing materials for this module.) One can find additional useful material for running this module in Moler (2011).

To start the module, we ask the students to imagine a world without Google or other search engines and to develop their own strategies for finding information on the Web. Usually, one of the first ideas is to compile an exhaustive list of every Web page. We use this to introduce the idea of a crawler to navigate Web pages, and this

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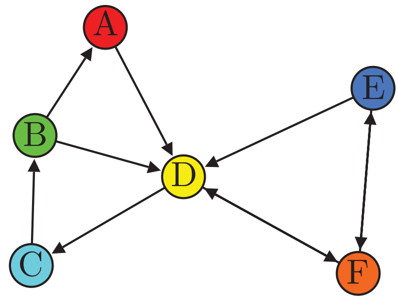

Fig. 2. An example of a directed network whose nodes we ask students to rank in order of their importance. This network is strongly connected (so any Markov chain on it is ergodic), so we can ignore the problem of dead-end nodes (i.e., nodes with an out-degree of 0). However, we did discuss the notion of dead ends on many occasions that we ran our module on ranking Web pages. The ranking of the nodes in this graph from largest to smallest PageRank score (in parenthesis) is as follows: D (0.3077), F (0.2051), B (0.1538), C (0.1538), E (0.1026), and A (0.0769). (color online)

leads naturally to the notion of representing the Web as a network with directed edges (the hyperlinks) between nodes (the Web pages).

We ask the students to think about how to figure out whether a Web page is relevant for the information that one seeks, and this leads almost immediately to the issue of how one should rank Web pages in order of their importance. One possibility that the students quickly bring up is that one can develop rankings based on the textual content of a page. (In one case, we had a good discussion about how we would try to use an automated method to distinguish the Amazon rain forest from Amazon.com.) We let the students know that the first Web search engines used to be "curated" by hand, which limited how much of the Web could be explored. This limitation provided an incentive for people to seek algorithmic methods to rank pages, which is what we want the students to explore. We ask the students to develop ideas for how to use the network structure of the Web to rank pages. (The difficulty of this transition in the discussion varied strongly between different groups of students.) Most of the time, the first structure-based ranking that the students proposed is to rank Web pages according to the number of incoming hyperlinks (i.e., according to their in-degree). We discuss whether someone can cheat this system (as well as simple text-based systems) to improve the ranking of a page and whether better methods are available.

In Figure 2, we show an example network that we use to help guide our discussion of how to rank Web pages. This example is particularly useful for moving beyond ideas for ranking based on the text on a page, as we can pretend that no such text exists (or that it is otherwise impossible to distinguish Web pages based on their text). We give the students a handout with this network (see the SOM), and we ask them to rank the nodes in order of their importance (and also to indicate how they have defined "important").

Motivated by the fact that people often seem to explore the Web by "randomly" following hyperlinks—as we point out to the students, who has not done this on Wikipedia?—we ask the students whether they can develop a ranking method

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based on this idea. We use phrasing along the following lines: If we have a large number of monkeys—it can be very compelling to refer to random walkers as "monkeys" (Hopkin, 2003)—who are clicking on hyperlinks randomly, what ranking would we obtain based on the number of times each page is visited? It can also be useful to discuss why Wikipedia is a "monkey trap," in the sense that many Wikipedia pages have high rankings in Google searches. (We occasionally discussed having one random walker versus having a large number of random walkers.) Using these questions, we introduce the rationale behind PageRank. *Crucially*, we try to avoid words like "eigenvalue" and "eigenvector" (and "ergodic," "Markov," etc.), though we do attempt to get the students to approximate (by hand) the PageRank eigenvector for a network like the one in Figure 2. They just do not know that what they are computing is called an eigenvector.

The example network in Figure 2 is very instructive. Different choices for how to measure node importance (e.g., in-degree versus out-degree) lead to different rankings, and we have interesting discussions regarding which ranking is "correct." These ideas could also be used to develop a module that focuses on centralities more generally—e.g., intuition related to the notion of "betweenness" sometimes comes up in Modules 2 (Small Worlds) and 3 (Networks and Diseases)—as well as how one might change the notion of importance depending on the question that one wants to answer. An interesting feature of the network in Figure 2, which is worth asking the students to try to prove, is that nodes B and C have the same PageRank score.

To compute the rankings, the students count the number of times the nodes are visited on different walks through the network. We and the students use two primary techniques for this calculation: (1) start from a uniform distribution and iteratively count the number of walkers on each node, or (2) try to identify relative orderings for the ranking of different nodes without calculating individual probabilities. The maximum out-degree in the example network is 2, which makes it easy to simulate a random walk by flipping a coin to decide which edge to follow in a given step. The students soon realize that one can get to node D from almost everywhere, and that it is indeed the most visited node in a (conventional) random walk. This then makes it the most important node in this ranking scheme. The students then realize that the nodes (C and F) that have incoming edges from D must come next in the rankings, and they discuss how to break the tie. Students also note that nodes C and B are always visited the exact same number of times (as long as we do not stop the walk before a monkey has had the chance to leave C). In some cases, we were able to get the students to calculate the fraction of times, in a walk with a finite number of steps, that each node was visited (as this gives an approximation of each node's PageRank score) rather than only determining the rank order. It was also useful to explore how these fractions change as one increases the length of a walk. We were consistently able to impart the intuition that the "better-linked" pages rather than the most-linked ones were the most important ones for the application at hand.

When there is enough time, we discuss that a monkey gets "trapped" on any Web page that has an out-degree of 0. This problem can be illustrated by adding a dead-end node G to the network in Figure 2. We ask the students to think about how one can change one's ranking methodology to be able to deal with this situation. This gives the opportunity to discuss the idea of a random walk with "teleportation"

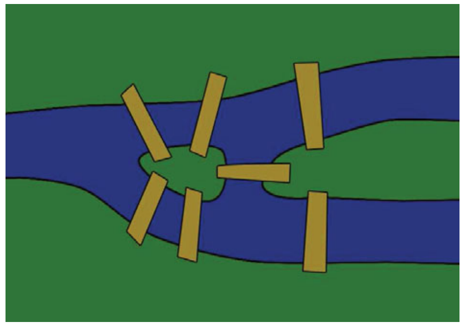

Fig. 3. *The Bridges of Königsberg Puzzle*. Is it possible to create a walk that crosses each bridge exactly once and also returns to its starting point? (color online)

(e.g., at each node, one follows an edge with a probability p or otherwise chooses some other node in the network via a random process). Once the dead-end node has been added, the graph is no longer "strongly connected," and one can use this situation to illustrate that even a small perturbation of a network can change its properties in a fundamental manner.

As part of this module, we sometimes discuss clever scientific ways to use Google—such as trying to measure the similarity between two football players by examining how often they show up together in Google searches (Lee et al., 2010).

Appendix E: Introduction to graph theory

This module, which was designed by Puck Rombach, provides an introduction to graph theory through the discussion of some famous mathematical "puzzles." In contrast to the other modules, it focuses on the theoretical (or "pure") side of mathematics. It is also arguably our most popular and successful module.

Graph theory's deep connection with networks makes it an area of pure mathematics that can resonate with students. The problems that we discuss with the students happen to also be relevant for applications (and we mention them when we are asked about them), but that is not our focus. We seek to show the students how to formulate and prove theorems, and we want to convey our excitement for and the value of abstract mathematical ideas for their own sake. This module also illustrates that university mathematics need not require any numbers or calculations, which is an important difference from the kind of mathematics with which students are familiar from schools.

Graph theory is wonderful because its basic ideas and many of its interesting problems can be explained in a few minutes in a way that allows people with little or no mathematical background (such as children) to understand them. Two examples that we discuss with the students are the Bridges of Königsberg puzzle (see Figure 3) and the Water, Gas, and Electricity puzzle (see Figure 4). Both of these puzzles are unsolvable, which one can demonstrate by systematically exhausting all possible solutions. We ask the students to try to find solutions to these puzzles and then—when they cannot—to try to prove that no solutions exist.

Excitingly, both puzzles hint at deep and general theorems. The Bridges of Königsberg puzzle (Euler, 1741) is unsolvable because it is both necessary and

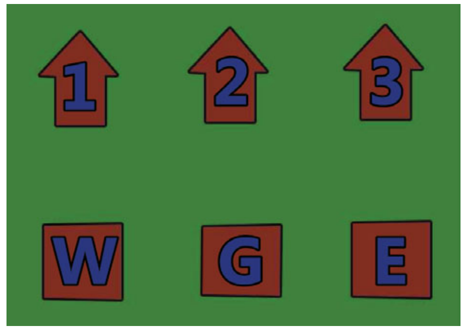

Fig. 4. *The Water, Gas, and Electricity Puzzle*. Is it possible to connect every house to all of water, gas, and electricity such that no lines cross? (color online)

sufficient that every node has an even degree for a graph to have an Euler cycle (i.e., a circuit that runs along every edge exactly once and returns to its starting point). In this case, no nodes have an even degree. Proving the necessity is easy—every time that an island (i.e., a node) is "entered," it must also be exited—but proving sufficiency is more difficult. This problem also allows us to discuss the notion of necessary and sufficient conditions for results to be true. The Water, Gas, and Electricity puzzle is unsolvable because $K_{3,3}$ (the complete bipartite graph on two sets of three nodes) is not planar.[11]

Another family of problems that we discuss at length with the students concerns graph coloring. Graph-coloring problems are easy to explain, but they can be excruciatingly difficult to solve. The most famous graph-coloring result is the Four-Color Theorem (Diestel, 2005), which we explore in detail with the students. We ask them the following question: How many colors does one need to color a map such that two countries that share a border are not colored with the same color? (Maps can be represented as planar graphs and vice versa, so map coloring is the same as planar graph coloring.) We provide paper and markers, ask the students to draw maps and color the countries, and examine the colorings that the students draw.[12] This allows us to determine quickly if they understand the definition of "coloring." We keep challenging them to try to construct maps that require more colors. The students find through empirical observation and discussion that none of them has been able to draw a map that needs more than four colors, and we eventually let them know that it impossible to construct a planar map of countries that is not four-colorable.[13] In one memorable incident, a student insisted (despite our statement that it was impossible) that he was going to construct a map that could not be four-colored, he kept working on that problem for the rest of the day, and then he insisted as the students were leaving that he was going to continue working at it and get it to work.

[11] A "planar" graph is a graph that can be drawn on a plane such that no edges cross each other.

[12] The same basic activity would of course be suitable for adults, but we realize that it might be more suitable to ask some adults to label the countries with numbers rather than to literally color the countries using markers.

[13] This was proved in the 1970s by Kenneth Appel and Wolfgang Haken (Appel & Haken, 1976), but it required exhaustive computer searches and hundreds of pages of analysis. There is not (yet) an elegant proof for this theorem.

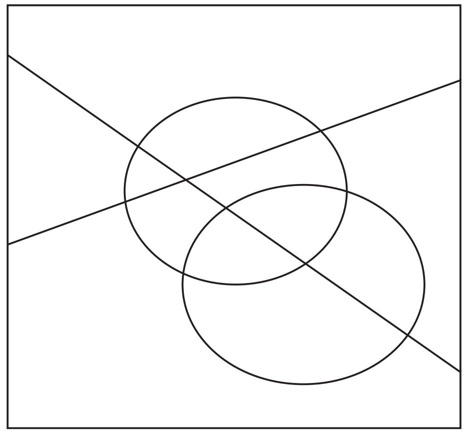

Fig. 5. An infinite-border map, for which one can prove a Two-Color Theorem.

To encourage students to get their hands dirty with mathematical proofs, we then discuss a *Two-Color Theorem* (see the discussions in Gardner, 1984 and the SOM), which is considerably simpler. We consider maps with the special rule that borders cannot end. That is, every border must either go off of the page in all directions (imagining that it goes to infinity) or it must connect to itself in a cycle. To make things simpler—although the result holds without this simplification—we do not allow borders to cross themselves. In Figure 5, we show an example of this special "infinite-border" map (IB-map). We teach the students how to prove this Two-Color Theorem using induction (Gardner, 1984), which we explain is a common approach in mathematics. (The proof is in the SOM.)

It might also be useful to ask the students to consider coloring maps that are drawn on objects that are more complicated than globes. For example, a map on a Möbius strip can require as many as six colors, and a map on a torus can require as many as seven colors. At the end of some sessions, we showed the students a Möbius strip that we constructed using six differently colored strips of paper that all touch each other.

If there is extra time or if it is useful to discuss something different, we also go through the proof of Hall's Marriage Theorem (HMT). This theorem gives a necessary and sufficient condition for marrying each member of one group of people to a member of another group of people, given that some of the latter have conditions that constrain who they are willing to marry.[14] The setting for HMT is enjoyable to explain, and it usually elicits a few laughs from the students. (HMT can also be discussed without bringing up marriage, though that is the traditional setting of the problem.)

Student reaction to this module has been overwhelmingly positive. To our great pleasure, most students have enjoyed solving problems for their own sake without having to be encumbered by any external importance. Most of the questions are

[14] Conveniently, half of the 2012 Nobel Prize in Economics was awarded for theoretical work on generalizing HMT ("The Prize," 2012). We used this as part of our response to a skeptical question about HMT ever being useful—the reaction to our answer was wonderful—though we also stressed that that did not matter, as we were only concerned with the fact that proving the theorem is interesting.

very challenging and difficult to answer straightaway. When running this module, we encourage the students to think like mathematicians. If they find a problem to be too hard, then we encourage them to consider a simpler but related question that they can try to answer first. If students do not know an answer, it is good to ask them to indicate what they *do* know. For particularly keen students, of course, this module can be scaled up to make it more challenging.

Appendix F: Do your friends have more friends than you do?

Two decades ago, Scott Feld wrote a well-known article that discusses why, on average, a given person's friends tend to have more friends than that person (Feld, 1991). Because this can be explained using simple mathematical arguments on some networks (such as the configuration model; see Bollobás, 2001; Newman, 2010), we decided that this idea would make a compelling module.

This module, which was designed by Lucas Jeub, starts by asking the students whether or not they have heard about this result. On the one occasion that we ran the module, none of the students had heard about anything like this (which is not terribly shocking), and we actually want students to express skepticism about this kind of result. Our plan in such a situation is ask the students to come up with an argument of why such a result cannot be true followed by guiding them through the intuition for the correct result that the mean number of friends of one's friends is larger than one's own mean number of friends.

To illustrate this result, we ask the students to think about their Facebook friends and the number of friends of their most popular Facebook friend. We also thought about getting the students to write down the number of people in the room that they consider to be friends, as this can of course help illustrate the same phenomenon. However, there is a major flaw in using this specific scenario to illustrate the key idea: the students were rather shy about saying how many friends they had.

In this module, we use the configuration model as a toy situation to illustrate the key phenomenon. We ask the students to write down their number of friends on a piece of paper.[15] We tabulate the numbers and ask the students to try and construct a network with the given degree sequence, and the idea of the configuration model then comes naturally by considering the set of all possible networks that one can construct from this information.

When we ran the module, we were hoping to get into a discussion of whether the fact that social networks are rather different from the configuration model (e.g., because of triadic closure) changes whether a random person in a network has fewer friends on average than do their friends, and we thereby wanted to get into some issues regarding the structure of social networks. However, although we were able to get the students to understand the intuitive arguments using the configuration model, our attempts to scale up from this point were unsuccessful. An even bigger issue is that the calculations necessary to verify the key result are tedious even for

[15] As indicated above, it would have been better to use a less personal mechanism to illustrate this example. For example, one can ask each student to choose an arbitrary positive integer bounded above by the number of people in the room and to just pretend that that number is his/her number of friends.

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small networks. We ended up finding something else to discuss with the remaining time and decided that this module needed to go back to the drawing board.

A while after we ran this module, Steve Strogatz presented an excellent explanation of Scott Feld's result in a *New York Times* article (Strogatz, 2012). We think that his explanation can serve as a useful springboard for a good module, and we suggest starting from there to try to develop a module with this theme. We believe that this theme has the potential to make an excellent module.

Appendix G: Structural balance (the ad hoc module)

This module was not "designed" by anybody, though we ran it once on an ad hoc basis instead of Module 6 (on why a random person in a social network has, on average, fewer friends than his/her friends) when the latter had not worked out as well as we would have liked earlier in the day. We needed something at the last minute, as we did not want to try Module 6 again without first redesigning it.

Motivated by a brilliant colloquium by Steve Strogatz on the previous day, we decided that structural balance would make an excellent topic. We covered the basic ideas behind structural balance, including whether or not three mutually antagonistic connections in a triad should be considered as balanced. We also guided the students through work by Cartwright and Harary (Cartwright & Harary, 1956) and asked them to consider this in real life as well as in Massively Multiplayer Online Role-Playing Games (MMORPGs) (Szell et al., 2010). (For some students, it might not be entirely clear which of these is more real.)

In our original brainstorming for sessions to design, we had actually considered developing a module on structural balance. The ad hoc module based on that idea was surprisingly successful given the lack of formal design and preparation, so we feel that structural balance would make a good module to develop more carefully. One can of course bring up alliances and conflicts in war and among schoolmates, and one can also discuss games like Risk in addition to MMORPGs.