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The realization of infinitely many universes in cosmology

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Abstract: It is shown that, for certain classes of cosmological model which either postulate or give rise to infinitely many universes, only a measure zero subset of the set of possible universes above a given size can in fact be physically realized. It follows that claims to explain the fine tuning of our universe on the basis of such models by appeal to the existence of all possible universes fail.

Introduction

There is a widespread consensus that modern cosmology has demonstrated that the universe which we inhabit is remarkably finely tuned for life.[1] This fact has prompted renewed interest in theological arguments from design.[2] A counter-strategy often adopted by those who wish to deny design has been to postulate the existence of many universes in which the constants of nature, and/or the initial conditions, are chosen randomly for each. Whilst some authors seem to think that 'vastly many' universes will do the trick,[3] others are drawn to postulate an infinite ensemble 'characterized by all conceivable combinations of initial conditions and fundamental constants'.[4] Perhaps an infinite and exhaustive ensemble is in a sense simpler than some limited number. Be that as it may, we are then not supposed to be surprised to find ourselves in our particular member of the infinite ensemble of all possible universes, since we could only exist in a universe very like ours. The existence of all possible universes is taken to explain ours by the simple maxim, 'Everything that can happen will happen, somewhere sometime'.

The hypothesis of infinitely many universes has been criticized on a number of grounds in the literature. For example, Swinburne argues that the hypothesis (even of an exhaustive infinity) is not simple, and is therefore of low prior probability, especially when compared with theism. Another criticism of a philosophical kind, raised by Ian Hacking, is that the hypothesis does not raise the probability that *this* universe is fine tuned.[5] Opponents of Hacking's view utilize the fact that we can only observe a fine-tuned universe.[6] Other criticisms include the non-observability, even in principle, of other universes.

In this article I describe what I believe to be a flaw in two versions of the infinitely many universes hypothesis. The argument is straightforward, relying on a simple mathematical property of the real line, and relates to the question as to whether the models in question can indeed be exhaustive.

The realization of infinitely many universes

There are a number of ways in which infinitely many universes might be conceived to arise, and George Gale provides a helpful classification of what he terms multi-world theories (MWTs).[7]

(i) 'Spatial MWTs'. Here many universes are envisaged as the simultaneous existence of infinitely many regions (sub-universes) in a single encompassing space. An hypothesis along these lines, in which the 'embracing' infinite universe is of low density and 'open', was suggested by George Ellis in the 1970s and is described by Ellis and Brundrit.[8] Nowadays the hypothesis is given credence by inflationary models whose bubble domains correspond to sub-universes in a single space-time.

(ii) 'Temporal MWTs'. The paradigm here is that the many universes arise as consecutive 'bounces' of a single, oscillating 'closed' space-time. John Wheeler has speculated that the various constants of nature and initial conditions could be randomly recycled at each bounce.[9]

(iii) 'Other-dimensional MWTs'. This involves adopting a realist approach to universes which do not even belong to our space-time, and, although such universes are deemed to arise through physical processes, they are more akin to the concept of 'possible worlds' in philosophy. One way, as noted by Gale, of achieving many universes in this category is as alternative branches of space-time, all deemed to exist owing to quantum splitting (this scenario is based on Hugh Everett's many-universes interpretation of quantum mechanics). Another way, which should perhaps be included in this category, is as 'baby' universes connected by 'worm holes' to 'parent' universes at singularities, since such baby universes create new space (this possibility is discussed by Stephen Hawking,[10] and, with slightly more technical detail by Guth[11]).

My critique relates only to the first two of these categories. However, it is worth noting in passing that McMullin makes the bold claim, tantalizing because he adduces no evidence, that MWTs based on Everett's branching worlds theory 'do not provide the range of alternative initial cosmic conditions or alternative physical laws that this version of an anthropic explanation of the initial parameter constraint would require'.[12] Further exploration of this point (indeed of the same point for 'baby' universes as well) would be welcome.

My critique of (i) and (ii), then, arises from the following fundamental, yet trivial, theorem: if the real line is divided into finite intervals of given minimum length, then there are, at most, countably infinitely many such intervals. This result easily generalizes to higher dimensions, so that there are only countably infinitely many regions of finite size above a given volume in space. With this theorem as basis, the argument runs as follows:

(1) There is an uncountable infinity of possible universes above a given size M.
(2) There can be at most countably many non-overlapping regions of size M in a single space.
(3) Hence, the universes realized in many universe cosmologies which postulate either a single 'containing' space or a single sequence of universes form a measure zero subset of the set of possible universes.
(4) Hence, the existence of many universes does not guarantee that there will be even one life-supporting universe.

Here (1) follows from the simple fact that 'size' is measured using the real number system; (2) is the fundamental theorem; and (3) follows from the fact that a countable subset of the real numbers, being the union of its individual points, is of measure zero.[13] Then (4) follows straightforwardly from (3).

It is clear from my expression of the 'fundamental theorem' that this argument applies directly to the Ellis–Brundrit type of simultaneously existing sub-universes of a single space. Why then have I included Wheeler-type consecutive universes in (3) above? The situation is similar, because now only countably infinitely many universes above a given *duration* can be included in the sequence.

It should be noted that, once the regions are defined, we are not at liberty to redefine them, so that new regions represent other universe sizes, because the original realization is meant to represent individual universes with particular parameter choices.

The fact that a particular realization of an infinite universe is of measure zero seriously undermines the existence of many universes as an explanation for fine tuning. If the probability of any sub-universe being finely tuned for life were finite, then, we are told, the probability that an infinite ensemble would contain life-bearing sub-universes would be 1 – this is the way the argument is usually formulated. Moreover, the number of life-bearing universes would then be infinite. This conclusion does, however, rely on the assumption that universe parameter sets are chosen randomly for each member of the ensemble, so that there is indeed a finite probability that any chosen universe will be life supporting. If, in fact, all possible universes exist – if 'everything that can happen, does happen, somewhere sometime' – then there is no need to invoke probability at all. If all possible universes exist, those with parameters sufficiently fine tuned for life, including our own, exist *ex hypothesi*. The proportion of them that are life-bearing is equal to the probability that an individual universe is life-bearing.

But if the set of realizable universes above a given size is of measure zero on the set of possible universes, then there is no guarantee that the realized set includes any life-bearing universes at all. For it to do so we require an additional hypothesis, for example, that the realized set contains the same proportion of life-bearing universes as the set of all possible universes.

In the Wheeler case we are again in the position that only a countable sequence of universes of duration above a certain minimum can be realized. Here too the extra hypothesis is required, that this sequence includes a finite proportion of life-bearing universes.

The use of probability in this context is in fact highly problematic. As for many of the usual fine-tuned parameters, it may be the case that the size or duration of a life-supporting universe must lie within a finite range (e.g. on the grounds that a too large universe may have expanded too fast for galaxies to form). We then have the problem of determining what the probability is that a parameter whose possible values lie in an infinite range actually fall in a finite range. If the infinite range is taken as the limit of a finite-range uniform distribution, as that range tends to infinity, then the answer will be zero. However, the actual probability distribution to use is unknown, and choice of a uniform distribution arbitrary. It was considerations of this kind which led Neil Manson to the pessimistic conclusion that no inference, either to many universes or a designer, can be made from the fine tuning of the universe.[14]

If the set of finely-tuned universes is of measure zero on the space of all possible universes, say because a particular parameter is of measure zero on its set of possible values, then the probability of any sub-universe being finely-tuned is zero. Thus the probability that any member of the infinite ensemble is life-bearing is zero. The number of life-bearing universes within the ensemble is then $0 \times \infty$, which is undefined. The appeal to infinitely many universes as an explanation for fine tuning therefore fails.

This point about the set of fine-tuned universes being of measure zero, and therefore nullifying the explanatory power of an infinite cosmos, was overlooked in a seminal paper by Collins and Hawking.[15] These authors showed that the set of asymptotically isotropic universes was of measure zero on the set of all spatially homogeneous universes. Deeming asymptotic anisotropy necessary for life, they went on to appeal to an infinite ensemble of universes to explain why we inhabit an asymptotically isotropic universe. But if the life-bearing universes form a set of measure zero, merely postulating an infinite ensemble is not enough to get us to an explanation for fine tuning, a point originally noted by Earman![16] Earman, discussing features supposedly varying across the sub-universes of the Ellis–Brundrit model, comments: 'But if the feature in question is unusual with a vengeance – measure zero – then the probability that it will be exhibited in some

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mini-world in the Ellis model is zero' (though this statement might be a bit too strong – see below).

The main argument of this paper does not appeal to particular features, such as asymptotic isotropy, which are required to be fine tuned. It appeals solely to the fact that only countably infinitely many universes above a given minimum size, out of a set of uncountably infinitely many possibilities, can be realized in a single space or sequence.

Objections

In this section I imagine a sceptical interlocutor posing some objections to my argument.

Objection 1

Are you not assuming that measure zero implies impossibility? The probability of choosing exactly $\frac{1}{2}$ by random choice on the interval [0,1] is zero, but this does not mean that $\frac{1}{2}$ does not exist!

Response

No, I am not assuming that measure zero entails impossibility. In fact it doesn't, as pointed out by Kingman and Taylor who note that, on a frequency interpretation of probability, an event E of measure zero is such that $r(n)/n$ converges to zero as n tends to infinity, where $r(n)$ is the number of times E occurs in n repetitions of the experiment: 'Thus E is not necessarily the impossible event $\varnothing$.'[17] Lawrence Sklar makes the same point in the context of statistical mechanics, referring to a set of points in phase space:

> **That a set has probability zero in the standard measure hardly means that the world won't be found to have its total situation represented by a point in that set. After all, every phase point is the member of an infinity of sets of measure zero, such as the set of that point by itself.[18]**

Rather, what I say is, 'The number of life bearing universes … is undefined'.[19] The point is that having infinitely many universes, rather than only 1, does not necessarily help, if the probability of a life-supporting choice of parameters is zero. The problem with one universe as a brute fact is that its parameters are so special, seemingly 'designed for life'. Infinitely many universes give an alternative explanation to design if the probability of life-supporting values for the parameters is finite. But if that probability is zero, we are not necessarily any further on.

In fact, this is exactly what Collins and Hawking found. For one particular requirement for life-bearing, namely asymptotic isotropy, the universes exhibiting this feature form a measure zero set on the set of all possible universes. These authors then assumed without further warrant that infinitely many universes would explain the specialness of this one.

Let me repeat just to clarify this. Consider first the standard argument for many universes. Suppose the probability of life-supporting parameters is finite. Then, if only one universe exists the probability that it will support life is negligible, being this small finite value. But if there are infinitely many universes the probability that at least one will support life is 1, given the further assumption of random choice. Now consider what happens if the probability of life-supporting parameters is zero. If there is only one universe, the probability that it will support life is zero. If there are finitely many, this probability is still zero (*pace* van Inwagen). If there are infinitely many, we do not know what the probability is that it will contain any life-bearing universes – certainly none are guaranteed. Perhaps this moves us a bit further on, but we need a yet further assumption to get a definitive explanation for life-bearing universes.

Objection 2

If your argument is right it looks as though 'fine tuning' plays no essential role in the argument. Suppose that there are finite upper and lower bounds on the values which constants can take in life-supporting universes, but no bounds on the values which these constants can take in universes in general. Then it looks as though the set of life-supporting universes will be of measure zero in the set of possible universes.

Response

This is just the point I was trying to make about fine-tuning arguments in general! It is indeed very difficult to quantify fine-tuning arguments for this sort of reason. The parameters might look incredibly fine tuned, e.g. for the sake of argument initial expansion rate right to 1 part in 10^{55}, and we are impressed by this. But any finite range would actually do, e.g. right to within a (large) factor, say a hundred million. The problem is that we don't know what probability distributions to take for these parameters.

One aspect of fine tuning which genuinely seems to involve a probability (and thereby provide a counter to Manson's argument) is initial entropy. Penrose[20] argues that the probability that a universe chosen at random possesses the order that ours does is:

$$1 \text{ in } 10^{10^{123}}$$

Objection 3

Does not the argument show that a single infinite universe cannot exist (since a single universe can be divided into finite regions), and also that a single finite universe cannot exist (since the set of possible sizes is uncountably infinite)? Isn't the problem therefore more likely to lie with the mathematics than the argument for infinitely many universes?

Response

The claim is not that a measure zero set cannot exist (see response to Objection 1), and certainly a single infinite universe and a single finite universe are both of measure zero. Rather, it is really a question of whether postulating infinitely many universes gets you much beyond the problems fine tuning poses if there is only one universe. If all possible universes could be realized, this would guarantee one (indeed infinitely many) like ours. If only a limited subset can be realized there is no such guarantee.

Objection 4

Even if sound, your argument is not as significant as you claim, because you have chosen a very unusual version of many universes to focus on, namely an infinite space containing an infinite number of equal size finite regions.

Response

I would dispute that the version of the many-worlds hypothesis chosen is unusual – it is basically sub-universes within a single space-time. Such a model is like that originally proposed by Ellis, and described in Ellis and Brundrit.[21] In the original proposal these were regions of a single all-embracing infinite, open universe. The regions had varying initial conditions and physical constants. Such universes could now be seen to arise as bubbles in some inflationary models. These universes are essentially finite non-overlapping regions, as is required for the main argument of this paper to carry. The universes are not of equal size, only of finite size above a given minimum (which can be as small as you please – and I cannot really see why one should be worried at the exclusion of infinitesimally small universes). The argument of the paper also applies to Wheeler-type sequential universes, so, in fact, it applies to two important classes of many-universe model.

Conclusion

The paper has shown that only a measure zero subset of possible universes can be realized by 'putting together' such universes in a single all-encompassing space-time. It follows that for two important many-universe cosmologies, namely a single space containing possible universes as sub-regions and a single sequence of universes, only a measure zero subset of possible universes will be realized. It follows that such cosmologies cannot guarantee the existence of even a single life-supporting universe.[22]

Notes

1. The classic text is J. D. Barrow and F. J. Tipler *The Anthropic Cosmological Principle* (Oxford: Oxford University Press, 1986).

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2. E.g. Richard Swinburne 'The argument from the fine-tuning of the universe', in John Leslie (ed.) *Physical Cosmology and Philosophy* (New York NY: Macmillan, 1990), 154–173.
3. E.g. Peter van Inwagen *Metaphysics* (Boulder, CO: Westview Press, 1993), 142–145.
4. Brandon Carter 'Large number coincidences and the anthropic principle in cosmology', in M. S. Longair (ed.) *Confrontation of Cosmological Theories with Observational Data* (Dordrecht: Reidel, 1974), 291–298.
5. See Ian Hacking 'The inverse gambler's fallacy: the argument from design. The anthropic principle applied to Wheeler universes', *Mind*, **96** (1987), 331–340.
6. For a recent discussion see the following debate: Rodney D. Holder 'Multiple universes as an explanation for fine-tuning', and Phil Dowe 'Multiple universe explanations are not explanations', *Science and Christian Belief*, **11** (1999), 65–66 and 67–68 respectively. However, Hacking has been supported recently by Roger White 'Fine-tuning and multiple universes', *Noûs*, **34** (2000), 260–276.
7. George Gale 'Cosmological fecundity: theories of multiple universes', in Leslie *Physical Cosmology and Philosophy*, 189–206.
8. G. F. R. Ellis and G. B. Brundrit 'Life in the infinite universe', *Quarterly Journal of the Royal Astronomical Society*, **20** (1979), 37–41.
9. E.g. J. A. Wheeler in C. W. Misner, K. S. Thorne, and J. A. Wheeler *Gravitation* (San Francisco CA: W. H. Freeman, 1973), ch. 44.
10. Stephen Hawking *Black Holes and Baby Universes and Other Essays* (London: Bantam, 1993), 115–125.
11. Alan H. Guth *The Inflationary Universe: The Quest for a New Theory of Cosmic Origins* (London: Jonathan Cape, 1997), 253–269.
12. See E. McMullin 'Indifference principle and anthropic principle in cosmology', *Studies in History and Philosophy of Science*, **24** (1993), 359–389, 380.
13. See J. F. C. Kingman and S. J. Taylor *Introduction to Measure and Probability* (Cambridge: Cambridge University Press, 1966), 88–89.
14. See Neil A. Manson 'There is no adequate definition of ''fine-tuned for life''', *Inquiry*, **43** (2000), 341–351.
15. C. B. Collins and S. W. Hawking 'Why is the universe isotropic?', *Astrophysical Journal*, **180** (1973), 317–334.
16. John Earman 'The SAP also rises: a critical examination of the anthropic principle', *American Philosophical Quarterly*, **24** (1987), 307–317.
17. Kingman and Taylor *Introduction to Measure and Probability*, 269.
18. Lawrence Sklar *Physics and Chance: Philosophical Issues in the Foundations of Statistical Mechanics* (Cambridge: Cambridge University Press, 1993), 182.
19. It would appear, however, that Earman has overlooked this point in the quotation I gave from 'The SAP also rises'.
20. Roger Penrose *The Emperor's New Mind* (Oxford: Oxford University Press, 1989), 344.
21. Ellis and Brundrit 'Life in the infinite universe'.
22. I am grateful to two anonymous referees for *Religious Studies* whose comments and objections to my argument have been invaluable in helping me clarify my thoughts and revise the paper.