J. Plasma Physics (2001), *vol.* 65, *part* 4, *pp.* 305–317. © 2001 Cambridge University Press 305
DOI: 10.1017/S0022377801001088 Printed in the United Kingdom

Continuous solutions of the hydrodynamic approach to cosmic-ray propagation

CHUNG-MING KO

Department of Physics, Institute of Astronomy, and Center for Complex Systems, National Central University, Chung-Li, Taiwan 320, Republic of China
(cmko@phy.ncu.edu.tw)

(Received 14 August 2000)

Abstract. A hydrodynamic approach is employed to study a cosmic-ray–plasma system, which comprises thermal plasma, cosmic rays and two oppositely propagating Alfvén waves. The hydrodynamic approach is a good approximation in dealing with the structure or dynamics of the system. In this paper, we concentrate on the steady-state structures of the system, in particular, structures with continuous (or smooth) profiles. Three mechanisms are responsible for the energy exchange between different components. They are work done by plasma flow via pressure gradients, cosmic-ray streaming instability and stochastic acceleration. The interplay between these mechanisms generates several morphologically different structures. They may be divided into two categories: one looks like the test-particle picture and the other looks like a modified shock. Very often the profiles are non-monotonic, which is in sharp contrast to systems with only thermal plasma and cosmic rays, whose flow velocity (and cosmic-ray pressure) profiles are always monotonically decreasing (and increasing).

1. Introduction

Propagation of energetic charged particles or cosmic rays in a magnetized thermal plasma has been studied extensively in the past few decades. Ignoring collisions, the basic interaction between cosmic rays and the thermal plasma is mediated by the embedded magnetic field. When cosmic rays encounter hydromagnetic waves or irregularities, they are scattered by the waves. They advect and diffuse through the thermal plasma. If waves of different phase speeds are present, the cosmic rays diffuse in momentum space. The process is called stochastic acceleration. Moreover, when the cosmic rays stream through the plasma, they excite hydromagnetic waves by the so-called cosmic-ray streaming instability. These waves in turn scatter the cosmic rays. The coupling between the cosmic rays and waves depends on the magnitude of the waves. Thus advection, real-space and momentum-space diffusion of the cosmic rays are all determined by the magnitude of the waves. For cosmic-ray propagation, see e.g. Skilling (1975a,b) and Schlickeiser (1989).

When the energy densities of the cosmic rays and waves are comparable to the energy density of the thermal plasma, the back-reaction on the plasma can no longer be ignored. A self-consistent model is needed. The mass density of cosmic rays is considered as negligible, because a few of them can produce a significant amount of energy density. Therefore the mass-conservation equation of the thermal plasma is not affected. However, the momentum equation is modified by the pressure gradient of cosmic rays and waves.

Actually, the cosmic rays and the thermal plasma are just components of a plasma system. The artificial separation helps us to understand the role played by the high- and the low-energy components, but the separation itself is rather arbitrary. (The separation becomes an important issue when one considers injection problems.)

In general, there are three approaches to the problem:

(i) in the kinetic theory approach, both cosmic rays and the thermal plasma are described by phase-space distribution functions;

(ii) in the hybrid approach, the cosmic rays are described by a phase-space distribution function,while the thermal plasma is considered as a fluid;

(iii) in the hydrodynamic approach, both cosmic rays and the thermal plasma are considered as fluids.

The hybrid approach has been studied extensively numerically, and some analytical solutions have been found (Malkov 1997a,b). The kinetic theory approach is a formidable task, even when one is armed with present computing capacity. We adopt the hydrodynamic approach in this paper because it is the simplest among the three approaches. Nevertheless, we deem that it is still a good approximation for studying the structure and dynamics of the cosmic-ray–plasma system. We notice that there are limitations or shortcomings of this approach (see e.g. Heavens 1984; Achterberg et al. 1984; Jones and Kang 1990; Jones and Ellison 1991; Ko 1995). The origin of some of its problems was nicely elucidated by Malkov (1997a,b). He showed that the hydrodynamic approach is a singular limiting case of the more general hybrid approach. Some of the problems can be attributed to the limiting process.

In general, the hydrodynamic approach contains the so-called closure parameters, for example, the ratio of the cosmic-ray energy density to its pressure (or the polytropic index), the coupling strength between the waves and the cosmic rays, etc. (see e.g. Duffy et al. 1994; Jiang et al. 1996; Ko 1998). In principle, some of these parameters can be computed by the hybrid or the kinetic theory approach. However, in the hydrodynamic approach, they are just prescribed quantities.

The hydrodynamic approach was started as a two-fluid model for cosmic-ray-modified shock problems (Axford et al. 1977, 1982; Drury and Völk 1981). Later, Alfv́en waves were added and the model became a three- or four-fluid model (McKenzie and Völk 1982; Ko 1992). In this paper we use the four-fluid model put forward by Ko (1992). The four fluids are thermal plasma, cosmic rays and two oppositely propagating Alfvén waves. Since there are two Alfvén waves with different phase velocities, stochastic acceleration can be addressed naturally. We must point out that the aforementioned model may be standing on shaky ground, when the two oppositely propagating Alfvén waves interact strongly. When the waves interact strongly, the system will develop eventually into the strong turbulence regime. The four-fluid model comes from propagation equations derived from quasilinear theory under the assumption of weak turbulence. Under what conditions is the model valid? This is a difficult question and is outside the scope of the present paper. It probably involves some estimations of the time scales set by the turbulence cascade, the wave generation (say by cosmic-ray streaming instability), and the cosmic-ray acceleration (say by stochastic acceleration). Bearing all these in mind, we still adopt the four-fluid model because it is 'relatively simple'.

Much work has been done on the steady state of one-dimensional cosmic-ray-modified shocks (or cosmic-ray–plasma systems). Drury and Völk (1981) and Axford et al. (1982) worked out the structure of the two-fluid model (cosmic rays and thermal plasma) in detail. Ko et al. (1997) examined the structure and efficiency of the modified shock with a simple phenomenological model of injection. McKenzie and Völk (1982) worked out the structure of the shock with backward-propagating Alfvén wave, which is a three-fluid model. To study stochastic acceleration consistently, Ko (1992) added both forward and backward propagating Alfvén waves to form a four-fluid model. The mathematics of the full system becomes very tedious, and little analytical work has been done.

Having said that, some simplified versions devoted to stochastic acceleration have been studied quite thoroughly. Ko (1992) showed that a spatially homogeneous system always tends towards a state with only one wave. However, such unidirectional wave systems are susceptible to cosmic-ray-driven magneto-acoustic instability (McKenzie and Webb 1984; Zank 1989; Ko and Jeng 1994). The unstable mode can be identified as the slow magneto-acoustic mode.

Jiang et al. (1996) proposed a nonlinear test-particle picture version of the model, in which the thermal plasma was considered as an 'energy and momentum reservoir'. They found that stochastic acceleration is often the dominant acceleration mechanism of cosmic rays. However, in some parameter regimes, it may just act as a trigger or a catalyst, and the work done by the background plasma flow becomes the major contributor.

Ko (1998) worked out the efficiency of shock acceleration in the nonlinear test-particle picture in a shocked background. According to his definition of efficiency, the efficiency can be negative in some parameter regime (a small region though), i.e. it is possible that cosmic-ray pressure far downstream of the shock may be less than the pressure without the shock. Moreover, Ko (2001) considered two flows with the same upstream and downstream speeds. One transforms from upstream to downstream smoothly but rapidly, while the other changes from upstream to downstream via a discontinuity (a shock or a subshock). The gain in cosmic-ray pressure in the shocked flow is always smaller than the gain in the continuous flow, i.e. in a certain sense, the shock decelerates the cosmic rays.

In this paper we present some numerical solutions to the full four-fluid model. The solutions are all continuous solutions. We should like to find out the relative importance of the three energy exchange mechanisms in the cosmic-ray–plasma system: work done by plasma flow, cosmic-ray streaming instability and stochastic acceleration. In Sec. 2 we describe the four-fluid model. Numerical solutions are presented in Sec. 3. Section 4 provides a summary and discussion.

2. Four-fluid model

Based on the cosmic-ray propagation equation of Skilling (1975a,b) and the wave energy exchange equation of Dewar (1970); Ko (1992) worked out a four-fluid model for the cosmic-ray–plasma system. The model consists of thermal plasma, cosmic rays and two Alfvén waves (which propagate along the magnetic field in opposite directions). All components are treated as fluids, i.e. they are described by an energy density or a pressure. The cosmic rays and waves are massless fluids, and the mass density of the system is provided solely by the thermal plasma. The governing equations of the model are the total mass and momentum equations, and energy equations of various components (i.e. kinetic energy and thermal energy of plasma, cosmic-ray energy, and wave energies), supplemented by the equations for the magnetic field.

In one-dimensional geometry and assuming that the magnetic field is parallel to the plasma flow, the model becomes (assuming no dissipation)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho U) = 0, \tag{1}$$

$$\rho\frac{\partial U}{\partial t} + \rho U\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}(P_{\rm th} + P_{\rm c} + P_{\rm w}^{+} + P_{\rm w}^{-}), \tag{2}$$

$$\frac{\partial E_{\rm k}}{\partial t} + \frac{\partial F_{\rm k}}{\partial x} = -U\frac{\partial}{\partial x}(P_{\rm th} + P_{\rm c} + P_{\rm w}^{+} + P_{\rm w}^{-}), \tag{3}$$

$$\frac{\partial E_{\rm th}}{\partial t} + \frac{\partial F_{\rm th}}{\partial x} = U\frac{\partial P_{\rm th}}{\partial x}, \tag{4}$$

$$\frac{\partial E_{\rm c}}{\partial t} + \frac{\partial F_{\rm c}}{\partial x} = [U + (e_{+} - e_{-})V_{\rm A}]\frac{\partial P_{\rm c}}{\partial x} + \frac{P_{\rm c}}{\tau}, \tag{5}$$

$$\frac{\partial E_{\rm w}^{\pm}}{\partial t} + \frac{\partial F_{\rm w}^{\pm}}{\partial x} = U\frac{\partial P_{\rm w}^{\pm}}{\partial x} \mp e_{\pm}V_{\rm A}\frac{\partial P_{\rm c}}{\partial x} - \frac{P_{\rm c}}{2\tau}, \tag{6}$$

where ρ and U are the density and velocity of the plasma; $P_{\rm k}$, $E_{\rm k}$ and $F_{\rm k}$ are the kinetic pressure, kinetic-energy density and kinetic-energy flux of the plasma; $P_{\rm th}$, $E_{\rm th}$ and $F_{\rm th}$ are the thermal pressure, thermal-energy density and thermal-energy flux of the plasma; $P_{\rm c}$, $E_{\rm c}$ and $F_{\rm c}$ are the pressure, energy density and energy flux of the cosmic rays; $P_{\rm w}^{\pm}$, $E_{\rm w}^{\pm}$ and $F_{\rm w}^{\pm}$ are the pressure, energy density and energy flux of the waves, and $\pm$ denote the forward- and backward-propagating waves. $V_{\rm A}$ is the Alfvén speed and is given by $V_{\rm A} = B/\sqrt{\mu_0\rho}$. In one-dimensional problems, the magnetic field B is spatially uniform when the field is in the same direction as the spatial coordinate (because $\boldsymbol{\nabla}\cdot\mathbf{B} = 0$). In (5) and (6), the terms $e_{\pm}V_{\rm A}\partial P_{\rm c}/\partial x$ and $P_{\rm c}/\tau$ represent cosmic-ray streaming instability and stochastic acceleration (or second-order Fermi process) respectively. The energy fluxes are given by

$$F_{\rm k} = E_{\rm k}U, \tag{7}$$

$$F_{\rm th} = (E_{\rm th} + P_{\rm th})U, \tag{8}$$

$$F_{\rm c} = (E_{\rm c} + P_{\rm c})[U + (e'_{+} + -e'_{-})V_{\rm A}] - \kappa\frac{\partial E_{\rm c}}{\partial x}, \tag{9}$$

$$F_{\rm w}^{\pm} = E_{\rm w}^{\pm}(U \pm V_{\rm A}) + P_{\rm w}^{\pm}U. \tag{10}$$

To close the system, we assume polytropic relations between the pressure and energy density of various components,

$$E_{\rm k} = \tfrac{1}{2}P_{\rm k} = \tfrac{1}{2}\rho U^2, \tag{11}$$

$$E_{\rm th} = \frac{P_{\rm th}}{\gamma_{\rm g} - 1}, \tag{12}$$

$$E_{\rm c} = \frac{P_{\rm c}}{\gamma_{\rm c} - 1}, \tag{13}$$

$$E_{\rm w}^{\pm} = 2P_{\rm w}^{\pm}; \tag{14}$$

and we also need a model for $e_\pm$, τ and κ. Assuming that the scattering frequency of cosmic rays (by waves) is proportional to the wave pressure, we have the following simple model (Ko 1992):

$$e_\pm = e'_\pm = \frac{P_{\rm w}^\pm}{P_{\rm w}^+ + P_{\rm w}^-}, \tag{15}$$

$$\frac{1}{\tau} = 16\alpha \frac{V_{\rm A}^2}{c^2} \frac{P_{\rm w}^+ P_{\rm w}^-}{P_{\rm w}^+ + P_{\rm w}^-}, \tag{16}$$

$$\kappa = \frac{c^2}{3\alpha(P_{\rm w}^+ + P_{\rm w}^-)}, \tag{17}$$

where c is the speed of light, and α indicates the strength of coupling. α, $\gamma_{\rm g}$ and $\gamma_{\rm c}$ can be considered as closure parameters.

In steady state, there are six integration constants. They are the magnetic flux

$$\Phi_{\rm B} = B, \tag{18}$$

the mass flux

$$\Phi_{\rm m} = \rho U, \tag{19}$$

the entropy constant

$$A = P_{\rm th}\rho^{-\gamma_{\rm g}}, \tag{20}$$

the total energy flux

$$F = F_{\rm k} + F_{\rm th} + F_{\rm c} + F_{\rm w}^+ + F_{\rm w}^-, \tag{21}$$

the total momentum flux

$$G = P_{\rm k} + P_{\rm th} + P_{\rm c} + P_{\rm w}^+ + P_{\rm w}^-, \tag{22}$$

and the wave action

$$W_{\rm A} = \left[F_{\rm c} + \frac{(U+V_{\rm A})^2}{V_{\rm A}} E_{\rm w}^+ - \frac{(U-V_{\rm A})^2}{V_{\rm A}} E_{\rm w}^-\right]. \tag{23}$$

Therefore the model can be reduced to a set of two-dimensional autonomous ordinary differential equations.

3. Numerical results

In this section, we seek steady-state solutions of the four-fluid model; in particular, we consider only one-dimensional (with magnetic field and plasma flow parallel to each other) systems. We adopt the coupling model described in the previous section (equations (15)–(17)); thus the systems are translation-invariant (i.e. they do not depend on x). The system can be described by a set of two-dimensional autonomous ordinary differential equations. For instance, $P_{\rm w}^+$ and $P_{\rm w}^-$, or U and $P_{\rm c}$, are convenient running variables. In principle, we may employ standard techniques to analyse the autonomous set of ordinary differential equations and classify all the solutions (as in Jiang et al. 1996). However, the mathematics in this case is very tedious, and instead we use a numerical method to study the system. Specifically, we search for continuous (smooth) physically admissible solution (i.e. we avoid solutions with subshocks).

https://doi.org/10.1017/S0022377801001088 Published online by Cambridge University Press

A physically admissible solution (or physical solution) satisfies the following requirement and boundary conditions. First of all, the pressures (and energy densities) of the solution must be non-negative. Secondly, in one-dimensional geometry, the 'most natural' boundary condition is that the solution approaches uniform states both far upstream ($x \to -\infty$) and far downstream ($x \to \infty$). (Generally speaking, if the solution approaches periodic states both far upstream and/or far downstream, it can also be classified as a physical solution. However, it can be shown that there is no physically admissible periodic solution. Basically, periodic solutions are either centres or limit cycles, and in two-dimensional autonomous systems, they must enclose a fixed point. In our case, at least one of the pressures P_c, $P_w^{\pm}$ must vanish at the fixed points because of the stochastic acceleration. Hence, even if periodic solutions existed, some pressures would be negative in part of the orbit. Thus periodic upstream or downstream states are excluded.)

Cosmic-ray–plasma systems without waves or with only one wave have been studied thoroughly (Drury and Völk 1981; Axford et al. 1982; McKenzie and Völk 1982; Ko et al. 1997). The plasma flow has three types of profiles: (i) uniform, (ii) monotonically decreasing and continuous, and (iii) monotonically decreasing with a subshock. We should point out that for systems with only one wave (i.e. unidirectional wave systems), only continuous profiles can be considered, because a subshock generates both waves downstream.

In this paper, therefore, we concentrate on flow profiles of systems with both forward and backward waves. Furthermore, we consider continuous and super-Alfvénic flows only (i.e. $M_A = U/V_A > 1$ everywhere). We note that uniform states are not physically allowable in two-wave systems because of the stochastic acceleration.

The magnetic field, velocity, density, pressures and length are normalized to B_0, U_0, ρ_0, P_0 and L_0, where $B_0^2/\mu_0 = \rho_0 U_0^2 = P_0$ and $L_0 = c^2/\alpha P_0 U_0$.

To find the steady-state solutions of the set of equations (1)–(6) and $\nabla \cdot \mathbf{B} = 0$, eight parameters are required. We choose three integration constants Φ_B, Φ_m and F, and values of the five quantities U, P_{th}, P_c, P_w^+ and P_w^- at $x = 0$. Moreover, we have to assign values to γ_g and γ_c (e.g. $\gamma_g = \frac{5}{3}$, and $\gamma_c = \frac{4}{3}$), and we take $\alpha = c^2/P_0 U_0 L_0$ (we do not need to set a numerical value for α, since it is not necessary to give numerical values to the normalization constants P_0, U_0 and L_0). We seek physically admissible solutions. We integrate the set of autonomous ordinary differential equations forward and backward from $x = 0$ (by the fourth-order Runge–Kutta method), and require that the solutions approach uniform states as $x \to \pm\infty$ (of course, we also require all pressures to be non-negative). We note that a more intuitive way to integrate the set of equations is to specify values at the far-upstream region (i.e. the inflow boundary). However, we have not taken this approach, because we have demanded that the far-upstream state be uniform (which causes some difficulty in integration from the far-upstream region). Of course, if we wish to find the values at the inflow boundary, we can easily compute them from the solutions we get. We work out several typical solutions (Figs 1–6), but we do not claim that we exhaust all generic profiles.

The parameters used in the figures are summarized in Table 1. The last three columns are computed quantities, not input parameters. In each figure, part (a) shows the profiles of U, V_A, P_{th}, P_c and $P_w^{\pm}$, while part (b) shows the strength of the energy transfer mechanisms of the system (those terms on the right-hand sides of (5) and (6)). In the figures, the stochastic acceleration P_c/τ is denoted by f_s, the

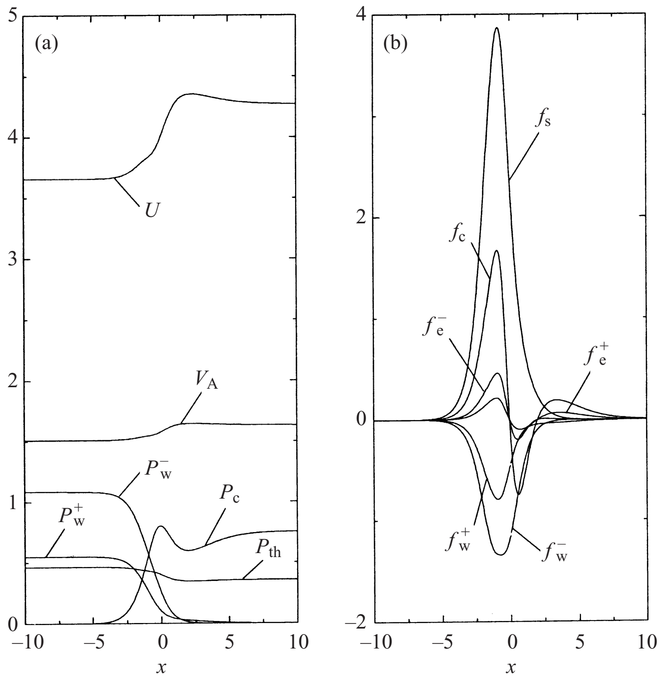

Figure 1. Structure of cosmic-ray–plasma system: (a) flow and pressure profiles; (b) energy transfer mechanisms. In (b), the stochastic acceleration P_c/τ is denoted by f_s, the cosmic-ray streaming instability $e_\pm V_A\, dP_c/dx$ by $f_e^\pm$, and the work done by plasma flow on cosmic rays ($U\, dP_c/dx$) and waves ($U\, dP_w^\pm/dx$) by f_c and $f_w^\pm$ respectively. The parameters used in this figure are $\Phi_B = 1.0$, $\Phi_m = 1.6$, $F = 31.26$, $U(0) = 4.0$, $P_{th}(0) = 0.4$, $P_c(0) = 0.8$, $P_w^+(0) = 0.1$ and $P_w^-(0) = 0.25$ (also see Table 1).

Table 1. Parameters used in Figs 1–6. Note that $\gamma_g = \frac{5}{3}$, $\gamma_c = \frac{4}{3}$ and $\alpha = c^2/P_0U_0$.

Figure	Φ_B	Φ_m	F	$U(0)$	$P_{th}(0)$	$P_c(0)$	$P_w^+(0)$	$P_w^-(0)$	A	G	W_A
1	1.0	1.6	31.26	4.0	0.4	0.8	0.1	0.25	1.842	7.95	12.82
2	1.0	1.6	26.36	4.0	0.4	0.8	0.001	0.2	1.842	7.801	6.340
3	1.0	1.6	26.30	4.0	0.4	0.8	10^{-6}	0.2	1.842	7.800	6.250
4	1.0	4.0	63.53	4.0	1.0	0.8	0.4	0.01	1.0	18.21	35.65
5	1.0	4.0	56.04	4.0	1.0	0.35	0.5	0.01	1.0	17.86	31.76
6	1.5	1.0	70.40	10.0	0.02	10^{-8}	0.4	0.2	0.9283	10.62	34.33

cosmic-ray streaming instability $e_\pm V_A\, dP_c/dx$ by $f_e^\pm$, and the work done by plasma flow on cosmic rays ($U\, dP_c/dx$) and waves ($U\, dP_w^\pm/dx$) by f_c and $f_w^\pm$, respectively.

As illustrated in the figures, the cosmic-ray–plasma system with two waves has numerous different profiles or structures. They may be roughly divided into two categories. On the one hand, the profiles of the flow and the thermal pressure of Figs 1(a) and 2(a) are rather uniform. They behave as in the test-particle picture, where the plasma acts as an 'energy and momentum reservoir'. On the other hand, the flow profiles of Figs 3(a)–6(a) resemble a modified shock. These two categories of solutions have one more difference. Both waves vanish in the far-downstream region of the 'test-particle-like' solutions (Figs 1(a) and 2(a)), while one wave survives in the far-downstream region of the 'modified-shock-like' solutions (Figs 3(a)–6(a)).

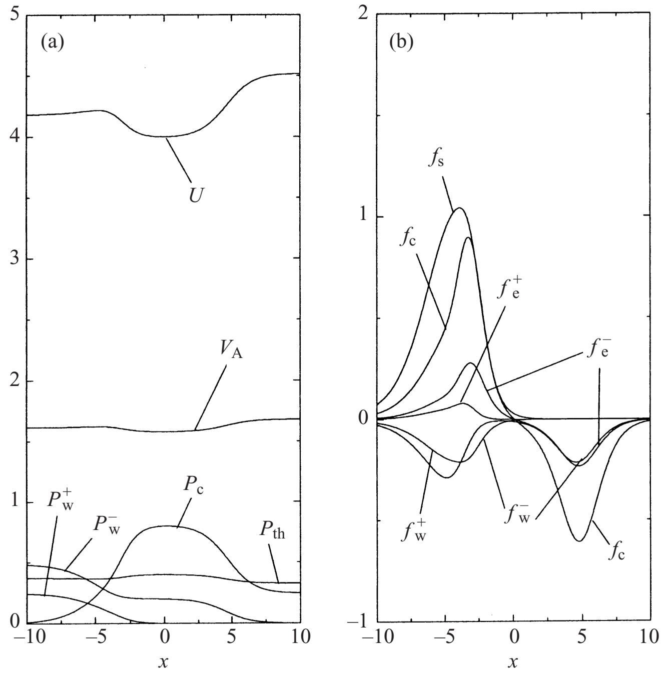

Figure 2. The same as Fig. 1, but with parameters $\Phi_B = 1.0$, $\Phi_m = 1.6$, $F = 26.36$, $U(0) = 4.0$, $P_{th}(0) = 0.4$, $P_c(0) = 0.8$, $P_w^+(0) = 0.001$ and $P_w^-(0) = 0.2$ (also see Table 1).

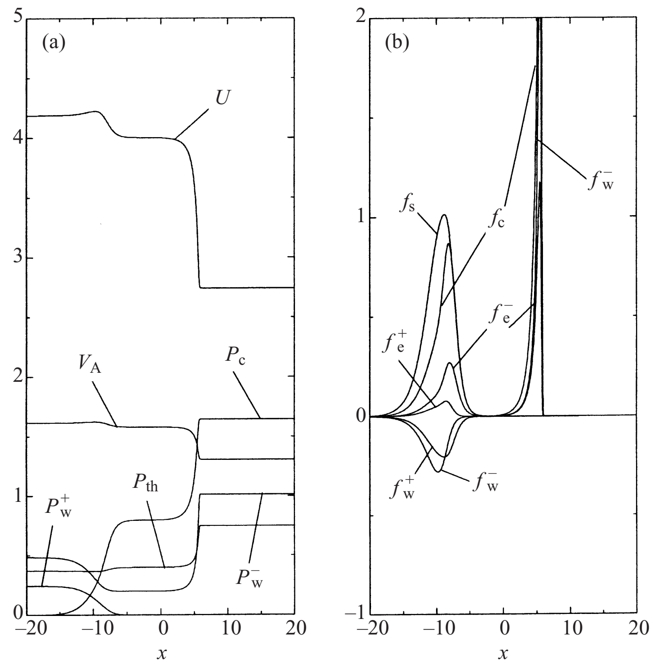

Figure 3. The same as Fig. 1, but with parameters $\Phi_B = 1.0$, $\Phi_m = 1.6$, $F = 26.30$, $U(0) = 4.0$, $P_{th}(0) = 0.4$, $P_c(0) = 0.8$, $P_w^+(0) = 10^{-6}$ and $P_w^-(0) = 0.2$ (also see Table 1).

https://doi.org/10.1017/S0022377801001088 Published online by Cambridge University Press

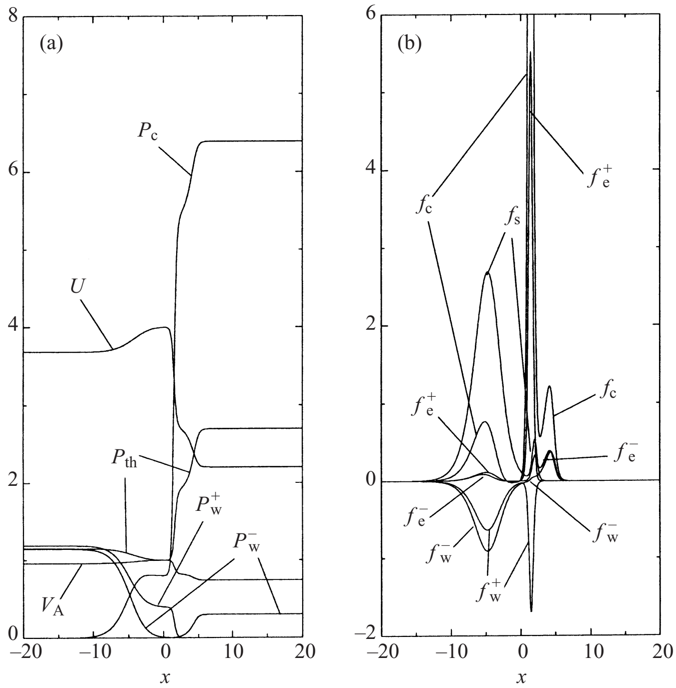

Figure 4. The same as Fig. 1, but with parameters $\Phi_B = 1.0$, $\Phi_m = 4.0$, $F = 63.53$, $U(0) = 4.0$, $P_{th}(0) = 1.0$, $P_c(0) = 0.8$, $P_w^+(0) = 0.4$ and $P_w^-(0) = 0.01$ (also see Table 1).

Since the flows of Figs 1(a) and 2(a) are rather uniform, we had expected that both pressure profiles would show features similar to the nonlinear test-particle picture in a uniform background (Jiang et al. 1996). The result in Fig. 1(a) does, but Fig. 2(a) does not. (In Fig. 2(a), the far-downstream P_c is much less than the peak P_c, while in the nonlinear test-particle picture, the far-downstream P_c is always the largest.) In Fig. 3(a), for x larger than, say 0, the profiles closely resemble those profiles of a system with only a backward wave (i.e. $P_w^+ = 0$), but the upstream state is totally different (when P_w^+ is not small). Figure 4(a) shows a peak in U and a valley in P_w^-, while Figs 5(a) and 6(a) show the opposite features. Although the thermal pressures of Figs 5(a) and 6(b) differ by orders of magnitude, the other profiles are similar.

After experimenting with different parameters, we conclude that in many cases the profiles are non-monotonic. This is in sharp contrast with systems without waves or systems with only one wave, in which all profiles are monotonic.

Finally, as illustrated in Figs 1(b)–6(b), very often the work done by plasma flow on cosmic rays $U\,dP_c/dx$ and the stochastic acceleration P_c/τ are the dominant energy transfer mechanisms. Moreover, among these two, the work done by plasma flow on cosmic rays is far more important in regions of the modified shock (Figs 3(b)–6(b)).

4. Summary and discussion

In this paper, we have studied a four-fluid cosmic-ray–plasma system. The model comprises thermal plasma, cosmic rays, and forward- and backward-propagating

https://doi.org/10.1017/S0022377801001088 Published online by Cambridge University Press

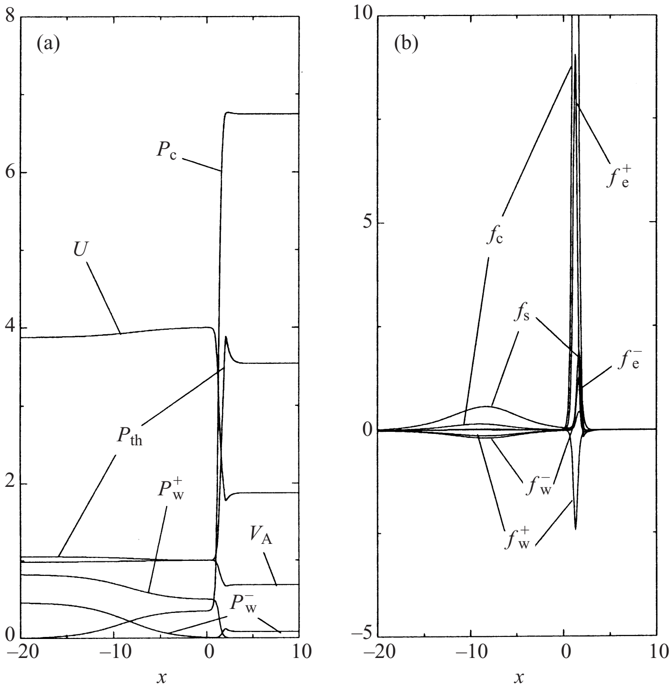

Figure 5. The same as Fig. 1, but with parameters $\Phi_B = 1.0$, $\Phi_m = 4.0$, $F = 56.04$, $U(0) = 4.0$, $P_{th}(0) = 1.0$, $P_c(0) = 0.35$, $P_w^+(0) = 0.5$ and $P_w^-(0) = 0.01$ (also see Table 1).

Alfvén waves. The governing equations are given in Sec. 2 (or see Ko 1992). The steady-state one-dimensional problems can be reduced to a two-dimensional autonomous system of ordinary differential equations. In principle, we can give a rigorous, analytical, treatment of the system, but the mathematics is very tedious. Hence we try to understand some properties of the system using a numerical method instead.

We seek continuous physically admissible solutions and we do not consider solutions with subshocks. In addition, we consider super-Alfvénic flow only ($M_A = U/V_A > 1$). A physically admissible solution approaches uniform states both far upstream and far downstream. Several possible solutions are shown in Figs 1–6. There are many parameters in the system, for example the magnetic flux Φ, the mass flux J, the entropy constant A, the total energy flux F, the total momentum flux G and the wave-action integral W_A (see Sec. 3 for the parameters used). The solutions can be crudely divided into two categories: 'test-particle-like' (Figs 1 and 2) and 'modified-shock-like' (Figs 3–6). The parameter space is very large, and we cannot claim that we have exhausted all possible qualitatively different solutions.

Recall that in systems without waves or in systems with only one wave, the flow velocity is a monotonically decreasing function of x, while the cosmic-ray pressure is a monotonically increasing function (Drury and Völk 1981; Axford et al. 1982; McKenzie and Völk 1982; Ko et al. 1997). (Basically, the reason for monotonicity is these systems can be reduced to a one-dimensional autonomous ordinary differential equation.) In systems where the thermal plasma is dominant (the so-called nonlinear test-particle picture), $P_w^{\pm}$ are monotonic while P_c may not be (Jiang et al. 1996; Ko 1998). As shown in Sec. 3, all the profiles (either velocities or pressures) of the full four-fluid model (systems with both forward and backward waves and

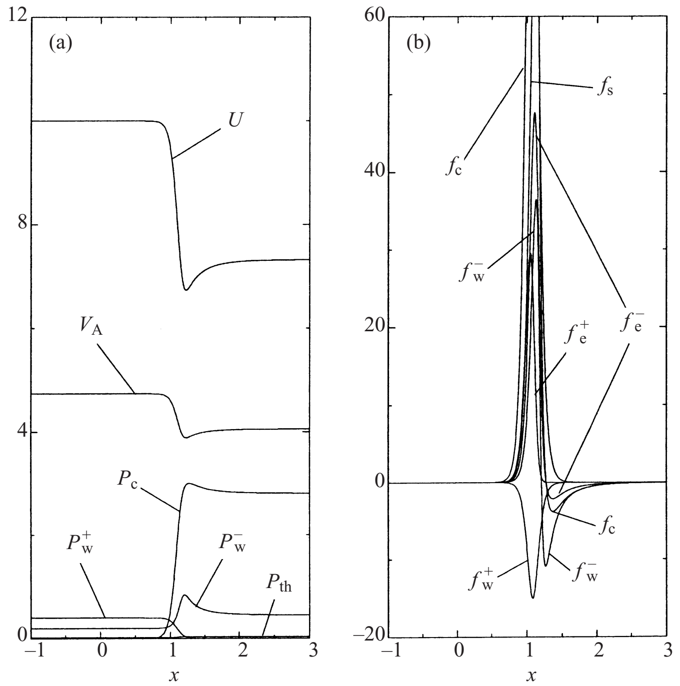

Figure 6. The same as Fig. 1, but with parameters $\Phi_B = 1.5$, $\Phi_m = 1.0$, $F = 70.40$, $U(0) = 10.0$, $P_{th}(0) = 0.02$, $P_c(0) = 10^{-8}$, $P_w^+(0) = 0.4$ and $P_w^-(0) = 0.2$ (also see Table 1).

account taken of the back-reaction of cosmic rays and waves upon plasma flow) do not necessarily show monotonicity (cf. Zank et al. (1993), who showed that the velocity profile can be non-monotonic when injection is considered). In fact, in most cases, non-monotonicity is the rule rather than an exception.

The rich morphology of the four-fluid model is the result of the interplay among various energy transfer mechanisms. (Of course, one may take the position that the profiles dictate the strength of the energy transfers. It is a question of 'chicken- and-egg'. Since the model is constructed in a self-consistent way, the question of which comes first does not really matter.) We can identify three basic energy transfer mechanisms in the four-fluid model:

(a) work done by plasma flow against pressure gradients ($U\,dP_c/dx$, $U\,dP_w^{\pm}/dx$);

(b) cosmic-ray streaming instability ($e_{\pm}V_A\,dP_c/dx$);

(c) stochastic acceleration (P_c/τ).

The first two are facilitated by pressure gradients. The last two exchange energy among cosmic rays and waves only. The first two can accelerate or decelerate cosmic rays, while the last one can only accelerate cosmic rays.

In all physically admissible solutions we sought, the far-upstream cosmic-ray pressure vanishes, and the far-downstream cosmic-ray pressure attains a finite value (not necessarily the maximum) (see Figs 1(a)–6(a)). Cosmic rays are always accelerated. Which of the aforementioned mechanisms is the dominant mechanism for cosmic-ray acceleration?

Cosmic-ray streaming instability is subordinate to the work done by plasma, because we are considering super-Alfvénic flows ($U > V_A$ and $e_\pm \leqslant 1$). The stochastic acceleration and the work done by plasma are comparable in the 'upstream' region, but the work done by plasma always wins in the 'downstream' region. Note that the work done by plasma may accelerate or decelerate cosmic rays as well in the 'downstream' region, but it is the dominant energy transfer mechanism anyway. Stochastic acceleration works only if all three pressures P_c and $P_w^\pm$ are present. If any one of them vanishes, this will kill the mechanism. The mechanism channels the energy contained in waves to cosmic rays. Subsequently, at least one wave withers 'rapidly' and the mechanism shuts down in or before the 'transition region' (say, $x \approx 0$).

In 'modified-shock-like' solutions (Figs 3–6), we may regard the stochastic acceleration as a trigger of the acceleration of cosmic rays. The major phase of acceleration is rendered by the plasma flow (against the cosmic-ray pressure gradient).

A slight change in the relative strength of the energy transfer mechanisms may cause a large change in the structure of the system. This is best illustrated in Figs 2 and 3. In these two figures, the integrals (Φ_B, Φ_m, A, F, G, W_A) are more or less the same, the variables at $x = 0$ are the same, except that $P_w^+ = 10^{-3}$ in Fig. 2 while $P_w^+ = 10^{-6}$ in Fig. 3. The 'upstream' (say, $x < 0$) profiles of the two figures are very similar, but the 'downstream' (say, $x > 0$) profiles are very different (for example, $P_w^- = 0$ in Fig. 2 while $P_w^- \neq 0$ in Fig. 3).

Furthermore, if we set $P_w^+ = 0$ at $x = 0$, the system becomes a unidirectional system (cf. McKenzie and Völk 1982). The 'downstream' (say, $x > 0$) profiles are almost identical to Fig. 3(a), but the 'upstream' (say, $x < 0$) profiles are totally different (for example $P_w^+ \neq 0$ and $P_c = 0$ in Fig. 3 while $P_w^+ = 0$ and $P_c \neq 0$ in the unidirectional system).

Although we only talk about continuous solutions in this paper, solutions with a genuine subshock are expected to exist (an extrapolation from two-fluid models – Drury and Völk 1981; Axford et al. 1982; Ko et al. 1997). They are more difficult to find, but we anticipate that their structures ought to be qualitatively different from those of two-fluid models. As the downstream states of two-fluid models are uniform, the downstream states of four-fluid models should not be, because both forward and backward waves are generated by the subshock. Stochastic acceleration prevents uniform states.

Finally, we should like to point out a difference between the physically admissible solutions of the nonlinear test-particle picture (where the plasma is considered as an 'energy and momentum reservoir' – Jiang et al. 1996) and those of the four-fluid model (in which the back-reactions are taken into account). In the nonlinear test-particle picture, all physically admissible solutions (with both waves) have $P_c = 0$ far upstream and $P_w^\pm = 0$ far downstream. As shown in Figs 3(a)–6(a) (the 'modified-shock-like' solutions), the four-fluid model has solutions with $P_w^- \neq 0$ far downstream. What role the back-reaction plays to have such an effect warrants further investigation.

Acknowledgements

The author has benefited greatly from discussions with C. M. Lin. This work is supported in part by the National Science Council, Taiwan, under Grants NSC 89-2112-M-008-012 and NSC 90-2112-M-008-014.

References

Achterberg, A., Blandford, R. D. and Periwal, V. 1984 *Astron. Astrophys.* **132**, 97.

Axford, W. I., Leer, E. and Skadron, G. 1977 In: *Proceedings of 15th International Cosmic Ray Conference, Plovdiv*, Vol. 2, p. 273.

Axford, W. I., Leer, E. and McKenzie, J. F. 1982 *Astron. Astrophys.* **111**, 317.

Dewar, R. L. 1970 *Phys. Fluids* **13**, 2710.

Drury, L. O'C. and Völk, H. J. 1981 *Astrophys. J.* **248**, 344.

Duffy, P., Drury, L. O'C. and Völk, H. J. 1994 *Astron. Astrophys.* **291**, 613.

Heavens, A. F. 1984 *Mon. Not. R. Astron. Soc.* **210**, 813.

Jiang, I.-G., Chan, K.-W. and Ko, C.-M. 1996 *Astron. Astrophys.* **307**, 903.

Jones, F. C. and Ellison, D. C. 1991 *Space Sci. Rev.* **58**, 259.

Jones, T. W. and Kang, H. 1990 *Astrophys. J.* **363**, 499.

Ko, C.-M. 1992 *Astron. Astrophys.* **259**, 377.

Ko, C.-M. 1995 *Adv. Space Res.* **15**, 149.

Ko, C.-M. 1998 *Astron. Astrophys.* **340**, 605.

Ko, C.-M. 2001 *Adv. Space Res.* **26**, 1863.

Ko, C.-M. and Jeng, A.-T. 1994 *J. Plasma Phys.* **52**, 23.

Ko, C.-M., Chan, K.-W. and Webb, G. M. 1997 *J. Plasma Phys.* **57**, 677.

McKenzie, J. F. and Völk, H. J. 1982 *Astron. Astrophys.* **116**, 191.

McKenzie, J. F. and Webb, G. M. 1984 *J. Plasma Phys.* **31**, 275.

Malkov, M. A, 1997a *Astrophys. J.* **485**, 638.

Malkov, M. A. 1997b *Astrophys. J.* **491**, 584.

Schlickeiser, R. 1989 *Astrophys. J.* **336**, 243.

Skilling, J. 1975a *Mon. Not. R. Astron. Soc.* **172**, 551.

Skilling, J. 1975b *Mon. Not. R. Astron. Soc.* **173**, 255.

Zank, G. P. 1989 *J. Plasma Phys.* **41**, 89.

Zank, G. P., Webb, G. M. and Donohue, D. J. 1993 *Astrophys. J.* **406**, 67.