A.A.S. **4**, II, 339-349 (2009)

REVIEWS

Actuarial Mathematics for Life Contingent Risks. By David C. M. Dickson, Mary R. Hardy and Howard R. Waters (Cambridge University Press, 2009. 493pp. ISBN: 9780521118255)

This is the fifth title in the International Series on Actuarial Science published by CUP in conjunction with the Institute and Faculty of Actuaries.

While the authors have many decades of teaching experience in three continents they all hail from a common Heriot-Watt University heritage. They have blended their contributions with great skill and the reader will not be conscious changing style across the different chapters. The style is formal — written in the 'regal we' rather than the passive. In some chapters this does not make for easy reading and can become irritating.

The text is designed for a two semester course. Chapters 1 to 7 encompass a 'single life' introductory course — life insurance background, survival models, life tables including selection, insurance benefits, annuities, premiums and policy values — for the first semester. Chapters 8 and 10 to 14 cover more advanced topics — multiple state models, interest rate risk, emerging costs for traditional and equity-linked insurance, option pricing and embedded options — for a follow — on course. Chapter 9 on pension funds can be added to this course.

The first course is similar to Bowers *et al.* (1997). The survival models chapter discusses the equivalence of the events $T_x \leq t$ and $T_0 \leq x + t \mid T_0 > x$ rather than using the term 'consistency condition', i.e. making the model for T_0 give probabilities that are consistent with the model for T_x. The differential and integral results that link μ_x to survival probabilities are derived and used, but the opportunity to refer to μ_x as a transition (probability) rate and give a preview of results in the later multiple state models chapter is not taken. Numerical solutions of the integral equations for $\mathring{e}_x$ are discussed in examples and exercises. The Makeham model of mortality

$$\mu_x = 0.0001 + 0.00035(1.075)^x$$

that is used for the majority of examples and exercises in the text is introduced.

Chapter 3 introduces select life tables. While elsewhere the use of software is highlighted, an opportunity to encourage the use of software to explore and compare mortality experiences, for example using the Standard Tables Program (STP) or Table Manager is missed. Similarly the fact that the life table itself is merely an aid to the computation of probabilities is not discussed.

Chapter 4 describes the valuation of insurance benefits. The use of the integer part (or floor) function, $K_x = \lfloor T_x \rfloor$ rather than a step function and the min function, $Y = a_{\overline{\min(K_x,n)}|}$ rather than

$$Y = \begin{cases} a_{\overline{K_x}|} & K_x = 0, 1, 2, \ldots\ldots n-1 \\ a_{\overline{n}|} & K_x = n, n+1, n+2\ldots\ldots \end{cases}$$

will tend to make the ideas less immediately accessible to those with limited mathematics exposure. Backwards recursions for the computation of values in a spreadsheet are described.

Chapter 5 describes the valuation of annuity benefits. The development mirrors that of Chapter 4. While useful in some contexts the introduction of the alternative of specification of random variables using indicator random variables, for example

$$Y = a_{\overline{(K_x+1)}|} = I(T_x > 0) + v\,I(T_x > 1) + v^2\,I(T_x > 2) + v^3\,I(T_x > 3) + \ldots\ldots$$

will confuse the less mathematically able.

Chapter 6 discusses net and gross premium calculation. The development is in terms of the future loss random variable. This allows not only for the presentation of the usual equations of value via expectations but also the use of the portfolio percentile premium principle which is not found in more traditional texts. The discussion of new business strain and diversification has interesting insights not found in other texts. The suggestion that the Central Limit Theorem can be used if the number of policies is 'greater than around 30' seems dangerous given the skewness that is present in many cases.

Chapter 7 introduces both gross and net premium policy values via the future loss random variable. Innovative features include profit presented as profit from all sources not just mortality, a discussion of asset shares and the numerical solution of Thiele's differential equation.

Chapter 8 is a comprehensive introduction to multiple state models. The development links back to the earlier discussions using the two state alive-dead model but introduces a wide variety of other models. The emphasis is on constructing and using equations to find numerical solutions rather than manipulating differential equations — integrating factors are used but not mentioned by name.

Chapter 9 describes pension fund mathematics. Naturally commutation functions are absent, but there is a helpful discussion of valuation assumptions and funding plans not usually found in similar texts.

Chapter 10 discusses interest rate risk. It includes a discussion of the term structure and how the earlier premium calculation methods can be adapted. A cash flow model is used to illustrate diversifiable and non-diversifiable risk. These concepts are illustrated in examples.

Chapters 11 and 12 describe profit testing. Two examples — term assurance with annual premiums, and endowment assurance with death and disability benefits and monthly premiums are used to illustrate profit testing. Separate reserve, premium and profit test bases are discussed and used in the examples.

Equity-linked policies are illustrated using a policy with an enhanced death benefit and a guaranteed maturity benefit. The example is extended by using stochastic investment returns and the results compared with the deterministic profit test. The ideas are then extended to show how to allow for non-diversifiable risk in pricing and reserving via Value-at-Risk and Conditional Tail Expectation. The discussion is written in a lively and accessible way.

Chapter 13 is an introduction to option pricing. Although the examples are insurance orientated, there is nothing 'new' in this chapter. As the authors point out it could be skipped by those who have already studied other standard texts.

Chapter 13 sets up the necessary infra-structure for Chapter 14 on the embedded options in insurance contracts. The guarantees in the examples in Chapter 12 are presented as options implicit in the equity-linked insurance. The discussion of pricing and reserving using options considers in turn guaranteed minimum maturity benefits and guaranteed minimum death benefits. There is then an extensive discussion of how the costs of guarantees can be recovered from regular rather than single premiums. Comparisons are made between the costs of stochastic reserving from Chapter 12 and the purchase of options.

Numerical answers to selected exercises are provided at the end of each chapter. The value of the text to students would be considerably enhanced by an online resource with more extensive and detailed solutions.

One of the declared aims of the text is to foster 'general business awareness in the life insurance context'. This has limited success, in part because such discussions are much abbreviated. For example, a newcomer might gain the impression that insurance companies are 'entitled' to profits at the expense of the policyholder rather than receiving payment for providing a service of pooling and managing risks with all that that implies.

The series aims to publish books that reflect changes and developments in the curriculum. This book certainly fulfils that objective. Bowers *et al.* (1997) was the first text to eschew the use of commutation functions in the face of increasing use of spreadsheets. This text continues that trend and also plays down the use of life tables and compound interest tables. However it does not take the natural next step of linking the topics discussed to the more comprehensive modelling software that has been developed in the last decade.

Bowers, N.L., Gerber, H.U., Hickham, J.C., Jones, D.A. & Nesbitt, C.J. (2nd Edition 1997). *Actuarial Mathematics*. The Society of Actuaries, Schaumburg, Illinois.

CMI Tables Program (STP) for Windows (Version 3.3 2009). Institute of Actuaries, London.

Table Manager Version 3.0. Society of Actuaries. Schaumburg, Illinois.

Philip Cooper

Nonlife Actuarial Models, Theory, Methods and Evaluation. By Yiu-Kuen Tse (Cambridge University Press, 2009. 544pp. ISBN: 9780521764650)

Based on the UK CT6 (previously 106) exam or the North-American Society of Actuaries exam C, Non-Life Actuarial Models puts a more mathematical framework in place for those students who want to know more than just how to pass the exam. Starting from basic concepts of distribution theory it progresses on to mixture distributions, aggregate loss models, ruin theory and how each of these can be put in to practice in real life.

The book is split into 4 sections: *Loss distributions, Risk and ruin, Credibility, and Model construction and evaluation.*

The loss distribution section discusses the two components of modelling any claim losses; that is claim frequency and claim severity. The book quickly moves on from these concepts, bringing them together to discuss both individual risk models and the collective risk model.

The section on risk and ruin discusses a subject that we are all familiar with; the discussion of risk and what is meant by the term risk together with its measurement and the likelihood of ruin. A variety of risk measures are discussed alongside the definition of a coherent risk measure, a concept not widely known in the actuarial profession. In the chapter on ruin theory the author constructs the surplus random variable and probes the sensitivity of this random variable to time, initial surplus and claim distribution.

Section three deals with the important aspect of data and the level of credibility to which we can hold such data. This section is dealt with in a very statistical setting, asking the question how recent, more relevant, experience

https://doi.org/10.1017/S1748499500000762 Published online by Cambridge University Press