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THE WILKIE MODEL FOR RETAIL PRICE INFLATION REVISITED

BY W. S. CHAN AND S. WANG

ABSTRACT

A first order autoregressive model was proposed in Wilkie (1995) for the retail price inflation series as a part of his stochastic investment model. In this paper we apply time-series outlier analysis to the data set and a revised model is derived. It significantly alleviates the problem of leptokurtic and positive skewed residual distribution as found in the original model. Finally, ARCH models for the original series and the outlier-adjusted data are also considered.

KEYWORDS

ARCH Models; Price Inflation; Stochastic Asset Models; Time-Series Outliers; Wilkie Model

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1. INTRODUCTION

1.1 The stochastic model of retail price inflation is a core part of the Wilkie composite model. It provides, directly or indirectly, inputs to other component variables. The most updated version of the Wilkie model can be found in Wilkie (1995).

1.2 A first order autoregressive process is employed to model the dynamics of price inflation. However, the problem of upward skewness and leptokurtosis (heavy 'tail') of the residuals has caused some concerns for many authors (see Kitts, 1990; Geoghegan *et al.*, 1992; and Huber, 1997). Clarkson (1991) suggests a non-linear model to tackle the problem of positive skewed residuals. Wilkie (1995, p.800) considers ARCH models to explain the fat-tailed residuals.

1.3 In this paper we apply the recently developed time-series outlier detection technique to the price inflation series. A revised model is obtained and the normality of the residuals is significantly improved.

1.4 The plan of the rest of the paper is as follows. In Section 2 we describe the data, and the original model is fitted. Time-series outlier analysis is performed in Section 3. ARCH modelling results are summarised in Section 4. Finally, some concluding remarks are given in Section 5.

2. THE ORIGINAL MODEL

2.1 The annual Retail Prices Index (RPI) series is taken from the June values of:

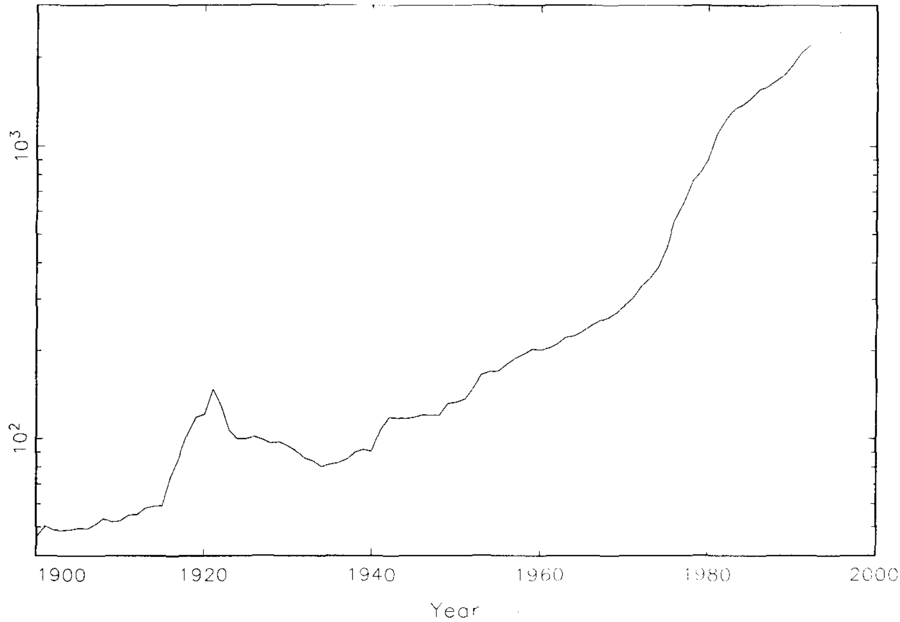

Figure 2.1. Retail prices index, 1896-1996

(a) 1896-1913: Board of Trade Wholesale Price Indices (Total Index), Table Prices 5 of Mitchell & Deane (1962);
(b) 1914-1947: 'All Items' Cost of Living Index, Table 84 of Central Statistical Office (1991);
(c) 1948-1990: 'All Items' Retail Prices Index, Table 1 of Central Statistical Office (1991);
(d) 1991-1993: 'All Items' General Index of Retail Prices, Table 18.7 of Central Statistical Office (1994); and
(e) 1994-1996: 'All Items' Retail Prices Index, Table 18.7 of Office of National Statistics (1997).

2.2 The series is rebased to 1923. The RPI series, $Q(t)$, is plotted in Figure 2.1 with a vertical logarithmic scale.

2.3 The price inflation series is defined by:

$$I(t) = \ln Q(t) - \ln Q(t-1)$$

and its graph is given in Figure 2.2.

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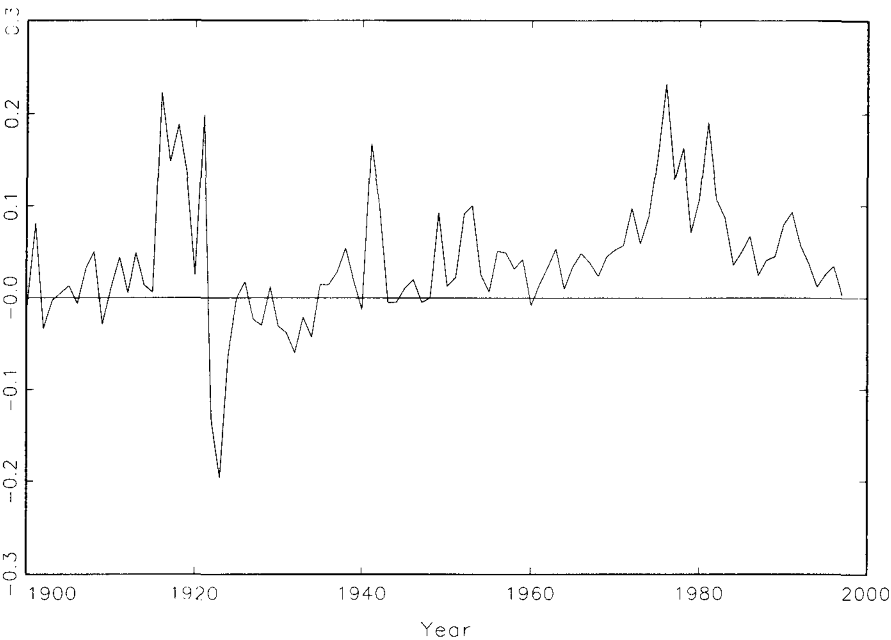

Figure 2.2. Annual force of inflation, 1896-1996

2.4 It appears that there were some price disturbances in the post-war 1919-22 period. The Second World War again induced a peak in 1940. The oil crisis in 1973-74 also caused exceptionally high inflation, with the year 1975 being the most seriously affected.

2.5 The original model proposed for the price inflation series $I(t)$, is (Wilkie, 1986, 1987, 1992, 1995):

$$I(t) = QMU + QA(I(t-1) - QMU) + QE(t)$$

$$QE(t) \sim \text{i.i.d. } N(0, QSD^2).$$

2.6 The fitted parameters, based on the series from 1896 to 1996, were obtained from the SCA computer system (Liu & Hudak, 1994) as:

$$QMU = 0.0404 \quad QA = 0.5010 \quad QSD = 0.0572.$$

2.7 The values are very similar to the results given by Wilkie (1995, p.784) for the period 1914-94.

2.8 Diagnostic checking of the residuals was performed. The Ljung & Box

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(1978) portmanteau statistic for testing independence of the residuals is 29.6 (critical value for $\alpha = 5\%$ is $\chi^2_{19,0.95} = 30.14$), which is marginally insignificant. The coefficients of skewness and kurtosis of the residuals are $\sqrt{b_1} = 0.03$ and $b_2 = 7.75$ respectively, indicating a slightly positive skewed and strongly leptokurtic residual distribution. Furthermore, the Jarque & Bera (1981) statistic is 93.88 (critical value for $\alpha = 5\%$ is $\chi^2_{2,0.95} = 5.99$), which also shows highly significant non-normality.

2.9 Wilkie (1995, p.783) is aware of the aberrant observations in 1919-21. The estimates of the AR(1) parameters are sensitive to these extreme values. Therefore, he suggests starting the series in 1923. However, spurious residuals are still found in 1940, 1975 and 1980 from the AR(1) model for 1923-94 (see Wilkie, 1995, p.785). In this paper we try to accommodate the outliers in the model instead of omitting the period with suspicious observations.

3. The Revised Model

3.1 Time-series observations are often influenced by interruptive events such as strikes, outbreaks of wars, sudden political or economic crises, unexpected heat or cold waves, or even unnoticed errors of typing and recording. The consequences of these interruptive events create spurious observations, which are inconsistent with the rest of the series. Such observations are usually referred to as outliers.

3.2 There are many ways to describe the effect of an outlier. Four commonly used outlier models and their effects are briefly discussed in Appendix A. They are additive outlier (AO), innovational outlier (IO), level shift (LS) and temporary change (TC).

3.3 The search for the location and type of an outlier in a contaminated time series is known as an outlier detection problem in time-series literature. It was first studied by Fox (1972) and considered by many other authors (see Muirhead, 1986; Ljung, 1993 and references therein). Time-series outlier detection techniques have been extensively applied to diverse fields, ranging from medicine (Thomas *et al.*, 1992) to financial markets (Guerard, 1990).

3.4 In this paper we apply the outlier detection method developed by Chen & Liu (1993) to the price inflation series. A brief description of the approach is given in Appendix B. The detection procedures are implemented in the programming package SCA (Liu & Hudak, 1994). Only highly significant outliers with *t*-ratios greater than 3.5 (as recommended by Liu & Hudak, 1994, p.746) are considered.

3.5 Several outliers are detected and the results are summarised in Table 3.1.

3.6 The First and Second World Wars might have created some anomalies in the price inflation dynamic which lasted for a few years. There was a 'temporary change' in the inflation process after the oil crisis. Note that the outliers detected by the test correspond to those informally observed in ¶2.4.

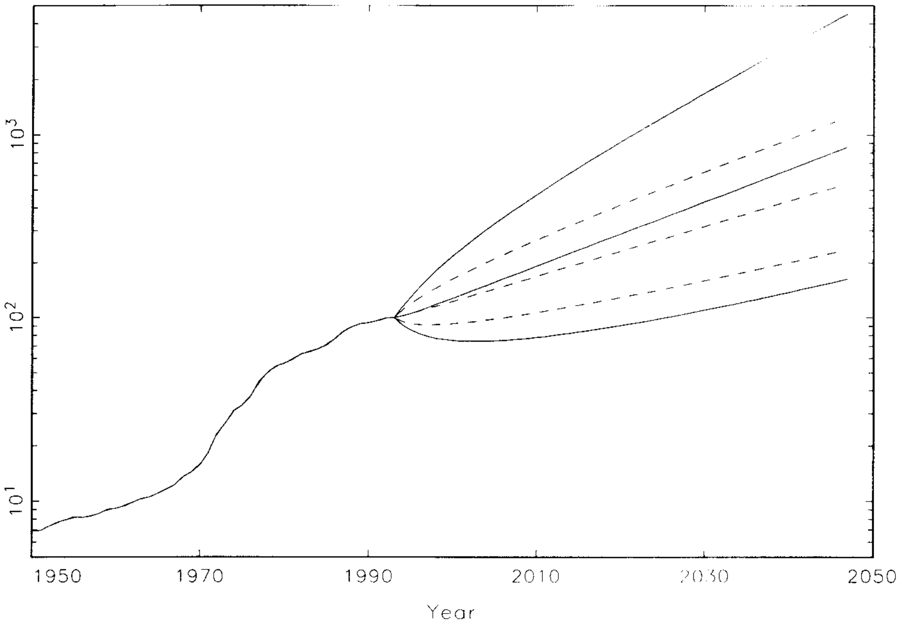

Figure 3.1. Retail price index, 1950-96, and forecast medians and forecast bounds for the original model (solid lines) and for the revised model (dashed lines), 1997-2050

Table 3.1. Outlier detection results for $I(t)$, 1896-1996

Time	Year	Type	t-value
20	1915	TC	6.02
25	1920	IO	4.07
26	1921	TC	-7.56
45	1940	IO	4.08
80	1975	TC	5.12

3.7 The revised model is specified as:

$$I(t) = QMU + QA(I(t-1) - QMU) + \frac{QTA}{1-\delta\beta}P(t,20) + \frac{QIA}{1-QA\beta}P(t,25)$$

$$+\frac{QTB}{1-\delta\beta}P(t,26) + \frac{QIB}{1-QA\beta}P(t,45) + \frac{QTC}{1-\delta\beta}P(t,80) + QE(t)$$

$$QE(t) \sim \text{i.i.d.N}(0, QSD^2)$$

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Table 3.2. Comparison of different models for inflation

		Model A	Model B	Model C	Model D
(1)	Model characteristics				
	Order	AR(1)	AR(1)	AR(1)	AR(1)
	Period	1896-1996	1896-1996	1923-1994	1919-1982
	Outlier adjustment	No	Yes	No	No
(2)	Key parameters				
	QMU	0.0404	0.0316	0.0473	0.0374
	QA	0.5010	0.3475	0.5773	0.5070
	QSD	0.0572	0.0372	0.0427	0.0618
(3)	RPI forecasts (base year = 1996)				
	Year				
	2000 Lower limit	79.3	91.5	85.7	76.1
	Median	113.5	111.8	114.5	112.4
	Upper limit	162.5	136.6	153.1	166.1
	2025 Lower limit	93.8	134.6	130.5	77.1
	Median	310.9	246.3	371.2	285.6
	Upper limit	1,030.7	450.6	1,055.9	1,058.3
	2050 Lower limit	162.6	236.5	283.1	118.7
	Median	853.7	542.7	1,211.1	727.6
	Upper limit	4,482.6	1,245.4	5,182.1	4,458.9

where $\delta = 0.7$, $P(t,T)$ is a pulse function (that is, $P(t,T)$ assumes the value 1 when $t = T$ and is 0 otherwise) and the backwards shift operator β is defined by $\beta^s X(t) = X(t - s)$.

3.8 The estimated parameters for 1896-1996 are obtained:

$$QMU = 0.0316 \quad QA = 0.3475 \quad QSD = 0.0372$$

$$QTA = 0.202 \quad QIA = 0.152 \quad QTB = -0.253 \quad QIB = 0.152 \quad QTC = 0.171.$$

3.9 Removing the outlier effects, which are mostly positive, produces a smaller long-term mean. The revised *QMU* value is 22% less than the original estimate reported in ¶2.6. Chan (1995a) shows that a large temporary change (TC) could push the first-lag autocorrelation as well as the *QA* value towards the dampening parameter $\delta = 0.7$. On the other hand, the outlier-adjusted *QA* estimate is free of such bias. Its value drops from the original 0.5010 to 0.3475. There is also a 35% reduction in *QSD* using the revised model.

3.10 Extensive diagnostic testing of the revised model was carried out. The Ljung & Box (1978) portmanteau statistic is 21.3, which should be compared with $\chi^2_{19,0.95} = 30.14$ at the 5% level. There is no evidence of non-independence of the residuals. The skewness and kurtosis coefficients of the residuals are 0.32 and 3.69, respectively. The Jarque & Bera (1981) test statistic is 3.70, which should be compared with $\chi^2_{2,0.95} = 5.99$ at 5% level. In conclusion, the

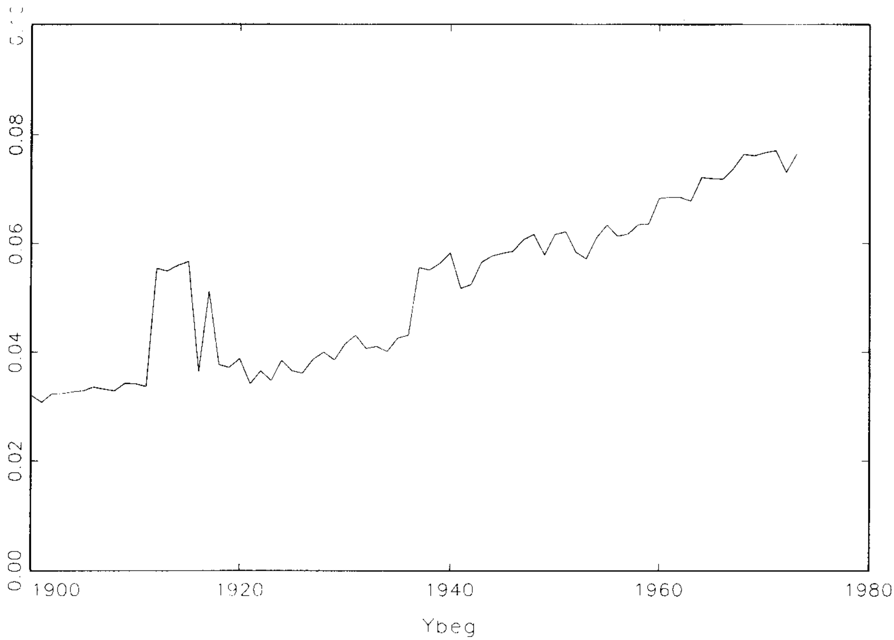

Figure 3.2. *QMU* estimates for the periods Ybeg-1996

diagnostic tests indicate neither non-independence nor non-normality of the residuals.

3.11 In Figure 3.1, we show the forecast median of $Q(t)$ by the original model, starting with the conditions in June 1996, on a logarithmic scale, along with its forecast bounds. The bounds are obtained by the mean plus and minus two standard deviations, using the formulae in Wilkie (1995, Appendix E.2). The results for the revised model are also plotted in the same graph with dashed lines. Conditional on no further outliers occurring, the revised model produces lower forecast medians and much narrower bounds.

3.12 We now compare, in Table 3.2, different models for inflation. Model A and Model B are discussed in ¶2.6 and ¶3.8, respectively. Model C and Model D are taken from Wilkie (1995, p.784). One can see that both the estimates of *QMU* and *QA* are rather sensitive to outliers and the period used. They also produce quite different RPI forecasts, especially in long range forecasts.

3.13 Finally, we study the stability of the estimates of the parameters. Periods of Ybeg-1996 are considered. The starting year (Ybeg) is rolled from 1896 to 1972. It creates 77 periods. The last period, 1972-96, has 25 observations, which is minimum for computing reasonable AR(1) estimates. In

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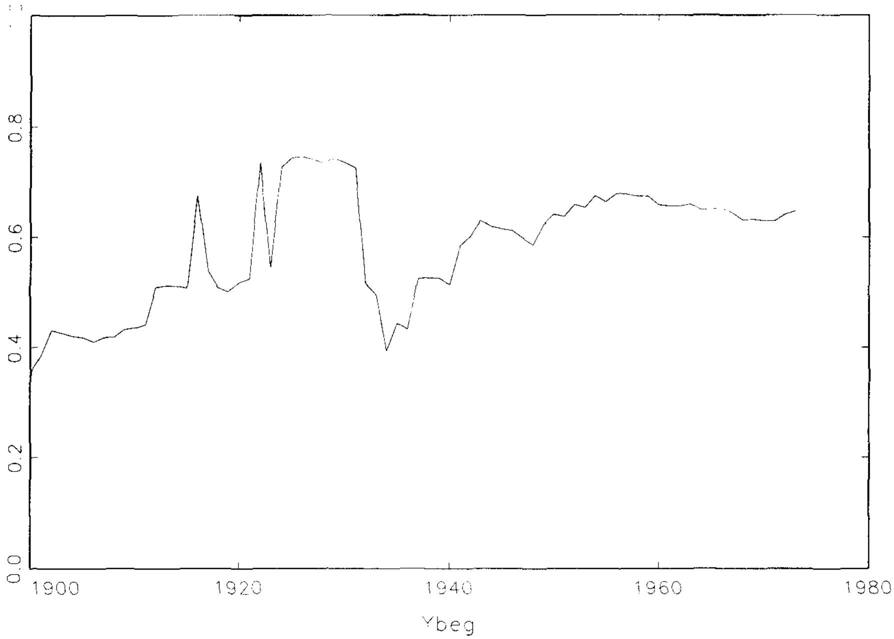

Figure 3.3. *QA* estimates for the periods Ybeg-1996

each period we estimate the *QMU* and *QA* parameters after performing outlier detection and adjustment for the series. The results are plotted in Figures 3.2 and 3.3. There is an upward trend in the long-term mean, while the *QA* values are quite stable at around 0.6 after the Second World War.

4. ARCH MODELS

4.1 Wilkie (1995, p.799) considers ARCH models for the price inflation. Three series:

$$QE(t)\text{-squared} = \{QE(t)\}^2;$$

$$I(t)\text{-squared} = \{I(t) - QMU\}^2; \text{ and}$$

$$IH(t)\text{-squared} = \{QA(I(t-1) - QMU)\}^2$$

were calculated for the 1896-1996 data set. The $QE(t)$-squared series has a significant first-lag autocorrelation (r_1 = 0.35). The cross-correlation coefficient between $QE(t)$-squared and $I(t-1)$-squared is 0.046, while the coefficient between $QE(t)$-squared and $IH(t)$-squared is 0.013. Engle (1982) proposes a Lagrange multiplier (LM) test for ARCH disturbances. McLeod & Li's (1983) portmanteau Q-test, calculated from the squared residuals, can also be used to specify ARCH processes. The first order LM and Q statistics for the $QE(t)$-squared series are 20.5 and 21.2 respectively. They should be compared with $\chi^2_{1,0.95} = 3.84$ at 5% level. According to these tests and indicators, there is evidence to suggest that an ARCH model would be useful for the series.

4.2 We consider the following AR(1)-ARCH(1) model:

$$I(t) = QMU + QA(I(t-1) - QMU) + QE(t)$$

$$QE(t) = \sqrt{QH(t)}\ \ QU(t)$$

$$QH(t) = QHA + QHB\{QE(t-1)\}^2$$

where $QU(t)$ is i.i.d. N(0, 1). Hamilton (1994, p.657) provides a review of various representation of ARCH models. We will estimate this model first for the unadjusted inflation series and then for the outlier-adjusted series.

4.3 The model in ¶4.2 states that the conditional variance $QH(t)$ is a stochastic process. Its expected value is the intercept (QHA) plus a fraction (QHB) of the last year's squared residual. The value of QHA must be positive; this is necessary because, without it, there is a possibility of $\{QE(t-1)\}^2$ being exactly zero, both $QH(t)$ and $QE(t)$ collapsing to zero, and $I(t)$ moving deterministically thereafter. Furthermore, since $QH(t)$ is a positive process, QHB must be non-negative.

4.4 Note that the ARCH model employed here is different to that used by Wilkie (1995, p.799). Wilkie's ARCH model, the conditional variance $QH(t)$, depends on the deviation of the last period's inflation from a parameter QHC, which is not necessarily equal to the long-term mean QMU.

4.5 The estimated parameters are:

$$\underset{(0.009)}{QMU = 0.0443} \qquad \underset{(0.086)}{QA = 0.5097} \qquad \underset{(0.0003)}{QHA = 0.00179} \qquad \underset{(0.209)}{QHB = 0.5455.}$$

The corresponding standard errors of the estimates are given in parentheses.

4.6 Despite the ARCH model giving an improvement on the log likelihood, the residuals of the model are still not normally distributed after allowing this form of heterogeneity. The residuals remain positive skewed and fat-tailed. The Jarque-Bera statistic is 112.7 (critical value for 5% is $\chi^2_{2,0.95} = 5.99$), which is highly significant at the 5% level. It is therefore appropriate to investigate

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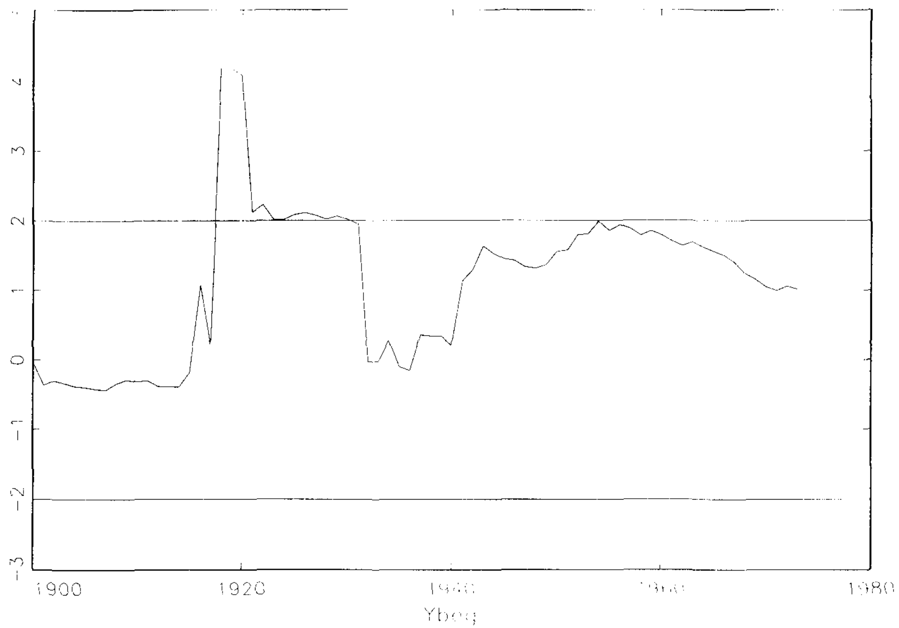

Figure 4.1. Signal-to-noise ratio $r_1/sd(r_1)$, of the first order autocorrelation of $QE^*(t)$-squared series calculated from the AR(1) model for $I^*(t)$, Ybeg-1996

whether an ARCH model applied to the outlier-adjusted series might be more satisfactory.

4.7 The outlier-adjusted price inflation series is defined by:

$$I^*(t) = I(t) - \frac{QTA}{1-\delta\beta}P(t,20) - \frac{QIA}{1-QA\beta}P(t,25) - \frac{QTB}{1-\delta\beta}P(t,26)$$
$$-\frac{QIB}{1-QA\beta}P(t,45) - \frac{QTC}{1-\delta\beta}P(t,80).$$

For convenience, we shall add an asterisk to the corresponding series to signify that they are calculated from the outlier-adjusted series.

4.8 The $QE^*(t)$-squared series does not show any significant autocorrelations. The cross-correlation coefficient between $QE^*(t)$-squared and $I^*(t-1)$-squared is 0.012. On the other hand, the cross-correlation coefficient between $QE^*(t)$-squared and $IH^*(t)$-squared is 0.003. Both coefficients are not significant. The first-lag LM and Q statistics for the $QE^*(t)$-squared series are 1.26 (p = 0.202) and 1.40 (p = 0.236), respectively. There is no evidence to suggest that an ARCH model would be useful.

4.9 It is interesting to note that the ARCH tests and indicators behave distinctly under the contaminated series (see ¶4.1) and under the outlier-adjusted

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series (see ¶4.8). The original model produces extreme residuals. These values are further magnified in the $QE(t)$-squared series. It is well known that multiple outliers could create spurious autocorrelations, which might lead to erroneous ARCH identification (see Chan, 1995b). Therefore, the ARCH tests and indicators may not be reliable under the influence of outliers.

4.10 Given that the tests in ¶4.8, showing no evidence of ARCH effects, may be unreliable, we will proceed to re-estimate the AR(1)-ARCH(1) parameters, using the outlier-adjusted series. The results are:

$$\underset{(0.006)}{QMU^* = 0.0279} \quad \underset{(0.122)}{QA^* = 0.3491} \quad \underset{(0.0003)}{QHA^* = 0.00116} \quad \underset{(0.162)}{QHB^* = 0.1560.}$$

The Jarque-Bera statistic for the residuals is 4.10 ($p = 0.129$), indicating normality. However, the t-ratio for the estimate QHB^* is 0.96, which confirms the complications of the tests in ¶4.8; the ARCH effects are insignificant.

4.11 We now repeat the rolling computations introduced in ¶3.13 to examine the usefulness of the AR(1)-ARCH(1) model for different periods. The signal-to-noise ratio $r_1/sd(r_1)$, of the first order autocorrelation of $QE^*(t)$-squared series is chosen as the ARCH indicator. Figure 4.1 gives the results. Only the periods with starting years Ybeg = {1917, 1918, ..., 1929} have significant ARCH effects. On the other hand, there is no evidence to suggest that an AR(1)-ARCH(1) model is useful for other periods.

5. Concluding Remarks

5.1. In this paper we study the issue of outliers in time-series modelling of retail price inflation. Several outliers are detected, and a revised model which explicitly incorporates the outliers is proposed. We also investigate ARCH effects in the presence of outliers, but we find that the usual ARCH tests and indicators are not robust to outliers.

5.2 The impact of the detected outliers in the price inflation on the other component variables of the Wilkie model will be a research topic of some interest. There are several advanced time-series models that may be useful in stochastic investment modelling. These include structural time-series models (Harvey, 1989), non-linear threshold models (Tong, 1990) and multivariate time-series models (Tiao & Tsay, 1989). Research in some of these topics is in progress.

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APPENDIX A

TIME SERIES OUTLIER MODELS

A.1 Suppose that an outlier-free time series Y_t has the stationary ARMA(p,q) representation:

$$\phi(\beta)Y_t = \theta(\beta)a_t$$

where β is the backwards shift operator such that $\beta^s Y_t = Y_{t-s,}$

$$\phi(\beta) = 1 - \phi_1\beta - \ldots - \phi_p \beta^p \theta(\beta) = 1 - \theta_1 \beta - \ldots - \theta_q \beta^q$$

and a_t is i.i.d. N(0,σ^2).

A.2 In general, outliers in time series can be viewed as the result of non-repetitive interventions. Thus, a contaminated time series Z_t consists of an outlier-free time series Y_t plus an exogenous intervention effect $\eta_t(T,\omega)$, i.e.:

$$Z_t = Y_t + \eta_t(T,\omega)$$

where T is the location of the outlier and ω is the magnitude of the outlier.

A.3 An additive outlier (AO) affects only the level of the given observation, and its corresponding exogenous intervention function is:

$$\eta_t(T,\omega) = \omega P(t,T)$$

where:

$$P(t,T) = \begin{cases} 1 & \text{if } t = T \\ 0 & \text{if } t \neq T. \end{cases}$$

A.4 An innovational outlier (IO) affects all observations Z_T, Z_{T+1},... beyond time T through the memory of the underlying ARMA system described by $\{\theta(\beta)/\phi(\beta)\}$:

$$\eta_t(T,\omega) = \frac{\theta(\beta)}{\phi(\beta)} \omega P(t,T).$$

A.5 A level shift (LS) with:

$$\eta_t(T,\omega) = \frac{1}{1-\beta} \omega P(t,T)$$

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is an event that affects a time series at a particular time point whose effect becomes permanent.

A.6 A temporary change (TC) is an event having an initial impact, and whose effect decreases exponentially according to a fixed dampening parameter say, δ:

$$\eta_t(T,\omega) = \frac{1}{1-\delta\beta}\omega P(t,T) \qquad \text{for } 0<\delta<1.$$

The AO and LS are two boundary cases of a TC, where $\delta = 0$ and $\delta = 1$ respectively. In practice, the value of δ often lies between 0.6 to 0.8 (Liu & Hudak, 1994, p.76). We employ $\delta = 0.7$ in this article as recommended by Chen & Liu (1993).

A.7 More generally, a time series might contain several, say m, outliers of different types, and we have the following general outlier model:

$$Z_t = Y_t + \sum_{j=1}^{m} \eta_t(T_j, \omega_j).$$

APPENDIX B

OUTLIER DETECTION AND ADJUSTMENT

B.1 Chen & Liu (1993) suggest a method for modelling time series with outliers. Their approach consists of three-stage iterative procedure based on detection, estimation and adjustment.

B.2 *Detection*

B.2.1 The outlier-free time series Y_t in ¶A.1 can be written as a linear combinations of the current and past innovations, i.e.:

$$Y_t = \pi(\beta)a_t$$

where $\pi(\beta) = \theta(\beta)/\phi(\beta) = 1 - \pi_1\beta - \pi_2\beta^2 \ldots$

B.2.2 The fitted residuals $\hat{e}_t = (Z_t - \hat{Z}_t)$, which may be contaminated with outliers, can be expressed as:

$$\begin{aligned} \text{AO:} &\quad \hat{e}_t = \omega\pi(\beta)P(t,T) + a_t \\ \text{IO:} &\quad \hat{e}_t = \omega P(t,T) + a_t \\ \text{LS:} &\quad \hat{e}_t = \omega\{\pi(\beta)/(1 - \beta)\}P(t,T) + a_t \\ \text{TC:} &\quad \hat{e}_t = \omega\{\pi(\beta)/(1 - \delta\beta)\}P(t,T) + a_t. \end{aligned}$$

B.2.3 Alternatively, we can employ a general time series regression to represent the equations in ¶B.2.2:

$$\hat{e}_t = \omega D(i,t) + a_t \qquad \text{for } i = \text{AO, IO, LS and TC}$$

where $D(i,t) = 0$ for all i and $t<T$, $D(i,t) = 1$ for all i and $t = T$, and

$$\begin{cases} D(\text{AO},t) &= -\pi_{t-T} \\ D(\text{IO},t) &= 0 \\ D(\text{LS},t) &= 1 - \sum_{j=1}^{t-T}\pi_j \\ D(\text{TC},t) &= \delta^{t-T} - \sum_{j=1}^{t-T-1}\delta^{t-T-j}\pi_j - \pi_{t-T} \end{cases}$$

for $t > T$.

B.2.4 The maximum value of the standardised t-statistic for the slope (outlier effects) of the above regression can be used for detecting outliers, i.e.:

$$\mathcal{T} = \max_{1 \le T \le n} \{\tau(\mathrm{AO}, T), \tau(\mathrm{IO}, T), \tau(\mathrm{LS}, T), \tau(\mathrm{TC}, T)\}$$

where:

$$\tau(i, T) = \frac{\hat{\omega}(i, T)}{\sqrt{\mathrm{Var}[\hat{\omega}(i, T)]}}$$

with:

$$\hat{\omega}(i, T) = \frac{\sum_{t=T}^{n} \hat{e}_t D(i, t)}{\sum_{t=T}^{n} [D(i, t)]^2}$$

and

$$\mathrm{Var}[\hat{\omega}(i, T)] = \frac{\hat{\sigma}_a^2}{\sum_{t=T}^{n} [D(i, t)]^2}$$

for i = AO, IO, LS and TC.

B.2.5 For a given location, these standardised statistics follow a normal distribution approximately. An outlier is detected if $\mathcal{T}$ is greater than a critical value $\mathcal{C}$. We employ $\mathcal{C} = 3.5$ (as recommended by Liu & Hudak, 1994) in this paper.

B.3 *Estimation*

B.3.1 With the type and the location of an outlier, we can jointly re-estimate the model parameters and the outlier effects.

B.4 *Adjustment*

B.4.1 After the estimation, one can adjust the outlier effects on the observations and the residuals using equations in ¶A.2 and ¶B.2.2, respectively.

B.5 The detection-estimation-adjustment cycle is repeated for the adjusted series until no new outliers are found.

B.6 Finally, the model is re-estimated for the ARMA parameters and all outlier effects simultaneously.