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THE ROLE OF TRANSITORY AND PERSISTENT SHOCKS IN THE CONSUMPTION CORRELATION AND INTERNATIONAL COMOVEMENT PUZZLES

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We study a two-country model with changes in the technological growth rate. Such changes are attributed to transitory and persistent shocks in the growth rate of technology. Cases are considered in which agents in two countries do not have enough information to distinguish between the two types of shocks; gradually, however, the persistence of the shock is recognized through the learning process. Utilizing a set of parameters obtained from U.S. and European productivity growth rates, it is then shown that (i) when persistent shocks affect the two countries identically, there is no consumption-correlation puzzle, and the international comovement puzzle becomes imperceptible; and (ii) even when persistent shocks affect the two countries differently, imperfect information plays an important role in explaining both the consumption-correlation puzzle and the international comovement puzzle (provided transitory shocks are strongly internationally correlated and are relatively larger than persistent shocks).

Keywords: Two-Country Model, International Real Business Cycle Model, Learning, Kalman Filter

1. INTRODUCTION

What happens to the international economy when the growth rate of technology changes in one country? Following Backus et al. (1992), a number of studies on international business cycles using a modern methodology, which is characterized by an approach based on maximization principles incorporated with rational expectations, have attempted to identify and quantify the sources of fluctuations in the international economy. Subsequent studies have also attempted to explain the discrepancies between the model implications and the observed data (for example, cross-country correlations of consumption and output), with varying

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degrees of success. Among others, two features have drawn particular interest: the consumption-correlation puzzle and the cross-country correlation puzzle. The former refers to the fact that the cross-country consumption correlations are weaker than the cross-country output correlations in data, whereas the standard international real business cycle (IRBC) model predicts the opposite. The latter puzzle states that the cross-country correlation coefficients in labor inputs and investments are positive in the data, but negative in the standard models' prediction. An influential paper by Baxter and Crucini (1995) emphasizes the importance of market structure (whether asset markets are complete or incomplete) as well as the importance of the persistence of shocks (whether shocks are transitory or permanent).[1] The latter finding leads us to our first research question: What happens to the international economy if the shock affects the *growth rate* of technology, instead of the *level*?[2]

Two additional questions stem from the similarity of shocks and the ability of the agent to recognize different shocks. There are, in theory, three types of changes in the level of technology due tobecause of the three types of shocks to the growth rate of technology. The shocks may be (i) transitory, (ii) persistent, or (iii) permanent. However, in reality, it is not obvious whether economic agents can identify the type of shock hitting the economy immediately after observing the change in the growth rate. Given our intuition—evoked by Baxter and Crucini (1995)—that an economic agent who recognizes the type of shock will behave differently in response to different types of shocks, the questions arrived at are as follows: (1) To what extent are the responses of the perfectly informed (about the type of shock) agent and those of the imperfectly informed agent different when they see a change in the growth rate of technology? (2) How do those differences affect the international economy? To answer these questions, we introduce a learning process for agents who do not have perfect information. Instead of realizing the type of shock instantaneously, such agents can only gradually recognize the persistence of the shock through the learning process. We then compare the responses of informed and uninformed agents to the different types of shocks.

Although a number of recent papers, including Erceg et al. (2006), Edge et al. (2007), Gilchrist and Saito (2007), and Boz et al. (2011), study learning models using the Kalman filter, most of them focus mainly on learning under an univariate unobserved shock process; i.e., the observable change in the growth rate of technology is attributed to two independent and unobservable shocks, namely transitory and persistent shocks. In contrast, in this paper we consider a learning model under a bivariate unobserved shocks process. Here the independent and unobservable transitory and persistent shocks to one country's growth rate are possibly correlated with the transitory and persistent shocks to the other country's growth rate, respectively. The reason for assuming a bivariate shock process is that despite the fact that finding the impulse responses under the learning mechanism involves some technical difficulties, the actual productivity data—constructed in the spirit of Backus et al. (1992)—support the bivariate process.

We show that, under certain conditions, the inclusion of the learning mechanism and of the shocks to the growth rate of technology allow us to reduce these two puzzles. With regard to the consumption-correlation puzzle, we obtain two remarkable conclusions. First, regardless of the information structure, the puzzle becomes extremely weak when we assume a common persistent shock that affects the two countries identically plus transitory shocks that are idiosyncratic to each country. Second, it is shown that learning plays a substantial role in replicating a positive and high cross-country correlation in outputs and a low cross-country correlation in consumptions, when a persistent shock affects the two countries differently. As for the international correlation puzzle, our conclusion is also twofold. With identical common persistent shocks in the model, the learning mechanism does not help us understand the international comovements. In the absence of such shocks, however, learning plays an important role in explaining the comovements, provided that transitory shocks are highly internationally correlated and are relatively larger than the persistent shocks.

The rest of this paper is organized as follows. Section 2 reviews the literature on the puzzles. A simple two-country model with possible technology growth change is described in Section 3. Also, using the data for the growth rates of productivity in the United States and the European aggregate,[3] we specify the time-series process of technological growth. The learning mechanism is explained in Section 4. Our exercises in Section 5 include finding impulse responses under both perfect information and imperfect information. A slightly different shock process that is also supported by data is presented in Section 6. Stochastic simulations and important implications for international comovements of macrovariables are presented in Section 7. Section 8 concludes.

2. A REVIEW OF THE LITERATURE

With complete asset markets and risk-sharing by households in two countries, standard IRBC models predict a high correlation between the two countries' consumptions. As for the consumption-correlation puzzle, in order to explain the existing differences between the model implications and the observed data, Stockman and Tesar (1995) consider both traded and nontraded goods in their model. It is then shown that the model with a taste shock that alters the relative weights on households' preference over these goods better captures the correlation observed in the data. An empirical study by Lewis (1996) confirms the significance of nontraded goods in reducing the puzzle when the households' utility function includes both traded and nontraded goods that are not separable, although this is not necessarily true without capital market restrictions such as a tax on the acquisition of foreign assets. Work by Boileau (1996) elucidates a low consumption correlation by presenting a model with a nonmarket sector and human capital; the latter opens up a channel of international externalities in production because the level of human capital in one country is influenced by the two countries' physical capital stocks. From an empirical point of view, tests proposed by Canova and Ravn (1996) can reject the risk-sharing conditions for the long and medium run, but not for the short run. This conclusion is somewhat in line with that of Pakko (2004), who finds that this puzzling feature appears mostly in the business cycle frequencies (medium run), utilizing the frequency domain to analyze the correlation coefficients across countries. More recently, studies have focused on an individual's behavior incorporated into novel models. For example, Fuhrer and Klein (2006) illuminate households' habit formation and Luo et al. (2010) examine imperfect information models, "robustness" [Hansen and Sargent (2008)], and "rational inattention" [Sims (2003)] and then show that "rational inattention" can explain the observed data.

The other prominent puzzle, the international correlation puzzle, is also challenged by a number of models. For example, Maffezzoli (2000) utilizes a standard IRBC model with complete markets, yet sheds light on human capital. Kehoe and Perri (2002), on the other hand, introduce an endogenous friction that arises from the assumption that "international loans are imperfectly enforceable." They succeed particularly in generating positive cross-country correlations in investment and labor input. Eliminating international financial markets completely, Heathcote and Perri (2002) replicate cross-country correlations that are observed in output, consumption, investment, and employment data. With regard to cross-country correlations in labor input and investment, Baxter and Farr (2005) find that the variable capital utilization rate helps explain the observed data, without the assumption of unrealistically large technology shocks. Table 1 summarizes cross-country correlation coefficients found in the data[4] as well as correlations predicted by models. Note that, unlike the prominent models of Backus et al. (1995), Baxter and Crucini (1995), and Kollman (1996), some new models, including the sunspot model of Xiao (2004), succeed in replicating a positive cross-country correlation in labor input and investment. Our model differs from their work in that we consider the role of learning in the presence of persistent and transitory shocks in solving these two puzzles.

3. MODEL

3.1. Households, Firms, and Market Structure

Our model is a standard IRBC model. In particular, preferences, production functions, capital accumulation equations, and the incompleteness of the market structure are identical to that of Baxter and Crucini (1993, 1995) and Baxter (1995). Households in two countries, "Home" and "Foreign," respectively, maximize their utility,

$$E_0 \sum_{t=0}^{\infty} \vartheta_t \frac{1}{1-\sigma} \left[C_t^{\theta} L_t^{1-\theta} \right]^{1-\sigma}; \quad E_0 \sum_{t=0}^{\infty} \vartheta_t \frac{1}{1-\sigma} \left[C_t^{*\theta} L_t^{*1-\theta} \right]^{1-\sigma},$$

where C_t and C_t^* are Home and Foreign consumptions (hereafter, we denote variables with an asterisk as foreign variables); L_t is leisure; and $0 < \vartheta_t < 1$ is a

TABLE 1. Cross-country correlations

	Data		BKK (1995)	BC (1995)	Kollman (1996)	Xiao (2004)	BF (2005)	HP (2002)
	BKK(1995)	US–EU	CM	IM	IM	IM	IM	IM
$\rho\,(y, y^*)$	0.66	0.40	−0.21	0.54	0.10	0.35	0.48	0.24
$\rho\,(c, c^*)$	0.51	0.17	0.88	−0.28	0.38	0.15	0.20	0.85
$\rho\,(i, i^*)$	0.53	0.47	−0.94	−0.50	−0.12	0.20	0.11	0.35
$\rho\,(n, n^*)$	0.33	0.47	−0.94	−0.56	−0.12	0.37	0.96	0.14
σ_c/σ_y	0.75	1.06	0.42	1.05	0.47	0.1	0.83	0.51
σ_i/σ_y	3.27	2.86	10.99	2.98	3.35	4.6	2.01	2.04
σ_n/σ_y	0.61	0.69	0.50	0.45	0.36	0.9	0.18	0.28

Notes: (1) " BKK," " BC," " BF," and " HP" are Backus et al. (1995), Baxter and Crucini (1995), Baxter and Farr (2005) (from their Table 2), and Heathcote and Perri (2002) (from their Table 2), respectively. (2) The values in the " Data" columns are taken from Table 11.2 of Backus et al. (1995), where the correlations are United States and European aggregate (first column), and computed from the quartely data for the United States and European aggregate, from 1980.I to 2010.III (second column). See Appendix E for data details. (3) " CM" stands for complete markets and " IM" stands for incomplete markets. (4) $\rho\,(x, x^*)$ stands for the cross-country correlation of variable x. (5) σ_x/σ_y is the standard deviation of x relative to that of y (output).

discount factor, which we shall discuss in the following section. Each household is endowed one unit of time, from which hours of work can be selected. Thus, each household's time constraint is

$$1 - L_t - N_t \geq 0; \quad 1 - L_t^* - N_t^* \geq 0,$$

where N_t is labor input. Production functions in the two countries are of the Cobb–Douglas form,

$$Y_t = K_t^{\alpha} \left(N_t X_t\right)^{1-\alpha}; \quad Y_t^* = K_t^{*\alpha} \left(N_t^* X_t^*\right)^{1-\alpha},$$

where X_t is the level of labor-augmented technology that follows the stochastic process specified later and K_t is capital stock.

Capital accumulation equations are

$$K_{t+1} = (1-\delta) K_t + \phi \left(\frac{I_t}{K_t}\right) K_t; \quad K_{t+1}^* = (1-\delta) K_t^* + \phi \left(\frac{I_t^*}{K_t^*}\right) K_t^*,$$

where δ is the rate of depreciation and $\phi(\cdot)$ is the adjustment cost function, which is assumed to be $\phi(\cdot) > 0$, $\phi'(\cdot) > 0$, and $\phi''(\cdot) < 0$. Households in both countries can sell and buy riskless (one-period discount) bonds. Let P_t^B be the time-t price of the bond that pays one unit of a consumption good in the next period, and let B_t be the time-t quantities of bonds held by Home households. Therefore, the Home and Foreign budget constraints are given by

$$P_t^B B_{t+1} + C_t + I_t = Y_t + B_t; \quad P_t^B B_{t+1}^* + C_t^* + I_t^* = Y_t^* + B_t^*.$$

The bond-market-clearing condition is

$$\pi B_t + (1-\pi) B_t^* = 0, \tag{1}$$

where π is the relative population (i.e., relative size) of Home.

3.2. Growth Rates and Time Series Process of Shocks

Data. Figure 1 illustrates quarterly growth rates (in terms of percentage) of productivity of the United States and the European aggregate, from 1980.I through 2010.III.[5] Summarized in Table 2, average productivity growth rates are similar: 0.38 (1.51% annually) for the United States and 0.35 (1.41% annually) for Europe. In addition, the standard deviation of the U.S. productivity growth rate, 0.74, is only slightly larger than that of the European productivity growth rate, 0.67. When the Hodrick–Prescott filter [Hodrick and Prescott (1997), hereafter HP filter] is applied to the data with a smoothing parameter $\lambda = 1,600$, as seen in Figure 1, the trend components defined by the HP filter are alike, although the correlation coefficient of the two processes is a mere 0.42. The trend components are, by construction of the HP filter, persistent: the autocorrelation for the U.S. trend is 0.989, whereas that for Europe is 0.996. The cyclical components in the two series

TABLE 2. Productivity and decomposition by the HP filter

	Growth rate (data)		HP trend			HP cycle	
	Mean	S.D.	S.D.	Correlation	Autocorrelation	S.D.	Correlation
U.S.	0.38	0.74	0.017	—	0.989	0.69	—
EU	0.35	0.67	0.027	0.42	0.996	0.59	0.19

Notes: (1) "S.D." stands for the standard deviation. (2) The numbers in the column under "Autocorrelation" are the autocorrelation coefficients of the HP trend.

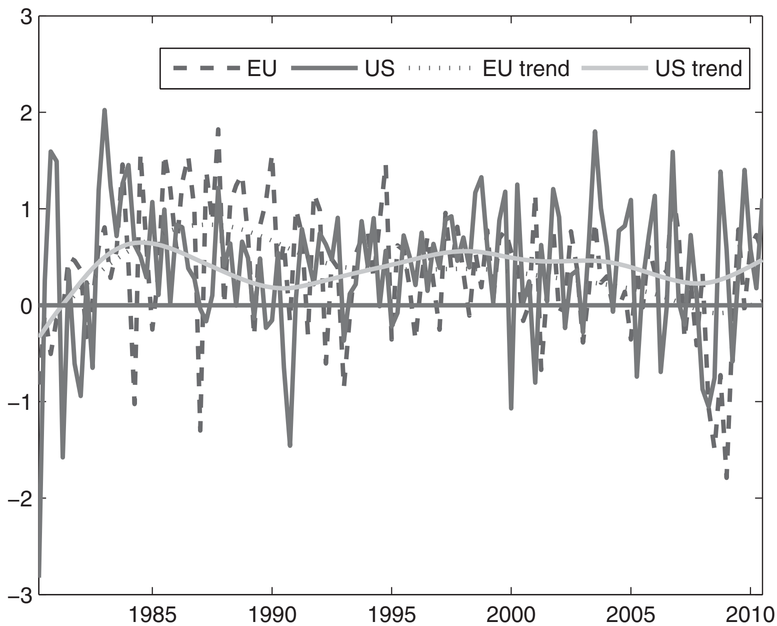

FIGURE 1. Growth rates of productivity (1980.I–2010.III) and the HP trend: the United States (solid line) and Europe (dashed line).

are also of the same magnitude, and they are positively correlated. Hence, by applying the HP filter, we detect similar persistent components in both series, and it may be a common component.

Instead of applying the HP filter for the decomposition, one can utilize a factor analysis to see if there is a common factor in the data. Roughly speaking, the factor model allows us to decompose the multivariate data into common factors and idiosyncratic shocks. Although the number of common factors is unknown a priori, Bai and Ng (2002) propose information criteria that consistently estimate the number of (common) factors in the data.[6] As Table 3 reports, we find one

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TABLE 3. Approximate factor model: $X_t = \Lambda F_t + e_t$

	Number of common factors	
Criterion	0	1
IC_{p1}	4.78	4.15
IC_{p2}	4.78	4.16

Notes: (1) X, Λ, F, and e are the vector of productivity growth rates, the factor loading matrix, the common factor, and the vector of idiosyncratic shocks, respectively. (2) Idiosyncratic shocks are allowed to be mutually correlated. (3) "IC_{p1}" and "IC_{p2}" are the information criteria that can consistently estimate the number(s) of factors. The number of factors that minimizes these criteria is a consistent estimate of the number of factors.

factor in the growth rate of productivity for the United States and Europe using Bai and Ng (2002) information criteria.

Having confirmed a common factor in the productivity growth rates of the two countries, one may question whether there is a common trend in the levels of the two countries' productivity. In other words, there is a possibility that the levels of productivity are cointegrated. To conduct a popular residual-based cointegration test defined by Phillips and Ouliaris (1990), the OLS estimates of the log levels of productivity ($\ln x_t^{US}$ and $\ln x_t^{EU}$ for the United States and Europe, respectively) and the first-order autoregressive (AR1) estimates of the residual are given as

$$\ln x_t^{US} = \underset{(0.031)}{1.169} + \underset{(0.024)}{0.900} \ln x_t^{EU} + \widehat{e}_t,$$

$$\widehat{e}_t = \underset{(0.019)}{0.998}\widehat{e}_{t-1} + \widehat{u}_t,$$

where standard errors are provided in parentheses.

When the long-run variance is estimated by the autoregressive spectral density with the length of lags being selected by the Akaike information criterion (AIC), the test statistics are $Z_t = -0.91$ and $Z_\rho = -2.98$, whereas the 5% critical values are -3.42 and -21.5, respectively, thereby failing to reject the null hypothesis of no cointegration.[7]

Specification. Following the observations described in the preceding subsection, we assume that the levels of technology in Home and Foreign evolve as follows:[8]

$$\frac{X_t}{X_{t-1}} = \gamma_t = \mu_t \varepsilon_t; \qquad \frac{X_t^*}{X_{t-1}{}^*} = \gamma_t^* = \mu_t \varepsilon_t^*,$$

where

$$\frac{\mu_t}{\mu} = \left(\frac{\mu_{t-1}}{\mu}\right)^{\rho} v_t;$$

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μ_t is the persistent component of the growth rate γ_t; μ is the steady-state level of μ_t; and $|\rho| \leq 1$, or equivalently, with the definition $d_t = \mu_t/\mu$,

$$d_t = d_{t-1}^{\rho} v_t.$$

The natural logarithms of shocks (denoted by $\widetilde{\varepsilon}_t$, $\widetilde{\varepsilon}_t^*$, and $\widetilde{v}_t$) are i.i.d. (independently and identically distributed):[9]

$$\widetilde{v}_t \sim \text{i.i.d.}\,(0, Q)\,; \quad \begin{bmatrix} \widetilde{\varepsilon}_t \\ \widetilde{\varepsilon}_t^* \end{bmatrix} \sim \text{i.i.d.}\,(0, \mathrm{R}),$$

where Q is a positive scalar and R is a (not necessarily diagonal) positive definite matrix. Note that persistent shocks are domestically and internationally uncorrelated with transitory shocks, whereas persistent shocks in two countries, as well as transitory shocks in both countries, may be correlated.

The (log-linearized) shock process is then described as

$$\begin{bmatrix} \widetilde{\gamma}_t \\ \widetilde{\gamma}_t^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \widetilde{d}_t + \begin{bmatrix} \widetilde{\varepsilon}_t \\ \widetilde{\varepsilon}_t^* \end{bmatrix}, \qquad \textbf{(2)}$$
$$\widetilde{d}_t = \rho \widetilde{d}_{t-1} + \widetilde{v}_t,$$

where $\widetilde{\gamma}_t$ and $\widetilde{\gamma}_t^*$ are the mean-subtracted growth rates of technology (in terms of percentage) in the United States and Europe, respectively.

It is important to keep in mind that this bivariate system of the shock process can deal with three types of shocks: purely transitory ($\rho = 0$), persistent ($0 < \rho < 1$), and permanent ($\rho = 1$). In the literature of structural changes, an outlier in the observed variable γ_t (hence, a level shift in the level of technology) is modeled in a way that is caused by a large and infrequent value of $\widetilde{\varepsilon}_t$, whereas a level shift in γ_t (hence, a change in the slope of the technology level's trend) is caused by a permanent ($\rho = 1$), large, and infrequent value of $\widetilde{v}_t$.[10]

Given the fact that macrovariables are growing at the rate γ_t for Home and γ_t^* for Foreign, we transform the variables as follows:

$$y_t = \frac{Y_t}{X_t}, \quad k_t = \frac{K_t}{X_{t-1}}, \quad i_t = \frac{I_t}{X_t}, \quad c_t = \frac{C_t}{X_t}, \quad b_t = \frac{B_t}{X_{t-1}}.$$

because the levels of capital stock and bond holdings are determined one period in advance, they are divided by the level of technology in the previous period.

3.3. Solving the Model

As pointed out by Schmitt-Grohé and Uribe (2003) and Boileau and Normandin (2008), among others, a two-country model with an incomplete asset market structure creates a problem, which is that the (deterministic) steady state is not

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unique (i.e., indeterminacy). Given that we analyze the model with changing technological growth rates based upon the dynamics of linearized equations around the deterministic steady state, indeterminacy would undermine the plausibility of the exercise, because it forces us to choose one steady state over many other possible steady states arbitrarily. To avoid this problem, we employ an Uzawa (1968)-type endogenous discount factor,

$$\vartheta_{t+1} = \beta\left(c_t, L_t\right)\vartheta_t,$$
$$\beta\left(c_t, L_t\right) = \left[1 + c_t^{\theta} L_t^{1-\theta}\right]^{-\zeta},$$

with $\vartheta_0 = 1$ and $\zeta \geq 0$. With this endogenous discount factor, households in the two countries choose ϑ_{t+1}, in addition to other controllable variables, c_t, L_t, N_t, b_{t+1}, i_t, and k_{t+1}. The Home household's maximization problem is then

$$\begin{aligned}\mathcal{L} = \max E_0 \sum_{t=0}^{\infty}\Bigg[\vartheta_t\Bigg(& u\left(c_t, L_t\right) + w_t\left(1 - L_t - N_t\right)\\ &+ \lambda_t\left\{(1-\delta)k_t - \left[\gamma_t k_{t+1} - \phi\left(\gamma_t \frac{i_t}{k_t}\right)k_t\right]\right\}\\ &+ p_t\left[F\left(k_t, N_t\right) + \frac{b_t}{\gamma_t} - P_t^B b_{t+1} - c_t - i_t\right]\Bigg)\\ &+ q_t[\vartheta_{t+1} - \beta(c_t, L_t)\vartheta_t]\Bigg],\end{aligned}$$

subject to the transversality condition $\lim_{t\to\infty}\vartheta_t p_t b_{t+1} = 0$; w_t, λ_t, p_t, and q_t are Lagrange multipliers (shadow prices of leisure, existing capital, output and the discount factor, respectively).

The Euler equation associated with this maximization problem is

$$\begin{aligned}\beta\left(c_t, L_t\right)E_t\Bigg\{\frac{\lambda_{t+1}}{\lambda_t}\Bigg[&\phi\left(\gamma_{t+1}\frac{i_{t+1}}{k_{t+1}}\right) - \phi'\left(\gamma_{t+1}\frac{i_{t+1}}{k_{t+1}}\right)\gamma_{t+1}\frac{i_{t+1}}{k_{t+1}}\\ &+ (1-\delta) + \gamma_{t+1}\phi'\left(\gamma_{t+1}\frac{i_{t+1}}{k_{t+1}}\right)\frac{\partial F\left(k_{t+1}, N_{t+1}\right)}{\partial k}\Bigg]\Bigg\} = \gamma_t,\end{aligned}$$

implying that the marginal cost of not using capital at t equals the expected marginal benefit of increasing capital at $t+1$.

Households in the Foreign country solve their maximization problem, which is analogous to Home's. General equilibrium in this two-country model is given by the bond market clearing condition (1), together with the firs-order conditions and the transversality conditions.[11]

3.4. Parameter Values

Following the literature of the IRBC, and assuming quarterly series data, parameters are chosen as presented in Table 4. All parameters except capital's share, α,

TABLE 4. Calibration parameters

Description	Parameter	Value
Capital's share	α	0.36
Steady-state discount factor	β	0.99
(Nonstochastic) steady-state growth rate	γ	1.004
Depreciation rate	δ	0.025
Relative risk aversion	σ	2
Relative size of Home	π	0.5
Interest rate (annual)	r	0.065
Elasticity of adjustment cost with respect to the investment-capital ratio	$-[(\gamma i/k)\phi''(\gamma i/k)]$ $\phi'(\gamma i/k)$	1/15
(Nonstochastic) steady-state labor input	N	0.2
Variance ratio, defined as Q/variance $(\widetilde{\varepsilon}_t)$	Φ	0.07,0.006, or 0.001
Persistence	ρ	0.8 or 0.85
Correlation of transitory shocks	ρ_R	0, 0.2, 0.5, 0.8, 0.95, 0.99 or 1

are taken from Baxter and Crucini (1995). To be consistent with the measurement of productivity, which Backus et al. (1992) and Boileau and Normandin (2008) utilize, we set $\alpha = 0.36$.

4. LEARNING

Under imperfect information, agents in Home and Foreign observe $[\widetilde{\gamma}_t, \widetilde{\gamma}_t^*]$, which are the percentage deviations of the growth rate from its initial steady state. But they do not observe the orthogonal components, $\widetilde{d}_t$ and $[\widetilde{\varepsilon}_t, \widetilde{\varepsilon}_t^*]$, separately. Hence, the inference about the current state of the persistent component of technology growth is employed based on the observations

$$\widetilde{d}_{t|t} \equiv E[\widetilde{d}_t | \widetilde{\gamma}_t, \widetilde{\gamma}_t^*, \widetilde{\gamma}_{t-1}, \widetilde{\gamma}_{t-1}^* \ldots].$$

Similarly, the inference about the transitory shock is given by

$$\widetilde{\varepsilon}_{t|t} \equiv E[\widetilde{\varepsilon}_t | \widetilde{\gamma}_t, \widetilde{\gamma}_t^*, \widetilde{\gamma}_{t-1}, \widetilde{\gamma}_{t-1}^* \ldots], \quad \widetilde{\varepsilon}_{t|t}^* \equiv E[\widetilde{\varepsilon}_t^* | \widetilde{\gamma}_t, \widetilde{\gamma}_t^*, \widetilde{\gamma}_{t-1}, \widetilde{\gamma}_{t-1}^* \ldots].$$

Assume that agents update inferences based on the steady-state Kalman filter. Then vectors $x_{t|t} = \widetilde{d}_{t|t}$ and $e_{t|t} = [\widetilde{\varepsilon}_{t|t}\ \widetilde{\varepsilon}^*_{t|t}]'$ are given by

$$x_{t|t} = (I - KH)\,Fx_{t-1|t-1} + Kz_t,$$
$$e_{t|t} = z_t - Hx_{t|t},$$

where

$$F = \rho,\ z_t = \begin{bmatrix} \widetilde{\gamma}_t \\ \widetilde{\gamma}^*_t \end{bmatrix},$$

K is the steady-state Kalman gain, and H is a coefficient matrix (vector) that will be explained later. In contrast to the models of one country (in which the steady-state Kalman gain is scalar, and hence readily computed), our model requires a 1×2 matrix as the Kalman gain. Natural questions to be asked are whether the steady-state Kalman gain is unique and stable; and if so, whether it can be computed analytically. As is shown in the Appendix, the answer to both questions is affirmative.

To choose the values for the parameters $\Phi \equiv Q/\text{variance}\,(\widetilde{\varepsilon}_t)$, ρ and R, we estimate the process of technological growth, (2), assuming that the shocks $\widetilde{\varepsilon}_t, \widetilde{\varepsilon}^*_t$, and $\widetilde{\upsilon}_t$ are normally distributed. The method of maximum likelihood is utilized, and the log-likelihood function is computed by Kalman filtering:

$$\begin{bmatrix} \widetilde{\gamma}_t \\ \widetilde{\gamma}^*_t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \widetilde{d}_t + \begin{bmatrix} \widetilde{\varepsilon}_t \\ \widetilde{\varepsilon}^*_t \end{bmatrix},$$
$$\widetilde{d}_t = \underset{(0.189)}{0.772}\widetilde{d}_{t-1} + \widetilde{\upsilon}_t, \tag{3}$$
$$\widehat{R} = \begin{bmatrix} \underset{(0.085)}{1.092^2} & \underset{(0.128)}{0.103} \\ 0.103 & \underset{(0.079)}{0.934^2} \end{bmatrix}, \quad \widehat{Q} = \underset{(0.152)}{0.266^2},\ \widehat{\Phi} = \underset{(0.084)}{0.069},$$

where standard errors (obtained by the delta method) are provided in parentheses; $\widehat{\Phi} \equiv \widehat{Q}/[\text{trace}\left(\widehat{R}\right)/2]$ and $H = [1\ \ 1]'$.

Table 5 summarizes the estimated results.[12] Based on the estimates, we set $\Phi = 0.07$, $\rho = 0.8$, and

$$R = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}.$$

5. THE IMPULSE RESPONSES

5.1. A Transitory Shock

Figure 2 shows the effects of a transitory shock under perfect information (dashed lines) and imperfect information (solid lines). The deviations are in terms of percentages. It is important to recognize that the effects of a transitory shock under

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TABLE 5. Estimation of productivity growth rate model:

$$\begin{bmatrix} \widetilde{\gamma}_t \\ \widetilde{\gamma}_t^* \end{bmatrix} = \begin{bmatrix} 1 \\ b \end{bmatrix} \widetilde{d}_t + \begin{bmatrix} \widetilde{\varepsilon}_t \\ \widetilde{\varepsilon}_t^* \end{bmatrix}$$

$$\widetilde{d}_t = \rho \widetilde{d}_{t-1} + \widetilde{v}_t$$

	Unrestricted model		Restricted model	
	Estimate	SE	Estimate	SE
b	3.0948	3.3035	1	NA
ρ	0.8625	0.1380	0.7715	0.1891
σ_ε	1.1345	0.0755	1.0916	0.0848
σ_{ε^*}	0.9183	0.0811	0.9340	0.0788
$\sigma_{\varepsilon\varepsilon^*}$	0.1902	0.1267	0.1033	0.1283
σ_v	0.0798	0.1135	0.2661	0.1520
Implied Φ	0.0060	0.0173	0.0686	0.0838
LL	−360.0056		−361.3665	

Notes: (1) Entries below SE are standard errors that are computed by the delta method. (2) LL stands for the value of the log-likelihood. (3) The likelihood ratio test statistic is 2.7218 and the 10% critical value for χ^2 with degrees of freedom of 1 is 2.71.

perfect information are the same as the effects of a permanent shock to the level of technology, which is analyzed in Baxter and Crucini (1995). A positive transitory (one-time) shock to the growth rate of Home technology raises consumption in Home, but lowers consumption in Foreign. When households in both countries know that the shock is transitory, Home output increases whereas Foreign output slightly decreases, after an initial rise for a short period of time. The latter occurs because labor input in Foreign increases on impact, as explained in Baxter and Crucini (1995),[13] because of the incompleteness of the market structure (i.e., this economy has only riskless bonds). The Home household provides less labor response to the shock (because an increase in the growth rate of technology causes a large wealth effect), which results in the Home household's choice of more leisure. The question is: Why does the household reduce labor input more under imperfect information than under perfect information? Similar questions can be raised regarding the Foreign investment responses. Why is Foreign investment of greater magnitude under imperfect information than under perfect information, when Foreign consumption and output seem to respond less under imperfect information? This is explained by the following argument. Imperfectly informed households consider two possibilities: (i) a positive transitory shock hits Home; (ii) a common shock hits both, but Foreign is hit by a negative transitory shock as well. Upon observing positive technological growth in Home with no change in Foreign,

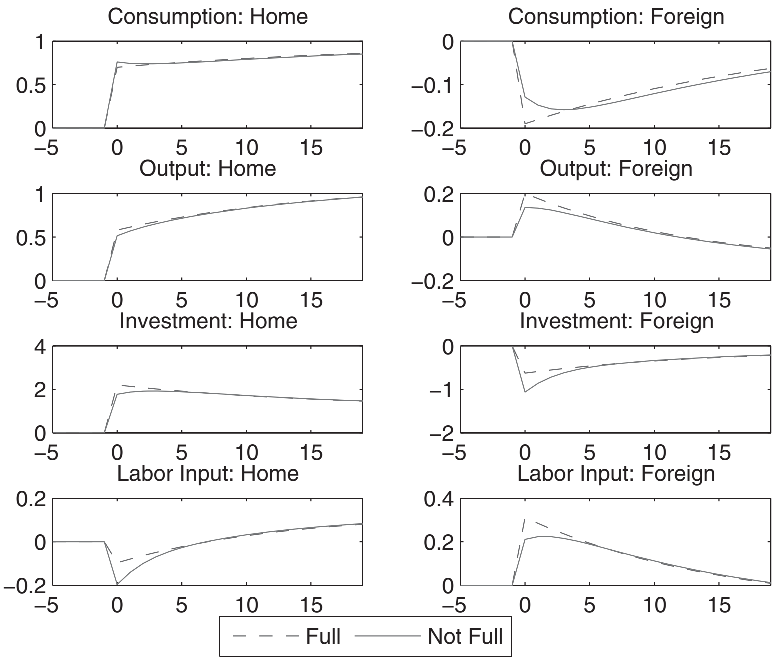

FIGURE 2. Responses (percent deviations) to a transitory shock: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.07$, $\rho_R = 0.1$.

and knowing that the transitory shocks are not very highly correlated across two countries, households think both possibilities are likely. If (ii) is considered to be true, as we will see in the next subsection, Home households reduce their labor supply because of the larger wealth effect, whereas Foreign households decrease investment because of the presumed negative transitory shock and positive persistent shock.[14] Figure 3 illustrates the Hicksian decomposition of Home's labor response to a transitory shock to Home. Comparing the interest rate, wage, and wealth effects, one can realize that the largest difference in the responses of labor input between perfect and imperfect information is attributed to the wealth effect.

It is, however, very uncertain whether imperfect information under the current ρ_R may be a solution to the international correlation puzzle per se. From Figure 2, we observe that for some variables, a transitory shock can widen the difference between oppositely moving responses of Home and Foreign.

5.2. A Common Persistent Shock

Figure 4 illustrates the responses to a persistent shock.[15] Facing a common persistent shock, households under perfect information in both Home and Foreign increase their consumption. They also increase leisure a great deal (hence the

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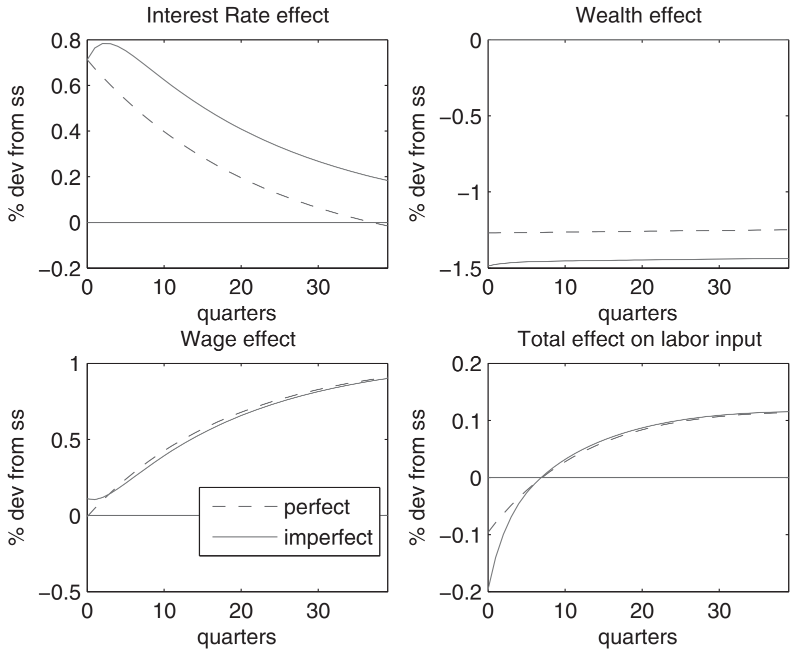

FIGURE 3. Hicksian decomposition of Home's labor response to a transitory shock to Home: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.07$, $\rho_R = 0.1$.

decrease in labor input) because of the wealth effect stemming from a persistent and large positive shock. Households without perfect information, however, do not increase their leisure. This can be understood by the following argument: Seeing the technological growth rates in both countries go up, households are unable to know immediately whether two transitory shocks are affecting two countries simultaneously or a common persistent shock is hitting the two countries. If the former is the case, households do not increase their leisure, because the wealth effect is not great enough.[16] Instead, in the latter case, a large wealth effect makes households decrease their labor input, at least for the first several periods after the shock.

Figures 5 and 6, which show the Hicksian decomposition of the labor responses, affirm this argument. First, Figure 5 reveals that the disparity in the responses under different information structures on the impact of a persistent shock is mainly due to the wealth effect. Both the interest rate and wage effects, on the other hand, contribute only slowly to the disparity in the responses under different information structures, but those effects do not create significant differences in the responses to the persistent shock on impact. Hence, immediately after the persistent shock, imperfectly informed households do not decrease their labor supply. But why is the wealth effect on the imperfectly informed households smaller (in absolute value)? The second observation provides an answer to this question. Recall that

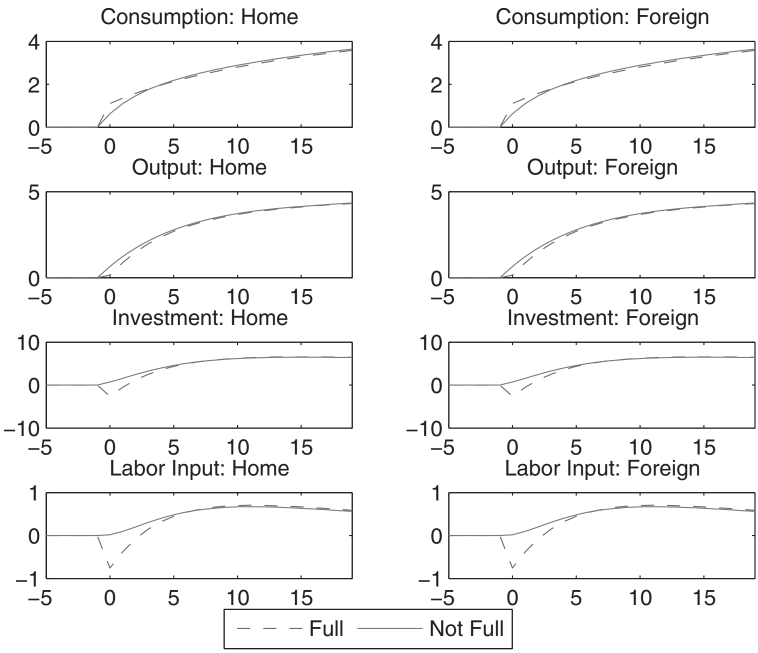

FIGURE 4. Responses (percent deviations) to a persistent shock that affects the two countries identically: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.07$, $\rho_R = 0.1$.

the imperfectly informed households do not know whether two transitory shocks hit the two countries simultaneously or a common persistent shock hits the two countries. Figure 6 compares the Hickisian decomposition of perfectly informed Home households' labor response in these two cases. It is clear that for perfectly informed households, the wealth effect generated by a common persistent shock is greater than that generated by the two transitory shocks. Therefore, we conclude that when they do not know the type of shock, households' response to a persistent shock falls between the responses to known simultaneous transitory shocks and the responses to a known persistent shock. Using the same argument, one can answer why the responses of not-fully-informed households to a transitory shock are shown to be of different magnitudes than those of fully informed households.

6. COMOVEMENTS OF MACROVARIABLES WHEN A PERSISTENT SHOCK HITS TWO COUNTRIES DIFFERENTLY

6.1. Setup

It is not surprising that a common persistent shock and mutually correlated transitory shocks generate comovements in macrovariables. But what if a persistent shock affects two countries differently? Does the comovement still appear?

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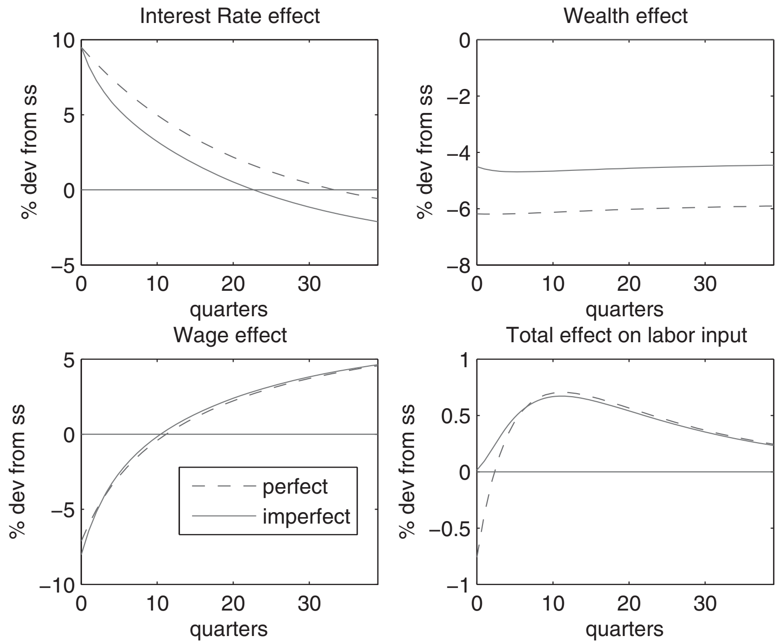

FIGURE 5. Hicksian decomposition of Home's labor response to a common persistent shock: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.07$, $\rho_R = 0.1$.

An important fact is that the process of technological growth, (2), imposes a restriction such that the common persistent shock affects the two countries identically, because the coefficient of the persistent component $\widetilde{d}_t$ is unity for both countries. One of the interesting features of our specification of technological growth is that we can estimate one of the coefficients of $\widetilde{d}_t$ (i.e., one element in H) without encountering an identification problem.[17] In fact (as shown in Table 5), by allowing one of the coefficients to be a free parameter to be estimated, we obtain

$$\begin{bmatrix} \widetilde{\gamma}_t \\ \widetilde{\gamma}_t^* \end{bmatrix} = \begin{bmatrix} 1 \\ \underset{(3.303)}{3.095} \end{bmatrix} \widetilde{d}_t + \begin{bmatrix} \widetilde{\varepsilon}_t \\ \widetilde{\varepsilon}_t^* \end{bmatrix},$$

$$\widetilde{d}_t = \underset{(0.138)}{0.862} \widetilde{d}_{t-1} + \widetilde{v}_t, \tag{4}$$

$$\widehat{R} = \begin{bmatrix} \underset{(0.076)}{1.134^2} & \underset{(0.127)}{0.190} \\ 0.190 & \underset{(0.081)}{0.918^2} \end{bmatrix}, \quad \widehat{Q} = \underset{(0.114)}{0.080^2}, \quad \widehat{\Phi} = \underset{(0.017)}{0.006}.$$

Noticing that (4) is an unrestricted version of (2), we conduct the likelihood ratio test in order to determine whether imposing the restriction of identical coefficients on $\widetilde{d}_t$ is justified. The test statistic is 2.72, whereas the 10% critical value of χ^2

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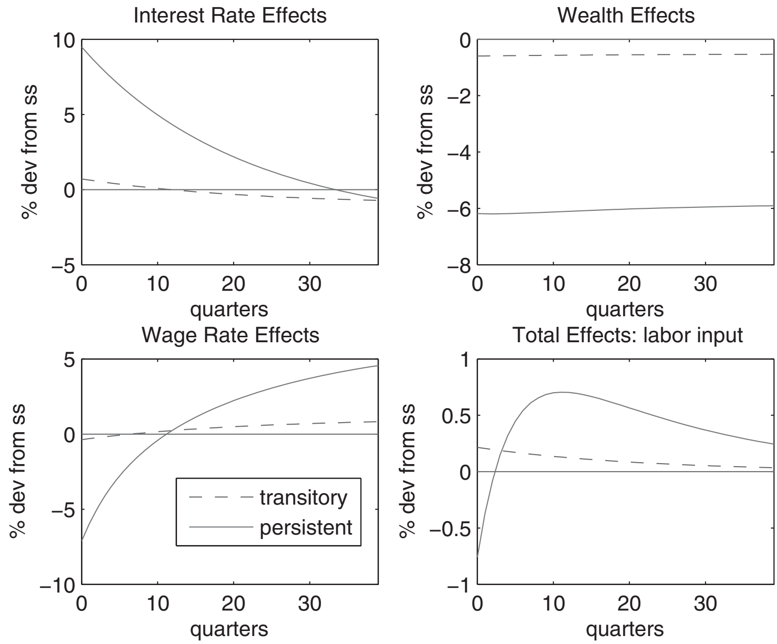

FIGURE 6. Hicksian decomposition of perfectly informed Home households' labor response to (a) simultaneous transitory shocks to Home and Foreign (dashed lines); and (b) a common persistent shock (solid lines). $\Phi = 0.07$, $\rho_R = 0.1$.

with one degree of freedom is 2.71. The null hypothesis of identical coefficients is therefore rejected at a 10% level. Because rejection at the 10% level is not strong evidence against the null hypothesis of "identical coefficients," we consider both the identical and the nonidentical coefficients to be worth exploring, although we deem the latter somewhat more plausible than the former. Even in this case—the persistent component does not affect the two countries' technological growth rate identically—our model, incorporated with the learning mechanism, can explain how the comovements are generated. Let $\rho = 0.85$, $H = [\,1\ 3.1\,]'$, $\Phi = 0.006$, and $\rho_R = 0.2$; we can compute impulse responses in the same way as we did in the previous section. In order to reduce puzzling features in international comovements, we argue in this section that two parameters, Φ and ρ_R, are essential elements, together with the assumption of imperfectly informed households.

6.2. Persistent Shocks

As seen in Figure 7, a persistent shock produces responses different from those in the previous case, where a common persistent shock affects the two countries identically. Because a persistent shock greatly increases the growth rate of

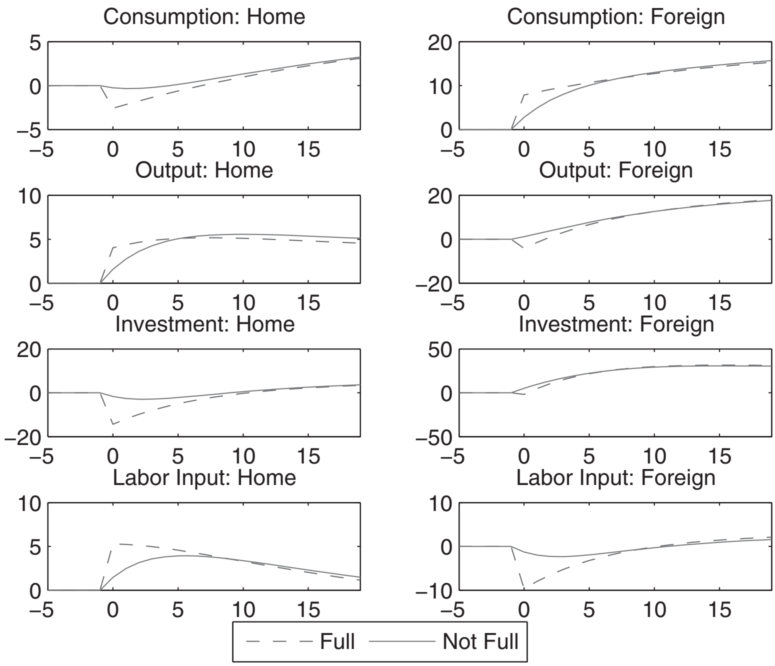

FIGURE 7. Responses (percent deviations) to a persistent shock that affects the two countries differently: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.006$, $\rho_R = 0.2$.

Foreign relative to the growth rate of Home—as if a positive persistent shock hit only Foreign—responses to such a shock tend to create countermovements of macrovariables, at least for a short period of time.

Remarkably, despite a persistent positive shock, output decreases in the Foreign country under perfect information. This is because a strong technology shock relative to Home allows Foreign households to consume more goods without working more, thereby decreasing Foreign output for a while. On the other hand, households with imperfect information respond very differently. Because they do not know the persistence of the shock, the wealth effect on their labor supply is limited: for the first several periods, Foreign households do not decrease their labor supply as much as they would under perfect information, and Home households do not increase their labor supply much. Consequently, a negative cross-country labor supply correlation is largely alleviated. Such different responses of labor inputs under imperfect information also moderate negative cross-country correlations in outputs, consumption, and investment. This implies that the learning mechanism helps us understand the comovements in macrovariables, even though persistent shocks affect the two countries differently.

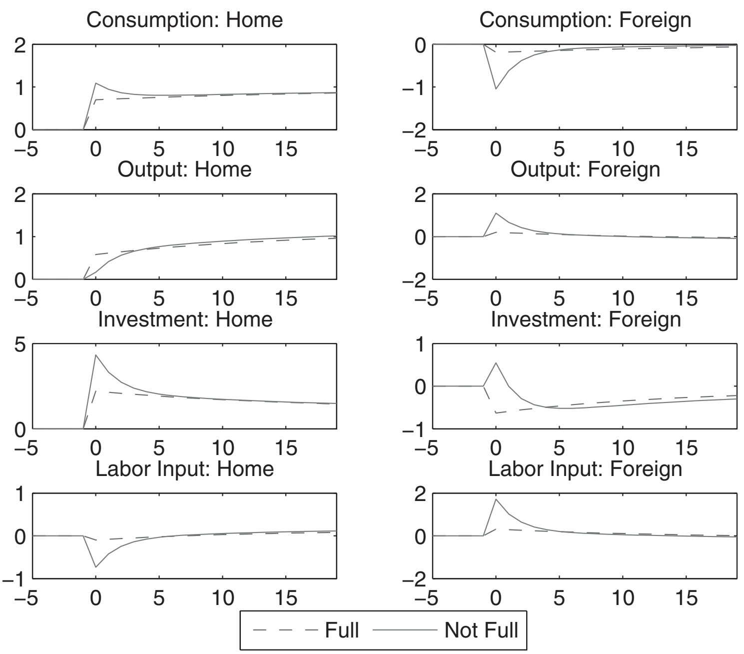

FIGURE 8. Responses (percent deviations) to a transitory shock that affects the two countries differently: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.006$, $\rho_R = 0.95$.

6.3. Transitory Shocks and Their International Correlation

When shocks are generated by the set of parameters in the previous subsection, a perfectly informed household's responses to a transitory shock are pretty much the same as those in Figure 2. In this case, a transitory shock creates very little difference between perfectly informed households' responses and imperfectly informed households' responses. Is there any role for ρ_R in widening or narrowing the difference? One example is the following. Suppose now that ρ_R is very high, say, $\rho_R = 0.95$, and that a transitory shock affects only Home's technology. Seeing an increase in technological growth only in Home, households with imperfect information can consider two scenarios: (1) a transitory shock increases Home technological growth; or (2) despite a persistent shock that decreases both Home and Foreign technological growth rates to different degrees, two positive transitory shocks—possibly internationally correlated—increase both Home and Foreign growth rates of technology. For those imperfectly informed households, the first scenario (i.e., the truth) is very unlikely because the correlation of transitory shocks is very high, which means that a positive transitory shock to Home should be almost always accompanied by a positive transitory shock to Foreign. In this case, a high ρ_R widens the gap between the perfectly informed households' response and the

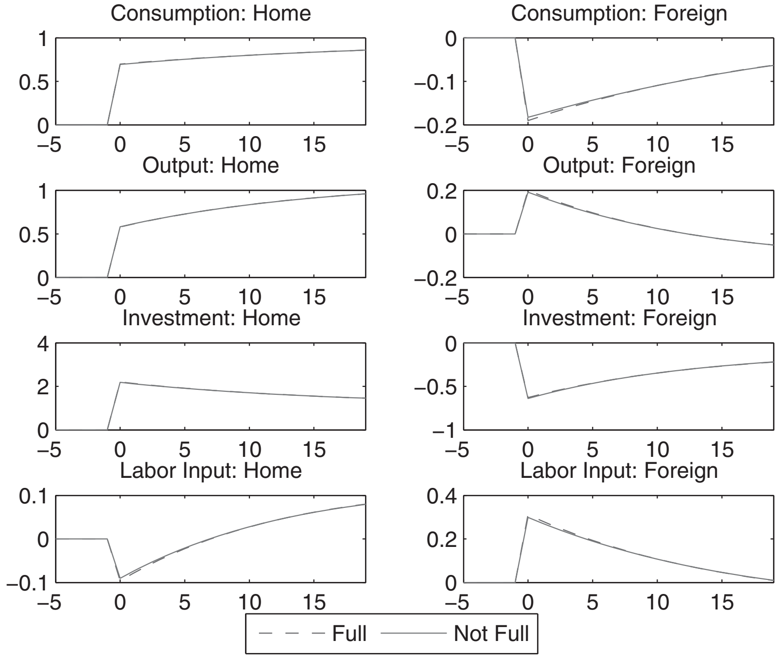

FIGURE 9. Responses (percent deviations) to a transitory shock that affects the two countries differently: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.001$, $\rho_R = 0.2$.

imperfectly informed households' response to a transitory shock. (See Figure 8 for the impulse responses when $\rho_R = 0.95$.) However, for two reasons, care must be taken in regard to the preceding discussion. First, because a high ρ_R means that transitory shocks are likely to hit the two countries simultaneously, the response to a single transitory shock only hitting one country may not be informative.[18] Second, even with a low ρ_R, imperfectly informed households may be confused, for the same reason discussed in Section 5.1. Therefore, how ρ_R alters the importance of learning should become clear after we conduct a simulation study.

Whether or not households are perfectly informed, a high correlation between transitory shocks is expected to enhance the international comovements of the macrovariables. This is because such highly correlated transitory shocks can offset the oppositely moving responses created by the persistent shock that affects the two countries differently.

6.4. Signal-to-Noise Ratio

Decreasing the variance ratio, Φ, makes households believe that most of the shocks are transitory. Intuitively, this is because most variations in technological growth are attributed to transitory shocks, so that households without perfect information

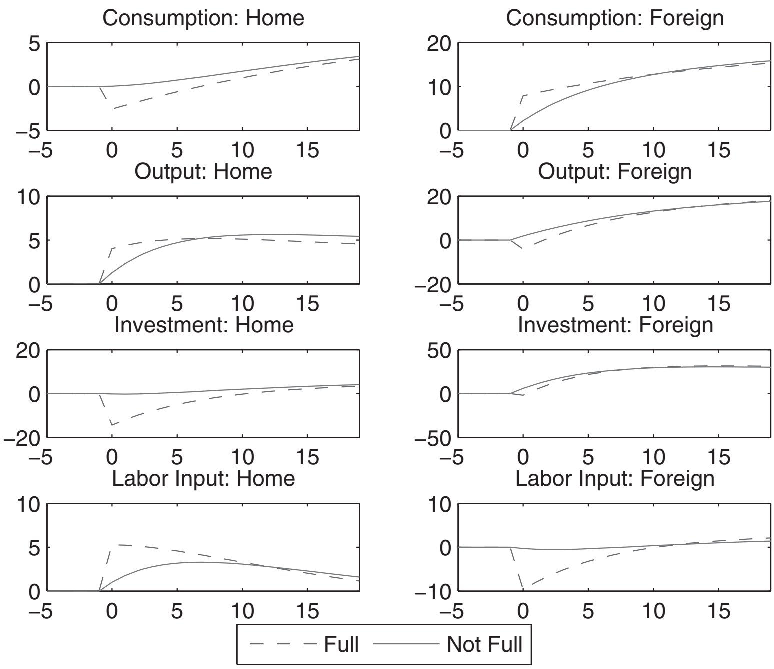

FIGURE 10. Responses (percent deviations) to a persistent shock that affects the two countries differently: Imperfect information (solid lines) and perfect information (dashed lines). $\Phi = 0.001$, $\rho_R = 0.2$.

consider most of the shocks to be transitory. Hence, if the actual shock is indeed transitory, this change negates the role of learning. However, if the shock is persistent, the importance of learning is strengthened, especially for the variables that respond in opposite directions to a transitory shock under perfect information, i.e., consumption, investment, and labor input. This phenomenon can be observed in Figures 9 and 10, which use the same set of parameters as Figure 7, except for $\Phi = 0.001$, instead of 0.006 in Figure 7. Because persistent shocks tend to generate international comovements, we may expect that a lower Φ is a solution to the puzzles. Yet, on the other hand, a lower Φ itself indicates very small persistent shocks; therefore, we need to advance to the following simulation section in order to see whether the puzzles can be solved.

7. SIMULATIONS

Table 6 reports the output, consumption, investment, and labor input correlations across the two countries, together with relative volatilities of those variables.[19] We compute correlation coefficients for (i) the international correlations in transitory shocks, $\rho_R = 0$, 0.2, 0.5, 0.8, 0.95, 0.99, and 1, and (ii) the persistent shocks

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TABLE 6. Simulation with shocks

	Perfect information							Imperfect information						
ρ_R	0	0.2	0.5	0.8	0.95	0.99	1.00	0	0.2	0.5	0.8	0.95	0.99	1.00
(a) $b = 1$, $\Phi = 0.07$														
$\rho\,(y, y^*)$	0.71	0.78	0.87	0.95	0.99	1.00	1.00	0.67	0.75	0.86	0.95	0.99	1.00	1.00
$\rho\,(c, c^*)$	−0.18	−0.02	0.27	0.65	0.91	0.98	1.00	−0.11	0.04	0.32	0.69	0.91	0.98	1.00
$\rho\,(i, i^*)$	−0.12	0.03	0.31	0.67	0.91	0.98	1.00	−0.42	−0.29	0.00	0.48	0.84	0.97	1.00
$\rho\,(n, n^*)$	0.00	0.13	0.38	0.71	0.92	0.98	1.00	−0.53	−0.44	−0.20	0.29	0.77	0.95	1.00
σ_c/σ_y	0.98	0.92	0.82	0.73	0.69	0.68	0.67	1.07	1.01	0.91	0.82	0.77	0.76	0.75
σ_i/σ_y	3.22	3.00	2.70	2.40	2.26	2.22	2.21	3.02	2.76	2.38	2.01	1.82	1.77	1.76
σ_n/σ_y	0.51	0.47	0.43	0.38	0.36	0.36	0.35	0.43	0.39	0.32	0.26	0.22	0.21	0.21
(b) $b = 3.1$, $\Phi = 0.006$														
$\rho\,(y, y^*)$	0.26	0.36	0.49	0.59	0.64	0.65	0.66	0.46	0.58	0.72	0.79	0.75	0.69	0.66
$\rho\,(c, c^*)$	−0.49	−0.38	−0.17	0.11	0.28	0.34	0.35	−0.46	−0.35	−0.14	0.13	0.29	0.34	0.35
$\rho\,(i, i^*)$	−0.34	−0.21	0.05	0.38	0.60	0.67	0.69	−0.62	−0.50	−0.24	0.18	0.51	0.64	0.69
$\rho\,(n, n^*)$	−0.79	−0.77	−0.73	−0.68	−0.65	−0.65	−0.65	−0.89	−0.86	−0.80	−0.71	−0.66	−0.64	−0.65
σ_c/σ_y	1.05	0.97	0.85	0.74	0.68	0.67	0.66	0.98	0.90	0.79	0.69	0.65	0.65	0.66
σ_i/σ_y	3.51	3.23	2.85	2.49	2.31	2.27	2.26	3.15	2.88	2.52	2.21	2.15	2.20	2.26
σ_n/σ_y	0.77	0.72	0.66	0.61	0.59	0.58	0.58	0.74	0.70	0.64	0.60	0.58	0.58	0.58
(c) $b = 3.1$, $\Phi = 0.001$														
$\rho\,(y, y^*)$	0.44	0.57	0.72	0.85	0.90	0.92	0.92	0.52	0.65	0.81	0.93	0.97	0.95	0.92
$\rho\,(c, c^*)$	−0.49	−0.34	−0.04	0.40	0.70	0.79	0.82	−0.48	−0.32	−0.03	0.40	0.69	0.79	0.82
$\rho\,(i, i^*)$	−0.49	−0.33	−0.02	0.45	0.78	0.89	0.92	−0.57	−0.41	−0.10	0.39	0.75	0.88	0.92
$\rho\,(n, n^*)$	−0.66	−0.58	−0.42	−0.21	−0.07	−0.03	−0.02	−0.72	−0.61	−0.40	−0.09	0.07	0.05	−0.02
σ_c/σ_y	1.13	1.03	0.88	0.75	0.69	0.67	0.66	1.11	1.01	0.86	0.73	0.67	0.66	0.66
σ_i/σ_y	3.59	3.25	2.79	2.35	2.14	2.08	2.07	3.48	3.15	2.69	2.26	2.07	2.04	2.07
σ_n/σ_y	0.59	0.54	0.46	0.40	0.37	0.36	0.36	0.58	0.53	0.46	0.40	0.37	0.36	0.36

Notes: (1) $\rho\,(x, x^*)$ stands for the cross-country correlation of variable x. (2)σ_x/σ_y is the standard deviation of x relative to that of y (output). (3) Each entry of the table is a mean of the simulation of the length of the data T=250 with 1,000 replications.

that affect the two countries identically and differently. The HP filter with the smoothing parameter of 1600 is used for all the generated series.

As seen in the preceding section, when the persistent shock affects two countries identically, consumptions, investments, and labor inputs in the two countries are negatively or very weakly correlated, for both perfect and imperfect information cases. A general tendency, however, is that higher cross-country correlations in macrovariables are generated by higher international correlations in transitory shocks (higher ρ_R). As is naturally conjectured from Baxter and Crucini (1995) and the previous section in this paper, there is no consumption-correlation puzzle for all values of ρ_R, regardless of the information structure. Except for small differences in the correlations of labor inputs (for which perfect information does a slightly better job of explaining the labor input comovement), there is no major difference between the perfect and imperfect information cases.

To explain the international comovement puzzle, however, it seems that a high correlation of transitory shocks is necessary.[20] As the second row in Table 6a shows, a high ρ_R results in higher cross-country correlations for these variables.

Is there any benefit in assuming imperfect information? Let us consider the case where the persistent shocks affect the two countries differently, as the likelihood ratio test indicates. As shown in Table 6b, both perfect and imperfect information fail to replicate positive cross-country correlations in labor input. Despite the fact that cross-country output correlation is always higher under imperfect information than under perfect information, imperfect information does not generate major differences. This is consistent with our discussion in the previous section: whereas learning plays an important role when ρ_R is extremely high and when one transitory shock affects only Home, this situation is quite implausible. In addition to a higher cross-country output correlation, a significant role of imperfect information can be found when we decrease the value of the variance ratio, Φ.

As Table 5 shows, the standard deviation of $\widetilde{v}_t$ is very imprecisely estimated. In other words, the variance ratio, Φ, may be smaller than 0.006.[21] Table 6c assumes exactly the same set of parameters as Table 6b, except for $\Phi = 0.001$, which in Table 6b is 0.006. Remarkably, with imperfect information, the simulation reported in Table 6c succeeds in generating positive cross-country correlations in labor input for higher values (but not 1[22]) of ρ_R, which perfect information fails to generate. As explained in the previous section, these results are obtained from the fact that decreasing the variance ratio makes imperfectly informed households believe that most shocks are transitory, thereby magnifying the difference between responses under perfect information and under imperfect information, if the true shock is persistent. Because persistent shocks create international comovements in macrovariables, particularly in consumption, investment, and labor input, Table 6c replicates positive international correlations, more clearly than in Table 6b.

How the international comovement puzzle can be eliminated is now apparent. A small Φ makes the role of learning substantial, especially when the shock is persistent. Then, the countermoving responses arising from the persistent shocks, which affect the two countries, can be offset by highly (but not perfectly) internationally correlated transitory shocks. In particular, when a persistent shock affects the two countries' technological growth rates differently, (i) a low signal-to-noise ratio Φ, such as 0.001, and (ii) a high international correlation in the transitory shocks ρ_R, such as 0.95, are necessary for the economy with imperfect information to eliminate the puzzle.

In summary, we find the following. (1) With persistent shocks that affect the two countries identically, there is no consumption-correlation puzzle, and the international comovement puzzle becomes imperceptible. (2) When persistent shocks affect the two countries differently, imperfect information plays an important role in reducing both the consumption-correlation puzzle (as imperfect information generates higher cross-country output correlations), and the international comovement puzzle (if transitory shocks are highly internationally correlated), provided Φ is low and ρ_R is high.

8. CONCLUDING REMARKS

A two-country model with persistent and transitory shocks to the growth rate of technology is studied. In particular, we consider the role of learning, which stems from the assumption that agents in the economy do not have adequate information about the persistence of the shock. Assuming that the learning process is given by the Kalman filter, we show that the steady state Kalman gain for the general process of shocks (including correlations) is unique, is stable, and can be readily computed.

After we estimate the process of technological growth rates for the United States and Europe in order to find reasonable sets of parameters, our simulations, as well as impulse-response figures, reveal two notable implications. First, the consumption-correlation puzzle disappears when the growth rate is allowed to change by a common persistent shock and transitory shocks. In this case, the international comovement puzzle almost disappears as well. Although this point may be seen as an obvious statement, given the shock process in the model, it should be noticed that the shock process is found in the data for the United States and the European aggregate, in the same spirit as Backus et al. (1992) and Boileau and Normandin (2008). Second, when the persistent shocks are not affecting two countries' growth rates in the same way, the learning mechanism helps us understand the higher cross-country output correlation relative to the cross-country consumption correlation. In addition, international comovements in macrovariables are explained by an imperfect information assumption, together with transitory shocks that are highly internationally correlated (much more highly than is observed in the data) and are relatively larger than persistent shocks.

NOTES

1. The significance of shocks to the growth rate in explaining the difference between IRBC models and observed data has been pointed out by many studies. For example, Rabanal et al. (2010), who assume that total factor productivity (TFP) for the United States and the rest of the world are

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co-integrated, succeed in resolving the anomaly that real exchange rates are inexplicably volatile. Aguiar and Gopinath (2007) emphasize a role of permanent shocks in emerging economies. Another recent study by Garcia-Cicco et al. (2010) finds that a small-open economy real business cycle model has limited empirical appeal for emerging economies, even though permanent shocks are assumed.

2. In this case, a transitory (one-time) shock to the *growth rate* corresponds to a permanent shock to the *level*. A permanent shock to the growth rate is often referred to as a structural change (break) in the slope of the trend. The effects on a closed economy are carefully analyzed by Pakko (2002).

3. Countries used to construct the European aggregate data are Austria, Finland, France, Germany (West Germany, before 1990), Italy, and the United Kingdom. See Appendix E for details.

4. In the first column, the correlation coefficients are taken from Backus et al. (1995). The coefficients in the second column are computed using data from 1980. I through 2010.III for the United States and Europe.

5. The data for productivity are constructed in the spirit of Backus et al. (1992), especially following Boileau and Normandin (2008). See Appendix E.

6. See Appendix D for details.

7. To get the long-run variance, we first run OLS:

$$\Delta\widehat{e}_t = \widehat{b}_0\widehat{e}_{t-1} + \sum_{i=1}^{k}\widehat{b}_i\,\Delta\widehat{e}_{t-k} + \widehat{u}_t,$$

and then the long-run variance is

$$s^2 = \frac{T^{-1}\sum_{t=k+1}^{T}\widehat{u}_t^2}{\left(1-\sum_{i=1}^{k}\widehat{b}_i\right)^2},$$

where k is the length of the lags, selected by the AIC.

8. To be consistent with our production function, the log level of technology is $\ln X_t = \ln x_t/(1-\alpha)$, where x_t is productivity as defined by Boileau and Normandin (2008).

9. The i.i.d. assumption is not well suited for structural changes, yet impulse responses are (by construction) well suited for the analysis of structural changes.

10. As Perron (1989) shows for the univariate case, it is difficult to distinguish between a stationary growth rate (i.e., a random walk in the level of technology) and a structural change (i.e., a very infrequent shock).

11. See Appendix B for more details.

12. The means of the technological growth rates are removed before the process is estimated. The growth rates are multiplied by 100 so that they are expressed as percentages.

13. By the Hicksian decomposition [King (1990)], Baxter and Crucini (1995) show that a positive interest rate effect, which raises the incentive to work more, dominates a negative wage effect, which discourages labor input with a low wage level. Thus, Foreign households provide more labor supply. For details of the Hicksian decomposition, see Online Appendix Section 8: http://www.clas.wayne.edu/multimedia/usercontent/File/Economics/wada/technical_revision_appendix.pdf.

14. As we will see in the next subsection, a persistent shock causes a decline in investment under the current parameter regarding the adjustment cost of investment. This occurs because large technological growth in the long run, which is associated with a persistent shock, does not require investment that would otherwise necessitate the adjustment cost.

15. A one-country model with a permanent change in the growth rate of technology, i.e., $\rho = 1$, together with an infrequent and large realization of shock v_t in our model, is studied in Pakko (2002).

16. Note that each country's labor response to the two transitory shocks affecting two countries simultaneously is different from the labor response to a transitory shock affecting only Home; the latter is the case analyzed in the previous subsection.

17. The Online Technical Appendix explains identifiability. In short, our model satisfies (i) minimum representation [i.e., observable and controllable; see p. 279 in Gourieroux and Monfort (1997)]

and (ii) invariance under multiplication of any nonsingular matrix, provided that one of the elements in H is normalized (to one).

18. It is still possible to know what would happen if the shocks are indeed highly correlated, and hit the two countries simultaneously. To do so, we consider an extreme case where transitory shocks are perfectly correlated across two countries. Because such shocks degenerate the variance–covariance matrix R in our specification, we need to modify the state-space form of the shock process as follows:

$$\begin{bmatrix} \widetilde{\gamma}_t \\ \widetilde{\gamma}_t^* \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ b & c \end{bmatrix} \begin{bmatrix} \widetilde{d}_t \\ \widetilde{\varepsilon}_t \end{bmatrix},$$

$$\begin{bmatrix} \widetilde{d}_t \\ \widetilde{\varepsilon}_t \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{d}_{t-1} \\ \widetilde{\varepsilon}_{t-1} \end{bmatrix} + \begin{bmatrix} \widetilde{v}_t \\ \widetilde{\varepsilon}_t \end{bmatrix}.$$

Suppose now that b and c are different. In such a case, as is explained in the Online Appendix, the households in the two countries immediately know the persistence of the shock, i.e., whether the shock is persistent or transitory. The intuition is as follows. By comparing the size of the shocks in Home and Foreign, if the Foreign shock is b times larger than the Home shock, then it must be persistent. If, instead, the Foreign shock is c times larger than the Home shock, then the households realize that it is a transitory shock. Learning does not play any role in this case. Indeed, there is no difference in the responses under perfect and under imperfect information.

When b and c are identical, comparing the size of the shock does not help identify the type of the shock. The households, however, know that both persistent and transitory shocks are common to the two countries, and hence, they solve the signal-extraction problem for the univariate process. In this case, learning does play some role. For further details, see the Online Appendix (Section 7).

19. Appendix C demonstrates how the level of the responding macrovariables can be computed.

20. The role of the parameter for the international correlation in transitory shocks, ρ_R, can be interpreted in the following way. Recall that under perfect information, a transitory shock to Home affects home output, consumption, and investment positively and labor input negatively, whereas it also affects Foreign consumption and investment negatively, and output and labor input positively. It is easy to understand why a high ρ_R raises the correlation of outputs. But why does the same thing happen to consumption, investment, and labor input? To understand this, notice that the magnitude of the countermoving responses in the two countries is different. For example, according to Figure 2, Home investment jumps upward more than 2% on impact, but Foreign investment declines less than 1%. When the shocks to Home and Foreign are correlated, the less than 1% decline in Foreign investment will be cancelled out by a positive transitory shock to Foreign, which is likely associated with a shock to Home. Not only will the decline be cancelled out, but Foreign investment will also increase because of the shock, which itself will cause more than a 2% increase in Foreign investment. As a result, investments in both countries increase, creating a comovement.

21. Although standard errors indicate the possibility that $\Phi = 0$, our likelihood ratio test rejects this hypothesis. It is well known that large standard errors in the maximum likelihood estimation, utilizing the state space model, are not uncommon [e.g., Shumway and Stoffer (2006)]. Hence, we consider Φ to be quite small, but not zero.

22. See footnote 18 for details.

23. Anderson et al. (1996), for example, argue for alternative methods to solve ARE (as well as this method). If all eigenvalues of Λ are distinct, then we do not have to use the Schur decomposition. The matrix consisting of eigenvectors of Λ is used as U. See Anderson and Moore (1979, p. 161).

REFERENCES

Aguiar, Mark and Gita Gopinath (2007) Emerging market business cycles: The cycle is the trend. *Journal of Political Economy* 115(1), 69–102.

Anderson, Brian D.O. and John B. Moore (1979) *Optimal Filtering*. Englewood Cliffs, NJ: Prentice-Hall.

Anderson, Evan W., Lars Peter Hansen, Ellen R. McGrattan, and Thomas J. Sargent (1996) Mechanics of forming and estimating dynamic linear economies. In Hans M. Amman, David A. Kendrick, and John Rust (eds.), *Handbook of Computational Economics,* vol. 1, pp. 171–252. Amsterdam: North Holland.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1992) International real business cycles. *Journal of Political Economy* 101, 745–775.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1995) International business cycles: Theory and evidence. In Thomas F. Cooley (ed.), *Frontiers of Business Cycle Research*, pp. 331–356. Princeton, NJ: Princeton University Press.

Bai, Jushan and Serena Ng (2002) Determining the number of factors in approximate factor models. *Econometrica* 70(1), 191–221.

Baxter, Marianne (1995) International trade and business cycles. In Gene M. Grossman and Kenneth Rogoff (eds.), *Handbook of International Economics*, vol. 3, pp. 1801–1864. Amsterdam: North Holland.

Baxter, Marianne and Mario J. Crucini (1993) Explaining saving and investment correlations. *American Economic Review* 83, 416–436.

Baxter, Marianne and Mario J. Crucini (1995) Business cycles and the asset structure of foreign trade. *International Economic Review* 36(4), 821–854.

Baxter, Marianne and Dorsey D. Farr (2005) Variable capital utilization and international business cycles. *Journal of International Economics* 65, 335–347.

Boileau, Martin (1996) Growth and the international transimission of business cycles. *International Economic Review* 37(4), 737–756.

Boileau, Martin and Michel Normandin (2008) Closing international real business cycle models with restricted financial markets. *Journal of International Money and Finance* 27, 733–756.

Boz, Emine, Christian Daude, and C. Bora Durdu (2011) Emerging market business cycles revisited: Learning about the trend. *Journal of Monetary Economics*, 58(6–8), 616–631.

Canova, Fabio and Morten O. Ravn (1996) International consumption risk sharing. *International Economic Review* 37(3), 573–601.

Edge, Rochelle M., Thomas Laubach, and John C. Williams (2007) Learning and shifts in long-run productivity growth. *Journal of Monetary Economics* 54, 2421–2438.

Erceg, Christopher J., Luca Guerrieri, and Christopher Gust (2006) SIGMA: A new open economy model for policy analysis. *International Journal of Central Banking* 2(1), 1–50.

Fuhrer, Jeffrey C. and Michael W. Klein (2006) Risky habits: On risk sharing, habit formation, and the interpretation of international consumption correlations. *Review of International Economics* 14(4), 722–740.

Garcia-Cicco, Javier, Roberto Pancrazi, and Martín Uribe (2010) Real business cycles in emerging countries? *American Economic Review* 100, 2510–2531.

Gilchrist, Simon and Masashi Saito (2007) Expectations, asset prices, and monetary policy: The role of learning. In John Y. Campbell (ed.), *Asset Prices and Monetary Policy*, pp. 45–102. Chicago: University of Chicago Press.

Golub, Gene H. and Charles F. Van Loan (1996) *Matrix Computations,* 3rd ed. Baltimore, MD: Johns Hopkins University Press.

Gouriéroux, Christian and Alain Monfort (1997) *Time Series and Dynamic Models*. Cambridge, UK: Cambridge University Press.

Hansen, Lars Peter and Thomas J. Sargent (2008) *Robustness*. Princeton, NJ: Princeton University Press.

Harvey, Andrew C. (1989) *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge, UK: Cambridge University Press.

Heathcote, Jonathan and Fabrizio Perri (2002) Financial autarky and international business cycles. *Journal of Monetary Economics* 49, 601–627.

Heston, Alan, Robert Summers, and Bettina Aten (2011) Penn World Table Version 7.0. Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.

Hodrick, Robert J. and Edward C. Prescott (1997) Postwar US business cycles: An empirical investigation. *Journal of Money, Credit and Banking* 29, 1–16.

Kehoe, Patrick J. and Fabrizio Perri (2002) International business cycles with endogenous incomplete markets. *Econometrica* 70(3), 907–928.

King, Robert G. (1990) Value and capital in the equilibrium business cycle program. In Lionel McKenzie and Stefano Zamagni (eds.), *Value and Capital: Fifty Years Later*, pp. 279–309. London: MacMillan.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo (1988) Production, growth and business cycles: I. The basic neoclassical model. *Journal of Monetary Economics* 21, 195–232.

King, Robert G. and Mark W. Watson (1998) The solution of singular linear difference systems under rational expectations. *International Economic Review* 39(4), 1015–1026.

King, Robert G. and Mark W. Watson (2002) System reduction and solution algorithm for singular linear difference systems under rational expectations. *Computational Economics* 20, 57–86.

Kollman, Robert (1996) Incomplete asset markets and the cross-country consumption correlation puzzle. *Journal of Economic Dynamics and Control* 20, 945–961.

Laub, Alan J. (1979) A Schur method for solving algebraic Riccati equations. *IEEE Transactions on Automatic Control* 24(6), 913–921.

Lewis, Karen K. (1996) What can explain the apparent lack of international consumption risk-sharing? *Journal of Political Economy* 104(2), 267–297.

Luo, Yulei, Jun Nie, and Eric R. Young (2010) Robust Control, Informational Frictions, and International Consumption Correlations. Research working paper RWP10-16, Federal Reserve Bank of Kansas City.

Maffezzoli, Marco (2000) Human capital and international real business cycles. *Review of Economic Dynamics* 3, 137–165.

Pakko, Michael R. (2002) What happens when the technology growth trend changes? Transition dynamics, capital growth, and the "New Economy." *Review of Economic Dynamics* 5, 376–407.

Pakko, Michael R. (2004) A spectral analysis of the cross-country consumption correlation puzzle. *Economics Letters* 84, 341–347.

Perron, Pierre (1989) The great crash, the oil price shock and the unit root hypothesis. *Econometrica* 57, 1361–1401.

Phillips, Peter C.B. and Sam Ouliaris (1990) Asymptotic properties of residual based tests for cointegration. *Econometrica* 58, 165–193.

Rabanal, Pau, Juan F. Rubio-Ramirez, and Vicente Tuesta (2010) Cointegrated TFP processes and international business cycles. *Journal of Monetary Economics* 58(2), 156–171.

Schmitt-Grohé, Stephanie and Martín Uribe (2003) Closing small open economy models. *Journal of International Economics* 61, 163–185.

Shumway, Robert H. and David S. Stoffer (2006) *Time Series Analysis and Its Applications With R Examples*, 2nd ed. New York: Springer.

Sims, Christopher A. (2003) Implications of rational inattention. *Journal of Monetary Economics* 50(3), 665–690.

Stockman Alan C. and Linda L. Tesar (1995) Tastes and technology in a two-country model of the business cycle: Explaining international comovements. *American Economic Review* 85(1), 168–185.

Uzawa, Hirofumi (1968) Time preference, the consumption function and optimum asset holdings. In James N. Wolfe (ed.), *Value, Capital and Growth: Papers in Honor of Sir John Hicks*, pp. 485–504. Edinburgh: University of Edinburgh Press.

Xiao, Wei (2004) Can indeterminacy resolve the cross-country correlation puzzle? *Journal of Economic Dynamics and Control* 28, 2341–2366.

APPENDIX A: THE KALMAN GAIN

A.1. THE SHOCK PROCESS AND THE FILTERING

For the argument regarding steady-state Kalman gain that we shall see later, it is convenient to write the process of shocks as follows:

$$\underbrace{\begin{bmatrix} \widetilde{\gamma}_t \\ \widetilde{\gamma}_t^* \end{bmatrix}}_{z_t} = \underbrace{\begin{bmatrix} 1 \\ b \end{bmatrix}}_{H} \underbrace{\widetilde{d}_t}_{x_t} + \underbrace{\begin{bmatrix} \widetilde{\varepsilon}_t \\ \widetilde{\varepsilon}_t^* \end{bmatrix}}_{e_t},$$

$$\underbrace{\widetilde{d}_t}_{x_t} = \underbrace{\rho}_{F} \underbrace{\widetilde{d}_{t-1}}_{x_{t-1}} + \underbrace{\widetilde{v}_t}_{v_t},$$

and

$$\widetilde{v}_t \sim \text{i.i.d.}\,(0, Q)\,, \begin{bmatrix} \widetilde{\varepsilon}_t \\ \widetilde{\varepsilon}_t^* \end{bmatrix} \sim \text{i.i.d.}\,(0, R),$$

where b is a known constant.

The Kalman filter yields

$$\begin{aligned} x_{t|t} &= Fx_{t-1|t-1} + K_t(z_t - HFx_{t-1|t-1}) \\ &= (I - K_t H)Fx_{t-1|t-1} + K_t z_t \end{aligned}$$

and

$$e_{t|t} \equiv z_t - Hx_{t|t},$$

where $a_{t|t} = E\,[a_t|t]$ for any variable a, and K_t is the Kalman gain.

Let the mean squared error matrix of the forecast be

$$P_{t|t-1} \equiv E[(x_t - x_{t|t-1})(x_t - x_{t|t-1})'],$$

where $x_{t|t-1} = E\,[x_t|t-1]$ is a one-step-ahead forecast of x_t.

A.2. THE EXISTENCE AND UNIQUENESS OF THE KALMAN GAIN

We shall show that

1. A unique steady-state Kalman gain K (i.e., $\lim_{t\to\infty} K_t = K$) exists;
2. Any nonnegative symmetric $P_{0|-1}$ converges to a unique P, which is stable.

LEMMA 1. *The pair of matrices (F, H) is detectable.*

Proof. It is sufficient to show that the pair of matrices (F, H) is observable, as the observability implies detectability [Harvey (1989, p. 116)]. The observability requires

$$\text{Rank}[H', F'H'] = 1.$$

Because H is the identity matrix, this requirement is satisfied. ■

LEMMA 2. *The pair of matrices (F, G) is stabilizable.*

Proof. It is sufficient to show that the pair of matrices (F, G) is controllable, as the controllability implies stabilizability [Harvey (1989, p. 116)]. The controllability requires

$$\text{Rank}\,[G,\ FG] = 1.$$

Because $1 = \text{Rank}\,(Q) = \text{Rank}(GG') \leq \min \text{Rank}\,[G, G'] = \text{Rank}\,(G)$, this requirement is satisfied. ■

THEOREM 1. *Anderson and Moore (1979, p. 77). If the pair of matrices (F, H) is detectable and the pair (F, G) is stabilizable for any G with $GG' = Q$, then*

1. *For any nonnegative symmetric initial condition $P_{0|-1}$, one has*

$$\lim_{t\to\infty} P_{t+1|t} = P,$$

with P independent of $P_{0|-1}$ and satisfying the steady-state algebraic matrix Riccati equation (ARE)

$$P = F[P - PH'(HPH' + R)^{-1}HP]F' + GG'. \tag{A.1}$$

2. *The filter is asymptotically stable: The steady-state Kalman gain*

$$K = PH'(HPH' + R)^{-1}$$

satisfies

$$|\lambda_i(F - FKH)| < 1,$$

for all i, where $\lambda_i\,(F - FKH)$ is the ith eigenvalue of $F - FKH$.

Remark 1. The solution to the ARE is unique, under the assumptions given in the preceding. Hence, the Kalman gain is unique.

A.3. SOLVING THE ALGEBRAIC RICCATI EQUATION AND THE COMPUTATION OF THE KALMAN GAIN

Following Laub (1979), we use the Schur decomposition to solve the ARE.[23]

THEOREM 2 (Laub (1979, Theorems 3, 4, and 6)). *Let the pair of matrices (F, H) be detectable and the pair (F, G) be stabilizable for any G with $GG' = Q$. Assume that F and R are nonsingular. Let $\Lambda \in \mathbf{R}^{2\times 2}$ be*

$$\Lambda = \begin{bmatrix} F' + H'R^{-1}HF^{-1}GG' & -H'R^{-1}HF^{-1} \\ -F^{-1}GG' & F^{-1} \end{bmatrix}.$$

Then

1. *(Schur Decomposition) Let Λ have eigenvalues λ_1, λ_2. There exists a unitary similarity transformation V such that $V'\Lambda V$ is upper triangular with diagonal elements λ_1, λ_2 in that order.*
2. *(Changing the Order of Diagonal Elements) One can find an orthogonal similarity transformation*

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}$$

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such that $U'\Lambda U = S$ is quasi-upper triangular:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix}.$$

The diagonal elements of S are λ_1, λ_2 in any order, but S_{11} has only stable eigenvalues.

3. *U_{11}, defined in the preceding, is invertible and $P = U_{21}U_{11}^{-1}$ solves the ARE (A.1).*

The steps to compute the Kalman gain are the following:

1. Compute Λ.
2. Compute the Schur decomposition of Λ. For example, one can use a Matlab function "`schur`."
3. Reorder the diagonal elements of S from smallest to largest. Get the corresponding orthogonal matrix U. For example, Algorithm 7.6.1 in Golub and Van Loan (1996) with the Givens rotation can be used.
4. Compute $P = U_{21}U_{11}^{-1}$. This is the unique solution to the ARE (A.1).
5. The steady-state Kalman gain is then $K = PH'(HPH' + R)^{-1}$.

APPENDIX B: MODEL SOLUTION AND IMPULSE RESPONSES

B.1. MODEL SOLUTION

A standard method for solving the rational expectations model in order to analyze the local dynamics around the steady state is given by King et al. (1988). We use King and Watson's (1998, 2002) system reduction method to solve our rational expectations model:

$$\underbrace{A}_{20\times 20} E_t \underbrace{Y_{t+1}}_{20\times 1} = \underbrace{B}_{20\times 20} \underbrace{Y_t}_{20\times 1} + \underbrace{C}_{20\times 2} \underbrace{X_t}_{2\times 1} + \underbrace{D}_{20\times 2} E_t \underbrace{X_{t+1}}_{2\times 1}, \tag{B.1}$$

where

$$Y_t = \left[\, c_t \; L_t \; k_t \; N_t \; i_t \; p_t \; w_t \; \lambda_t \; q_t \; c_t^* \; L_t^* \; k_t^* \; N_t^* \; i_t^* \; p_t^* \; w_t^* \; \lambda_t^* \; q_t^* \; b_t \; P_t^B \,\right]',$$

$$X_t = \left[\, \widetilde{\gamma}_t \; \widetilde{\gamma}_t^* \,\right]'.$$

The solutions to our two-country model can be described as follows:

$$Z_t = \Pi S_t, \tag{B.2}$$

$$S_t = MS_{t-1} + g\eta_t, \tag{B.3}$$

where

$$Z_t = \left[\, Y_t' \; X_t' \,\right]',$$

$$S_t = \left[\, k_t \; k_t^* \; b_t \; \widetilde{d}_t \; \widetilde{\varepsilon}_t \; \widetilde{\varepsilon}_t^* \,\right]',$$

$$\underbrace{\Pi}_{22\times 6}, \underbrace{M}_{6\times 6}, \underbrace{g}_{6\times 3}, \eta_{\mathrm{t}} = \left[\, \widetilde{\upsilon}_t \; \widetilde{\varepsilon}_t \; \widetilde{\varepsilon}_t^* \,\right]'.$$

B.2. IMPULSE RESPONSE

As is assumed by Gilchrist and Saito (2007), we consider a case of certainty equivalence. First, we define the filtered state vector

$$S_{t|t} = \left[\, k_t \;\; k_t^* \;\; b_{t|t} \;\; \widetilde{d}_{t|t} \;\; \widetilde{\varepsilon}_{t|t} \;\; \widetilde{\varepsilon}_{t|t}^* \,\right]'.$$

Suppose that a persistent growth rate shock occurs:

- Perfect information:

$$S_{1|1} = [\,0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0\,]'.$$

- Imperfect information:

$$S_{1|1} = \left[\,0 \quad 0 \quad 0 \quad \widetilde{d}_{t|t} \quad \widetilde{\varepsilon}_{t|t} \quad \widetilde{\varepsilon}_{t|t}^* \,\right]'.$$

Then, the impulse responses are computed by the state space:

$$Z_t = \Pi S_t,$$
$$S_t = M S_{t-1}.$$

Computational note:
Step 1.
Compute the steady-state Kalman gain and then compute $\{\widetilde{d}_{t|t}\}_{t=1}^T$, $\{\widetilde{\varepsilon}_{t|t}\}_{t=1}^T$, and $\{\widetilde{\varepsilon}_{t|t}^*\}_{t=1}^T$.
For every $t = 1$ to $T-1$, iterate the following steps.
Step 2.
Plug $\widetilde{d}_{t|t}$, $\widetilde{\varepsilon}_{t|t}$, and $\widetilde{\varepsilon}_{t|t}^*$ into the corresponding elements in S_t.
Step 3.
Compute $Z_t = \Pi S_t$.
Step 4.
Update $S_{t+1} = M S_t$.
For the perfect information case, one can omit steps 1 and 2.

APPENDIX C: DEVIATIONS FROM THE TRENDS

Following Pakko (2002), we compute the change in the growth rate of variable x_t as

$$\log x_t / x_{t-1} = \overline{\gamma}_x + \widetilde{\gamma}_{xt} + \widetilde{x}_t - \widetilde{x}_{t-1},$$

where $\overline{\gamma}_{xt}$ is the old (before the shock) growth rate; $\widetilde{\gamma}_{xt}$ is the change in the growth rate; and $\widetilde{x}_t$ is the (log or percent) deviation of x from its trend, which is obtained from the responses of systems (B.2) and (B.3). Therefore, a response of variable x (in level) around its old trend can be computed as

$$\sum_{j=1}^{t} \Delta \log x_j - t\overline{\gamma}_x,$$

whereas deviations from the new trend are $\widetilde{x}_t$.

APPENDIX D: THE APPROXIMATE FACTOR MODEL

The approximate factor model with k factors is, for $t = 1, \ldots, T$,

$$\underset{2\times 1}{X_t} = \underset{2\times k}{\Lambda}\underset{k\times 1}{F_t} + \underset{2\times 1}{e_t},$$

where

$$X_t = \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}, \quad e_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

and X_{1t}, X_{2t} are de-meaned growth rates of the United States and Europe, respectively. e_t is a zero-mean error vector that satisfies the conditions stated in Assumption C of Bai and Ng (2002).

The matrix notation for the model is then

$$\underset{T\times 2}{X} = \underset{T\times k}{F}\underset{k\times 2}{\Lambda'} + \underset{T\times 2}{e},$$

where

$$X = \begin{bmatrix} X_{11} & \cdots & X_{1T} \\ X_{21} & \cdots & X_{2T} \end{bmatrix}, \quad F = \begin{bmatrix} F_1' \\ \vdots \\ F_T' \end{bmatrix}, \quad e = \begin{bmatrix} e_{11} & \cdots & e_{1T} \\ e_{21} & \cdots & e_{2T} \end{bmatrix}.$$

The factor loading matrix can be estimated by

$$\min\,(2T)^{-1}\sum_{i=1}^{2}\sum_{t=1}^{T}(X_{it} - \lambda_i F_t)^2$$
$$= \min\,(2T)^{-1}\,\text{trace}[(X - F\Lambda')(X - F\Lambda')'],$$

with a normalization $F'F/T = I$. This is the same problem as the principal component problem: The solution is given by setting the column vectors of F as the eigenvectors of XX', and the estimate for Λ is then

$$\widehat{\Lambda} = X'F/T.$$

For $k = 0$ and 1, one can compute the information criteria:

$$\text{IC}_{p1}(k) = \ln\left[V\left(k, \widehat{F}^k\right)\right] + k\left(\frac{T+2}{2T}\right)\ln\left(\frac{2T}{T+2}\right),$$
$$\text{IC}_{p2}(k) = \ln\left[V\left(k, \widehat{F}^k\right)\right] + k\left(\frac{T+2}{2T}\right)\ln 2,$$

where

$$V\left(k, \widehat{F}^k\right) = \min_{\Lambda}\,(2T)^{-1}\sum_{i=1}^{2}\sum_{t=1}^{T}\left(X_{it} - \lambda_i^{k'}\widehat{F}_t^k\right)^2,$$
$$\widehat{k} = \text{argmin}\, IC_{pj}(k),$$

where $j = 1, 2$ is a consistent estimator of k.

TABLE E.1. Data sources

Variable	Explanation	Source
y_{it}^*	Nominal GDP: Gross Domestic Product: Expenditure approach, national currency, current prices, seasonally adjusted	QNA
p_{it}	Price level: Consumer price index, all items, base year = 2005	MEI
rgdpch_i	Real GDP per capita in 1995	PWT
pop_i	Total population in 1995	PWT
em_{it}	Civilian employment index: Employment, all persons, seasonally adjusted, base year = 2005	MEI
c_{it}^*	Nominal consumption: Private final consumption expenditure, national currency, current prices, seasonally adjusted	QNA
i_{it}^*	Nominal investment: Gross fixed capital formation, national currency, current prices, seasonally adjusted	QNA

Notes: QNA: OECD Quarterly National Accounts; MEI: OECD Main Economic Indicators; PWT: Penn World Table 7.0, Heston et al. (2011).

APPENDIX E: DATA

In the spirit of Backus et al. (1992), we construct the data for output, consumption, investment, labor input, and productivity for both the United States and the European aggregate, following Boileau and Normandin (2008), who use more recent data to study a two-country model with incomplete markets. Quarterly data from 1980:I through 2010:III are taken from the OECD Main Economic Indicators and the OECD Quarterly National Accounts. The European aggregate consists of Austria, Finland, France, Germany (West Germany, before 1990), Italy, and the United Kingdom. Table E.1 summarizes the variables and the data set we use.

1. *Output weights.* The output weights utilized in this paper are assumed to be constant over time. The base year Real GDPs for all the countries are computed by multiplying real GDP per capita in 1995 by total population in 1995. Both series are taken from the Penn World Table Version 7.0 [Heston et al. (2011, variables "pop" and "rgdpch")]. After computing all of the countries' real GDPs, we define the output weight for country i as the real GDP of country i relative to the sum of the real GDPs of all the countries (six European countries plus the United States). The output weight for country i is
$$\omega_i^y = \frac{\widetilde{y}_i}{\sum_j \widetilde{y}_j},$$
where $\widetilde{y}_i$ is real output:
$$\widetilde{y}_i = \text{rgdpch}_i \times \text{pop}_i.$$
2. *Price level.* The all-items consumer price index (CPI) for the base year 2005 from the OECD Main Economic Indicators is used.

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3. *Output, consumption, investment.* The following series are taken from the OECD Quarterly National Accounts: "Gross domestic product (expenditure approach), measured in national currency, current prices, seasonally adjusted" for output; "Private final consumption expenditure, measured in national currency, current prices, seasonally adjusted" for consumption; and "Gross fixed capital formation, measured in national currency, current prices, seasonally adjusted" for investment. After each European country's nominal variables are deflated by the corresponding price level, the European aggregates of output, consumption, and investment are computed as the weighted sums of the six European countries' real output, real consumption, and real investment, respectively. We use the output weights for the weighted sums. As for U.S. output, consumption, and investment, the U.S. output weight is multiplied by the U.S. real output, real consumption, and real investment, respectively.
Hence, real output, real consumption, and real investment for country i are computed as

$$y_{it} = \frac{y_{it}^*}{p_{it}}, \quad c_{it} = \frac{c_{it}^*}{p_{it}}, \quad i_{it} = \frac{i_{it}^*}{p_{it}},$$

where y_{it}^*, c_{it}^*, and i_{it}^* are nominal output, nominal consumption, and nominal investment (in terms of national currency), respectively, and p_{it} is the all-item consumer price index. Thus, the European aggregates of real output, real consumption, and real investment are

$$Y_t^* = \sum_{i=\text{Europe}} \omega_i^y y_{it}, \quad C_t^* = \sum_{i=\text{Europe}} \omega_i^y c_{it}, \quad I_t^* = \sum_{i=\text{Europe}} \omega_i^y i_{it};$$

and U.S. real output, real consumption, and real investment are

$$Y_t = \omega_{\text{US}}^y y_{\text{US},t}, \quad C_t = \omega_{\text{US}}^y c_{\text{US},t}, \quad I_t = \omega_{\text{US}}^y i_{\text{US},t}.$$

4. *Labor input.* Indices for employment, "Employment of all persons, seasonally adjusted, for the base year 2005," are taken from Labour Force Statistics in the OECD Main Economic Indicators. Then population in 1995 is multiplied by the employment index to compute labor input for each country. The weighted sum of the six European countries' labor input (with the weights being each country's population relative to the six countries' population) is the European aggregate of labor input. Hence, country i's labor input is

$$n_{it} = \text{em}_{it} \times \text{pop}_i,$$

where em_{it} is the civilian employment index ("Employment of all persons, seasonally adjusted, for the base year 2005" from Labour Force Statistics in the OECD Main Economic Indicators) and pop_i is the population in 1995. As for the European aggregate, employment is computed as

$$N_t^* = \sum_i \omega_i^n n_{it},$$

where

$$\omega_i^n = \frac{\text{pop}_i}{\sum_{j=\text{European}} \text{pop}_j};$$

and for the United States,

$$N_t = n_{\text{US},t}.$$

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5. *Productivity.* For both the United States and the European aggregate, we compute (the log of) productivity as follows:

$$\ln x_t = \ln Y_t - (1 - \alpha) \ln N_t,$$

where Y_t is real output, N_t is labor input, and $\alpha = 0.36$. The (log) level of technology, $\ln X_t$, is then $\ln X_t = \ln x_t / (1 - \alpha)$.

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