GPS Receiver Performance Enhancement via Inertial Velocity Aiding

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An integrated GPS/INS navigation system can employ inertial velocity information to produce a more robust system. For a stand-alone GPS receiver, decreasing the receiver tracking loop bandwidth reduces the probability of losing lock in a jamming or interference environment if vehicle dynamics are low. However, reduced bandwidth increases tracking errors when dynamics are present. Beyond a certain limit, it causes a serious degradation in the dynamic tracking loop performance. Providing inertial velocity aiding to the receiver tracking loops is an effective and popular treatment to help resolve this problem. In this paper, performance of the GPS receiver tracking loops using inertial velocity aiding will be investigated. Different types of tracking loops, from 1st to 3rd order, are covered. Following the discussion of the system architecture and derivation of the related transfer functions for the tracking loops, both with and without aiding, the system performance, including transient response, steady-state error, and noise bandwidth is evaluated.

KEY WORDS

1. Integration. 2. GPS. 3. Inertial Navigation.

1. INTRODUCTION. Inertial navigation methods have been used in a wide variety of navigation applications in the field of marine, aerospace and spacecraft technology. Global Positioning System (GPS) and inertial navigation systems (INS) have complementary operational characteristics, and the synergy of the systems is well known. The GPS receiver provides low frequency data (long-term) to the INS to allow INS errors, which drift at a slow rate, to be estimated. On the other hand, the INS provides high frequency data to the GPS for error mitigation. The goal of GPS/INS integration is to combine the features of both systems and so improve overall system performance and therefore safety.

The advantages of integration include: continued navigation during periods of GPS outage, and acquisition and re-acquisition of satellites as they come into view or re-appear after wing or tail shadowing, or masking by foliage or other natural or man-made obstructions. In highly dynamic manoeuvres, inertial velocity provides the GPS tracking loops with additional information not available to a stand-alone receiver.

There are several options for integrating the GPS and INS sensors. Essentially, there are two mechanisms of integration: open loop (feed-forward) versus closed loop (feed-back), one filter (tightly-coupled) versus two filters (loosely-coupled). Figure 1 shows the system architecture of a tightly-coupled GPS/INS integrated system that offers a single navigation solution. The integrated navigation filter supplies a Kalman filter estimate of a combined set of GPS/INS error parameters; that is: three position

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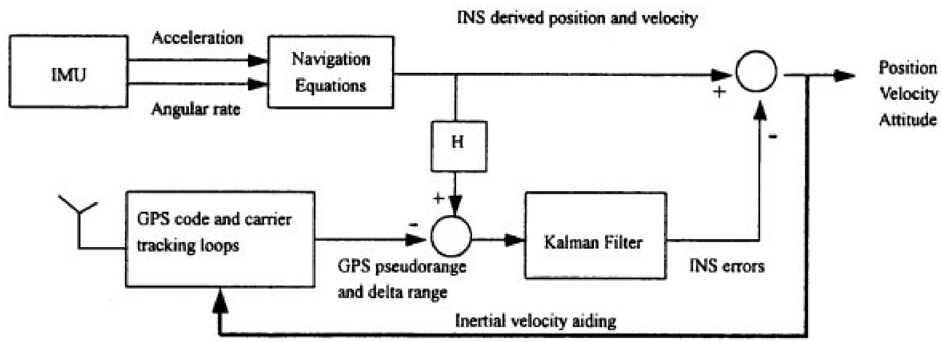

Figure 1. Tightly coupled GPS/INS integrated systems.

errors, three velocity errors, three attitude errors, three accelerometers biases, three gyro biases, one clock error, one clock rate error, and so on. More detailed information can be found in Phillips and Schmidt (1996). Useful GPS data are obtained only while the carrier-tracking and/or code-tracking loops are locked onto the desired signals. Loops with narrow bandwidths can improve the SNR (signal-to-noise ratio) tolerance and enhance resistance to jamming or interference but can also lead to loss of tracking under highly dynamic manoeuvres. The use of inertially derived velocity to aid tracking loops can substantially reduce loop bandwidths required without the penalty of increased dynamic tracking errors. In general, inertial aiding enhances fast acquisition of initial tracking, provides propagation of the navigation solution, replaces a satellite measurement, and assists continuous tracking of a satellite.

The receiver performance when using a specific second-order tracking loop has been preliminarily addressed by He and Chen (1998). In this paper, complete mathematical derivation is implemented and performance enhancement is evaluated for the velocity-aided GPS receiver tracking loops.

2. SYSTEM ARCHITECTURE OF THE TRACKING LOOPS.

A typical GPS receiver contains two tracking loops simultaneously. A carrier-tracking loop tracks the carrier phase and the code-tracking loop tracks the signal code to within a small fraction of the chip duration. Most GPS receiver designs have a mode of operation that employs a non-coherent delay-locked loop (DLL) for code tracking and a Costas phase-locked loop (PLL) for tracking the Doppler-shifted carrier (the frequency change due to vehicle dynamics). The pseudo-ranges obtained from the code tracking provide a position fix, while the pseudo-range rate (or delta range) estimates obtained from the Costas loop provide a velocity fix, which is accomplished by counting the number of Doppler frequency shifts of carrier cycles that occur over a finite time interval.

The carrier-tracking loop will be more sensitive to dynamics due to the fact that it tracks a much higher frequency signal than the code-tracking loop. If the carrier-tracking loop loses lock during a highly dynamic manoeuvre, the other receiver loops will subsequently lose lock. The other receiver loops will not see the full dynamics, and they typically will be aided from either the carrier loop or an external navigation source. When the vehicle is in a low jamming/interference environment, the carrier loop can provide aiding to the code loop; when the vehicle is in a medium or high

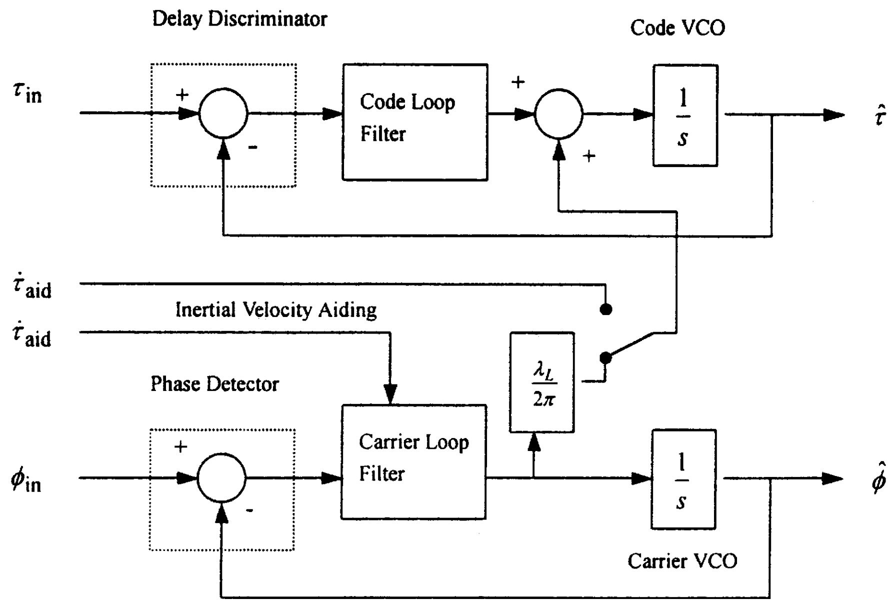

Figure 2. Simplified GPS receiver tracking loops (Bye, *et al.*, 1998).

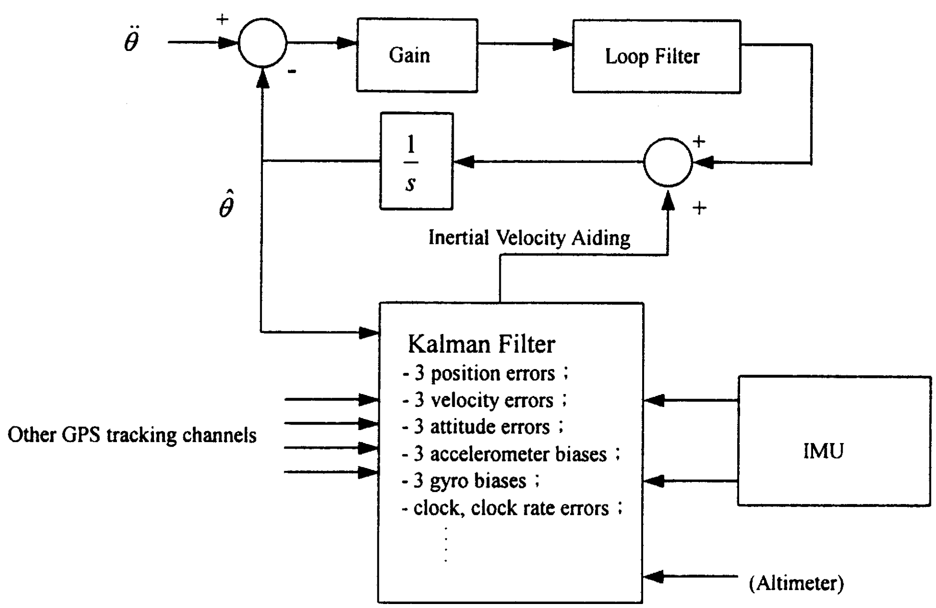

Figure 3. Inertial velocity aiding to the tracking loop for a tightly coupled GPS/INS integrated system (Sennott, 1995).

jamming/interference environment, or undertakes a highly dynamic movement, the carrier loop may not be able to function properly. If the GPS receiver is unable to maintain lock with the carrier loop, code loop tracking can be maintained by replacing the carrier-based velocity information with the inertial aiding signal to prevent loss of signal lock. This system architecture is shown as in Figure 2.

https://doi.org/10.1017/S0373463300001260 Published online by Cambridge University Press

The sections of Figure 1 and 2 showing inertial velocity aiding of receiver tracking loops from the navigation Kalman filter are expanded in Figure 3.

3. TRANSFER FUNCTIONS. A generalised block diagram of an inertial aided tracking loop that is applicable for analysis of either carrier or code loops is shown in Figure 4. This figure shows how inertial velocity aiding influences the

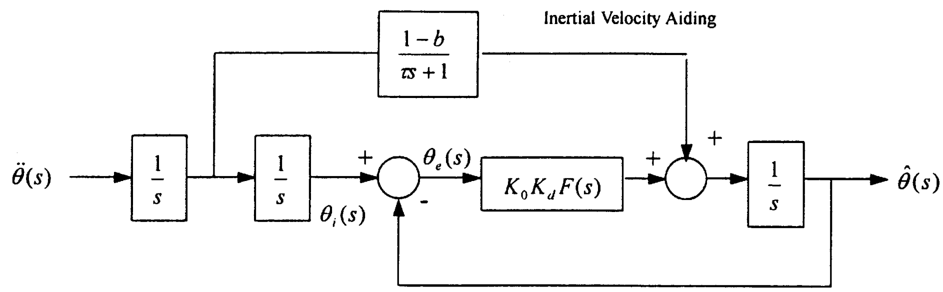

Figure 4. Block diagram for the receiver tracking loop with inertial velocity aiding (He and Chen, 1998).

tracking loop. Notice that if θ represents the position, then the aiding originates from the integrated acceleration or velocity. The closed-loop loop transfer function of the tracking loop without aiding can be obtained from

$$H(s) = \frac{\hat{\theta}(s)}{\theta_i(s)} = \frac{K_0 K_d F(s)}{s + K_0 K_d F(s)} = \frac{G(s)}{s + G(s)}, \tag{1}$$

and the transfer function of the inertial aided tracking loop becomes:

$$H(s) = \frac{\hat{\theta}(s)}{\theta_i(s)} = \frac{\left(\frac{1-b}{\tau s+1}\right)s + K_0 K_d F(s)}{s + K_0 K_d F(s)} = \frac{\left(\frac{1-b}{\tau s+1}\right)s + G(s)}{s + G(s)}, \tag{2}$$

where: $G(s) = K_0 K_d F(s)$, K_0 represents the gain factor of the voltage-controlled oscillator (VCO) and K_d is the phase-detector/delay discriminator gain factor. From Equation (2), it can be seen that the output signal of an inertial aided loop is a combined signal from the low-pass filter $\frac{G(s)}{s+G(s)}$ and the high-pass filter, $\frac{s}{s+G(s)}$ $\frac{1-b}{\tau s+1}$, where: b is the velocity scale factor error and τ is sensor or processing lags/delayed time. The additional term in the closed-loop transfer function, i.e.$(1-b)/(\tau s+1)$, represents the imperfections in the measurement process. In the case of $b = 1$, aiding information cancels out and it becomes an unaided case.

The transfer function $F(s)$ represents the loop filter of a tracking loop. The loop filter is a low-pass filter used to suppress the noise and high-frequency signal components from the phase-detector/delay discriminator and provide a dc-controlled signal for the VCO. A first-order tracking loop is obtained if $F(s) = 1$, that is, if the loop filter is omitted. The loop gain is then simply $K = K_0 K_d$. The second-order loops are widely applied because of their simplicity and good performance. There are

Table 1. Loop filters for different orders of receiver tracking loops.

1st order	2nd order (lag)	2nd order (active)	2nd order (passive)	3rd order
1	$\frac{1}{\tau_1 s+1}$	$\frac{\tau_2 s+1}{\tau_1 s}$	$\frac{\tau_2 s+1}{\tau_1 s+1}$	$\left(\frac{\tau_2 s+1}{\tau_1 s}\right)^2$

for $\tau_1 = 0$ for $\tau_2 = 0$

for $\tau_1 = \tau_2$

usually three options for selecting the loop filters for a second-order loop, a (simple) lag filter, an active (lag-lead) filter or a passive (lag-lead) filter. The addition of a simple lag filter to a first-order loop does not affect the noise bandwidth and, therefore, the second-order loop with lag filter is sometimes referred to a modified first-order loop rather than a genuine second-order loop. The two second-order tracking loops will be nearly the same if $\tau_2 K \gg 1$ (or $1/K \ll \tau_2$) in the passive filter. There are applications in which a higher order loop is necessary. Third order loops are insensitive to acceleration and fourth order loops are insensitive to jerk (rate of change of acceleration). It is rare that a loop is constructed with an order higher than third (Gardner, 1979) and therefore these will not be covered in the present work. To ensure stable tracking, it is common practice to build loop filters with equal numbers of poles and zeros. An nth-order loop with all-loop poles at $s = 0$ and $n-1$ arbitrary zeros uses (Gardner, 1979):

$$G(s) = \frac{K}{s}\left(1 + \frac{a_2}{s} + \frac{a_3}{s^2} + \ldots + \frac{a_n}{s^{n-1}}\right)$$
$$H(s) = \frac{K(s^{n-1} + a_2 s^{n-2} + \ldots a_n)}{s^n + K(s^{n-1} + a_2 s^{n-2} + \ldots + a_n)}. \tag{3}$$

Table 1 shows the loop filters of interest in this paper, including a first-order filter, a third-order filter and three second-order filters. The closed-loop transfer functions for different types of tracking loops can be obtained when substituting the loop filters into Equations (1) and (2). The results are summarised in Table 2. By checking the characteristic polynomials, the case with velocity aiding simply has one more multiplier term $(\tau s + 1)$ compared to that of un-aided case. Therefore, stability regions of the tracking loop are identical for systems with and without aiding ($\tau > 0$).

4. TRANSIENT RESPONSE.

Different approaches can be performed for obtaining the transient response. Here, the closed-loop transfer functions in Table 2 will be expressed in forms of ordinary differential equations (ODE), which can be numerically solved by the fourth-order Runge–Kutta method.

The transfer functions in Table 2 can be rewritten in the following form:

$$H(s) = \frac{c_{n-1} s^{n-1} + c_{n-2} s^{n-2} + \ldots + c_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}, \tag{4}$$

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Table 2. Closed-loop transfer functions of the tracking loops.

Loop Description	Unaided	Velocity Aided
1^{st} order	$\frac{K}{s+K}$	$\frac{(1-b+\tau K)s+K}{\tau s^2+(\tau K+1)s+K}$
2^{nd} order (lag)	$\frac{K}{\tau_1 s^2+s+K}$	$\frac{(1-b)\tau_1 s^2+(1-b+\tau K)s+K}{\tau_1\tau s^3+(\tau_1+\tau)s^2+(\tau K+1)s+K}$
2^{nd} order (active)	$\frac{(\tau_2 s+1)K}{\tau_1 s^2+\tau_2 Ks+K}$	$\frac{[(1-b)\tau_1+\tau_2\tau K]s^2+(\tau_2 K+\tau K)s+K}{\tau_1\tau s^3+(\tau_2\tau K+\tau_1)s^2+(\tau_2 K+\tau K)s+K}$
2^{nd} order (passive)	$\frac{(\tau_2 s+1)K}{\tau_1 s^2+(\tau_2 K+1)s+K}$	$\frac{[(1-b)\tau_1+\tau_2\tau K]s^2+[(1-b)+\tau_2 K+\tau K]s+K}{\tau_1\tau s^3+(\tau_2\tau K+\tau_1+\tau)s^2+(\tau_2 K+\tau K+1)s+K}$
3^{rd} order	$\frac{(\tau_2^2 s^2+2\tau_2 s+1)K}{\tau_1^2 s^3+\tau_2^2 Ks^2+2\tau_2 Ks+K}$	$\frac{[(1-b)\tau_1^2+\tau\tau_2^2 K]s^3+(2\tau\tau_2 K+\tau_2^2 K)s^2+(\tau K+2\tau_2 K)s+K}{\tau\tau_1^2 s^4+(\tau_1^2+\tau\tau_2^2 K)s^3+(2\tau\tau_2 K+\tau_2^2 K)s^2+(\tau K+2\tau_2 K)s+K}$

which can be described by the following nth-order linear ordinary differential equation:

$$\frac{d^n x}{dt^n}+a_{n-1}\frac{d^{n-1} x}{dt^{n-1}}+a_{n-2}\frac{d^{n-2} x}{dt^{n-2}}+\ldots+a_0 x = c_{n-1}\frac{d^{n-1} u}{dt^{n-1}}+c_{n-2}\frac{d^{n-2} u}{dt^{n-2}}+\ldots+c_0 u. \quad (5)$$

The above ODE can also be equivalently rewritten as a set of n first-order equations and then expressed in state-space representation

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{Ax}+\mathbf{Bu}\\ \mathbf{y} &= \mathbf{Cx}+\mathbf{Du}\end{aligned}, \quad (6)$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (7a)$$

$$\mathbf{C} = [c_0 \quad c_1 \quad \cdots \quad c_{n-2} \quad c_{n-1}];\ \mathbf{D} = 0, \quad (7b)$$

if the state vector is defined as:

$$\mathbf{x} = [x \quad \dot{x} \quad \cdots \quad x^{(n-2)} \quad x^{(n-1)}]^T.$$

The unit step response is employed for testing the transient behaviour. There are several parameters open to the designers. The total number of parameters depends on the loop filters to be selected. Two of the parameters are in the inertial information (b and τ); one in the first-order loop (K); two in the second-order loop with lag filter (K and τ_1); three in the other two second-order loops and the third-order loop (K, τ_1, and τ_2). What we are concerned with here is the influence of inertial velocity aiding,

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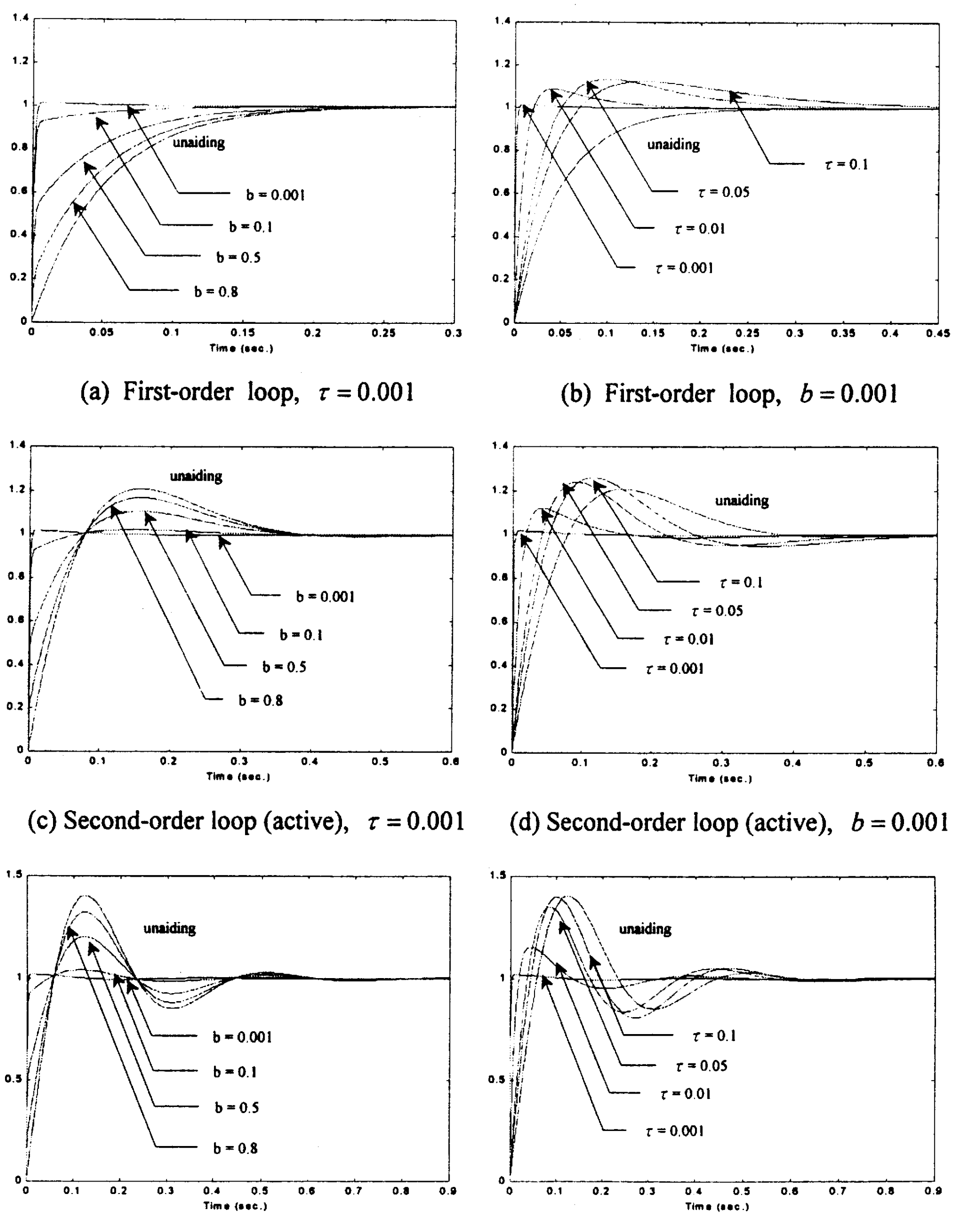
hence, adjusting b and τ while fixing all the other parameters will be investigated in the present work. Select $K = K_0 K_a = 20$ and $\tau_1 = \tau_2 = 0{\cdot}1$, so that the system transient performance is governed by τ and b. Five curves, including four for the aided case and one for the un-aided case, are shown in each plot of Figure 5.



(a) First-order loop, $\tau = 0.001$

(b) First-order loop, $b = 0.001$

(c) Second-order loop (active), $\tau = 0.001$

(d) Second-order loop (active), $b = 0.001$

(e) Third-order loop, $\tau = 0.001$

(f) Third-order loop, $b = 0.001$

Figure 5. Influence of b and τ on transient responses for velocity aided loops.

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In Figure 5, parameters τ and b, will be selected fixed in turn, while varying the other one by four different values. It can be seen that the inertial aiding reduces the rise time and maximum overshoot for all types of filters when fixing $\tau = 0{\cdot}001$ and varying b. The scale factor error b represents the quality of an inertial navigation system. A small b value represents a good quality inertial system. A system with $b = 0{\cdot}001$ represents roughly the level of CEP performance of today's one nautical mile per hour inertial systems without special calibration (Hemesath, 1980). Even when b is very large, e.g. $b = 0{\cdot}8$, inertial information still improves the transient response performance to some extent.

Caution should be paid on the selection of τ. Aided velocity will move the system from under-damped to over-damped. The maximum overshoot increases when τ increases up to a certain value (approximately 0·05 for the first-order loop and 0·1 for the second-order loop with active filter in the present example) and starts to decrease after that. Therefore, velocity aiding reduces the rise time but could result in a larger maximum overshoot than the un-aided case. In general, selecting τ no more than 0·001 will provide a satisfactory result.

5. STEADY-STATE ERROR. A small steady-state error is usually desired and is considered to be the criterion of good tracking performance. If the error should become so large that the VCO skips cycles, the loop is considered to have lost lock. The transfer functions for the tracking errors are

(a) for un-aided case

$$E(s) = \frac{\theta_e(s)}{\theta_i(s)} = 1 - H(s) = \frac{s}{s + G(s)}, \tag{8}$$

(b) for velocity-aided case

$$E(s) = \frac{\theta_e(s)}{\theta_i(s)} = 1 - H(s) = \frac{s}{s + G(s)} \frac{\tau s + b}{\tau s + 1}. \tag{9}$$

It can be seen that the effect of aiding is to modify the loop transfer function by the attenuation factor

$$A(s) = \frac{\tau s + b}{\tau s + 1} \tag{10}$$

This factor represents the imperfections in the measurement process.

The steady-state errors can be evaluated by means of the final value theorem of the Laplace transforms

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s[1 - H(s)], \tag{11}$$

and the results are:

(a) for unaided case

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} \frac{s^2\, \theta_i(s)}{s + G(s)}, \tag{12}$$

(b) for velocity-aided case

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} \frac{s^2\, \theta_i(s)}{s + G(s)} A(s). \tag{13}$$

When $s \rightarrow 0$, the transfer function for the attenuation factor becomes

$$A(0) = b. \tag{14}$$

Since the scale factor error b is usually relatively small compared to 1, the steady-state errors for those without aiding are $1/b$ (usually much larger than 1) times those with inertial velocity aiding. We conclude that the velocity aiding facilitates reduction of steady-state error. With perfect aiding (i.e. $\tau = b = 0$, which implies the attenuation factor is zero), the loop tracks all dynamics without error thereby implying theoretically that the bandwidth can be arbitrarily narrow. For a system with $b = 0{\cdot}001$ (which represents roughly the level of CEP performance of today's one nautical mile per hour inertial systems without special calibration) the steady-state error will be 0·001 times of that without aiding. The worst case is set as $b = 1$, for which the system becomes an unaided case.

Table 3. Error-response transfer functions.

Loop description	Un-aided	Velocity aided
1st order	$\frac{s}{s+K}$	$\frac{s}{s+K}\frac{\tau s+b}{\tau s+1}$
2nd order (lag)	$\frac{\tau_1 s^2+s}{\tau_1 s^2+s+K}$	$\frac{\tau_1 s^2+s}{\tau_1 s^2+s+K}\frac{\tau s+b}{\tau s+1}$
2nd order (active)	$\frac{\tau_1 s^2}{\tau_1 s^2+\tau_2 Ks+K}$	$\frac{\tau_1 s^2}{\tau_1 s^2+\tau_2 Ks+K}\frac{\tau s+b}{\tau s+1}$
2nd order (passive)	$\frac{\tau_1 s^2+s}{\tau_1 s^2+(\tau_2 K+1)s+K}$	$\frac{\tau_1 s^2+s}{\tau_1 s^2+(\tau_2 K+1)s+K}\frac{\tau s+b}{\tau s+1}$
3rd order	$\frac{\tau_1^2 s^3}{\tau_1^2 s^3+\tau_2^2 Ks^2+2\tau_2 Ks+K}$	$\frac{\tau_1^2 s^3}{\tau_1^2 s^3+\tau_2^2 Ks^2+2\tau_2 Ks+K}\frac{\tau s+b}{\tau s+1}$

6. EQUIVALENT NOISE BANDWIDTH. The design procedure for either the carrier loop or the code loop is to select a bandwidth that produces tracking errors under maximum dynamics approximately equal to the lock limit of the loop. If inertial information is delivered into the tracking loops, the required minimum bandwidth can be reduced without the penalty of increasing dynamic errors. The single-side equivalent noise bandwidth, in Hz, for a tracking loop with transfer function is expressed as:

$$B_n = \frac{1}{|H(0)|^2}\int_0^{\infty} |H(j\omega)|^2\, df, \tag{15}$$

where: $\omega = 2\pi f$ and the magnitude of the frequency response is

$$|H(j\omega)| = [H(j\omega)H(-j\omega)]. \tag{16}$$

Calculation of the type of integration given by Equation (15) may be complicated. R. S. Philips' table of integrals for definite integrals can be employed (Brown and Hwang, 1997)

$$I_n = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{c(s)c(-s)}{a(s)a(-s)}\, ds, \tag{17}$$

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Table 4. Table of Integrals.

$$I_n = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{c(s)c(-s)}{a(s)a(-s)} ds$$
$$c(s) = c_{n-1} s^{n-1} + c_{n-2} s^{n-2} + \ldots + c_0$$
$$a(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0$$
$$I_1 = \frac{c_0^2}{2a_0 a_1}$$
$$I_2 = \frac{c_1^2 a_0 + c_0^2 a_2}{2a_0 a_1 a_2}$$
$$I_3 = \frac{c_2^2 a_0 a_1 + (c_1^2 - 2c_0 c_2) a_0 a_3 + c_0^2 a_2 a_3}{2a_0 a_3 (a_1 a_2 - a_0 a_3)}$$
$$I_4 = \frac{c_3^2(-a_0^2 a_3 + a_0 a_1 a_2) + (c_2^2 - 2c_1 c_3) a_0 a_1 a_4 + (c_1^2 - 2c_0 c_2) a_0 a_3 a_4 + c_0^2(-a_1 a_4^2 + a_2 a_3 a_4)}{2a_0 a_4(-a_0 a_3^2 - a_1^2 a_4 + a_1 a_2 a_3)}$$

where:

$$c(s) = c_{n-1} s^{n-1} + c_{n-2} s^{n-2} + \ldots + c_0,$$
$$a(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0.$$

Table 4 provides description for such type of integration up to fourth order in more detail. Using the transfer functions in Table 2 and implementing calculation of Equations (15) and (17), analytical solutions of tracking loop bandwidths can be obtained, for both with and without velocity aiding. After calculation, the results are summarized in Table 5. The analytical solutions are complicated but, from another viewpoint, very general, too. Abundant information can be gained for the analysis of noise bandwidth based on these analytical solutions.

Parameters $K = K_0 K_d = 10$ and $\tau_1 = \tau_2 = 0{\cdot}1$ are selected for investigation to compare the bandwidths of un-aided and aided loops. When these three parameters are fixed, the bandwidths of un-aided loops are obtained, which are 2·5 Hz, 5 Hz and 12·5 Hz for the first-, second- (active), and third-order tracking loops, respectively. For the aided loops, the bandwidths are now governed by the quality of inertial aiding, that is, two aiding parameters: the velocity scale factor error b and processing lags/delayed time τ. With one of these two values reduced, the bandwidth increases. The bandwidth is reduced at a relatively smooth rate, with b increasing from 0 to 1 and is very sensitive when τ is small, where a small increase in τ results in large reduction of bandwidth. This information tells us that a small τ ($< 0{\cdot}001$) needs to be selected for obtaining a sufficient large bandwidth. When $b = 1$, it becomes an un-aided case, and the bandwidth is equal to that of an un-aided loop. We have seen that the external velocity aiding helps widen the bandwidth. A second-order loop with active filter is presented as an example for illustration. The bandwidths varying with b and τ for the aided loops are shown in Figure 6, which is a three-dimensional plot for better illustration of the influence of parameters b and τ. When the two aiding parameters are both small, $b = \tau = 0{\cdot}001$, the bandwidths will extend up to approximately 255 Hz, which is a much larger extension than the un-aided loops. Therefore, the tracking loops can be operated with very narrow bandwidths to improve SNR and so enhancing resistance to jamming or interference.

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Table 5. Equivalent noise bandwidths, B_n(Hz).

Loop description	Un-aided	Velocity aided
1st order	$\frac{K}{4}$	$\frac{K^2\tau^2+(3-2b)K\tau+(1-b)^2}{4\tau(K\tau+1)}$
2nd order (lag)	$\frac{K}{4}$	$\frac{K^2\tau^3+(3-2b)K\tau^2+[(1-b)^2+\tau_1 Kb^2]\tau+(1-b)^2\tau_1}{4\tau(K\tau^2+\tau+\tau_1)}$
2nd order (active)	$\frac{\tau_2^2 K+\tau_1}{4\tau_1\tau_2}$	$\frac{(\tau_1+\tau_2^2 K)K\tau^3+[(3-2b)\tau_1\tau_2+\tau_2^3 K]K\tau^2+[\tau_1^2 b^2+(3-2b)K\tau_1\tau_2^2]\tau+[\tau_1^2\tau_2(1-b)^2]}{4\tau\tau_1\tau_2(K\tau^2+K\tau_2\tau+\tau_1)}$
2nd order (passive)	$\frac{K(\tau_2^2 K+\tau_1)}{4\tau_1(\tau_2 K+1)}$	$\frac{(\tau_1+\tau_2^2 K)K^2\tau^3+(1+\tau_2 K)[(3-2b)\tau_1 K+\tau_2^2 K^2]\tau^2}{4\tau\tau_1(1+\tau_2 K)[K\tau^2+(1+\tau_2 K)\tau+\tau_1]}$ $+\frac{[\tau_1(1-b)^2+\tau_1^2 Kb^2+4\tau_1\tau_2 K(1-b)+(3-2b)(\tau_1\tau_2^2 K^2)]\tau+(1-b)^2\tau_1^2(1+\tau_2 K)}{4\tau\tau_1(1+\tau_2 K)[K\tau^2+(1+\tau_2 K)\tau+\tau_1]}$
3rd order	$\frac{\tau_2^2 K(2\tau_2^3 K+3\tau_1^2)}{4\tau_1^2(2\tau_2^3 K-\tau_1^2)}$	$\frac{(2\tau_2^3 K+3\tau_1^2)\tau_2^2 K^2\tau^4+[4\tau_2^6 K^3+\tau_1^2 K^2\tau_2^3(10-4b)-2\tau_1^4 K(1-b)]\tau^3}{4\tau\tau_1^2(2\tau_2^3 K-\tau_1^2)[K\tau^3+2\tau_2 K\tau^2+\tau_2^2 K\tau+\tau_1^2]}$ $+\frac{[2\tau_2^7 K^3+(11-8b)\tau_1^2\tau_2^4 K^2+(4b+2b^2-6)\tau_1^4\tau_2 K]\tau^2}{4\tau\tau_1^2(2\tau_2^3 K-\tau_1^2)[K\tau^3+2\tau_2 K\tau^2+\tau_2^2 K\tau+\tau_1^2]}$ $+\frac{[2\tau_2^3 K(3-2b)+\tau_1^2(4b^2+2b-3)]\tau_1^2\tau_2^2 K\tau}{4\tau\tau_1^2(2\tau_2^3 K-\tau_1^2)[K\tau^3+2\tau_2 K\tau^2+\tau_2^2 K\tau+\tau_1^2]}$ $+\frac{(2\tau_2^3 K-\tau_1^2)\tau_1^4(1-b)^2}{4\tau\tau_1^2(2\tau_2^3 K-\tau_1^2)[K\tau^3+2\tau_2 K\tau^2+\tau_2^2 K\tau+\tau_1^2]}$

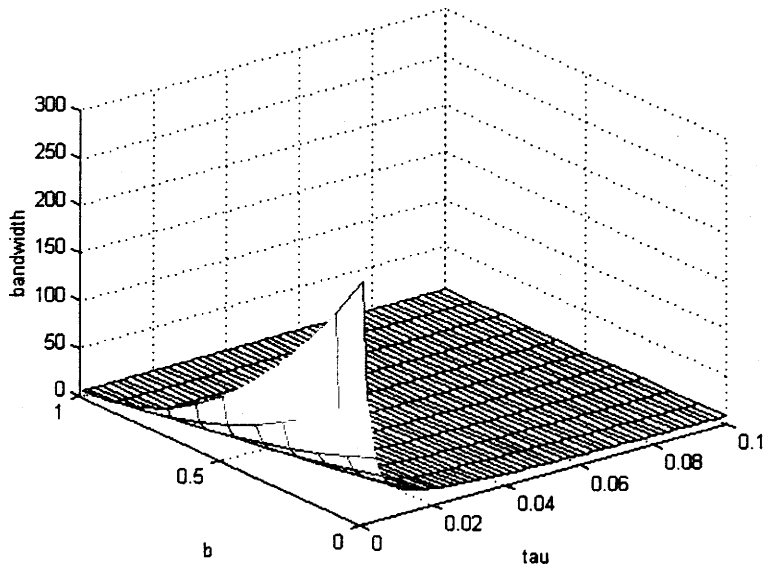

Figure 6. Bandwidth wideness due to inertial velocity aiding for a second-order tracking loop with active filter.

7. CONCLUSIONS. The performance enhancement of a GPS receiver created by external velocity aiding has been investigated. The tracking loops from first through third order have been covered in this discussion. The system architecture of the tracking loop under external velocity aiding has been established, and the mathematical derivation for the closed-loop transfer functions, error transfer functions, and noise bandwidths, has been performed and analytical solutions provided. Several numerical examples, including the transient response and noise bandwidth, are provided for illustrating the benefits of aiding the GPS tracking loop with inertial sensors. Substantial performance improvements, such as steady-state error reduction, faster transient response and bandwidth extension, are obtained through inertial velocity aiding to the GPS receiver.

ACKNOWLEDGEMENTS

The author gratefully acknowledges the support of the National Science Council of the Republic of China under Grant NSC 89-2211-E-019-014.

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