Kantian Review, 19, 2, 305–313 © Kantian Review, 2014
doi:10.1017/S1369415414000077

On Kästner's Treatises

IMMANUEL KANT
Translated by Christian Onof and Dennis Schulting

Note from the Translators

The text of this translation is based on the Akademie text contained in volume 20 of *Kant's Gesammelte Schriften*, ed. Königlich Preussischen, later Deutschen Akademie der Wissenschaften zu Berlin (Berlin: de Gruyter, 1900–). We used bold type where in the original spaced type (*Sperrdruck*) is used. For easy cross-reference, the Akademie page numbers have been inserted in square brackets in the text below. All other insertions in square brackets are ours. For clarification we sometimes give the original German word or phrase in parentheses. We have also consulted the translated excerpts in Allison (1973), the Italian translation by Claudio La Rocca (Kant 1994), and the French translation by Michel Fichant (Kant 1997).

We want to thank Claudio La Rocca for providing us with the Italian translation, contained in his edition of Kant's essay against Eberhard (Kant 1994). We also thank Piter van Tuinen and Philip Westbroek for their assistance with the translation of the text fragment from Raphson's *De spatio reali* and an anonymous referee for *Kantian Review* for helpful comments on the main text. In addition, we thank Wolfgang Ertl and Thomas Land for their comments on the translation, and Wolfgang also for clarifying the meaning of the scholastic concept of 'res'.

[AA 20: 410] Pieces from a Kästner or Klügel[1] can provide value to any collection, without their exactly having the intention to reveal the truth in matters where others would have been in error. The three treatises[2] by Councillor Kästner in this second volume[3] concern the manner in which the geometer can meet the demands which can be made on him because of [the issue of] the possibility of his object, its determination and the unprovable principles governing it, and are wholly limited to mathematics, which is not at all favourable to the assertions of Mr Eberhard; since precisely the contrast of its ability to the inability of metaphysics to meet these demands in any way (provided this happens with the certainty that one can request from all putative rational cognition) lets the latter appear in so much more a disadvantageous light.

On p. 391 of the second volume, it is correctly said that 'Euclid assumes the possibility of drawing a straight line and describing a circle [411] without proving it',[4] which amounts to saying: without proving this possibility **by means of inferences**; for the **description**, which occurs *a priori* by means of the imagination in accordance with a rule, and is called construction, is itself the proof of the possibility of the object. The mechanical **drawing** (*Zeichnung*) (p. 393) which presupposes it [i.e. the construction] as its model is hereby not at all taken into consideration. That, however, the possibility of a straight line and a circle cannot be demonstrated **mediately** through inferences, but only immediately through the construction of these concepts (which is not at all empirical), is due to the fact that among all constructions (exhibitions which are determined according to a rule in intuition *a priori*) some must be **the first**, such as the **drawing** (*Ziehen*) or the describing (in thought) of a straight line and the **rotation** thereof around a fixed point, where neither the latter can be derived from the former, nor these from any other construction of the concept of a magnitude. The constructions of other concepts of this sort in space are all derived in geometry, and this derivation Mr Kästner calls the **demonstration** of their possibility. Against this manner of assuming the possibility of that in regard to which one is immediately conscious of the ability to construct its concept, the *Critique* has nothing in the slightest to say. Rather it cites this as an [412] example for dogmatic metaphysics, in order to do the same for its own concepts, whereby it is noted: that, if no exhibition in the intuition (whether, as is the case with concepts of geometry, this be possible [merely] *a priori*, or also, as with those of physics, empirically) were added to the concept, then we could not make out by means of mere concepts that such a thing, as is thought under the concept of **magnitude**, or which corresponds to the concept of cause, is possible. This reservation and the demand, based upon it, that metaphysics provide for all its concepts a corresponding intuition (for which it already suffices when one connects according to a rule of combination which can also be exhibited in intuition, that which is given in some intuition), is here of utmost importance. For with all due respect for the principle of [non-]contradiction and without in the slightest offending against it, metaphysics can initially introduce *a priori* concepts, which can be formed in pure intuition (as in geometry); then such concepts as can at least be formed in experience (such as the concept of cause); and further such concepts which, to be

https://doi.org/10.1017/S1369415414000077 Published online by Cambridge University Press

sure, cannot without contradiction be formed in any conceivable example, [413] but in many other respects (e.g. practical) are very reasonable; lastly however let all enthusiastic delusion and alleged philosophical insight into which one in fact has no insight at all, creep in.[5] For all barriers to the freedom of waxing lyrical are removed, as soon as one frees the subtle reasoner (*Vernünftler*) from the obligation to prove the objective reality of his concepts of things, of which he claims to have theoretical knowledge, by means of intuition (which though is not a **seeing,**[6] but a representation of the particular insofar as it is not merely thought, but given for thought) without which guarantee he goes into raptures[7] among mere thought entities.—Very wisely but of little consolation for Mr Eberhard, Councillor Kästner says therefore (p. 402): '**I leave it undecided** whether outside geometry the possibility of exhibiting a thing *a priori* can be set forth in such a way that one thereby shows that there is no contradiction in its concept.' He adds, in a very apposite and illuminating way: 'Euclid would demand of Wolff (in respect of the possibility of a most perfect being)[8] [414] that he make (*machen*) a most perfect being; namely in precisely the sense in which Euclid makes[9] the icosahedron, in the understanding.'[10] The latter cannot mean that this solid figure is **in** the intellect, but it only means that according to a **rule**, which the understanding thinks, that concept is given a corresponding intuition *a priori* (in the imagination).

Thus the concept of '**decahedron**' does not contain a contradiction, but the mathematician does not hold it to be valid that, just because this **concept** is possible, its **object** is possible, but demands that one [415] exhibit it in intuition, as it is then shown that this concept does indeed not contradict itself but does contradict the conditions of the construction of a regular solid.[11]

The demand placed on the metaphysician would therefore be this one: he must **represent** (*vorstellig machen*) by means of some example what he means by reality, i.e. the absolutely positive[12] of things; but he can only get [this example] from objects of experience, of which everything which one can call real in them, is according to its essential constitution dependent on conditions, delimited and inseparably connected with negations, so that one cannot leave these out of the concept of reality without at the same time cancelling it out, hence no example (corresponding intuition) can be found for the concept of pure reality, still less for the idea of the connection in one entity of all, however heterogeneous reality; this demand therefore forces the metaphysician

https://doi.org/10.1017/S1369415414000077 Published online by Cambridge University Press

to admit that in this case as for the concept of a supersensible being in general, its possibility ([i.e.] the objective reality of its concept) can simply not be proven.

[416] The expression of Mr Kästner is therefore, if somewhat striking, meaningful and good, and the *Critique* can always grant him that in order to prove the possibility of a thing it is not enough that one find no contradiction in its concept, but one must be able to **construct** (*machen*) its object in the understanding, either, as in geometry, by means of pure intuition (in the construction of the concept), or, as in natural science, from the matter and in conformity with the rules that experience furnishes us.

That which Councillor Kästner, on pp. 403–19, expounds on the representation of space, is entirely meant for the mathematician, in order to determine the legitimate use that the latter makes of that representation, and is just as disadvantageous for Mr Eberhard as the preceding, as it is said on p. 405: 'Whatever one wants to call this concept of geometric space, **either pictorial or non-pictorial**, I leave that to whoever determines the meaning of these words. For me, it is abstracted from sensible representations.'[13] Mr Eberhard's whole exposition of space, however, revolves around those expressions, and it would be impossible for him to determine their meaning.

When Mr Kästner says: for him, as mathematician, the concept of [417] space is abstracted from sensible representations, this can likewise hold for the metaphysician; for without application of our sensible capacity of representation to actual objects of the senses, even that which may be contained *a priori* in these would not become known to us. This should however not be understood as if that representation of space were first to originate from and be generated through the sensible representation, which would contradict the properties of space, which in mathematical propositions are grasped *a priori*, (p. 406) 'not demonstrated through observation, measurement and weighing up (but *a priori*)'.[14]

Since that which is expounded from p. 407 to [p.] 419 merely concerns the use of the concept of the infinite **in geometry**, it lies outside the scope of this review. However, since it might seem to Mr Eberhard and others that this should equally have been a refutation of the infinity of space, of which the *Critique* says that it is inseparably attached to this representation, it is appropriate for a review of a journal which has made metaphysics its main [418] subject to point out the different uses of the concept of infinity in both sciences.

[419] Metaphysics must show how one can **have** the representation of space, geometry however teaches how one can **describe** a space, viz., exhibit one in the representation *a priori* (not by drawing). In the former, space is considered in the way it is **given**, before all determination of it in conformity with a certain concept of object. In the latter, one [i.e. a space] is **constructed** (*gemacht*). In the former it is **original** and only one (unitary) space, in the latter it is **derived** and hence there are (many) spaces, of which the geometer however, in accord with the metaphysician, must admit as a consequence of the foundational representation of space, that they can only be thought as parts of the unitary original space. Now one cannot name a magnitude, in comparison with which each assignable [unit] of the same type is only equal to a part of it, anything other than **infinite**. Thus, the geometer, as well as the metaphysician, represents the original space as infinite, in fact as infinitely given. For this is in itself specific to the representation of space (and in addition that of time), such as can be found in no other concept: that all spaces are only possible and thinkable as parts of one single space. Now when the geometer [420] says that a line could always be extended no matter how far one has drawn it, then this does not mean what is said in arithmetic about numbers, namely that one can always, and endlessly, increase them through the addition of other units or numbers (for the added numbers and magnitudes, which are expressed through it, are for themselves possible, without them having to belong, together with the previous ones, as parts, to one magnitude); rather, that a line can be extended to infinity means so much as: the space in which I describe the line, is greater than every one line which I may describe in it; and thus the geometer grounds the possibility of his task of increasing a space (of which there are many) to infinity on the original representation of a unitary, infinite, **subjectively given** space. Now that the geometrically and objectively given space is always **finite** agrees completely with this; for it is only given through its being **constructed** (*gemacht*). That, however, the metaphysically, i.e. originally, nonetheless merely subjectively given space, which (because there is no plurality thereof) cannot be brought under any concept which would be constructible, but to be sure contains the ground of the construction of all possible geometrical concepts, is **infinite** [421] only indicates that it consists in the pure form of the sensible mode of representation of the subject, as *a priori* intuition; hence in this, as singular representation, the possibility of all spaces, which goes to infinity, is **given**. With this also agrees entirely what Raphson,[15]

according to Councillor Kästner's quotation on p. 418, says, [namely] that the mathematician is always only concerned with an *infinito potentiali* [a potential infinite], and [that] *actu infinitum* (the metaphysically given [infinite]) *non datur a parte rei, sed a parte cogitantis* [an infinite in actuality[16] is not given on the side of the thing, but on the side of the thinker];[17] this latter mode of representation is however not thereby fabricated and false, but rather lies at the foundation of the infinitely progressing constructions of geometrical concepts, and leads metaphysics to the subjective ground of the possibility, i.e. the ideality, of space, with which and the [422] debate on this doctrine the geometer is absolutely not concerned; for he would have to get involved in a dispute with the metaphysician about how the difficulty that space, and everything that fills it, is infinitely divisible and nevertheless does not consist of infinitely many parts, is to be resolved.

In all this the reviewer finds Councillor Kästner fully in accord with the *Critique of Pure Reason*, also there where, on p. 419, he says of geometrical doctrines: 'Never does one infer from the image, but from that which the understanding thinks when considering the image (*beym Bilde denkt*).' He undoubtedly understands by the first the empirical drawing, by the second the pure intuition that accords with a concept, i.e. a rule of the understanding, namely the construction of it [i.e. of the concept], which is not an empirical exhibition of the concept. When, however, he cites the *Philosophisches Magazin*, as if herewith he had hit on and confirmed Mr Eberhard's opinion of the **pictorial** (*Bildlichen*) in contrast to the intelligible, then he is much mistaken. For he [i.e. Eberhard] understands by **pictorial** (*Bildlichen*) not something like the figure in space as geometry would view it, but space itself (although it is hard to understand, how one could form an **image** (*Bild*) of something external to oneself [423] without presupposing space); and his intelligible (*Intelligibeles*) is not for instance the concept of a possible object of the senses, but of something that the understanding must represent, not in space, but as its ground on the basis of which it [i.e. space] can first be explained. But he will readily be excused for this misunderstanding by anyone who has felt the difficulty of connecting a self-consistent concept with this expression that is used so diversely by Mr Eberhard.

Notes

1 Georg Simon Klügel (1739–1812) was a professor of mathematics in Halle, and a student of Kästner's. In the *Philosophisches Magazin* (see n. 3 below), he published *Grundsätze der reinen Mechanik*. Kant had a high regard for Klügel (cf. AA 11: 257).

2 The treatises concerned here are: *Was heißt in Euklids Geometrie möglich?*, *Ueber den mathematischen Begriff des Raums* and *Ueber die geometrischen Axiome*. See preceding introductory essay.

3 It concerns the second volume of the *Philosophisches Magazin*, which was published, in four volumes, by the Wolffian philosopher J. A. Eberhard (1739–1809) from 1788 to 1792. The second volume was published in 1790; Kästner's papers appear in the 'viertes Stück' on pp. 391–430. The primary aim of the journal was to attack the Critical philosophy from a broadly Leibnizian-Wolffian perspective. This provoked Kant into his one major public response, aside from his response in the *Prolegomena* to criticism concerning his idealism, to his critics in his *On a Discovery whereby any New Critique of Pure Reason is to be made Superfluous by an Older one* (1790), which was mainly directed at Eberhard. See further the Introduction above.

4 Kant paraphrases here. Kästner actually writes: 'Euclid's demands, αἰτηματα, are: To draw a line from each point to each point, to extend each bounded straight line as far as one wants, to describe a circle around every centre with every radius. / That is all that he accepts as possible, without proving that it is possible' (*Was heißt in Euklids Geometrie möglich?*, p. 391). Kästner quotes from Euclid's Postulates in book 1 of *The Elements*: 'I. A right line may be drawn from any one point to any other. II. A terminated right line may be produced to any length in a right line. III. A circle may be described from any centre, and with any distance from that centre as a radius' (Casey 1885).

5 In his Italian translation of the text (Kant 1994: 159 n. 5), La Rocca notes that Kant here plays on the double meaning of *Einsicht*, which he says can mean in German 'comprehension' but also 'intuition, vision'. In English, the term 'insight' also has the connotation of 'intuitive understanding'. Kant often lambasts the claim made by exalted (*schwärmerische*) thinkers that one has an immediate insight into or a vision of the truth of something. On Kant's critique of enthusiasm or exaltation in philosophy, see e.g. AA 8: 389–406.

6 Kant alludes here to the idea, propagated by exalted amateur philosophers such as Johann Georg Schlosser (1739–99), of a 'premonition (*praevisio sensitiva*) of that which is not an object of the senses at all, viz., an *intimation* of the super-sensible' (AA 8: 397; Kant 2002: 438). Interestingly, in this same essay in which he decries obscurantism in philosophy and which he published a few years later than when he wrote on Kästner, Kant defends a proto-Critical Plato against pseudo-Plato (the Plato of the apocryphal letters, which were translated into German in 1795 by the aforementioned Schlosser) in regard to the possibility of *a priori* intuition as a requirement to extend knowledge beyond concepts (AA 8: 391–3, esp. 391n). According to Kant, Plato's theory of *a priori* intuition (i.e. the indirect, ectypal cognition of the Ideas by means of anamnesis) must be seen in terms of enabling a 'regressive' explanation of the possibility of *a priori* knowledge, not as part of a 'progressive' claim that would extend our cognition, by means of the Ideas, beyond sensibility through any sort of intellectual vision or intimation (AA 8: 398). Thus, by using the term 'seeing' here, Kant markedly links any type of philosophy that makes a claim to being able to conceive of the particular merely by means of concepts directly to obscurantism, which is 'the death of all philosophy' (ibid.; Kant 2002: 438).

7 Kant uses here the verb 'herum zu schwärmen', alluding to *Schwärmerei* among certain contemporary esoteric philosophers, i.e. the tendency to adopt an exalted tone and to boast of direct philosophical insights that are unavailable to the uninitiated, not least those who profess adherence to more mundane academic philosophy. See further nn. 5 and 6 above.

8 The clause between parentheses is Kant's insertion.

9 In both occurrences here of the verb 'to make', this translates the German verb *machen* (which Kästner uses). As noted in our introduction, Kant interprets this as 'constructs'.

10 See Euclid, *Elements*, XI, def. xxvii; XIII, prop. 16.

11 We follow Moretto (2011: 269 n. 36) here, who substitutes 'solid' for 'body' (*Körpers*).

12 Kant has in mind here the connection between the traditional ontological notion of 'realitas' and the modal category of the 'absolute position' of things.

13 This quote is from Kästner's *Ueber den mathematischen Begriffs des Raums*, in *Philosophisches Magazin*, 2/4 (1790), 403–19 (Kant's emphasis).

14 This is a paraphrase of Kästner's text, rather than a direct quotation.

15 Kästner quotes the tract *De spatio reali, seu ente infinito, conamen mathematico-metaphysicum* by Joseph Raphson, in his *Analysis aequationum universalis* (London, 1690, 2nd edn 1702). Raphson (1648–1715) was an English mathematician who was in close contact with Newton. In aforementioned work, apart from the putative differentiation between the potential infinite and the actual infinite that Kant alludes to, Raphson discussed the method (now known as the Newton-Raphson method) for approximating the roots of an equation more than forty years before it was actually published under Newton's own name, in *Method of Fluxions* (1736).

16 Kant probably misreads *infinitum actu* here as meaning 'actual infinite' as distinct from 'potential infinite'; hence his insertion of 'the metaphysically given', thus putatively signifying a type distinction between metaphysically given space and determinate geometric space. But as in Fichant's French translation (Kant 1997), *infinitum actu* should be translated as 'infinite in act' or 'infinite in actuality' (Fichant translates it as 'un infini en acte'). But more importantly, as Fichant (1997b: 44–5) points out, what Raphson and presumably Kästner in his quoting of Raphson mean here is that it is the *potential infinite* that can in actual fact not be contained in the things themselves, but only exists 'on the side of the thinker' (*a parte cogitantis*). Fichant (1997b: 44) suggest that, also because he had not Raphson's text itself to hand but relied on Kästner's part quotation part paraphrase, Kant misreads 'such an infinite (*huiusmodi infinitum*)' in the Raphson quote (see n. 17 below, section 6) as referring to 'infinite in actuality (*infinitum actu*)' in the previous section 5, rather than to the potential infinite that Raphson mentions in section 4. However, apart from the fact that the Latin *huiusmodi* refers to the nearest noun (in this case, *infinitum actu*), we do not think that the reference itself is unambiguous, although Fichant is right to point out that from the following in section 6 it is clear that Raphson means the *potential* infinite, when he says that the infinite is in the mind rather than in the objects, thus exactly the opposite of how Kant reads it. At any rate, it seems clear that Raphson does not make a clear type distinction between a potential and an actual infinite, where the latter would be metaphysically and subjectively given, as Kant suggests, and the former concern geometric, objective space as constructible in the understanding (see further Fichant 1997b: 41–6). Raphson's position is best understood as a development of Henry More's conception of space as an infinite absolute immaterial reality. Following More's critique of Descartes's understanding of space as extension, Raphson argues for an 'infinite, immovable, immaterial' space (Koyré 1957: 191) from the following considerations about motion: 'Thus from every motion (extended and corporeal), even from the [only] possible ones, follows necessarily [the existence of] an immovable and incorporeal extended [entity], because everything which moves in the extension must necessarily move through extension' (Raphson, *De spatio reali*, ch. 4, p. 67; translation by Koyré 1957: 191). So if Raphson claims

that infinite space is not material or corporeal, this by no means entails that it is *in* the mind, but rather that it constitutes an immaterial (quasi-divine) reality. Cf. Fichant (1997b: 41–2).

17 Raphson writes the following (Kant's reference concerns specifically section 6): '1. The infinite is an ambiguous term. Hence, before it can be defined, we must first differentiate or, as you wish, analyse it. It is used by philosophers in a double sense. We therefore encounter the infinite as twofold: as potential (as one says) and as actual (*actuale*). From this division – which although sufficiently clear is not sufficiently carefully applied – arise many difficulties about the nature of the infinite. 2. Let us start with the potential infinite. We consider this first abstractly according to its nature. After that we shall illustrate its nature by applying certain distinctive examples. / 3. The first characteristic of its nature that presents itself is this: that in actuality (*actu*) it [the infinite] always becomes finite. Otherwise the actual would be infinite. / 4. Secondly, that according to its essence it is unlimited (*interminabile*), or that while unendingly forever and ever progressing it will never reach any limit of that progression. This is how the term "infinite" originated among lay people and why it can conveniently be defined as "interminable finite". / 5. It is obvious that from the above the infinite in actuality can never emerge. / 6. Such an infinite is not given on the side of the object, but merely on the side of the thinker. For every being that exists in actuality is that which exists and it has as much actuality (as the Scholastics say) as it has entity; it is therefore clear that the ground according to which it is called the infinite, that is, its unlimitedness, only exists in the thinker, and not in the object' (*Analysis aequationum universalis*, '*De spatio reali, seu ente infinito*', ch. 3, *De Infinito abstracte considerato*, pp. 37–8; our translation).

References

Allison, H. ed., (1973) *The Kant-Eberhard Controversy*. Baltimore, MD: Johns Hopkins University Press.

Casey, J. (1885) *The First Six Books of the Elements of Euclid*. Dublin: Hodges, Figgis.

Kant, I. (1994) 'Sulle trattazione di Kästner'. In *Contro Eberhard: La Polemica sulla Critica della Ragion Pura*. Ed. and trans. C. La Rocca. Pisa: Giardini, pp. 157–64.

—— (1997) 'Sur les articles de Kästner'. Ed. and trans. M. Fichant. *Philosophie*, 56, 13–19.

—— (2002) *Theoretical Philosophy after 1781*. Ed. and trans. H. Allison et al. Cambridge: Cambridge University Press.

Koyré, A. (1957) *From the Closed World to the Infinite Universe*. Baltimore, MD: Johns Hopkins University Press.

Moretto, A. (2011) 'Matematico'. In S. Besoli, C. La Rocca and R. Martinelli (eds), *L'universo kantiano: Filosofia, scienze, sapere*. Macerata: Quodlibet, pp. 261–313.