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Circularly polarized modes in magnetized spin plasmas

A. P. MISRA, G. BRODIN, M. MARKLUND and P. K. SHUKLA†

Department of Physics, Umeå University, SE-901 87 Umeå, Sweden
(mattias.marklund@physics.umu.se)

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Abstract. The influence of the intrinsic spin of electrons on the propagation of circularly polarized waves in a magnetized plasma is considered. New eigenmodes are identified, one of which propagates below the electron cyclotron frequency, one above the spin-precession frequency, and another close to the spin-precession frequency. The latter corresponds to the spin modes in ferromagnets under certain conditions. In the non-relativistic motion of electrons, the spin effects become noticeable even when the external magnetic field B_0 is below the quantum critical magnetic field strength, i.e. $B_0 < B_Q = 4.4138 \times 10^9$ T and the electron density satisfies $n_0 \geqslant n_c \simeq 10^{32}\,\mathrm{m}^{-3}$. The importance of electron spin (paramagnetic) resonance (ESR) for plasma diagnostics is discussed.

1. Introduction

During recent years there has been a rapid increase in the interest of quantum plasmas (see e.g. [1–15]). This has been stimulated by experimental progress in nanoscale plasmas [6], ultracold plasmas [16], spintronics [17], and plasmonics [18]. However, already more than 40 years ago, Iannuzzi [19] established the possibility for the existence of electron spin (paramagnetic) resonance (ESR) in a fully ionized low-temperature plasma, and predicted its importance, e.g. in the plasma diagnostics for measuring the particle density with a greater precession than the conventional technique, in determining the particle velocity spectrum perpendicular to the magnetic field, as well as to calculate the magnetic field in the propagation of electromagnetic (EM) waves (e.g. whistlers, Alfvén waves, shock waves, etc.) in plasmas. Recent investigation [20] along this line indicates that besides the currently prevalent laser methods, ESR technique can be successfully used for plasma diagnostics, e.g. measuring the electron densities in the microwave region. Furthermore, the importance of spin effects in plasmas has also been studied using kinetic plasma theory [11–13, 21, 22], with applications to wave propagation [11–13] as well as other phenomena [21, 22]. The hydrodynamic description of spin plasma waves can also be found in the literature (see e.g. [10, 14, 23]).

Although, whistler waves have been studied for almost a century, they are still a subject of intense research in view of its importance not only in space plasmas,

† Also at: Institut für Theoretische Physik IV, Fakultät für Physik and Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

but also in many astrophysical environments, e.g. in the atmosphere of neutron star envelope, in the coherent radio emission in pulsar magnetosphere, etc. Quasilinear theory [24] and simulation [25] show that whistler waves can be used to resonantly accelerate electrons. Furthermore, they can also be used to interpret the fine structure of Zebra-type patterns and fiber bursts in solar type II and IV radio bursts [26]. Thus, the occurrence of ESR might be useful for the electron acceleration in the propagation of EM radiation in the pulsar magnetosphere as well as for plasma diagnostics in the microwave region in laboratory experiments if the conditions favor.

In this paper, we will derive and analyze the dispersion relation for the propagation of circularly polarized (CP) EM (CPEM) waves in a magnetized spin plasma using a spin fluid model. Various fluid models are appropriate in different regimes (see e.g. [9, 10]). The model to be used contains the Bohm-de Broglie potential, the magnetic dipole force and includes the spin-precession dynamics as well as the spin magnetization current. Its basis can be found in, e.g. [10], and more rigorous foundation can be given starting from the kinetic theory presented in [13]. Specifically, we will focus our discussion to the ESR as well as the properties of spin modified whistler-like modes.

2. Weakly nonlinear evolution

The non-relativistic evolution of spin $-1/2$ electrons can be described by [10]

$$(\partial_t + \mathbf{v}_e.\nabla)\,\mathbf{v}_e = -\frac{e}{m_e}\left(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}\right) - \nabla P_e/m_e n_e + \frac{\hbar^2}{2m_e^2}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) + \left(\frac{2\mu}{m_e\hbar}\right)\mathbf{S}.\nabla\mathbf{B}, \tag{1}$$

$$(\partial_t + \mathbf{v}_e.\nabla)\,\mathbf{S} = -\left(2\mu/\hbar\right)\left(\mathbf{B} \times \mathbf{S}\right), \tag{2}$$

where n_e, m_e, $\mathbf{v}_e$, respectively, represent the number density, mass, and velocity of electrons, $\mathbf{E}$ ($\mathbf{B}$) is the electric (magnetic) field, and P_e is the electron pressure. Also, $\mathbf{S}$ is the spin angular momentum with its absolute value $|\mathbf{S}| = \hbar/2$; $\mu = -(g/2)\mu_B$, where $g \approx 2.0023193$ is the electron g-factor and $\mu_B \equiv e\hbar/2m_e$ is the Bohr magneton. The equations are then closed by the Maxwell equations:

$$\nabla \times \mathbf{E} = -\partial_t\mathbf{B}, \nabla.\mathbf{B} = 0, \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0\left(\varepsilon_0\partial_t\mathbf{E} - en_e\mathbf{v}_e + \left(2\mu/\hbar\right)\nabla \times n_e\mathbf{S}\right), \tag{4}$$

The above (1) and (2) apply when different spin states (i.e. spin-up and spin-down relative to the magnetic field) can be well represented by a macroscopic average. This may occur for very strong magnetic fields (or a very low temperature), where generally the lowest energy spin state is populated. Alternatively, when the dynamics on a time-scale longer than the spin-flip frequency is considered, the macroscopic spin state is well described by the thermodynamic equilibrium spin configuration, and the above model can still be applied. In the later case, the macroscopic spin state will be attenuated by a (thermodynamic) factor decreasing the effective value of $|\mathbf{S}|$ below $\hbar/2$. However, this case will not be considered further in the present paper. As a consequence, our studies will be focused on the regime of strong magnetic fields, as seen in astrophysical plasmas [27].

In what follows, we will assume the propagation of a CPEM waves to be of the form $\mathbf{E} = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})E(z,t)\exp(ikz - i\omega t) +$ c.c., along an external magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$, where $E(z,t)$ is the weakly modulated wave amplitude (i.e. we assume $|(1/f)\partial f/\partial z| \ll k, |(1/f)\partial f/\partial t| \ll \omega$, for all variables f), k (ω) represents the EM wavenumber (frequency) and c.c. denotes the complex conjugate. In the interaction of high-frequency (HF) EM waves with the HF electron plasma response, the use of cold plasma approximation is well justified in view of the fact that for large field intensities and moderate electron temperature, the directed velocity of electrons in the HF fields is much larger than the random thermal speed. Moreover, it can also be shown that the density perturbation associated with the high-frequency EM wave is zero. Thus, taking the curl of (1) and using (2)–(4) we readily obtain the following evolution equation for CPEM waves:

$$0 = \frac{e}{m_e}\partial_t \mathbf{B} + \frac{\varepsilon_0}{en_e}\partial_t\left(\partial_t^2\mathbf{B} + \frac{1}{n_e}\nabla n_e \times \partial_t\mathbf{E}\right) - v_{ez}\nabla\times\partial z\mathbf{v}_e + \frac{1}{e\mu_0}\partial_t\left[\frac{1}{n_e}\nabla\times(\nabla\times\mathbf{B})\right]$$
$$- \frac{2\mu}{e\hbar}\partial_t\left[\frac{1}{n_e}\nabla\times(\nabla\times n_e\mathbf{S})\right] + \frac{1}{m_e\mu_0 n_e}\nabla\times[(\nabla\times\mathbf{B})\times\mathbf{B}] + \frac{2\mu}{m_e\hbar}\nabla\times(S^a\nabla B_a)$$
$$- \frac{\varepsilon_0}{m_e n_e}\nabla\times(\partial_t\mathbf{E}\times\mathbf{B}) - \frac{2\mu}{m_e\hbar n_e}\nabla\times[(\nabla\times n_e\mathbf{S})\times\mathbf{B}]. \qquad (5)$$

The weakly nonlinear (5) is a useful result when considering the interaction between low-frequency (LF) and HF fields, where the LF fields are induced by the ponderomotive force. Equation (5) then needs to be complemented by equations for the LF dynamics, and naturally the number of dependent variables can be further reduced. However, before this line of research is pursued, a more thorough study of the linear theory should be made, as will be undertaken in the next section.

3. Linear theory of whistler waves

Introducing the variables $B_\pm = B_x \pm iB_y$, $E_\pm = E_x \pm iE_y$, etc., suitable for CP waves, and limiting ourselves to the linearized theory, we obtain respectively from the Faraday's law and the spin-evolution equation [15]:

$$B_\pm = \pm\frac{ik}{\omega}E_\pm,\ S_\pm = \mp\frac{2\mu\,|S_0|\,B_\pm}{\hbar(\omega\pm\omega_g)}. \qquad (6)$$

Using (6) to express the free current as well as the magnetization current in terms of the electric field, and using (4), the following linear dispersion relation is obtained for the CPEM modes

$$n_R^2 = 1 - \frac{\omega_{pe}^2}{\omega\,(\omega\pm\omega_c)} - \frac{g^2\omega_{pe}^2 k^2\,|S_0|}{4\omega^2 m_e(\omega\pm\omega_g)}, \qquad (7)$$

which can be rewritten as

$$n_R^2\left(1 + \frac{\omega_\mu}{\omega\pm\omega_g}\right) = 1 - \frac{\omega_{pe}^2}{\omega\,(\omega\pm\omega_c)}, \qquad (8)$$

where $n_R \equiv ck/\omega$ is the refractive index, and where the upper and lower sign, respectively, stand for the left-hand circularly polarized (LCP) and right-hand circularly polarized (RCP) waves. Also, $\omega_\mu = g^2\hbar/8m_e\lambda_e^2$ is a frequency which involves the spin correction due to plasma magnetization current and $\lambda_e \equiv c/\omega_{pe}$ is the electron skin-depth (inertial length scale). Moreover, $\omega_{pe} \equiv \sqrt{n_0 e^2/\varepsilon_0 m_e}$, $\omega_c = eB_0/m_e$ and $\omega_g = (g/2)\omega_c$ are respectively the electron plasma, cyclotron and the spin-precession frequency. In absence of the spin-magnetization, the well-known classical dispersion relation, namely $\omega^2 = c^2k^2 + \omega\omega_{pe}^2/(\omega \pm \omega_c)$ is recovered. A few comments are in order. The first and the second term in the right-hand side of (7) appear due to the displacement current and the free electron current. The term involving ω_μ appears even in absence of the external magnetic field, since the spin perturbation is due to the wave magnetic field and not due to the constant field B_0 which does not provide any magnetic dipole force. Note, however, that an unperturbed spin state with $|\mathbf{S}_0| = \hbar/2$, which has been used in the derivation, typically requires that the external magnetic field is strong. Thus, inclusion of the electron spin perturbation leads to a modification of the dispersion relation for transverse plasma oscillations. This modification is clearly substantial when $\omega < \omega_g \lesssim \omega_\mu$, i.e. when $\hbar\omega_c \gtrsim m_e c^2$ for $\omega_{pe} \lesssim \sqrt{2}\omega_c$, where c is the speed of light in vacuum. This corresponds to a regime of very strong magnetic field in which the external field strength approaches or exceeds the quantum critical magnetic field, i.e. $B_0 \gtrsim B_Q \equiv 4.4138 \times 10^9\,T$. In such a situation relativistic effects [28] might be important. On the other hand, for the non-relativistic motion of electrons we have $\hbar\omega_c < m_e c^2$, i.e. $B_0 < B_Q$ for $\omega_{pe} > \sqrt{2}\,\omega_c$. In this case, the density regime in which the magnetic field is 'non-quantizing' and does not affect the thermodynamic properties of the electron gas, is $n_0 \gg n_c \simeq 10^{32}\mathrm{m}^{-3}$ and the temperature $T_e \gtrsim T_B \simeq \hbar\omega_c/k_B$, where k_B is the Boltzmann constant. Thus, in the strong magnetic field and highly dense plasmas, the electron spin effect can no longer be neglected, rather it modifies the wave dispersion leading to new eigenmodes.

Inspecting now the term $\propto \hbar$ in (7), we note that

$$\frac{\hbar k^2}{m_e\omega} = \left(\frac{\hbar\omega_c}{m_e c^2}\right)\left(\frac{c^2k^2}{\omega^2}\right)\left(\frac{\omega}{\omega_c}\right). \tag{9}$$

So, the spin current can be much larger than the classical free current when $|J_{M\pm}/J_{C\pm}| \sim \hbar k^2/m_e\omega \gg 1$. This basically holds when either (i) $\hbar\omega_c \gtrsim m_e c^2$, $\omega < \omega_c, ck$ or (ii) $\hbar\omega_c < m_e c^2$, $\omega < \omega_c$, and $\omega \ll ck$ is satisfied. The case of $\omega > \omega_g > \omega_c$ is rather less important, as it does not give rise to wave propagation ($n_R^2 < 0$). Also, in the very LF regime $\omega \ll \omega_c$, the ion motion can be of importance, and we will therefore not consider that case. Thus, one important mode could be the RCP LF ($\omega < \omega_c$) EM waves (whistlers). Let us now see how the dispersion relation reduces when either of the two cases is considered. In the limit of $|J_{M\pm}/J_{C\pm}| \sim \hbar k^2/m_e\omega \gg 1$, the dispersion (8) reduces to

$$(\omega \pm \omega_g)(1 - \omega^2/c^2k^2) + \omega_\mu = 0, \tag{10}$$

from which one finds for $\omega \ll ck$ a purely spin-modified frequency $\omega_1 \approx \mp\omega_g - \omega_\mu$. Also, if $\omega_\mu \ll ck$ and $\omega_\mu \ll \omega_g$, we have $\omega_2 \approx \mp\omega_g$ and $\omega_3^2 \approx c^2k^2$. The frequencies $\omega_{1,2}$ may correspond to the spin waves in ferromagnets under certain conditions [29]. Figure 1 displays the modes for RCP waves obtained as numerical solutions of the dispersion equation. Evidently, there exist two eigenmodes apart from an HF ($\omega > \omega_g$) one, namely a mode close to the electron-cyclotron or spin-precession frequency, and the other one is the LF mode below the cyclotron frequency. In contrast to the HF modes, the spin modified LF modes propagate with the frequency

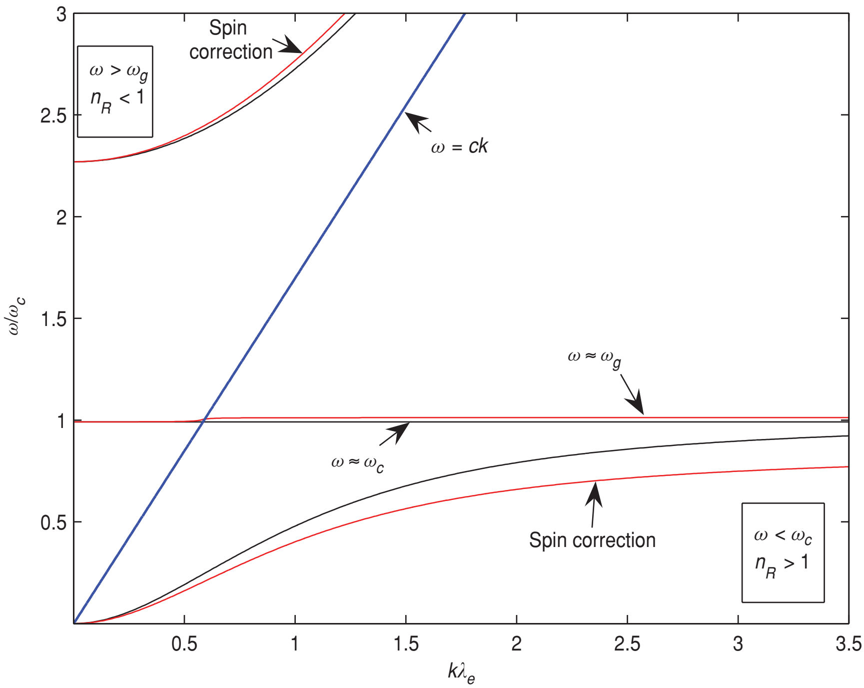

Figure 1. (Colour online) Different eigenmodes for RCP waves obtained as numerical solutions of the dispersion (7) are shown with respect to the normalized wavenumber and frequency for $B_0 = 5 \times 10^8\,\mathrm{T} < B_Q$, $n_0 = 7 \times 10^{36}\,\mathrm{m}^{-3} \gg n_c$.

below that in the classical case. On the other hand, in the very HF regime ($\omega \gg \omega_g$), $n_R^2 \approx 1 > 0$, and so we have $\omega^2 = c^2k^2$, which is the dispersion relation of a EM wave in vacuum. Evidently, since $n_R^2 < 0$ for $\omega > \omega_g$, there must exist an intermediate frequency $\omega(> \omega_g)$ at which the solution for n_R^2 must pass through a zero value, and becomes positive again. Thus, the cut-off frequencies for which $n_R^2 = 0$ are obtained as

$$\omega_{R,L} = \frac{1}{2}\left(\mp\omega_c + \sqrt{\omega_c^2 + 4\omega_{pe}^2}\right),$$

where $\mp$ stand for RCP and LCP waves respectively. Clearly, the RCP waves have lower cut-off frequency than the LCP modes. On the other hand, the resonances for the RCP waves ($n_R^2 \to \infty$) associated with both the orbital and the spin-gyration, occur (LCP mode has no resonance) as either $\omega \to \omega_c$ or $\omega \to \omega_g$, i.e. when the angular frequency of the wave electric field matches either due to electron cyclotron motion (cyclotron resonance) or due to the intrinsic spin of electrons (ESR). At the resonance, the transverse field associated with the RCP wave rotates at the same velocity as electrons gyrate around B_0. The electrons thus experiences a continuous acceleration from the wave electric field, which tends to increase their perpendicular energy. Therefore, it is not surprising that RCP waves propagating along the external magnetic field and oscillating at the cyclotron frequency or spin-precession frequency are absorbed by electrons. This may be the consequence to the recently developed experiment based on microwave absorption and the ESR to be successfully used

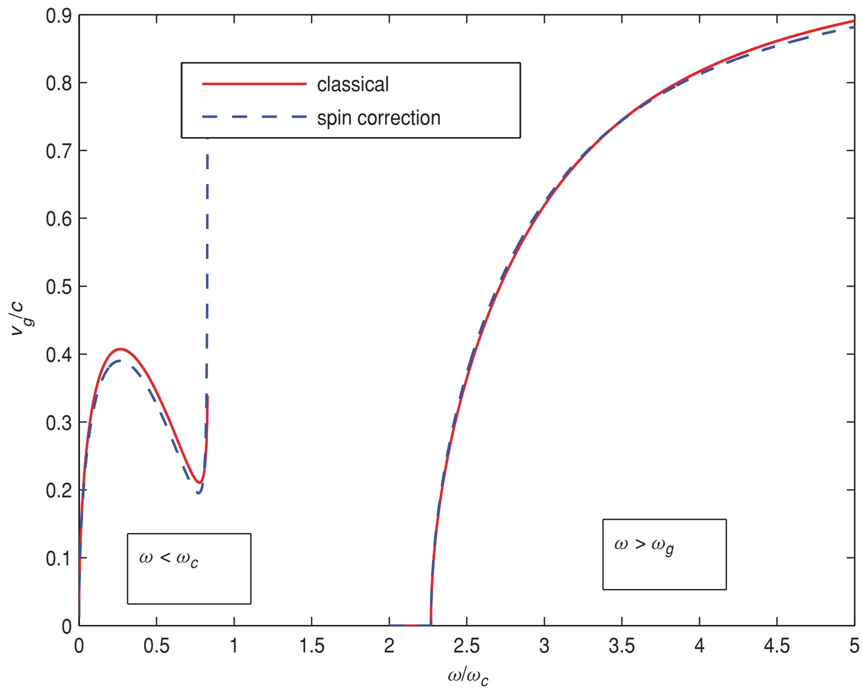

Figure 2. (Colour online) The normalized group speed given by (11) is plotted as a function of the normalized wave frequency for the RCP waves with the same set of parameters as in Fig. 1.

for plasma diagnostics [20]. On the other hand, since the spin effect is appreciable in the strongly magnetized dense plasmas, the ESR could well be relevant for the coherent EM radiation in pulsar magnetosphere or magnetized white dwarfs. Let us now see how the group speed ($v_g \equiv d\omega/dk$) of the CP waves is modified with the spin correction. We obtain from the dispersion relation (see (7))

$$v_g = \Lambda/\left(2\omega/\omega_{pe}^2 + \Gamma\right), \tag{11}$$

where Λ and Γ are given by

$$\Lambda = \frac{2c^2k}{\omega_{pe}^2} + \frac{g^2k\,|S_0|}{2m_e(\omega \pm \omega_g)}, \quad \Gamma = \frac{\omega_c}{(\omega \pm \omega_c)^2} + \frac{g^2k^2\,|S_0|}{4m_e(\omega \pm \omega_g)^2}. \tag{12}$$

Clearly, the LF ($\omega < \omega_c$) component of a pulse (whistlers) propagates more slowly than the HF ($\omega > \omega_g$) components as is evident from Fig. 2. It follows that by the time a pulse returns to a ground level it has been stretched out temporarily, because the HF component of the pulse arrives slightly before the LF components. This also accounts for the characteristic whistling down-effect observed at ground level. Moreover, the group speed v_g of the whistlers increases in the frequency regime $0 < \omega < \omega_c/2$, and decreases in the other subinterval $\omega_c/2 \lesssim \omega < \omega_c$ before it reaches the maximum nearer the resonance point. However, the group speed of the HF modes approaches the speed of light as ω ($> \omega_g$) increases and gets saturated

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at large ω. From Fig. 2 we also note that the spin force reduces the group speed in strongly magnetized dense plasmas.

4. Summary and discussion

To summarize, the dispersion relation for the propagation of HF CPEM waves is obtained in a magnetized spin plasma. The electron spin modifies the plasma current density and thereby introduces a correction term in the dispersion relation, which, in turn, gives rise a new CP HF mode. The spin effects are seen to be substantial in the very strong magnetic field ($B_0 \gtrsim B_Q \equiv 4.4138 \times 10^9 T$) and highly dense plasmas ($n_0 \geqslant n_c \simeq 10^{32} m^{-3}$) where the relativistic effects might be important. However, in non-relativistic plasmas, the spin of electrons can also be important in the case $B_0 < B_Q$ together with $n_0 \geqslant n_c$. In particular, when the spin current dominates over the classical free current the RCP EM waves resonantly interact with the electrons only at the spin-precession frequency. Such resonance should be helpful for particle acceleration in the coherent radio emission of the pulsar magnetosphere or magnetized white dwarfs. The study of the spin modified modes might also be important at least from the diagnostic points of view, since the observation of the propagation characteristics of the wave modes may be used in order to determine the physical parameters in plasmas [30]. Lastly, the electron spin resonance (ESR) could be an important consequence to the recently developed experiment for plasma diagnostics in the microwave region, if the conditions favor [20].

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