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Dispersive mixing: within or between pores?

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We review some of the processes leading to dispersion and mixing in porous media, exploring the differences between the travel time distribution of fluid particles within a pore throat and between pore throats of different size within the porous layer. A recent paper of Liu *et al.* has combined a model of these travel time distributions with a continuous time random walk to quantify the dispersion as a function of the Peclet number. We describe some further problems relating to dispersive mixing of tracer which may be amenable to this approach, including dispersion caused by macroscopic lenses of different permeability, dispersion of tracer which partitions between the fluid and matrix and the effects of buoyancy on mixing.

Key word: mixing

1. Introduction

When fluids move through porous media, the detailed flow field is complex owing to the intricate topology of the pore spaces between the solid matrix and the no slip condition on the matrix surface. This leads to a wide range of travel times of fluid particles as they follow different streamlines from one plane to a second parallel plane downstream. This process forms a progressively more convoluted interface when one fluid displaces a second miscible fluid through the porous layer. When the effects of molecular diffusion are included, the interaction with the mechanical dispersion leads to a gradually growing zone across which the average concentration of fluid varies from the upstream to the downstream value.

Experiments and numerical simulations suggest that the mixing depends on the Peclet number of the flow, $Pe = U\delta/D$, defined in terms of the mean Darcy speed U, the molecular diffusivity D and the typical length scale of the flow δ, often taken to scale with

the mean pore size. The Peclet number can be understood as the ratio of the molecular diffusion time to the advection time across the pore-spaces. High Pe corresponds to relatively fast flow, and it has been argued that the effective dispersion coefficient scales linearly with Pe, whereas for smaller Pe the dispersion coefficient follows a more complex set of power law scalings (Saffman 1959; Koch & Brady 1988).

In the paper by Liu *et al.* (2025), an argument is made that both the fluctuations in the fluid speed within individual pore spaces and also the fluctuations in the flow speed between different pore spaces, of different size, may both play a role in determining the dispersive mixing, and the authors propose that a continuous time random walk can be used to model these processes (cf. Berkowitz, Scher & Silliman 2000). In quantifying this dispersion, they build on the idealised model that porous media consists of a series of pore throats with a range of radii and lengths, which connect at junctions. The fluid particles arriving at a junction from one pore throat may separate into different pore throats as they move downstream.

Within a single pore throat, the low-Reynolds-number flow develops a Pouseille type of shear flow, with a characteristic mean travel time across the pore, as well as a travel time distribution for the fluid particles on different streamlines. With low flow rates, and hence low Pe, the fluid concentration diffuses so as to be uniform at each point along the pore throat. The travel time for the upstream fluid parcels and any associated tracer to migrate along a pore throat therefore depends inversely on the average speed along the pore throat, unless the flow is so slow that diffusion along the pore throat dominates. At higher flow rates, and hence higher Pe, the travel time distribution associated with the Pouseille flow controls the advance of the upstream flow along the pore throat. By combining the travel time distribution for the pore throats of a given size and hence Pe, with the distribution of pore throat sizes, the rate of dispersion of the fluid–fluid front can be calculated. If there is a wide range of pore throat sizes, then both the advective and diffusive limits may apply in the larger and smaller pore throats, respectively, whereas with a smaller range of sizes, one or other Pe number limit typically pertains. Liu *et al.* (2025) have explored the predictions of this model with an idealised series of distributions for the pore throat sizes, and they obtain results for the dispersion in good accordance with published experimental and numerical results. As well as providing an elegant idealised model which is able to characterise these asymptotic dispersion regimes, and which suggests that at larger Pe the dispersion scales as $D_m Pe \ln(Pe)$, the model presented by the authors promises to uncover the transient adjustment to these regimes.

2. Future directions

It would be of great interest to explore whether the approach can be developed for application to a number of more complex mixing problems in porous media. Presently the model relies on the matrix being statistically homogeneous and unbounded. In many geological formations, especially in sedimentary rocks, gradients of grain size and macroscopic layering reflect the time-dependence of the depositional processes forming the rock (Phillips 1991; Woods 2015) (figure 1*a*). As the scale of the fluid flow increases, progressively larger scale heterogeneities may be encountered as suggested by field-scale tracer dispersion data (Cala & Greenkorn 1986). If there are systematic vertical gradients of grain size or multiple layers formed of grains with different size distributions, this may lead to an effective vertical shear and spreading of the fluid–fluid interface at a rate proportional to time (Matheron & de Marsily 1980; Woods 2015). In addition, Eames &

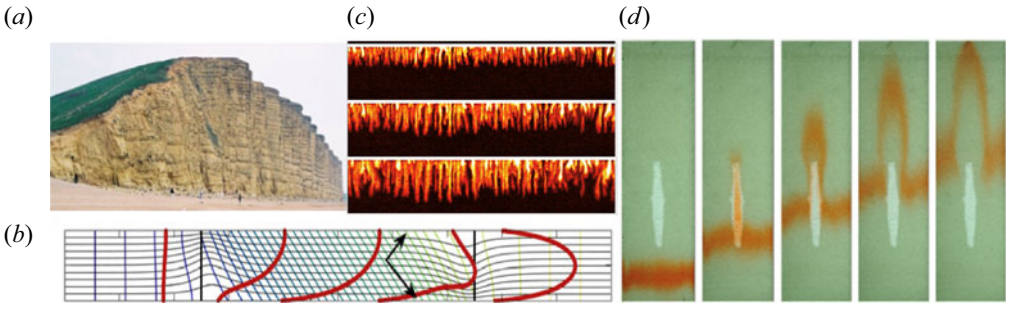

Figure 1. (*a*) Photograph of the Bridport Sandstone, a typical porous rock, with multiple layers. (*b*) Image of a line of tracer (brown) moving from left to right, from a layer of uniform permeability, through a cross-bedded layer, and back to a layer of uniform permeability, illustrating the distortion of the flow in passing between the different regions of rock, and the generation of shear (Bhamidipati & Woods 2020). (*c*) False colour image of the growth of buoyancy-driven fingers as a dense fluid migrates downwards through a bead pack, displacing a less-dense fluid (image courtesy of N. Mingotti). (*d*) Image of a bead-pack experiment, including a lens of high permeability. A band of red dye migrates along the pack, then rapidly moves through the lens and back into the pack, leading to large distortion of the band of dye (Woods 2015).

Bush (1999) have illustrated that if there are localised but large-scale heterogeneities of scale d and permeability ratio k_1/k_2 compared with the far-field distributed randomly in an unbounded layer, then a macro-dispersion of order $(ud/D)f(k_1/k_2)$ develops owing to the fluctuations of the flow field with mean Darcy speed u and where $f(k_1/k_2)$ is a function which depends on the permeability ratio, which they calculate. It would be interesting to see if this can be accommodated in Liu *et al.*'s framework with a multi-peak pore size distribution. Also, if the layer is bounded above and below by impermeable rock, such heterogeneities can in fact lead to large-scale shear and anomalous spreading of the front (figure 1*b*,*d*) again proportional to time (Bhamidipati & Woods 2020, 2021). It would be interesting to extend the Liu *et al.* model to accommodate such effects.

In some natural systems, tracers partition between the solid and the fluid. With rapid molecular diffusion across the pore spaces, this leads to tracer concentration gradients developing in the direction of the flow (Zhang, Hesse & Wang 2017). Physically, one may anticipate that as the Pe number increases, the mechanical dispersion of tracer will lead to the leading part of the tracer being carried by streamlines which are further from the matrix boundaries, thereby suppressing adsorption onto the matrix. The modelling approach of Liu *et al.* may provide a fruitful approach to explore such effects, perhaps by including a source or sink term in the model.

In other cases, there may be density differences across a migrating fluid-fluid interface, for example in geological carbon storage, where an aqueous solution saturated in CO_2 may form through dissolution of pure CO_2. CO_2-saturated solutions are more dense than an equivalent undersaturated solution, and the associated stabilising buoyancy force may act to regulate the mixing if the CO_2-saturated solution migrates through the aquifer (Menand & Woods 2005; Unwin *et al.* 2016). It would again be interesting to evolve the present pore–throat model (Liu *et al.* 2025) to explore these buoyancy regulated flow regimes especially for higher Pe flows. This may also open up new approaches for modelling the effect of high Pe on the mixing of fingers formed in the Rayleigh Taylor instability in a porous media (figure 1*c*; Clarke *et al.* 2025; De Paoli, Howland & Verzichio 2024).

Declaration of interest. The author has no conflicts of interest related to this work.

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