Direct Determination of Angular Velocity Using GPS

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Controlling a ship in a berthing operation is carried out mainly by the change of state, such as velocity and yaw rate (turn rate), although the value of the change of state is very small at berthing. Very high precision is, therefore, required to determine the velocity and angular velocity. A sensor that has an accuracy of $\pm 0.02°/s$ ($1\ \sigma$) is sought for determination of turn rate in a berthing system. Three-dimensional angular velocity can directly be determined, with 2 independent baselines of 3 GPS antennas, using instantaneous Doppler measurements or phase rate (temporal difference of phase) observations. This paper discusses the mathematical model for direct determination of angular velocity using GPS, and the comparison of the results of the angular velocity determination using the Doppler and phase rate. The precision of angular velocity determination is estimated using temporal difference of the attitude sensors (TSS and gyrocompass) on board a hydrographic sounding ship. The RMS values of the difference of yaw rate determination between the two systems were: $\pm 0.16°/s$ using phase rate and $\pm 0.31°/s$ using Doppler measurements with the separation of onboard antennas of *ca*. 1·34 m. 10 m baselines could satisfy the sensor requirements for angular velocity determination during berthing maneuvers.

KEY WORDS

1. GPS. 2. Marine. 3. Berthing.

1. INTRODUCTION. Direct determination of angular velocity (attitude rate) is highly desirable for use in state feedback control. Angular velocity is usually obtained by rate gyros. GPS offers another possibility for direct determination of angular velocity using either the phase rate or differentiating the attitude solutions. Cohen (1992) discussed the possibility of angular velocity determination by taking derivatives of the attitude matrix. Montgomery *et al*. (1994) gave experimental results for phase rate measurements (in single differencing between antennas), and provided the angular velocity estimation from such measurements.

Angular velocity is the rate of change of attitude angles, roll (ϕ), pitch (θ) and yaw (ψ), and the attitude of a rigid body platform, for instance a ship, is defined as the orientation of a body frame with respect to a (left-hand) local level reference frame. The attitude angles are the rotary motions about the x-, y- and z-axes of the body

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frame, respectively. The axes fixed in the ship (body frame) form a right-hand orthogonal system. The origin, O, is at the centre of gravity of the ship. The x-axis is along the centre plane of the ship, coincident with the longitudinal axis of inertia and its positive direction is forward. The z-axis is also in the centre plane of the ship, but normal to the x and is positive downward. The y-axis is normal to x and z and is positive to starboard. The transformation of coordinates from the local level reference frame to the body frame can be expressed with a matrix containing the attitude angles. The angular velocity is considered as the rate of change of these attitude angles. As the precision of attitude determination using GPS is scaled with the GPS antenna separation, longer baselines between the onboard antennas can yield better precision for attitude determination. The configuration of antennas also influences the precision of attitude determination (Ueno *et al.*, 1997).

Controlling a ship in a berthing operation is mainly by the change of state, such as velocity and turn rate, although the value of the change of state is very small at berthing. Therefore, very high precision is required to determine the velocity and angular velocity (Kobayashi, 1993). Takai and Ohtsu (1990) conducted a test on automatic berthing using a 50 m training ship. They used a rate gyro (precision $\pm 0{\cdot}302°/\mathrm{s}$) as a sensor for yaw rate determination. The present authors developed observation models using GPS phase and Doppler observations for an automatic berthing system and estimated the precision of angular velocity determination on board a hydrographic sounding ship.

2. MATHEMATICAL MODEL.

A single difference observation equation for GPS attitude determination is expressed by Equation (1) (Babineau and Santerre, 1997; Cohen, 1992). When the antennas are not connected to the same receiver or an external oscillator, one must estimate relative receiver clock parameters.

$$\Delta\Phi_i^j = \vec{\boldsymbol{b}}_i^T \boldsymbol{T}_3 \boldsymbol{R} \vec{\boldsymbol{e}}^j + \lambda \Delta N_i^j + c\Delta dT_i + \Delta\nu_i^j \tag{1}$$

where,

$\Delta\Phi_i^j$: single difference phase observation for the j-th satellite and the i-th baseline;

$\vec{\boldsymbol{b}}_i$: (3×1) baseline vector in the body frame ($i = 1, 2, \ldots, m$);

$\Delta\Phi_i^j$: (3×1) satellite-receiver unit vector in the local level reference frame ($j = 1, 2, \ldots, n$);

$\boldsymbol{T}_3 \boldsymbol{R}$: (3×3) transformation matrix that contains attitude angles;

c: velocity of light in a vacuum;

ΔdT_i: relative clock errors of the i-th baseline;

ΔN_i^j: GPS carrier phase ambiguity for the j-th satellite, the i-th baseline in single difference;

λ: carrier wavelength; and

$\Delta\nu_i^j$: phase residuals for the j-th satellite, the i-th baseline.

A satellite-receiver unit vector is expressed as follows:

$$\vec{e}^j = [e_x^j, e_y^j, e_z^j]^T_{local\ level} \tag{2}$$

Keeping in mind that $\vec{b}_i$, T_3 and ΔN_i^j are not function of time and taking the time derivatives of Equation (1) yields the observation equation for angular velocity determination as follows:

$$\Delta\dot{\Phi}_i^j = \vec{b}_i^T T_3 \dot{R} \vec{e}^j + \vec{b}_i^T T_3 R \dot{\vec{e}}^j + c\Delta d\dot{T}_i + \Delta\dot{\nu}_i^j \tag{3}$$

where,

$$R = R_1(-\phi)R_2(-\theta)R_3(\psi) \tag{4}$$

$$\dot{R} = \dot{R}_1(-\phi)R_2(-\theta)R_3(\psi) + R_1(-\phi)\dot{R}_2(-\theta)R_3(\psi) + R_1(-\phi)R_2(-\theta)\dot{R}_3(\psi) \tag{5}$$

The term $\dot{\vec{e}}^j$ expresses the temporal variation of direction cosines. $\Delta\dot{\Phi}_i^j$ is the phase rate (Doppler) for the *i*-th baseline and the *j*-th satellite in m/s and $\Delta d\dot{T}_i$ is the relative clock drift for the *i*-th baseline.

In order to linearise the observation equation (3), the terms related to angular velocity have to be separate to those related to the attitude angles. Time derivatives of the rotation matrices $\dot{R}_1(-\phi)$, $\dot{R}_2(-\theta)$, and $\dot{R}_3(\psi)$ are expressed as follows:

$$\left.\begin{aligned} \dot{R}_1(-\phi) &= \frac{\partial R_1(-\phi)}{\partial\phi}\dot{\phi} = S_1\dot{\phi} \\ \dot{R}_2(-\theta) &= \frac{\partial R_1(-\theta)}{\partial\theta}\dot{\theta} = S_2\dot{\theta} \\ \dot{R}_3(\psi) &= \frac{\partial R_1(\psi)}{\partial\psi}\dot{\psi} = S_3\dot{\psi} \end{aligned}\right\} \tag{6}$$

Substituting the corresponding terms of Equation (3) with Equation (6), yields:

$$\dot{R} = S_1(-\phi)R_2(-\theta)R_3(\psi)\dot{\phi} + R_1(-\phi)S_2(-\theta)R_3(\psi)\dot{\theta} + R_1(-\phi)R_2(-\theta)S_3(\psi)\dot{\psi} \tag{7}$$

Then, Equation (3) can be rewritten as:

$$\Delta\dot{\Phi}_i^j = \vec{s}_i^j\dot{\Theta} + \vec{b}_i^T T_3 R \dot{\vec{e}}^j + c\Delta d\dot{T}_i + \Delta\dot{\nu}_i^j \tag{8}$$

where,

$$\dot{\Theta} = [\dot{\phi}\ \dot{\theta}\ \dot{\psi}]^T \tag{9}$$

$$\vec{s}_i^j = [\vec{b}_i^T\, T_3\, M_1\, \vec{e}^j \quad \vec{b}_i^T\, T_3\, M_2\, \vec{e}^j \quad \vec{b}_i^T\, T_3\, M_3\, \vec{e}^j \tag{10}$$

with $M_1 = S_1 R_2 R_3$, $M_2 = R_1 S_2 R_3$, $M_3 = R_1 R_1 S_3$,

Next, linearise Equation (8) around the approximate angular velocity and take the partial derivatives with respect to the angular velocity $\dot{\Theta}$ (rad/s) and relative clock drift for each baseline. Design matrix A ($mn \times (m+3)$) for m baselines and n satellites can be obtained as follows:

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$$A = \begin{bmatrix} \vec{b}_1^T T_3 M_1 \vec{e}^1 & \vec{b}_1^T T_3 M_2 \vec{e}^1 & \vec{b}_1^T T_3 M_3 \vec{e}^1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ \vec{b}_1^T T_3 M_1 \vec{e}^n & \vec{b}_1^T T_3 M_2 \vec{e}^n & \vec{b}_1^T T_3 M_3 \vec{e}^n & 1 & 0 & 0 & \cdots & 0 \\ \vec{b}_2^T T_3 M_1 \vec{e}^1 & \vec{b}_2^T T_3 M_2 \vec{e}^1 & \vec{b}_2^T T_3 M_3 \vec{e}^1 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ \vec{b}_2^T T_3 M_1 \vec{e}^n & \vec{b}_2^T T_3 M_2 \vec{e}^n & \vec{b}_2^T T_3 M_3 \vec{e}^n & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & 0 \\ \vec{b}_m^T T_3 M_1 \vec{e}^1 & \vec{b}_m^T T_3 M_2 \vec{e}^1 & \vec{b}_m^T T_3 M_3 \vec{e}^1 & 0 & 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & 0 & \vdots & & \vdots \\ \vec{b}_m^T T_3 M_1 \vec{e}^n & \vec{b}_m^T T_3 M_2 \vec{e}^n & \vec{b}_m^T T_3 M_3 \vec{e}^n & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{11}$$

The single difference misclosure vector $\boldsymbol{W}$ contains phase rate or instantaneous Doppler observations.

$$W = \begin{bmatrix} \Delta\delta\overline{\Phi}_1^1 - \vec{s}_1^{\,1}\dot{\Theta}_0 - \vec{b}_1^T T_3 R \dot{\vec{e}}^1 \\ \vdots \\ \Delta\delta\overline{\Phi}_1^n - \vec{s}_1^{\,n}\dot{\Theta}_0 - \vec{b}_1^T T_3 R \dot{\vec{e}}^n \\ \Delta\delta\overline{\Phi}_2^1 - \vec{s}_2^{\,1}\dot{\Theta}_0 - \vec{b}_2^T T_3 R \dot{\vec{e}}^1 \\ \vdots \\ \Delta\delta\overline{\Phi}_2^n - \vec{s}_2^{\,n}\dot{\Theta}_0 - \vec{b}_2^T T_3 R \dot{\vec{e}}^n \\ \vdots \\ \Delta\delta\overline{\Phi}_m^1 - \vec{s}_m^{\,1}\dot{\Theta}_0 - \vec{b}_m^T T_3 R \dot{\vec{e}}^1 \\ \vdots \\ \Delta\delta\overline{\Phi}_m^n - \vec{s}_m^{\,n}\dot{\Theta}_0 - \vec{b}_m^T T_3 R \dot{\vec{e}}^n \end{bmatrix} \tag{12}$$

where $\Delta\delta\overline{\Phi}_i^j$ is the phase rate, which is a temporal difference of phase measurements between two consecutive epochs divided by its sampling interval, in single difference for the i-th baseline and the j-th satellite. The initial values for $\dot{\Theta}_0$ are obtained from the difference of attitude angles, $\Theta = [\phi \ \theta \ \psi]^T$ between epochs k and $k-1$:

$$\dot{\Theta}_0 = \frac{\Theta_k - \Theta_{k-1}}{\Delta T} \tag{13}$$

The term $\dot{\vec{e}}^{\,j}$ is approximated by the variation of the satellite unit vector between the two epochs k and $k-1$ divided by the sampling interval ΔT:

$$\dot{\vec{e}}_k^{\,j} \approx \frac{\vec{e}_k^{\,j} - \vec{e}_{k-1}^{\,j}}{\Delta T} \tag{14}$$

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When the instantaneous Doppler measurements are used, the observation $\Delta d\bar{\Phi}_i^j$ is substituted by the single difference Doppler measurements ΔD_i^j expressed in m/s.

The normal matrix N and least-squares solution of the angular velocity is as follows:

$$\boldsymbol{N} = \boldsymbol{A}^T \boldsymbol{A} \tag{15}$$

Angular velocity and clock drift is obtained using the following equation:

$$\hat{\boldsymbol{X}} = (\boldsymbol{A}^T \boldsymbol{A})^{-1} (\boldsymbol{A}^T \boldsymbol{W}) \tag{16}$$

The *a posteriori* variance factor and the estimated variance-covariance matrix are calculated as follows:

$$\hat{\sigma}_0 = \sqrt{\frac{\hat{\boldsymbol{V}}^T \hat{\boldsymbol{V}}}{n_{obs} - u}} \tag{17}$$

$$\hat{\Sigma}_{\hat{X}} = \hat{\sigma}_0^2 \boldsymbol{N}^{-1} \tag{18}$$

where u is the number of parameters to be estimated ($u = m+3$) and n_{obs} is the number of single difference observations ($n_{\text{obs}} = mn$). The residuals are calculated as follows:

$$\hat{\boldsymbol{V}} = \boldsymbol{A}\hat{\boldsymbol{X}} - \boldsymbol{W} \tag{19}$$

3. TESTS ON BOARD A HYDROGRAPHIC SHIP.

Tests were conducted to check the performance of the algorithms and the navigation solution in an environment close to reality, the berthing of a large ship. The tests on board a 35 m hydrographic sounding ship F. C. G. SMITH were conducted at Trois-Rivières, on the St. Lawrence River, about 130 km upstream (southwest) of Quebec City, on 17 November 1997. Three geodetic points were set up on the wharf beforehand using static GPS measurements. One of the points was used for relative positioning and velocity determination. Distance to the GPS reference station was about 200 m when the ship was at the wharf, and reached a maximum of about 0·35 km during the tests.

The tetrahedron antenna configuration (Figure 1) was constructed of a PVC (polyvinyl chloride plastic) pipe to support the antenna A and an aluminum base structure that allows installation of 3 other GPS antennas. The GPS receivers used were Ashtech Z-XII for antennas A and B and Ashtech LD-XII for antennas C and D. Four precision geodetic antennas from Ashtech were used on board. The baseline length was about 1·34 m. The antenna configuration was mounted on one of the ventilating structures to keep it level as shown in Figure 1. The three attitude angles (roll, pitch and yaw), and their rate of change, were calculated with 3 independent baselines (AB, AC and AD). Figure 2 shows the ship in a sounding operation.

Because of physical constraints, the tetrahedron antenna structure could not be installed on the longitudinal axis of the ship. The alignment to the ship's longitudinal axis was made as close as possible using handrails on the top of the bridge as reference. The offset from the longitudinal axis was calculated using the GPS measurement by the Canadian Coast Guard, whose receiver is installed on the mast above the pivotal point of the ship.

The ship made turns to draw elongated loops four times off the wharf. The duration of the test was about an hour. Observations were made with an elevation mask angle of 10°, and data interval was 1 second. Signals from the satellites were sometimes blocked by the ship's structure during the test, since the tetrahedron could not be installed at the ideal location. However, more than 5 satellites were available for most

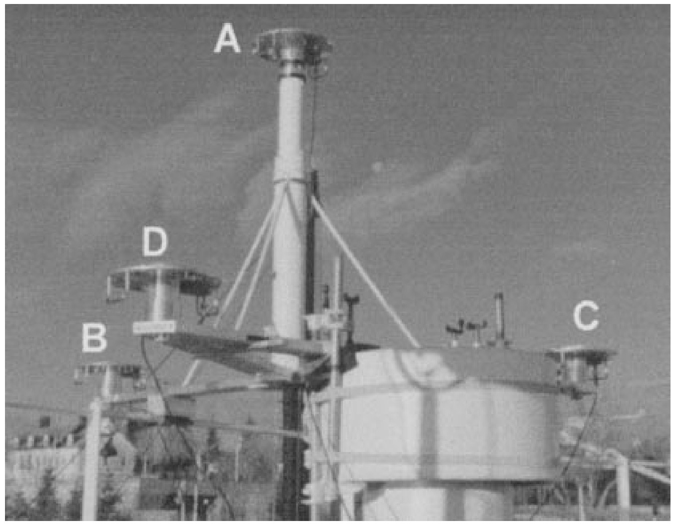

Figure 1. Antenna installation on board F. C. G. SMITH.

Figure 2. F. C. G. SMITH during a sounding operation.

Table 1. Technical description of sensors on board.

	Gyrocompass Anschuz Standard-20	TSS-335B
Reading	0·1°	0·01°
Precision	±0·5°	±0·1°
Data rate	6–7/s	20–21/s

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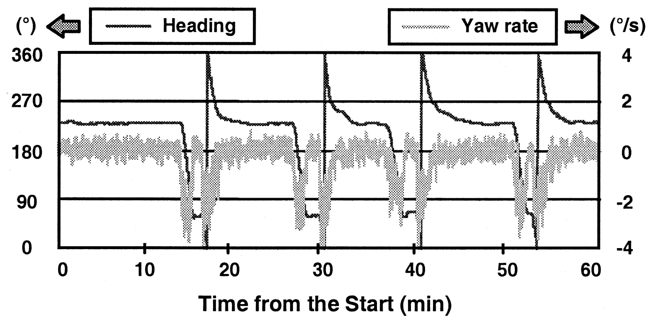

Figure 3. Yaw rate determination using Doppler.

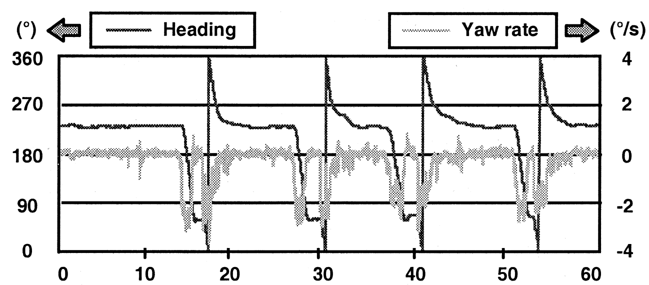

Figure 4. Yaw rate determination using phase rate.

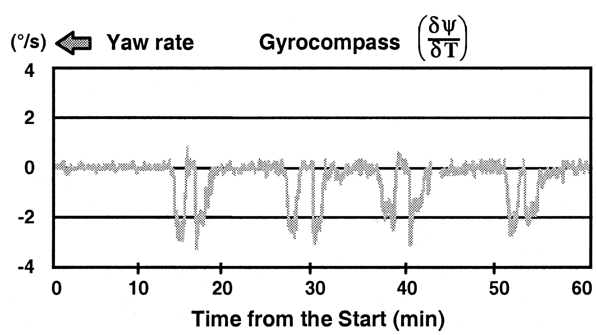

Figure 5. Temporal difference of heading by the gyrocompass.

of the time. The PDOP varied between 2·31 and 3·36. The number of common satellites available for all antennas varied from 4 to 7 during the test.

There was no sensor that provided direct measurement of angular velocity. However, making use of temporal difference of attitude from the gyrocompass and TSS (roll, pitch and heave) sensor on board, the accuracy of angular velocity determination was estimated. These onboard sensors were synchronised to the GPS time. Table 1 summarises the technical description of the TSS and gyrocompass.

Figure 3 shows the result of yaw rate calculations using instantaneous Doppler measurements for the tests. Figure 4 shows the same using the temporal difference of phase measurements. The root mean square (RMS) of the *a posteriori* variance factor of angular velocity determination were $\pm 0{\cdot}35$ cm/s for phase rate, $\pm 1{\cdot}30$ cm/s for

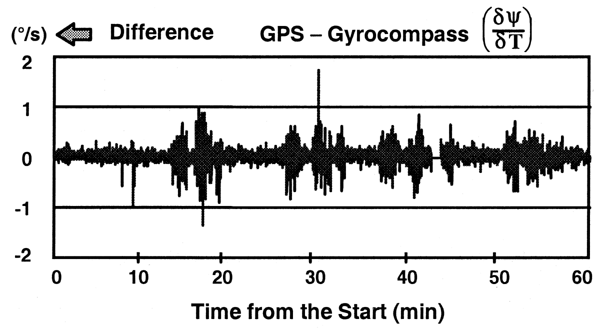

Figure 6. Difference of yaw rate between the GPS (phase rate) and temporal difference of heading by the gyrocompass.

Table 2. Difference of angular velocity determination between GPS and temporal difference on TSS/gyrocompass.

	Doppler		Phase rate	
Difference	Mean	RMS	Mean	RMS
Roll rate	0·04°/s	±0·6°/s	0·03°/s	±0·27°/s
Pitch rate	−0·02°/s	±0·63°/s	−0·63°/s	±0·23°/s
Yaw rate	0·02°/s	±0·31°/s	0·01°/s	±0·16°/s

instantaneous Doppler measurements. Doppler measurements were twice as noisy as the phase difference. Rate of change of heading, which is a temporal difference of heading (divided by the data rate) obtained by the gyrocompass, is shown in Figure 5. There is a similar trend in the results of yaw rate obtained by the direct calculation of yaw rate by GPS. Figure 6 shows the difference between the two methods. Difference of angular velocity between the temporal difference of the TSS sensor or gyrocompass and GPS angular velocity determination is summarised in Table 2. The RMS of the differences between GPS and external sensors was 0·316–0·327°/s for phase rate and 0·331–0·367°/s for Doppler measurements. The results obtained by Doppler measurements were about 2·35 times noisier than those by phase rate. Assuming that the accuracy of rate of change measurements from the TSS and gyrocompass is of the same order, the GPS yaw rate determination is slightly better using the tetrahedron antenna configuration.

4. CONCLUSIONS. Direct calculation of angular velocity is possible using GPS. The use of instantaneous Doppler and the temporal difference of phase observations were compared. The precision of yaw rate determination was estimated using the temporal difference of attitude sensors (TSS sensor and gyrocompass) on board a hydrographic sounding ship. The RMS values (1 σ) of the difference of yaw rate determination between GPS and the temporal difference of gyrocompass output was ±0·316°/s during the maneuvers using temporal phase difference with 1·34 m of GPS antenna separation. The RMS values of the difference using Doppler measurements was ±0·331°/s. The results obtained using Doppler measurements were about 2–3 times noisier than those by phase difference. The precision of the angular velocity solution scales with the baseline length as in the attitude determination from

Equations (8) and (10). The precision of the angular velocity solution scales with baseline length as in the attitude determination. Our test results showed that angular velocity determination with the baseline of 1·34 m does not meet the required accuracy. Extrapolating the results, and taking into account the error sources in kinematic mode, a 10 m baseline configuration using phase rate measurements could satisfy the required accuracy of angular velocity for ship's berthing.

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