J. Plasma Phys. (2018), *vol.* 84, 745840303 © Cambridge University Press 2018 1
doi:10.1017/S0022377818000648

Strongly driven surface-global kinetic ballooning modes in general toroidal geometry

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(Received 29 March 2018; revised 27 May 2018; accepted 29 May 2018)

Kinetic ballooning modes in magnetically confined toroidal plasmas are investigated putting emphasis on specific stellarator features. In particular, we propose a Mercier criterion which is purposely designed to allow for direct comparison with local flux-tube gyrokinetics simulations. We investigate the influence on the marginal frequency of the mode of a magnetic curvature which is inhomogeneous on the magnetic flux surface due to the fieldline-label dependence. This is a typical (surface) global effect present in non-axisymmetry. Finally, we propose an artificial equilibrium model that explicitly retains the fieldline-label dependence in the magnetic drift, and analyse the stability of the system by introducing a representation of the perturbations similar to the flux-bundle model of Sugama *et al.* (*Plasma Fusion Res.*, vol. 7, 2012, 2403094). The coupling of flux bundles is shown to have a stabilising effect on the most unstable local flux-tube mode.

Key words: fusion plasma, plasma instabilities

1. Introduction

In recent years, great effort has been devoted to the investigation of gyrokinetic instabilities that can cause turbulent transport in stellarators. In particular, analytical and numerical progress has been made for electrostatic instabilities, such as trapped electron modes (Proll, Xanthopoulos & Helander 2013; Faber *et al.* 2015), ion-temperature-gradient-driven modes (Plunk *et al.* 2014; Helander *et al.* 2015; Xanthopoulos *et al.* 2016; Zocco *et al.* 2016) and electron-temperature-gradient-driven modes (Jenko & Kendl 2002). Electromagnetic gyrokinetic instabilities have been explored much less. At present, our understanding is based on the use of numerical codes and is limited to a handful of works (Sugama & Watanabe 2004; Baumgaertel *et al.* 2012; Ishizawa *et al.* 2014, 2015; Mishchenko *et al.* 2015). This *status quo* is clearly not satisfactory, especially if we consider our lack of analytical insight. The state of affairs is different in the sphere of energetic particle physics, especially in tokamaks, where there is certainly no lack of analytically driven research (see the review of Chen & Zonca (2016) and references therein). For transport studies, a first step towards the reconciliation of analytics and numerics, for strongly driven

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kinetic ballooning modes (KBMs), was made in the work of Aleynikova & Zocco (2017). Here, quantitative agreement between electromagnetic gyrokinetic numerical simulations and a finite-β (where β is the ratio of kinetic to magnetic plasma pressure) diamagnetic modification of ideal magnetohydrodynamics (MHD) was found. The results of Aleynikova & Zocco (2017), however, only apply to a simple geometric setting, and an extension to more relevant geometries is required. In this article we complement the numerical work of Aleynikova *et al.* (2018) on the stellarator Wendelstein 7-X, and put forward an analytical formulation of the diamagnetic modification of ideal MHD used by Aleynikova & Zocco (2017) in a surface-global setting. Our analysis will then be local in the radial direction of the torus, but the equilibrium magnetic field is allowed to vary on the magnetic flux surface with respect to the fieldline as is possible in non-axisymmetric geometries. We show how to properly choose coordinates in such a way that the original derivation of the Mercier criterion (Mercier 1960; Mercier & Luc 1974) can be performed also in a stellarator geometry with the use of the ballooning transform of Connor, Hastie & Taylor (1979). We identify the metric elements that characterise surface-global effects and study how they impact the real frequency of the diamagnetically modified ideal MHD mode proposed by Aleynikova & Zocco (2017). Finally, a discrete description similar to the flux-bundle model of Sugama *et al.* (2012) is introduced. The effect of the fieldline-label dependence on the curvature drift is investigated within this framework and it is found to be stabilising. This stabilisation is related to a possible violation of the Mercier criterion.

2. Formulation

The equation for the divergence of the plasma current, when each term is ordered to accommodate linear ballooning modes, results in a second-order differential equation, in the fieldline-following variable l, for the potential ψ that defines the parallel component of the magnetic potential $-\mathrm{i}\omega A_\parallel = \nabla_\parallel \psi$. Here ω is the complex mode frequency, $\nabla_\parallel = \boldsymbol{b}\cdot\nabla$, with $\boldsymbol{b} = \boldsymbol{B}/B$, where $\boldsymbol{B}$ is the equilibrium magnetic field, and B is its modulus. The general form of this equation is equation (2.35) of Tang, Connor & Hastie (1980). This comes from a sound expansion of the gyrokinetic equation, $k_\parallel v_{\mathrm{thi}} \ll \omega \ll k_\parallel v_{\mathrm{the}}$. Here $v_{\mathrm{ths}} = \sqrt{2T_s/m_s}$ is the thermal speed for a species with temperature T_s and mass m_s. When a finite $\beta \sim \epsilon \equiv k_\parallel^2 v_{\mathrm{thi}}^2/\omega^2 \ll 1$ ordering is implemented, magnetic compressibility is retained, and the curvature and grad-B drifts are kept, consistent with the Grad–Shafranov equation, the relevant equation for kinetic ballooning modes is a simple diamagnetic modification of the ideal MHD ballooning equation (Roberts & Taylor 1965; Aleynikova & Zocco 2017)

$$\frac{B/B_a^2}{\beta_i}\frac{v_{\mathrm{thi}}^2}{\omega^2}\nabla_\parallel bB\nabla_\parallel\psi = -b\left[1-\frac{\omega_{*i}}{\omega}(1+\eta_i)\right]\psi - 2\frac{\omega_\kappa\omega_p}{\omega^2}\psi, \tag{2.1}$$

where B_a is a reference constant magnetic field, $\beta_i = 8\pi p_i/B_a^2$, $b = 0.5k_\perp^2 v_{\mathrm{thi}}^2/\varOmega_i(B)^2$, $\boldsymbol{k}_\perp$ is the wave vector (of perturbations) across the equilibrium magnetic field and $\varOmega_i(B) = m_i c/(eB)$ is the ion cyclotron frequency. In a surface-global setting, the $\boldsymbol{k}_\perp^2 = k_i k^i = k^j g_{ji} k^i$ term becomes the Laplacian operator in curvilinear geometry

$$b = -\frac{1}{2}\frac{\rho_i^2}{a^2}\frac{B_a^2}{B^2}\frac{1}{\sqrt{g}}\sum_{i,j=1}^{2}\frac{\partial}{\partial x^i}\sqrt{g}g^{ij}\frac{\partial}{\partial x^j}, \tag{2.2}$$

where $\rho_i = v_{\mathrm{thi}}/\Omega_i(B_a)$, and we introduced a triplet of contravariant coordinates $\boldsymbol{x} = (x^1, x^2, x^3)$. Each x^i is a scalar function of the Cartesian spatial coordinates (x, y, z). We then define the contravariant metric tensor $g^{ij} = \nabla x^i \boldsymbol{\cdot} \nabla x^j$ where $\nabla = \boldsymbol{e}^x\partial_x + \boldsymbol{e}^y\partial_y + \boldsymbol{e}^z\partial_z$ is the gradient in Cartesian coordinates, $\sqrt{g} = (\nabla x^1 \times \nabla x^2 \boldsymbol{\cdot} \nabla x^3)^{-1}$ is the determinant of the Jacobian matrix $J_i^j = \partial_i x^j$ and a is a reference length scale. The functions g^{ij} will soon be specified. The diamagnetic frequency is

$$\omega_{*s} = \frac{1}{2}\frac{v_{\mathrm{ths}}}{L_n}\frac{\rho_s}{a}\left(-\mathrm{i}\frac{\partial}{\partial x^2}\right), \tag{2.3}$$

where $L_n^{-1} = \mathrm{d}\ln n/\mathrm{d}x^1$. Then $\omega_{ps} = \omega_{*s}(1+\eta_s)$, with $\eta_s = \mathrm{d}\ln T_s/\mathrm{d}\ln n$, $\omega_p = \omega_{pi} - \omega_{pe}$ and $L_{p,s}^{-1} = L_n^{-1}(1+\eta_s)$, with $L_p^{-1} = L_{p,i}^{-1} + L_{p,e}^{-1}$. Notice that we have corrected a multiplicative factor 2 on the left-hand side of equation (2.1) of Aleynikova & Zocco (2017). We will now be more specific with the coordinate system.

We follow Xanthopoulos *et al.* (2009) and consider a modification of the Boozer system (Boozer 1982) that respects the field alignment:

$$(x^1, x^2, x^3) = (s, q(s)(\theta - \theta_0) - \zeta, \theta - \theta_0), \tag{2.4}$$

where $s = \Phi/\Phi_{\mathrm{edge}}$, with Φ the toroidal magnetic flux and Φ_{edge} its value at the last closed flux surface, and θ and ζ are the Boozer poloidal and toroidal angles, respectively. Thus, $\sqrt{g_B}B^2 = B_\theta\Psi'(s) + B_\zeta\Phi'(s)$, where Ψ is the poloidal magnetic flux and $\boldsymbol{B} = \Psi'(s)\nabla x^1 \times \nabla x^2$. The prime is a total derivative with respect to the explicit argument s, $q = \Phi'/\Psi' \equiv \iota^{-1}$, and θ_0 is the familiar free parameter of ballooning theory (Connor, Hastie & Taylor 1978; Connor *et al.* 1979; Hastie & Taylor 1981). We have

$$\begin{aligned}\omega_\kappa &= -\mathrm{i}\boldsymbol{v}_d \boldsymbol{\cdot} \nabla \\ &= -\mathrm{i}v_{\mathrm{thi}}^2\frac{B_a}{B}\frac{\hat{\boldsymbol{b}} \times \boldsymbol{\kappa}}{\Omega_i(B_a)} \boldsymbol{\cdot} \nabla \\ &= -\mathrm{i}v_{\mathrm{thi}}\rho_i\frac{B_a}{B}\sum_{i=1}^{2}\hat{\boldsymbol{b}} \times \boldsymbol{\kappa} \boldsymbol{\cdot} \nabla x^i\frac{\partial}{\partial x^i},\end{aligned} \tag{2.5}$$

where $\boldsymbol{\kappa} = \hat{\boldsymbol{b}} \boldsymbol{\cdot} \nabla\hat{\boldsymbol{b}}$. We now introduce our first assumption: $\partial_{x^1} \equiv 0$. That is, we are neglecting the radial structure of the mode under consideration. From this, it also follows that $\theta_0 \equiv 0$, since in ballooning theory it can be shown that θ_0 is proportional to the radial wavenumber. The effect of k_1 must be carefully considered for each non-axisymmetric machine under consideration, depending on the global shear of its configurations. For instance, a finite k_1 seems to be crucial to capture the most unstable KBM in Large Helical Device (LHD) (see (Ishizawa *et al.* 2014, figure 1)). In the case of W7-X, k_1 could have a less prominent role (Aleynikova *et al.* 2018).

By using equation (23) of Xanthopoulos *et al.* (2009), and the properties of Boozer coordinates, after some straightforward algebra, we obtain

$$\omega_\kappa\omega_p = -\frac{1}{4}\frac{\rho_i^2}{a^2}\frac{v_{\mathrm{thi}}^2}{L_p a}\left[B_s B\nabla_\parallel\left(\frac{1}{B^2}\right) + \frac{P'(s)}{B^2/2} + \frac{\partial_s B^2}{B^2} - \frac{a^2 B_a B}{2P'(s)}\nabla_\parallel\left(\frac{j_\parallel}{B}\right)\hat{s}\theta\right]\frac{\partial^2}{\partial(x^2)^2}, \tag{2.6}$$

where $j_\parallel = \hat{\boldsymbol{b}} \boldsymbol{\cdot} \boldsymbol{j}$, $\boldsymbol{j}$ is the plasma current and $\hat{s} = 2s_0 q'(s_0)/q(s_0)$ is the global magnetic shear at a given radial location s_0, and we used $\Psi_N' = \Psi'/(a^2 B_a) = \sqrt{s}/q$ (see also equation (141) of Xanthopoulos *et al.* (2009)). This form will be extremely important in order to derive a Mercier criterion, because of the explicit $\nabla_\parallel$. We finally choose $x^3 = \theta$, so that $\sqrt{g_N}a\nabla_\parallel = (B_a/B)\partial_\theta$, where $\sqrt{g_N} = 2qa^{-3}\sqrt{g_B}$ is the normalised Jacobian.

It is now possible to specify the form of (2.2). The metric elements entering (2.2) have first been presented by Cooper (1992) and have also been evaluated by Xanthopoulos *et al.* (2009). Then, we find it convenient to write

$$\begin{aligned} b &= -\frac{1}{2}\frac{\rho_i^2}{a^2}\frac{B_a^2}{B^2}\frac{1}{\sqrt{g_N}}\frac{\partial}{\partial x^2}\sqrt{g_N}g_B^{22}\frac{\partial}{\partial x^2} \\ &= -\frac{1}{2}\frac{\rho_i^2}{a^2}\frac{B_a^2}{B^2}\frac{1}{\sqrt{g_N}}\frac{\partial}{\partial x^2}\sqrt{g_N}[b_0+b_1\hat{s}\theta+b_2\hat{s}^2\theta^2]\frac{\partial}{\partial x^2}, \end{aligned} \tag{2.7}$$

with $b_0=(g_{ss}B^2-B_s^2)/(a^2B_a^2)$, $b_1=(B_\theta B_s-g_{s\theta}B^2)/(s_0a^2B_a^2)$ and $b_2=g^{ss}a^2/(4s_0)\equiv g_N^{ss}$. Since we are assuming (for perturbations!) $\partial/\partial x^1=\theta_0\equiv 0$, the g_B^{22} term is the only metric element left in the summation that defines b in (2.2). It is perhaps interesting to note that, in (2.7), the function b shows the same $\hat{s}\theta$ dependence that it would have in the well-known $\hat{s}-\alpha$ model: $b_{\hat{s}-\alpha}\propto k_2^2+k_1k_2\hat{s}\theta+k_2^2\hat{s}^2\theta^2$. However now the coefficients b_i are not constant, and we have a linear secular term even if $k_1\equiv 0$! This is purely geometric, and comes from the off-diagonal entries of the metric tensor. However, this term does not play a role in the formulation of the Mercier criterion.

2.1. *Local Mercier criterion and its validity*

Before analysing the properties of (2.1) when the x^2-variation of the eigenfunction is allowed, it seems reasonable to follow the analysis of Connor, Hastie and Taylor of the ballooning equation (Connor *et al.* 1979), and derive a Mercier criterion which is valid in a local flux-tube gyrokinetic context for stellarators. This is important since, historically, the derivation of the ballooning equation for stellarators has been based on a complicated minimisation of the ideal MHD potential (Correa-Restrepo 1978) à la Mercier (Mercier 1960; Mercier & Luc 1974) and its application to local flux-tube gyrokinetics is not straightforward. In some works on ideal MHD ballooning modes in stellarators, Hamada (1962) coordinates are used (Correa-Restrepo 1978). In others (Hegna & Nakajima 1998), Boozer coordinates are introduced but the field-following coordinate is not specified and the secular terms are expressed implicitly in terms of integrals on the local shear. The 'stellarator expansion' was used by Sugama & Watanabe (2004). The most explicit formulation of the Mercier criterion for stellarators is the one given in a not so very accessible article by Nührenberg & Zille (1987), where the authors use the toroidal angle as the field-following coordinate and do not order the global shear with the plasma β, unlike us. Additional instances of the use of a Mercier criterion in stellarators (Fu *et al.* 1992; Gardner & Blackwell 1992) lead to chapter 5 of the book of Bauer, Betancourt & Garabedian (1984), which, in turns, leads cyclically to the work of Mercier & Luc (1974). Since in this bibliographical odyssey, lasting more than 38 years (therefore nearly 4 times the original odyssey), we could not find a derivation of the indicial ballooning equation that: is based on the poloidal angle being the fieldline-following coordinates, relates to local flux-tube gyrokinetics and gives an explicit ordering for the global shear, we decided to present such calculation here. The equation we study is then

$$\begin{aligned} &\frac{1}{\sqrt{g_N}}\frac{\partial}{\partial\theta}\frac{b}{\sqrt{g_N}}\frac{\partial}{\partial\theta}\psi=-b\frac{\omega(\omega-\omega_{pi})}{\omega_A^2}\psi \\ &\quad -\frac{\rho_i^2}{2a^2}\frac{k_2^2v_{\mathrm{thi}}^2}{aL_p\omega_A^2}\left\{\frac{B_s}{aB_a}\frac{1}{\sqrt{g_N}}\frac{\partial}{\partial\theta}\frac{B_a^2}{B^2}+2\frac{P'(s)}{B^2}+\frac{\partial_sB^2}{B^2}-\frac{aB_a^2}{P'(s)}\frac{1}{\sqrt{g_N}}\frac{\partial}{\partial\theta}\left(\frac{j_\|}{B}\right)\hat{s}\theta\right\}\psi, \end{aligned} \tag{2.8}$$

with $\omega_A^2 = v_{\text{thi}}^2/(\beta_i a^2)$, $b = \rho_i^2 k_2^2 B_a^2/(2a^2 B^2) g_N^{22}$ and $g_N^{22} = b_0 + b_1 \hat{s}\theta + b_2 \hat{s}^2\theta^2$, where the b_i have been defined in the previous section.

We are now in the position to seek a solution of the type

$$\psi = z^\alpha \left(g_0 + \frac{g_1}{z} + \frac{g_2}{z^2} + \cdots \right), \tag{2.9}$$

where $z = \hat{s}\theta$, and we consider radial locations for which $\iota = n/m$, with m and n integers. Then, the functions g_i have the period of the equilibrium, and $\int_\Gamma \mathrm{d}g_i = 0$ if Γ in the path on integration along a closed fieldline. When the fieldline is chosen to be a high-order rational $\int_\Gamma (\cdots)\,\mathrm{d}\theta / \int_\Gamma\, \mathrm{d}\theta \approx (2\pi)^{-2} \int_0^{2\pi}\, \mathrm{d}\theta \int_0^{2\pi}\, \mathrm{d}\zeta (\cdots)$. The index α is a complex quantity which determines a necessary condition for marginal stability. Rigorously, the diamagnetic correction of (2.1) renders the original treatment of Connor, Hastie and Taylor extremely difficult. The problem has been studied by Connor, Tang & Allen (1984) by means of an asymptotic matching procedure. Here the authors consider the case $\omega - \omega_{pi} \ll \omega_A$, and solve (2.1) in two asymptotic regions: one defined by $\hat{s}\theta \sim 1$, the other $\hat{s}\theta \sim \omega_A^2/\omega(\omega - \omega_{pi}) \gg 1$. Asymptotic matching of the two solutions then provides a stability criterion that incorporates some diamagnetic effects. The authors also notice that, for (2.1), a necessary condition for ω to be imaginary in that $\mathrm{Re}[\omega] = \omega_{pi}/2$. This implies that, at marginality, $\omega(\omega - \omega_{pi}) = -\omega_{pi}^2/4$, and, more importantly, equation (2.1) is solved in an asymptotic expansion in $\omega_{pi}^2/4\omega_A^2 \ll 1$. Explicitly, we have

$$k_2^2 \frac{\rho_i^2}{aL_p} \beta_i \ll 16 \frac{L_p}{a}, \tag{2.10}$$

which, for a given β_i, determines the range of wavelength for which the $\omega - \omega_{pi} \ll \omega_A$ analysis of (2.1) is valid

$$\sqrt{\beta_i} k_2 \rho_i \ll 4L_p. \tag{2.11}$$

We consider this limit to apply and proceed order by order.

Then, to order $z^{\alpha+2}$, one obtains

$$\frac{\mathrm{d}}{\mathrm{d}\theta} \frac{g_N^{ss}}{\sqrt{g_N}} \frac{B_a^2}{B^2} \frac{\mathrm{d}g_0}{\mathrm{d}\theta} = 0, \tag{2.12}$$

and $g_0 = 1$. To order $z^{\alpha+1}$, we have

$$\frac{\mathrm{d}}{\mathrm{d}\theta} \left[\frac{g_N^{ss}}{\sqrt{g_N}} \frac{B_a^2}{B^2} \left(\frac{\mathrm{d}g_1}{\mathrm{d}\theta} + \alpha \hat{s} \right) - \frac{v_{\text{thi}}^2}{aL_p \omega_A^2} \frac{aB_a^2}{P'(s)} \frac{j_\parallel}{B} \right] = 0. \tag{2.13}$$

A constant of integration is chosen so that $\int_\Gamma \mathrm{d}\theta \mathrm{d}g_1/\mathrm{d}\theta = 0$. Then

$$\begin{aligned} \frac{\mathrm{d}g_1}{\mathrm{d}\theta} + \alpha\hat{s} &= \frac{v_{\text{thi}}^2}{aL_p\omega_A^2} \frac{aB_a^2}{P'(s)} \frac{\sqrt{g_N}}{g_N^{ss}} \frac{B^2}{B_a^2} \frac{j_\parallel}{B} \\ &\quad + \frac{\sqrt{g_N}}{g_N^{ss}} \frac{B^2}{B_a^2} \frac{\alpha\hat{s} - \dfrac{v_{\text{thi}}^2}{aL_p\omega_A^2} \dfrac{aB_a^2}{P'(s)} \displaystyle\int_\Gamma \mathrm{d}\theta \frac{\sqrt{g_N}}{g_N^{ss}} \frac{B^2}{B_a^2} \frac{j_\parallel}{B}}{\displaystyle\int_\Gamma \mathrm{d}\theta \frac{\sqrt{g_N}}{g_N^{ss}} \frac{B^2}{B_a^2}}, \end{aligned} \tag{2.14}$$

where each term is of the form of those of equation (43) of Connor *et al.* (1979).

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To order z^{α}, after integrating in $\int_{\Gamma} \mathrm{d}\theta$, we obtain

$$
\begin{aligned}
&(\alpha+1)\hat{s} \int_{\Gamma} \mathrm{d}\theta \frac{g_N^{ss}}{\sqrt{g_N}} \frac{B_a^2}{B^2} \left(\frac{\mathrm{d}g_1}{\mathrm{d}\theta} + \alpha \hat{s} \right) \\
&\quad + \frac{v_{\mathrm{thi}}^2}{a L_p \omega_A^2} \int_{\Gamma} \mathrm{d}\theta \left[\frac{B_s}{a B_a} \frac{\partial}{\partial\theta} \frac{B_a^2}{B^2} + \sqrt{g_N} \left(2\frac{P'(s)}{B^2} + \frac{\partial_s B^2}{B^2} \right) \right] \\
&\quad + \frac{v_{\mathrm{thi}}^2}{a L_p \omega_A^2} \frac{a B_a^2}{P'(s_0)} \int_{\Gamma} \mathrm{d}\theta \frac{j_{\parallel}}{B} \frac{\mathrm{d}g_1}{\mathrm{d}\theta} = 0,
\end{aligned}
\tag{2.15}
$$

and, again, each term of this equation resembles those of equation (44) of Connor *et al.* (1979). After using (2.14), one gets the indicial equation $\alpha(\alpha+1) + \mathcal{D} = 0$, with

$$
\begin{aligned}
\mathcal{D} = &\frac{v_{\mathrm{thi}}^2}{L_p a \omega_A^2 \hat{s}^2} \left\{ \left(\int_{\Gamma} \mathrm{d}\theta \frac{\sqrt{g_N} B^2}{g_N^{ss} B_a^2} \right) \int_{\Gamma} \mathrm{d}\theta \left[\frac{B_s}{a B_a} \frac{\partial}{\partial\theta} \frac{B_a^2}{B^2} + \sqrt{g_N} \left(2\frac{P'(s)}{B^2} + \frac{\partial_s B^2}{B^2} \right) \right] \right. \\
&+ \frac{v_{\mathrm{thi}}^2}{L_p a \omega_A^2} \left(\frac{a B_a^2}{P'(s_0)} \right)^2 \left[\left(\int_{\Gamma} \mathrm{d}\theta \frac{\sqrt{g_N} B^2}{g_N^{ss} B_a^2} \right) \int_{\Gamma} \mathrm{d}\theta \frac{\sqrt{g_N} B^2}{g_N^{ss} B_a^2} \left(\frac{j_{\parallel}}{B} \right)^2 \right. \\
&\left. \left. - \left(\int_{\Gamma} \mathrm{d}\theta \frac{\sqrt{g_N} B^2}{g_N^{ss} B_a^2} \frac{j_{\parallel}}{B} \right)^2 \right] \right\} - 2 \frac{v_{\mathrm{thi}}^2}{a L_p \omega_A^2 \hat{s}} \frac{a B_a^2}{P'(s)} \left[\left(\int_{\Gamma} \mathrm{d}\theta \frac{\sqrt{g_N}}{g_N^{ss}} \frac{B^2}{B_a^2} \right) \int_{\Gamma} \mathrm{d}\theta \frac{j_{\parallel}}{B} \right. \\
&\left. - \int_{\Gamma} \mathrm{d}\theta \frac{\sqrt{g_N}}{g_N^{ss}} \frac{B^2}{B_a^2} \frac{j_{\parallel}}{B} \right].
\end{aligned}
\tag{2.16}
$$

This implies that $\mathcal{D} = O(1)$ for

$$
\hat{s} \sim \beta_i \frac{a}{L_p} \sim \beta' \ll 1,
\tag{2.17}
$$

which is the condition that determines the ordering of the global shear for which the asymptotic form (2.9) is acceptable. The Mercier criterion that can be used for comparisons with flux-tube gyrokinetics in a stellarator, when conditions (2.11) and (2.17) apply, is then

$$
\mathcal{D} < 1/4
\tag{2.18}
$$

for stability, where $\mathcal{D}$ is defined by (2.16). This result is not new, as each term of (2.14) and (2.15) can be identified with the respective terms in equations (43) and (44) of Connor *et al.* (1979), where different coordinates were used. In the absence of an equilibrium parallel current, it is a limiting condition on the gradient of the plasma β plus a correction due to the covariant component of the equilibrium magnetic field. The usefulness of our result resides in its possible application to gyrokinetic numerical studies, which is now viable since we expressed the stability parameter $\mathcal{D}$ in terms of modified Boozer coordinates that commonly interface stellarator equilibria codes and gyrokinetic codes (Xanthopoulos *et al.* 2009). As a final remark, we notice the relation of our result, derived in modified Boozer coordinates, and the common concept of 'magnetic well'. Since the plasma volume enclosed in a magnetic surface is $V(s) = \int_0^s \mathrm{d}s \int_0^{2\pi} \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\zeta \sqrt{g_N}$, we have $\mathrm{d}^2V/\mathrm{d}s^2 = \int_0^{2\pi} \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\zeta\, \partial_s \sqrt{g_N}$. Had we expressed the curvature drive in (2.6) in terms of equilibrium poloidal and toroidal

https://doi.org/10.1017/S0022377818000648 Published online by Cambridge University Press

current fluxes, we would have been left with the non-secular component of the magnetic drift, $(\omega_\kappa\omega_p)_{NS}$, proportional to

$$(\omega_\kappa\omega_p)_{NS} \propto \sqrt{g}B\nabla_\parallel\left(\frac{B_s}{B^2}\right) + \sqrt{g}\frac{P'(s)}{B^2} + \frac{1}{B^2}[J\Psi'' - I\Phi''] - \partial_s\sqrt{g}, \tag{2.19}$$

where, indeed, J and I are the toroidal and poloidal current fluxes and Φ and Ψ are the toroidal and poloidal magnetic fluxes. This expression would replace the non-secular term at the second line of (2.8), and would result in an explicit dependence on $\mathrm{d}^2V/\mathrm{d}s^2$ for the Mercier index in (2.16). For negative pressure gradients, a positive $\mathrm{d}^2V/\mathrm{d}s^2$ then adds to the drive of pressure-driven instabilities, making them more unstable. Similarly, a negative $\mathrm{d}^2V/\mathrm{d}s^2$ has a stabilising effect (Johnson & Greene 1967). In the first case, the magnetic configuration is said to possess a 'magnetic hill', while is the second case it has a 'magnetic well'. Even if this nomenclature is somewhat intuitive, our expression seems more conclusive for what concerns the positive–definiteness of the driving terms (the radial derivatives at the first line of (2.16)): $\sqrt{g_N}(2P'(s) + \partial_s B^2)/B^2$. From this it is evident that a minimisation of the volume-averaged B^2 is beneficial. The same conclusion was drawn by Boozer (1981) (see discussion after (29)). We conclude this section by noticing that our expression for the Mercier index $\mathcal{D}$ in (2.16) agrees with equation (85) of Cooper (1992), only if the global shear is ordered to be as small as the equilibrium plasma pressure gradient. It is easy to see that this is imposed by the smallness of the global shear. The reason why the global shear has to be small, in our multiple scale asymptotic analysis of the ballooning equation, is explained well in the Introduction of § II of Connor *et al.* (1984). While the ballooning equation used to derive the Mercier index of Cooper (1992) does not agree with our starting point (however, see also the alternative improved version of Cooper, Singleton & Dewar (1996)), its application to ideal marginal stability is valid and agrees with our result.

3. Surface-global diamagnetism

Equation (2.8) implies that, in a local flux tube, a necessary condition for instability is $\mathrm{Re}[\omega] = \omega_{pi}/2$. This can be seen by multiplying the equation by the complex conjugate eigenfunction ψ^* and integrating by parts along the fieldline. The result is a second-order algebraic equation for the eigenvalue, ω, whose imaginary part is not zero only if, indeed, $\mathrm{Re}[\omega] = \omega_{pi}/2$. We now see how this changes in a surface-global setting. The Laplacian in curvilinear coordinates of (2.2) is more tractable if the metric elements are slowly varying in $x^2 : \partial_{x^2}\ln g_{ss} \sim \partial_{x^2}\ln g_{s\theta} \sim \partial_{x^2}\ln g^{ss} \ll \partial_{x^2}\ln\psi$, then (2.1) becomes

$$\frac{1}{\sqrt{g_B}}\frac{\partial}{\partial\theta}\frac{B_a^2}{B^2\sqrt{g_B}}\frac{\partial}{\partial\theta}\Omega = -\frac{B_a^2}{B^2}\frac{\omega(\omega-\omega_{pi})}{\omega_A^2}\Omega - \frac{v_{\mathrm{thi}}^2}{L_p a\omega_A^2}\frac{B}{B_a}[K_{AS}(\theta) + \epsilon_h K_h(\theta, x^2)]\Omega, \tag{3.1}$$

where $\Omega = \Omega_r + \mathrm{i}\Omega_i \equiv \partial_{x^2}^2\psi$ and the magnetic drift is formally split into an axisymmetric, K_A, and a non-axisymmetric component K_h, where ϵ_h is a constant. If we multiply by Ω^*, and integrate in $\mathrm{d}\theta$, we obtain a quadratic equation for ω:

$$\omega^2 + \mathrm{i}\zeta\omega_{pi}^{(0)}\omega + \lambda^2 = 0 \tag{3.2}$$

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with

$$\left.\begin{aligned}\zeta = \zeta_r + \mathrm{i}\zeta_i \equiv \frac{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} \varOmega^* \partial_{x^2}\varOmega}{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} |\varOmega|^2}, \\ \lambda^2 = \frac{v_{\mathrm{thi}}^2}{L_p a} \frac{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} [K_{AS}(\theta) + \epsilon_h K_h(\theta, x^2)] |\varOmega|^2}{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} |\varOmega|^2} \\ - \omega_A^2 \frac{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} \left|\frac{\partial \varOmega}{\partial\theta}\right|^2}{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} |\varOmega|^2},\end{aligned}\right\} \tag{3.3}$$

and

$$\omega_{pi}^{(0)} = \frac{1}{2}\frac{\rho_i}{a}\frac{v_{\mathrm{thi}}}{L_n}(1+\eta_i). \tag{3.4}$$

In the strongly driven case, $\lambda \gg |\zeta|\omega_{pi}^{(0)}$, the ideal MHD growth rate and a small real correction are found

$$\omega \approx \mathrm{i}\gamma_{\mathrm{MHD}} + \frac{\omega_{pi}^{(0)}}{2}\zeta_i. \tag{3.5}$$

The real correction to the ideal MHD growth rate is the frequency $\omega_{pi}^{(0)}/2$ times a surface-global factor. The result is then

$$\omega_r = \frac{\omega_{pi}^{(0)}}{2} \frac{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} (\varOmega_r \partial_{x^2}\varOmega_i - \varOmega_i \partial_{x^2}\varOmega_r)}{\displaystyle\oint \mathrm{d}x^2 \int_{-\infty}^{\infty} \mathrm{d}\theta \frac{\sqrt{g_B}B_a^2}{B^2} |\varOmega|^2}. \tag{3.6}$$

Let us now consider a trial function which is a rotation by an angle $k_2 x^2$ of a function $\hat{\varOmega}(\theta)$ defined on a flux tube

$$\varOmega(\theta, x^2) = \hat{\varOmega}(\theta)[\cos(k_2 x^2) + \mathrm{i}\sin(k_2 x^2)]. \tag{3.7}$$

Equation (3.6)) then reduces to the local result

$$\omega_r = \frac{\omega_{pi}}{2} \equiv \frac{1}{4}\frac{v_{\mathrm{thi}}}{L_n}(1+\eta_i)k_2\frac{\rho_i}{a}. \tag{3.8}$$

A less trivial fieldline-label dependence of the eigenfunction generates an effective surface-global diamagnetic frequency. Let us consider, for instance, a system with helical symmetry, thus

$$\varOmega(\theta, x^2) = \sum_{l=-M}^{M} \hat{\varOmega}_l(\theta)\{\cos[l(q\theta + x^2)] + \mathrm{i}\sin[l(q\theta + x^2)]\}. \tag{3.9}$$

The contribution to the surface-global real frequency of each helical harmonic M is proportional to

$$\varOmega_r \partial_{x^2}\varOmega_i - \varOmega_i \partial_{x^2}\varOmega_r = M\{\cos^2[M(q\theta + x^2)] + \sin^2[M(q\theta + x^2)]\} = M, \tag{3.10}$$

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thus

$$\omega_r^{(M)} = \frac{\omega_{pi}^{(0)}}{2} M, \tag{3.11}$$

and the marginal frequency is affected by the number of poloidal turns it takes the helix to close onto itself.

We conclude that, in a surface-global setting, for large pressure gradients, the real frequency of unstable KBMs (as described by diamagnetic MHD, equation (2.1)) can differ from the value $\omega_{pi}/2$ for purely geometrical reasons.

4. Lattice-drift model for KBMs

A further geometric effect that we expect to observe is associated with the x^2-dependence of the strength of the curvature drive, the term that multiplies ω_κ in (2.1). In (3.1), this term was formally separated into an axisymmetric and a non-axisymmetric part: $\omega_\kappa \propto K_{AS}(\theta) + \epsilon_h K_h(\theta, x^2)$. The effect of the x^2 dependence in ω_κ has been investigated for the case of the ion-temperature-gradient mode. In the work of Zocco *et al.* (2016), the authors performed an asymptotic expansion in $\epsilon_h \ll 1$. For finite ϵ_h, the authors introduced a discrete Fourier expansion of the ion-temperature-gradient driven (ITG) eigenvalue equation (Zocco, Xanthopoulos & Helander 2018). The non-axisymmetric term $\epsilon_h K_h(\theta, x^2)$ then generates a side-band coupling of the Fourier component of the eigenfunction. The eigenvalue equation is written in a matrix form, and a surface-global eigenvalue equation is given by setting to zero the determinant of the matrix, which strikingly resembles the equation of state of quantum electrons in a periodic crystal. The same approach is now possible for KBMs, however there is now a complication owing to the second-order derivative on the left-hand side of (2.1), which was neglected in the aforementioned ITG studies. In practice, we need to introduce an explicit form for ω_κ is (2.1), expand the eigenfunction using as a basis the functions used to construct ω_κ and study a system of coupled ballooning equations, rather than one ballooning equation, which is sufficient in the axisymmetric case, since ω_κ is a function of θ only. The careful reader might recognise that such approach is similar to the flux-tube-bundle model introduced by Sugama *et al.* (2012) and used numerically by Nunami, Watanabe & Sugama (2010). Thus, we proceed by neglecting the complications related to the x^2 dependence of the left-hand side of (2.1), and start with (3.1). We assume $B^2 \approx B_a^2$ and take $\sqrt{g_B} = \text{const}$. We add a small helical correction to the driving term found in concentric circular geometry

$$K_{AS}(\theta) + \epsilon_h K_h(\theta, x^2) = \cos\theta + \hat{s}\theta \sin\theta + \epsilon_h \{\cos[M(q\theta + x^2)] + \sin[M(q\theta + x^2)]\}, \tag{4.1}$$

where L_B is some effective average radius of curvature. The system is artificial but useful to build up some intuition to be used in the interpretation of either surface-global or flux-bundle numerical simulations. If we use $k_2 \to -\mathrm{i}\partial_{x^2}$, $\psi = \sum_m \psi_m \exp[2\pi x^2/a]$, equation (3.1) becomes

$$\begin{aligned} &\frac{1}{\beta_i}\frac{\partial}{\partial\theta}(1+\hat{s}^2\theta^2)\frac{\partial}{\partial\theta}\psi_m \\ &\quad = -\hat{\omega}\left[\hat{\omega} - \frac{\ell_c}{2L_n}\rho_* m(1+\eta_i)\right](1+\hat{s}^2\theta^2)\psi_m - \frac{\ell_c^2}{L_pL_B}(\cos\theta + \hat{s}\theta\sin\theta)\psi_m \\ &\qquad - \epsilon_h \frac{\ell_c^2}{2L_pL_B}\left\{\mathrm{e}^{\mathrm{i}Mq\theta}\left(1-\frac{M}{m}\right)^2\psi_{m-M} + \mathrm{e}^{\mathrm{i}Mq\theta}\left(1+\frac{M}{m}\right)^2\psi_{m+M}\right\}, \end{aligned} \tag{4.2}$$

https://doi.org/10.1017/S0022377818000648 Published online by Cambridge University Press

where $\hat{\omega}=\omega/(v_{\mathrm{thi}}/\ell_c)$, ℓ_c is a connection length and L_B an effective radius of curvature. The first two lines of (4.2) are simply the Fourier series expansion of the axisymmetric equation studied by Aleynikova & Zocco (2017). Non-axisymmetry is induced by the helical term. Let us consider a given $m_0 \sim \rho_*^{-1} \gg 1$, $m=m_0-\Delta m$ and $\hat{\omega}=\hat{\omega}_0+\mathrm{i}\hat{\gamma}_0+\delta\hat{\omega}\equiv\bar{\omega}+\delta\hat{\omega}$, where

$$\frac{\Delta m}{m_0}\sim\frac{\delta\hat{\omega}}{|\hat{\omega}^{(0)}|}\sim\epsilon_h\ll 1, \tag{4.3}$$

with $\hat{\omega}_0=(\ell_c/4L_n)m_0\rho_*(1+\eta_i)$ and $\hat{\gamma}_0=\mathrm{Im}[\bar{\omega}]$, where $\bar{\omega}$ is the solution of the quadratic equation for the axisymmetric problem $\bar{\omega}(\bar{\omega}-\omega_{pi})+\tilde{\lambda}^2=0$, and

$$\tilde{\lambda}^2=\frac{\ell_c^2}{L_pL_B}\frac{\displaystyle\int_{-\infty}^{\infty}\mathrm{d}\theta(\cos\theta+\hat{s}\theta\sin\theta)|\psi_{m_0}|^2}{\displaystyle\int_{-\infty}^{\infty}\mathrm{d}\theta(1+\hat{s}^2\theta^2)|\psi_{m_0}|^2}-\frac{1}{\beta_i}\frac{\displaystyle\int_{-\infty}^{\infty}\mathrm{d}\theta(1+\hat{s}^2\theta^2)\left|\frac{\partial\psi_{m_o}}{\partial\theta}\right|^2}{\displaystyle\int_{-\infty}^{\infty}\mathrm{d}\theta(1+\hat{s}^2\theta^2)|\psi_{m_0}|^2}. \tag{4.4}$$

Notice that in the subsidiary $\rho_*m_0\ll 1$ limit, the mode is purely growing and ψ_m is real. We now consider this limit. After using the finite-difference formula for the m-space derivatives, the imaginary part of the first-order correction reads

$$\mathrm{Im}[\delta\hat{\omega}]=\epsilon_h\frac{\ell_c^2}{L_pL_B\hat{\gamma}_0}\frac{\displaystyle\int_{-\infty}^{\infty}\mathrm{d}\theta\,\cos(q\theta)|\psi_{m_0}|^2\left(\frac{2}{\psi_{m_0}}\left.\frac{\partial^2\psi}{\partial m^2}\right|_{m_0}-1\right)}{\displaystyle\int_{-\infty}^{\infty}\mathrm{d}\theta(1+\hat{s}^2\theta^2)|\psi_{m_0}|^2}, \tag{4.5}$$

which is finite for $\rho_*m_0\ll 1$, and always negative if ψ_m has a maximum in m_0. This result proves that the helical correction to the axisymmetric ballooning mode is stabilising. Perhaps, the most important feature of (4.5) is that stabilisation occurs for any value of q, while the Mercier condition for stability, for concentric circular cross-sections, shows a strong dependence on q (Glasser, Green & Johnson 1976; Porcelli & Rosenbluth 1998)

$$\mathcal{D}=\frac{8\pi r}{\hat{s}^2B}\left|\frac{\mathrm{d}p}{\mathrm{d}r}\right|(1-q^2)<\frac{1}{4}. \tag{4.6}$$

Equation (4.5) then implies that a system can be ballooning unstable according to the Mercier criterion, but the surface-global effect could mitigate the instability.

5. Summary and discussion

In this article we studied several new aspects of kinetic ballooning modes in magnetically confined toroidal plasmas that stem from purely geometric properties of the confining magnetic field. This was done for large equilibrium plasma pressure gradients since, in this limit, analytical progress can be made. The surface-global formulation of the problem was presented. Here, physical quantities are kept radially local but variations in the fieldline-label coordinate are allowed for both equilibrium and perturbed fields. A novel form of the Mercier stability criterion, useful for quantitative comparison with stellarator flux-tube gyrokinetic codes was given. The use of modified Boozer coordinates led us to the conclusion that a minimisation of the average of the magnetic field magnitude square is beneficial for stability. We explain the relation between this result and the stabilising effect of magnetic wells on equilibrium configurations. For surface-global systems, we derived the general form equivalent to the necessary condition for instability of KBMs which constrains the frequency of the mode. It is found that purely geometric effects can result in mode frequencies that differ from the tokamak result $\mathrm{Re}[\omega] = \omega_{pi}/2$, where ω_{pi} is the total diamagnetic frequency of the ions. Finally, the effect of the coupling of several flux tubes covering a flux surface has been studied. This coupling has a stabilising effect on the local most unstable mode, and can lead to a possible violation of the Mercier criterion.

Acknowledgements

We thank P. Helander, J. Nührenberg, J. Connor, H. Sugama, M. Nunami and M. Nakata, for their comments on different aspects of this work. Part of this work was motivated by discussions with M. Romanelli in the framework of the ITPA for transport and confinement, and was carried out under the auspices of EUROfusion for the JET task T17-05 D3 'Gyrokinetic simulation in the pedestal (global effects)' and was completed during a visit at the National Institute for Fusion Science, Toki, Gifu, Japan. We thank F. Zonca and A. Mishchenko for their comments on the work on kinetic ballooning mode theory presented by K.A. at the 17th European Fusion Theory Conference, Athens, October 2017.

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