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ARTICLES

TECHNOLOGY ADOPTION DURING THE PROCESS OF DEVELOPMENT: IMPLICATIONS FOR LONG-RUN PROSPECTS

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Most papers studying the impacts of technology adoption on income trajectories assume that firms adopt frontier technologies when available. If these technologies are skill intensive, less-developed economies may fail in successfully implementing them and may become trapped in a low-growth equilibrium. Within a Schumpeterian growth model, we show that differences in adoption barriers and incentives to the accumulation of skills produce differences in the technology level that is optimal to adopt. If the economy is not overly distorted, copying nonfrontier technologies helps compensating for the scarcity of skills and increases the likelihood of copying frontier technologies in the long run. If distortions are significant, it may be optimal to copy less-advanced technologies even in the long run. If adoption is not a skill-intensive activity, then copying frontier technologies is always optimal; all economies achieve a high-growth equilibrium and only income differences persist in the long run.

Keywords: R&D Investment, Technology Adoption, Growth and Development, Technology Choice, Convergence and Polarization, Transitional Dynamics

The degree of technological complexity varies enormously across countries. While some developed economies (e.g., Japan and the United States) use complex technologies to produce their mix of products, other economies, (e.g., Kenya and Niger) produce using much less advanced ones. For example, in an index of technological sophistication created by the OECD that ranges from 1 (low sophistication) to 5 (high sophistication), Asian countries have an average score of 3.0 (with Hong Kong and Korea having a score of 3.9). Latin American countries have an average score of 2.2, while Sub-Saharan African countries only average 1.8. There are economies that have shown impressive technological upgrades during the past 20

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years. For example, China had an average score of 3.1 in 1995 that increased to 3.8 in 2007. However, there are other countries that have not produced significant improvements, such as Argentina, which has maintained a score of 2.1 during that same time span.[1]

Most of the literature that studies steady-state growth rates and income level differences in models of technology adoption make the strong assumption that firms will copy the most advanced technology available (the technological frontier). However, given the differences in the type of adopted technologies and adoption lags, several questions arise. For instance, why do we not observe a larger technology convergence among countries in a world of rising globalization? What are the implications of this behavior for long-run prospects? Should countries put efforts into adopting state-of-the-art technologies when available? Will poor countries eventually catch up with developed economies or will they remain in a poor growth path?

In order to answer these questions, we develop a Schumpeterian model that includes technology adoption choices. This model produces two types of equilibria: a high-growth equilibrium (the economy copies frontier technologies and grows at a high rate) and a low-growth equilibrium (it copies backward technologies and diverges from developed economies).

The framework used is a multisector growth model that is closely related to the models of Aghion and Howitt (1998) and Howitt (2000). Technological improvements result from costly and risky research and development (R&D), which is undertaken by R&D firms in different sectors of the economy. The argument of the paper builds on the following two assumptions. First, implementing technology uses an input that is produced domestically. We refer to this domestic input as skills. Entrepreneurs obtain skills through a learning function that depends on the stock of knowledge in the economy. Second, the productivity of these skills depends on the complexity of the adopted technology. In particular, the more complex the technology, the lower the productivity of a given skill level.[2] The firm chooses a level of technology and R&D investment. These decisions impact the level of technology in use and the probability of being successful in its adoption. Both decisions depend on deep parameters such as adoption barriers, the intensity with which adoption uses skills to implement new technologies, and the quantity and quality of education. We build the model in a way that permits a closed solution.

Our first contribution is to study the effects of relaxing the assumption of copying frontier technologies finding new paths of development and new steady-state equilibria. We show that copying less-sophisticated technologies permits economies with poor initial conditions to escape a low-growth equilibrium in the long run. This result arises as this relaxation expands the set of parameters (or the set of economic conditions) that are coherent with high growth in the long run. That is, equilibria that countries were not able to achieve should they have copied state-of-the-art technologies during their transitional dynamics are now reachable.

Second, we show that it is optimal to copy the technological frontier only under certain (restrictive) circumstances, namely when the implementation of better technologies is not skill intensive (or not dependent on a domestic input) and when the economy has few distortions thus providing enough incentives to produce skills and generate R&D investment. In all other cases, the optimal adoption choice is not the technological frontier.

Third, copying nonfrontier technologies can be a transitory or a permanent situation. The situation is only transitory if the economy has relatively low-adoption barriers, a healthy R&D environment, and a good education system, but only has a small stock of accumulated knowledge. In this case, a small stock of knowledge provides few skills to entrepreneurs so adopting state-of-the-art technologies is too costly. For this economy, it is optimal to adopt less-sophisticated technologies until it accumulates enough knowledge. Eventually, it will adopt frontier technologies. On the other hand, copying less-advanced technologies is a permanent situation if the economic environment is not R&D friendly enough. In this case, trying to copy frontier technologies is not an optimal decision even in the long run.

Fourth, the framework provides predictions regarding the path of adopted technology throughout the development process for countries starting their developing process. The optimal technology choice is a positive function of the economy's knowledge stock, the quality and quantity of education, and a negative function of the skill intensity in adopting the technology and the level of adoption barriers. Regarding transitional dynamics, the higher the adoption barriers and the skill intensity needed to adopt technologies, the longer that an economy will copy nonfrontier technologies.

The results presented in this paper are related to other findings in the literature. In relation to the importance of human capital for catching up, as early as 1952, Gerschenkron pointed out the importance of social capabilities for producing this outcome. Afterward, Nelson and Phelps (1966) state that adoption capacities depend on domestic conditions and in particular on the stock of human capital. On the empirical side, the paper by Benhabib and Spiegel (1994) shows that the rate of technology diffusion depends on the level of education. Regarding the literature on technology choice, Basu and Weil (1998) and Acemoglu and Zilibotti (2001) argue that difficulties in benefiting from adoption arise because technologies generated by developed economies are not appropriate for developing economies. In both models, developed economies' technologies are created for a mix of inputs that is not available in the developing economy. In Basu and Weil (1998), the developing economy is short of physical capital; in Acemoglu and Zilibotti (2001), the shortage is skilled labor. Research closely related to these works include Caselli and Coleman (2006) and Vandenbussche et al. (2006), which focus on the effects of having different relative endowments of skilled and unskilled labor on the choice of technology to produce the final output. Our results contribute to this literature by characterizing the path of adopted technologies of an economy in its transitional dynamics and in its steady state. While the aforementioned literature takes the relative endowments of skilled and unskilled labor as given, we endogenously determine the stock of skills (and knowledge) that will be available in the transition and in the steady state.

Our paper is also associated with studies on convergence and divergence in growth rates. If the activity of adoption does not intensively use skills to implement new technologies, the model produces convergence in growth rates independently of initial conditions and parameters. In fact, most of the studies on technological adoption find convergence in growth rates in the long run [Parente and Prescott (1994, 2002) and related literature]. The possibility for growth through adoption is key to explaining this convergence behavior as it permits laggard economies to benefit from technologies created elsewhere.[3] However, differences in long-run growth rates are possible when adoption intensively uses skills to implement new technologies. Our model can lead to a decreased capacity of copying foreign technologies and thus to the divergence in growth rates among economies.[4] In our model, R&D barriers may have effects on both, long-run growth rates and income levels. These barriers affect technological improvements through an additional channel: they impact R&D rewards, R&D investment, and thus the path of skill accumulation. Models with technological transfers that produce different growth rates in the long run are scarce.[5] Howitt (2000) presents a model in which a high-growth (convergence) equilibrium happens every time there is some R&D investment. Implicitly, this model has a constant technology adoption capacity, so that a low-growth equilibrium (or polarization) occurs only when the R&D sector fully disappears. Aghion et al. (2005) and Howitt and Mayer-Foulkes (2005) extend this model to emphasize two channels that can lead to growth traps. The first paper focuses on credit constraints that impede that the less-developed economy gets enough funding for financing R&D activities. The second paper, which is more closely related to our work, highlights the problems of skill acquisition, which is needed for R&D. Complementing this last paper, our work explicitly addresses a mechanism that links R&D productivity with skill accumulation. It also studies the conditions that explain when changes in the R&D environment affect (or not) long-run growth.

In a related view, Acemoglu et al. (2006) argue that technological advances depend on the economy's capacity to switch from an imitation strategy (investment based) to an innovation one (selection based). The key assumption is that innovation becomes more important as the economy approaches the technological frontier. While this work focuses on the economy's ability to generate adequate institutional arrangements that maximize technological change at each stage of development, our work focuses on studying the economic conditions that generate the sufficient level (and flow) of knowledge that maximizes technological change at each stage of development. In the language of Acemoglu et al. (2006), our paper studies the conditions that allow for switching from a strategy based on copying less-advanced technologies to one based on copying the frontier.[6] Finally, Chu et al. (2014) study optimal, intellectual property rights policies (IPRPs) based on a country's stage of development. The farther away from the technological frontier, IPRPs should be weak to foster adoption activities. Meanwhile, when the economy is closer to the technological frontier, IPRPs should be strong to facilitate innovation.

The paper is organized as follows. Sections 1 and 2 present the analytical model and the analysis of the aggregated equilibrium, respectively. Section 3 discusses the role of an optimal technology adoption target and its implications for the transition and the steady state. We also present in this section some comparative statics and dynamics through numerical exercises and a discussion on how different R&D conditions affect the optimal technology adoption target, the development path, and the long-run growth. Finally, Section 4 summarizes the results and offers some conclusions.

1. THE MODEL

The model follows Aghion and Howitt (1998) and, particularly, Howitt (2000). Consider one benchmark economy out of J small open economies. The benchmark economy is composed of two types of sectors: a homogeneous and competitive final goods sector and a broad intermediate sector producing different qualities of inputs. All of the action happens in the broad intermediate sector, which produces a continuum of intermediate inputs of mass one. The intermediate subsector is composed of two firm types: one R&D firm and many non-R&D ones. The R&D firm engages in R&D activities to improve the currently used technology. When successful, this firm produces a new quality of the intermediate input and obtains profits. The non-R&D firms are capable of producing the intermediate input with the current technology, but with a productivity disadvantage relative to the R&D firm that successfully introduced the new technology.

Technological progress is endogenous at the country level and is given through the adoption and implementation of better technologies. There is a technological frontier that grows at the exogenous rate g.

The economy is populated by two types of agents: a continuum of homogeneous entrepreneurs of mass one that live for two periods and a continuum of workers of mass one that live for one period. Entrepreneurs engage in R&D activities and workers inelastically supply their time endowment to the final goods producer. Both types of agents derive linear utility from the consumption of the final good. We further assume that markets are complete and that there is perfect access to foreign capital. The risk-free international bond has a constant interest rate equal to r_{B_t}. All risk is idiosyncratic. In this setting, consumption and production decisions are independent. Time is discrete: $t = 1, 2, 3, \ldots$.

1.1. Producers: Final Goods and Intermediate Firms

There is a single final and homogeneous good that is produced by a representative firm with the following technology:

$$Y_t = L^{1-\epsilon} \int_0^1 A_{it} x_{it}^{\epsilon} di. \tag{1}$$

The firm produces with a flow of labor L and intermediate inputs x_{it}. The mass of workers is constant for every period. The firm pays in equilibrium the marginal product of labor and the marginal product of inputs p_{it} given by

$$p_{it} = \epsilon A_{it} x_{it}^{\epsilon-1} L^{1-\epsilon}. \tag{2}$$

Every intermediate input i embeds a productivity level of A_i (we denote both, the subsector and the input by the subscript i). The higher the productivity embedded in input x_{it}, the higher the quantity of Y_t generated by one unit of x_{it}. Inputs are produced with the following technology:

$$x_{it} = \eta \frac{K_{it}}{A_{it}}. \tag{3}$$

The intermediate firm in subsector i only uses physical capital to produce the input. Physical capital is divided by the technology embedded in the intermediate input to account for how more advanced technology requires more capital to embed that technology in the input. Parameter η determines the productivity of firms in producing the inputs. R&D and non-R&D firms differ on the value of this parameter.

We assume that in every period there is a large number of non-R&D firms capable of producing the input with the currently available technology in subsector i. However, in some subsectors, there will also be an R&D firm that was successful in the previous period in improving the technology and will be able to produce the input this period, with a productivity advantage. In particular, the successful R&D firm can produce inputs with a parameter $\eta_{R\&D} = 1$, whereas a non-R&D firm can produce inputs with a parameter of $\epsilon < \eta_F < 1$.[7] Both types of firms compete *à la* Bertrand so that the equilibrium price of inputs corresponds to the marginal cost of the second most efficient producer. Thus, $p_{it} = A_{it} r_t / \eta_F$, where r_t corresponds to the cost of capital. Replacing p_{it} in equation (2), we obtain the equilibrium quantity supplied from each intermediate input given by $x_{it} = L(\epsilon \eta_F / r_t)^{\frac{1}{1-\epsilon}}$. Consequently, when the R&D firm is successful in improving the technology, this firm serves the whole market and obtains profits equal to

$$\pi_{it} = \left(\frac{1-\eta_F}{\eta_F} \right) A_{it} L \left(\frac{\epsilon\, \eta_F}{r_t^{\epsilon}} \right)^{\frac{1}{1-\epsilon}}. \tag{4}$$

Profits are proportional to the technology used in each sector. Perfect access to foreign capital ensures that $r_t = r_{B_t} + \delta$ in equilibrium, where δ corresponds to the depreciation rate of physical capital.

1.2. The R&D Market and R&D Firms

Howitt (2000) specifies that technological improvements arrive randomly at a rate $n\lambda$, where λ is a parameter indicating the productivity of the research technology and n corresponds to a production factor (typically R&D investment or labor used

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in research). When successful, a firm can operate with the technological frontier. We depart from this setting in three dimensions. First, the firm has to choose a technology adoption target, which will not necessarily be the technological frontier. Second, we redefine the factor of production, n, as a combination of effective units of R&D investment ($i_{it} \equiv I_{it}/A_{i,t}$) and of domestic skills ($s_{it} \equiv S_{it}/A_{i,t}$). Skills in our setting will be crucial and endogenous. Third, in contrast to Howitt's specification, we assume that the arrival rate of technological improvements shows decreasing returns to scale to R&D investment and skills, which provides a nontrivial solution for the technology target. We discuss all these features of the model when appropriate.

Entrepreneurs and R&D firms' choices. In every period t, a new entrepreneur is born in every subsector i that starts an R&D firm in order to improve the current technology. R&D is risky. If the R&D firm is successful in adopting a new technology, it will become the new producer of input i in the next period. However, if it is not successful, it will become a noninnovative firm.[8]

The entrepreneur has to acquire skills to engage in R&D activities. Skills S_{it} are produced through a learning production function, $S_{it} = f(\mu, \theta, H_t)$. We define learning as a broad concept that includes all activities that generate new skills, such as formal education, training, or any other activity that produces new capabilities. This learning function depends on the fraction of time that the entrepreneur spends learning, μ, the quality of the education system, θ, and the "content" H_t provided by this system. We assume that the content is a public good that is proportional by a factor of ξ to the stock of knowledge accumulated by the economy thus far. Moreover, we assume that the stock of knowledge H_t is the result of all R&D previously done by R&D firms. In this sense, it corresponds to intangible capital that provides skills for adopting and implementing foreign technologies in the future. We assume that the more knowledge the economy has accumulated, the more skills that agents can obtain by interacting with the other members of the society. Parameter ξ governs the degree of spillovers of this knowledge. In particular, we will consider the following learning function:

$$S_{it} = \mu_{it}^{\theta} \xi H_t; \qquad 0 < \mu < 1 \quad \text{and} \ \theta > 1. \tag{5}$$

The entrepreneur chooses the fraction of time devoted to obtaining skills for a given quality of the education system.

To copy a new technology, the firm has to perform two activities: First, it must incur a cost by searching/buying a better technology in the world market and, second, it must implement the technology to match the particular features of its productive sector. We assume that the cost of searching/buying the technology is proportional to the targeted technology at the factor κ.[9] To implement the technology, the firm has to further invest R&D resources.

The firm faces the maximization problem presented in equations (6) to (7), simultaneously choosing the fraction of time devoted to skills acquisition μ_{it}, the technology to adopt, which we call the technology (adoption) target, A^T_{it}, and R&D investment, I_{it}, for given state variables $A_{\max,t}$, A_{it}, and H_t.

$$\max_{I_{it}, A^T_{it}, \mu_{it}} \ \phi_{it} W_{it} - I_{it} - \kappa A^T_{it} \quad \text{subject to} \tag{6}$$

$$A_{\max,t} \geq A^T_{it} \tag{7}$$

$$I_{it} \geq 0. \tag{8}$$

$W_{it} = (\pi_{i,t+1})/(1 + r_B)$ corresponds to profits in present value if the R&D firm is successful[10]; ϕ_{it} corresponds to the probability that the R&D firm will be successful in implementing the new technology; $A_{\max,t}$ corresponds to the world technological frontier available at time t; and A_{it} corresponds to the technology currently in use in subsector i.

Equation (7) states that the adoption target has to be lower than or equal to the technology frontier. The probability of success is key and is defined in the next equation:

$$\phi_{it} \equiv \left(\frac{I_{it}}{A^T_{it}}\right)^{\alpha} \left[\left(\frac{S_{it}}{A^T_{it}}\right)^{\gamma} (1 - \mu_{it})\right] \lambda; \quad \alpha + \gamma < 1. \tag{9}$$

Its determinants are the following: First, it depends on R&D investment I_{it}. We scale this investment by the technology target of the R&D firm, A^T_{it}, to account for how increased technological complexity increases the amount of resources needed; α is an intensity parameter.

Second, the term $(S_{it}/A^T_{it})^{\gamma}$ accounts for the role that domestic skills play in the adoption activity. We include skills as a production factor to account for the long literature initiated by Nelson and Phelps that points out that human capital, in general, and education, in particular, play an important role in increasing the capacity for innovation and adoption of new technologies. Implementing a new technology is not an automatic process. It requires specific skills in order to understand the foreign technology and also to implement it in a suitable way to the specific sector.[11] However, to define the probability of being successful, the relevant measure of skills adjusts depending on the difficulty of the targeted technology. Again, we assume that the more advanced a technology is, the more complex it is to implement and the more skills it requires to be mastered. In other words, the skill base of a country may be insufficient to copy state-of-the-art technologies.

Third, skill requirements may be a key determinant for adopting new technologies or they may be nonessential. Parameter γ denotes the intensity by which skills are used in the adoption activity. In this setup, the parameter is equal for all countries. A value of $\gamma = 0$ implies that skills are not needed as an input for adopting and, consequently, does not affect the firm's adoption possibilities. As the value for this intensity increases, skills as an input for adoption become more relevant.[12] We further assume that the probability function shows decreasing returns to scale to R&D investment and the stock of skills: i.e., $\alpha + \gamma < 1$. This assumption contrasts with seminal Schumpeterian growth models that consider that the production function of new technologies exhibits constant returns to scale (CRS). We consider this to not only be a more realistic assumption, but also to be consistent with the empirical evidence [see, for example, Kortum (1993) and Cohen and Klepper (1996)].[13]

Fourth, the probability of successfully adopting a technology also depends on barriers, policies, institutions, and incentives to copy foreign technologies.[14] Parameter λ comprises all of these effects. This parameter reflects the kind of barriers emphasized by Parente and Prescott (1994) and lies within the range [0, 1]. The lack of barriers to adopting new technologies implies a value for $\lambda = 1$ and, conversely, maximum barriers imply a $\lambda = 0$. This parameter can vary across countries. Finally, the entrepreneur devotes the fraction $(1 - \mu_{it})$ of time to open her R&D firm. We define w_{it} as the present value of the R&D firms' profits per unit of technology as $w_{it} \equiv W_{it}/A^T_{it}$. This expression depends only on aggregate factors. Thus, all sectors face the same profits adjusted by the level of technology and $w_{it} = w_t$.

The maximization problem for any period t yields the first-order condition presented in equation (10) for the optimal fraction of time spent acquiring skills, μ_{it}. The entrepreneur faces the following problem: First, the more time she spends acquiring skills, the higher the probability of successfully implementing new technology, but the less time she can devote to implementing it. According to equation (10), the higher the quality of the education system, the more time that is optimal to spend at schools. On the other hand, if technologies require increased skills to be copied, the more time the entrepreneur needs to spend at schools to copy frontier technologies.

$$\mu_{it} = \mu = \frac{\gamma\theta}{1 + \gamma\theta}. \tag{10}$$

According to equation (10), at any time t, the entrepreneur devotes the same fraction of time to education. Combining equations (10) and (5), we obtain the level of skills obtained by an entrepreneur given by

$$S_{it} = S_t = \left[\frac{\gamma\theta}{(1 + \gamma\theta)}\right]^{\theta} \xi H_t. \tag{11}$$

The level of skills that an entrepreneur can obtain is limited by the quality of the education system and the stock of knowledge, H_t, of the economy. If the stock of knowledge does not flow efficiently to all agents in the economy through the educational institutions (low ξ), the process of acquiring skills may be difficult.

The next two equations present the first-order conditions for R&D investment and the technology target. In these equations, we replace the skill variable S_t by its optimal value as obtained in equation (11).

$$\alpha \left(\frac{I_{it}}{A_{it}^T}\right)^{\alpha} \lambda\beta \left(\frac{H_t}{A_{it}^T}\right)^{\gamma} A_{it}^T w_t \; = \; I_{it}, \tag{12}$$

$$(1-\alpha-\gamma) \left(\frac{I_{it}}{A_{it}^T}\right)^{\alpha} \lambda\beta \left(\frac{H_t}{A_{it}^T}\right)^{\gamma} w_t \quad \geq \; \kappa, \tag{13}$$

$$\text{where} \quad \beta = \frac{(\gamma\theta)^{\gamma\theta}}{(1+\gamma\theta)^{1+\gamma\theta}} \; \xi^{\gamma}. \tag{14}$$

For a given technology target and stock of knowledge, an increase in R&D investment not only increases the probability of success and thus expected benefits, but also increases R&D expenses. On the other hand, for a given R&D investment and stock of knowledge, a higher technology target increases profits if the firm is successful. However, it also reduces the probability of being successful and increases adoption costs. The firm equates marginal revenues with marginal costs [equation (12)].

The technology target has an upper limit bound at the technology frontier. Thus, from equation (13), we observe two cases of special interest, namely when it is optimal for the firm to copy frontier technologies and when it is not optimal to do so.

1.3. Case 1: Choosing Nonfrontier Technologies

An interior solution happens when equation (13) holds with equality meaning that it is optimal for an R&D firm to choose nonfrontier technologies. Dividing equation (12) by equation (13), we obtain a relationship between R&D investment and the technology target given by

$$\frac{I_{it}}{A_{it}^T} = \left(\frac{\alpha\kappa}{1-\alpha-\gamma}\right). \tag{15}$$

There is a constant relation between I_{it} and A_{it}^T that depends on the cost of adopting new technologies, κ, the intensity of using R&D resources, α, and skills in the adoption activity, γ. If the cost of copying a new technology increases, then it is optimal that the firm spends relatively less resources in more advanced technologies and spends relatively more resources on the implementation of them. Combining equation (15) with equation (13), we obtain an expression for the technology target chosen by the R&D firm.

$$A_{it}^T = A_i^T = H_t \left[\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha} \alpha^{\alpha} \beta\lambda w_t \right]^{1/\gamma}. \tag{16}$$

The technology target is an increasing function of the stock of knowledge, H_t, the quality and quantity of the education system, β, and profits (and scale) of the economy, w_t. That is, the better the education system, the better the skills acquired by entrepreneurs and the more likely that advanced technologies can be successfully introduced. Besides, the technology target is a decreasing function of both the adoption barriers and the intensity in which the implementation of new technology requires skills. Although there is state-of-the-art technology available in the world economy, economies with small stocks of knowledge will choose nonfrontier technologies.

By combining equation (16) with restriction (7), we obtain the knowledge threshold below which firms choose nonfrontier technologies. This threshold does not depend on sector variables and all sectors face the same condition.[15]

$$H_t \leq \underline{H}_t \equiv \frac{A_{\max,t}\kappa^{\frac{1-\alpha}{\gamma}}}{\left[(1-\alpha-\gamma)^{(1-\alpha)} w_t \lambda \beta \alpha^{\alpha}\right]^{\frac{1}{\gamma}}}. \tag{17}$$

First, the knowledge threshold increases with the technology frontier. Second, this threshold is dynamic as the technology frontier changes every period. That means, that surpassing the threshold in one period does not ensure that firms will be able to copy frontier technologies in the future. The economy must constantly increase its knowledge to continuously copy frontier technologies. Given this, if the economy has a small stock of knowledge and the growth rate of the technological frontier accelerates, it is more likely that firms will continue to copy less-advanced technologies for longer periods of time. The binding restriction for catching up is not simply having a small stock of absolute knowledge, H_t, but having a small stock of knowledge in relation to what is needed to copy more complex and advanced technologies H_t/A_{it}^T. Third, the threshold decreases with the quality and quantity of the education system as a better education system is more efficient in providing skills to entrepreneurs.

By combining equations (9), (15), and (16), we obtain the level of optimal investment and the probability of success in equilibrium as presented in equations (18) and (19), respectively:

$$I_{it} = I_t = H_t \left[\left(\frac{1-\alpha-\gamma}{\kappa} \right)^{1-\alpha-\gamma} \alpha^{\alpha+\gamma} \beta \lambda w_t \right]^{1/\gamma} \tag{18}$$

$$\phi_{it} = \phi_t = \frac{\kappa}{(1-\alpha-\gamma) w_t}. \tag{19}$$

In equilibrium, the R&D firm combines R&D investment with a technology target to maintain the probability of success constant [equation (19)].

What happens if technologies become more skill intensive? An increase in the adoption skill intensity parameter produces an increase in the equilibrium probability of success. The intuition is the following: as skill intensity increases, it is more difficult to copy advanced technologies given the stock of knowledge in the economy. As a consequence, R&D firms reduce both R&D investment and the technology adoption target. However, the reduction of the technology target is stronger than the reduction of R&D investment increasing the probability of success, but reducing expected profits $\phi_t A_t^T w_t$.

1.4. Case 2: Choosing Frontier Technologies

An R&D firm chooses to copy frontier technologies when entrepreneurs have enough skills to develop and implement state-of-the-art technology. This happens every period that $H_t > \underline{H}_t$ [equation (17)]. According to equation (12), and to the fact that $A_{it}^T = A_{\max,t}$, it chooses a level of R&D investment given by

$$I_{it} = I_t = A_{\max,t} \left[\left(\frac{H_t}{A_{\max,t}} \right)^{\gamma} \lambda\beta\alpha w_t \right]^{\frac{1}{1-\alpha}}. \tag{20}$$

In this case, the R&D firm invests a larger amount of R&D relative to the technological frontier ($I_{it}/A_{\max,t}$) compared to the case where it was copying nonfrontier technologies. The proof is presented in Appendix A. The reasoning behind this is that knowledge is no longer binding for copying state-of-the-art technologies. In fact, if available, the firm would rather copy technologies even more advanced than the existing $A_{\max,t}$. As a consequence, the firm invests more to compensate for the fact that it cannot target a technology higher than the technology frontier. In this way, it increases the probability of success and maximizes expected profits.

Investment relative to the technology target, i.e., the technological frontier, is no longer constant and depends on the relationship between the stock of knowledge and the technological frontier. By introducing equations (20) and (11) in the definition of the probability of success, we obtain the probability of success in equilibrium presented in the next equation. This probability evolves in line with technology-adjusted knowledge $H_t/A_{\max,t}$.

$$\phi_{it} = \phi_t = \left[(\alpha w_t)^{\alpha} \left(\frac{H_t}{A_{\max,t}} \right)^{\gamma} \lambda\beta \right]^{\frac{1}{1-\alpha}}. \tag{21}$$

2. AGGREGATE IMPLICATIONS

We now turn to the aggregate implications. Variables without subscript i denote the corresponding variable at the aggregate level, i.e., $X = \int_0^1 X_i di$. Variables in lowercase define variables in term of the technology frontier, i.e., $x_t \equiv X_t/A_{\max,t}$. The development path of the economy depends on the law of motion for the stock of knowledge and for aggregate productivity. Aggregate productivity is equivalent to the economy's average productivity A_t, which evolves as

$$A_{t+1} = A_t + \int_0^1 \phi_t \left(A_{it}^T - A_{it}\right) di \tag{22}$$
$$= A_t + \phi_t \left(A_t^T - A_t\right).$$

We assume that successes are uniformly and independently distributed across sectors. Thus, the probability of success ϕ_t corresponds to the actual fraction of R&D firms that are successful in implementing new technology at the aggregate level. It also measures the rate of creative destruction (rate of entry and exit of R&D firms) in equilibrium.

We model the accumulation of knowledge H_t as an externality resulting from R&D investment as in the following expression:

$$H_{t+1} - H_t = q \int_0^1 I_{it} di = q I_t, \tag{23}$$

where q is a spillover parameter that captures how much of R&D investment transforms in knowledge. We assume that when R&D firms research implementing new technology in their specific sector, they accumulate experience, which increases the economy's stock of knowledge. The stock of knowledge grows regardless of whether the R&D firm was successful or not.[16] This way of modeling the stock of knowledge follows a long tradition in this type of literature. See, for example, Klenow and Rodriguez-Clare (2005) who review the role played by this type of externalities in theoretical growth models and their empirical relevance in explaining differences in income per capita.

By dividing equations (22) and (23) by the technological frontier $A_{\max,t}$, we obtain the economy's average productivity and the stock of knowledge in relative terms.

$$a_{t+1} = \frac{a_t + \phi_t \left(a_t^T - a_t\right)}{1+g} \tag{24}$$

$$h_{t+1} = \frac{h_t + q i_t}{1+g}. \tag{25}$$

Again, we find two types of aggregate trajectories depending on whether R&D firms are choosing frontier or nonfrontier technologies.

2.1. Aggregate Trajectories and Steady State

Replacing the equilibrium values for the technology target in terms of the technology frontier a_t^T, the rate of creative destruction ϕ_t, and R&D investment in terms of the technology frontier i_t[17] in equations (24) and (25), we obtain the law of motion for the two variables of interest during the development path. As profits adjusted by the technology frontier w_t are constant, when dividing the minimum knowledge threshold by the technology frontier ($\underline{H}_t/A_{\max,t} \equiv \underline{h}_t = \underline{h}$), we obtain a constant knowledge threshold. As the world interest rate r_B is constant, profits adjusted by the level of technology are constant too (i.e., $w_t = w$) and depend only on parameters.

The law of motion for relative productivity is given by

$$a_{t+1} = \begin{cases} \dfrac{a_t + \frac{\kappa}{(1-\alpha-\gamma)w}\left(h_t\left\{\alpha^{\alpha}[(1-\alpha-\gamma)/\kappa]^{1-\alpha}\beta\lambda w\right\}^{1/\gamma} - a_t\right)}{1+g} & \text{if } h_t < \underline{h} \quad \textbf{(26a)} \\ \dfrac{a_t + \left[h_t^{\gamma}(\alpha w)^{\alpha}\lambda\beta\right]^{\frac{1}{1-\alpha}}(1-a_t)}{1+g} & \text{if } h_t \geq \underline{h}. \quad \textbf{(26b)} \end{cases}$$

The law of motion for relative knowledge is

$$h_{t+1} = \begin{cases} \dfrac{h_t + q\left[\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\lambda\beta w\right]^{\frac{1}{\gamma}} h_t}{1+g} & \text{if } h_t < \underline{h} \quad \textbf{(27a)} \\ \dfrac{h_t + q\left(h_t^{\gamma}\alpha\lambda\beta w\right)^{\frac{1}{1-\alpha}}}{1+g} & \text{if } h_t \geq \underline{h}. \quad \textbf{(27b)} \end{cases}$$

Defining h^{-}_{t+1} as the law of motion for knowledge accumulation when $h_t < \underline{h}$ [equation (27a)] and h^{+}_{t+1} when $h_t \geq \underline{h}$ [equation (27b)], we can characterize the function h_{t+1} in three dimensions: (i) h_{t+1} is continuous at the threshold of relative knowledge $\underline{\mathrm{h}}$; (ii) h^{-}_{t+1} is a linear function of h_t and h^{+}_{t+1} is a concave function of h_t; and (iii) the slope of h^{+}_{t+1} is lower than the slope of h^{-}_{t+1} and the slope of h^{+}_{t+1} is less than one when h_t tends to infinity. The corresponding proofs are presented in Appendix B.

Depending on the values of the parameters, the economy can follow two types of development paths: one leading to a high-growth equilibrium and one leading to a low growth one. Given (i), (ii), and (iii), the next three results define the different equilibria and discuss their characteristics.

RESULT 1. *If* $[q^{\gamma}\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\beta\lambda w]^{\frac{1}{\gamma}} > g$, *then the economy converges toward a high-growth steady state with a relative knowledge of* h_{ss} *regardless of the initial stock of relative knowledge* $H_1/A_{\max,1} \equiv h_1$ *in t=1.*

This converging case is plotted in Figure 1. The previous result implies that the function h^{-}_{t+1} is above the 45-degree line. As h_{t+1} is continuous at $\underline{\mathrm{h}}$, h^{+}_{t+1} is a concave function, and its slope is less than one when h_t tends to infinity, this

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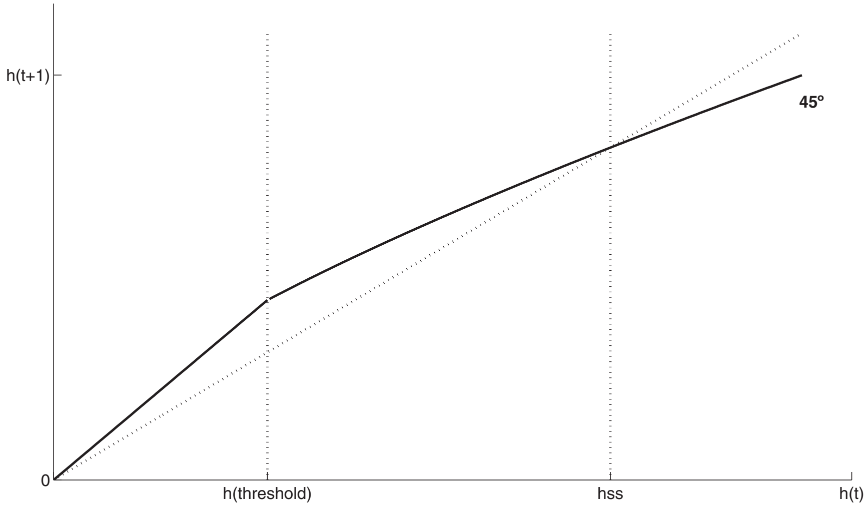

FIGURE 1. Law of motion of the stock of knowledge relative to the technological frontier. The convergence case.

function crosses the 45-degree line at the relative knowledge steady-state value $h_{ss} > \underline{h}$.

The intuition is the following: If this economy starts with $h_1 < \underline{h}$, then its developing process starts by copying nonfrontier technologies and accumulating knowledge at a higher rate than the world innovation rate, g. This economy reaches in finite time T the relative knowledge threshold $h_T = \underline{h}$.[18]

If $h_1 > \underline{h}$, then, by the concavity of the knowledge accumulation function [equation (27b)], knowledge is growing slower than the technological frontier. The economy loses knowledge in relative terms until reaching its steady-state value when it continues growing at the same rate as the technological frontier. The steady state for a high-growth economy is unique and is given by the expression in equation (29).

This result shows that the likelihood of achieving high growth in steady state depends on both domestic factors (e.g., the quality and quantity of the education system, adoption barriers) and global ones (e.g., the technological frontier's growth rate, the blueprint costs, the skill intensity required for the implementation of new technology). Thus, the higher the educational quality is, the more the spillovers R&D investment has, and the lower the adoption barriers and costs are, the easier is for an economy to accumulate new knowledge and keep pace with the expansion of the technological frontier. These factors allow the economy to produce the flow of knowledge needed to copy and implement the most advanced technologies. Note, however, that an economy with poor domestic factors can still copy advanced technologies in the long run if the cost of buying new technologies is sufficiently low or the technological frontier grows sufficiently slowly. For example, if technological progress occurs only sporadically (low g), an economy that slowly builds new knowledge can afford to only sporadically copy advanced technologies while remaining relatively updated.

The high-growth long-run equilibrium has the following properties:

(i) The economy targets $A_{\max,t}$, equivalently, $a_T = 1$ in the long run.
(ii) The economy grows at the growth rate of the technological frontier g.
(iii) Relative productivity and relative knowledge stock are given by[19]

$$a_{(ss)} = \frac{\left[\lambda\beta q^{\gamma}\left(\alpha w\right)^{\frac{\alpha(1-\alpha-\gamma)+\gamma}{1-\alpha}}\right]^{\frac{1}{1-\alpha-\gamma}}}{\left[\lambda\beta q^{\gamma}\left(\alpha w\right)^{\frac{\alpha(1-\alpha-\gamma)+\gamma}{1-\alpha}}\right]^{\frac{1}{1-\alpha-\gamma}} + g^{\frac{1-\alpha}{1-\alpha-\gamma}}} \tag{28}$$

$$h_{(ss)} = \left(\frac{q}{g}\right)^{\frac{1-\alpha}{1-\alpha-\gamma}} \left(\alpha w \lambda \beta\right)^{\frac{1}{1-\alpha-\gamma}}. \tag{29}$$

Equation (28) implies that all economies sharing this type of long-run equilibrium converge in growth rates, but differ in their relative productivity. Differences in adoption barriers and in the quality of the education system explain these level differences.

RESULT 2. *If* $[q^{\gamma}\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\beta\lambda w]^{\frac{1}{\gamma}} < g$, *then the economy follows a low-growth development path regardless of the initial stock of knowledge* h_1.

This diverging case is plotted in Figure 2. Now, the function for knowledge accumulation is below the 45-degree line for all values of h_t. The economy's absolute knowledge stock is increasing through time, but does so at a slower pace than the technological frontier. Independent of initial values, the economy loses capacity for copying advanced technologies during its transitional path and drifts away from developed countries in the long run.

The low-growth long-run equilibrium has the following properties:

(i) The economy copies nonfrontier technologies. The targeted technology relative to the technological frontier becomes increasingly laggard, that is $a_T \to 0$ in the long run. At any period t, the technology target is given by

$$A_t^T = H_t\left[\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha}\alpha^{\alpha}\beta\lambda w\right]^{1/\gamma}. \tag{30}$$

(ii) The economy grows at a positive rate, in absolute terms, but slower than the technological frontier. In particular, the growth rate of the economy equals the

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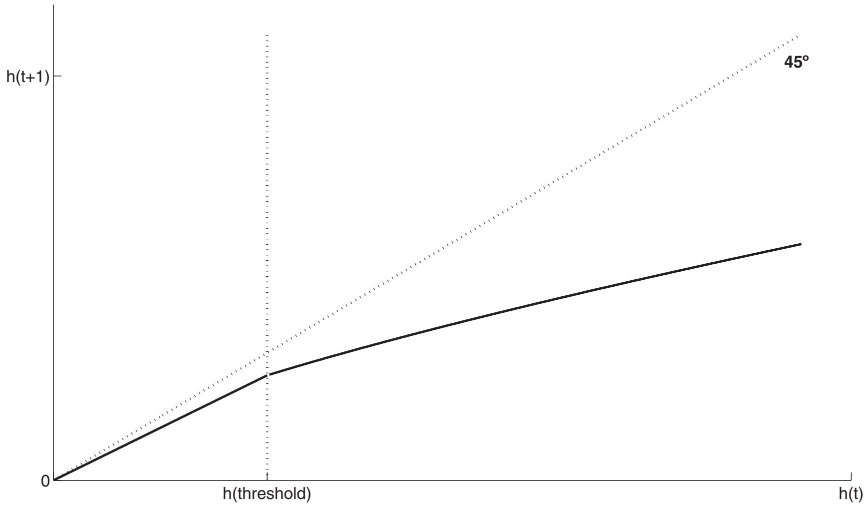

FIGURE 2. Law of motion of the stock of knowledge relative to the technological frontier. The polarization case.

growth rate of the stock of knowledge, which is given by

$$\begin{aligned}\frac{A_{t+1}-A_t}{A_t} &= q\left[\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{(1-\alpha-\gamma)}\alpha^{(\alpha+\gamma)}\beta\lambda w\right]^{\frac{1}{\gamma}} \\ &= \frac{H_{t+1}-H_t}{H_t} \\ &< g.\end{aligned} \tag{31}$$

The corresponding proof is presented in Appendix C.

(iii) The economy drifts away from the developed world, that is, its relative productivity level $a \to 0$ and its relative stock of knowledge $h \to 0$ in the long run.

In contrast to economies that are heading to a high-growth steady state and, therefore, grow at the same rate, economies in this low-growth equilibrium grow at different rates in the long run. In this equilibrium, the growth rate depends solely on domestic conditions. Therefore, we will observe different development patterns for these economies if they differ in their economic structure.

Second, these economies grow at a positive rate and produce new knowledge and agents acquire newer and better skills through time. They still benefit from external innovations as they are copying better technologies during the transition and in the steady state. However, these economies will never find it profitable to copy frontier technologies.

The technology target will vary between these economies. The level of technology that economies decide to copy will depend, among other factors, on the skills that entrepreneurs can attain, which in turn depend on the quality of the

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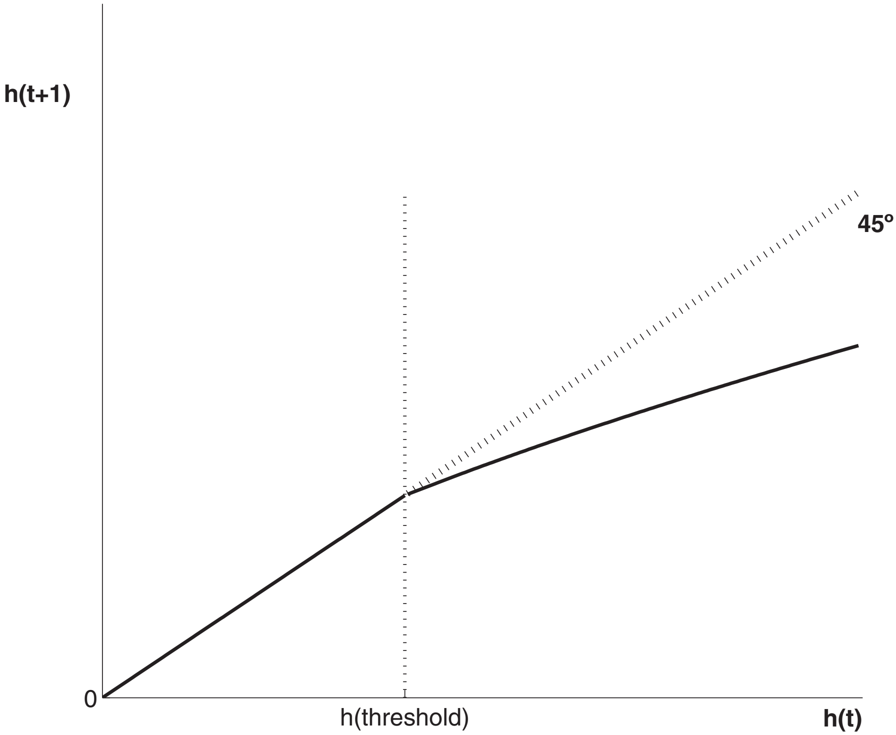

FIGURE 3. Law of motion of the stock of knowledge relative to the technological frontier.

education system. The different technology targets will produce different flows of R&D investments and different rates of technological change.

RESULT 3. *There is a particular case when* $[q^{\gamma}\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\beta\lambda w]^{\frac{1}{\gamma}} = g$. *This case is plotted in Figure 3.*

There are two cases of interest. Case 1: $h_1 < \underline{h_1}$. *In this case, the absolute stock of knowledge grows at the rate of the technological frontier* g. *Consequently, the relative stock of knowledge remains constant and* $H_t/A_{\max,t}=h_1$, $\forall\, t$.

As the economy does not have sufficient knowledge to copy frontier technologies, it copies a constant fraction of it. During this transition, the relative stock of knowledge remains constant, but relative productivity evolves according to equation (26a) with $h_t=h_1$, $\forall\, t$. In the long run, the economy reaches the relative productivity presented in equation (33).

The long-run equilibrium is characterized by the following properties:

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(i) The economy targets technologies that are proportional to the technological frontier. That is $A^T \propto A_{\max,t}$.

$$a_t^T = h_1 \left[\left(\frac{1-\alpha-\gamma}{\kappa} \right)^{1-\alpha} \alpha^\alpha \beta \lambda w \right]^{1/\gamma} \equiv c. \tag{32}$$

Equivalently, $a_T = c$, with $c < 1$ and constant.

(ii) The economy grows at the growth rate of the technological frontier g.

(iii) Relative productivity is given by[20]

$$a_{ss} = \frac{h_1}{\kappa^{(1-\alpha-\gamma)/\gamma}} \frac{\left[\alpha^\alpha (1-\alpha-\gamma)^{1-\alpha} \beta \lambda w\right]^{1/\gamma}}{g(1-\alpha-\gamma)w + \kappa}. \tag{33}$$

Case 2: $h_1 \geq \underline{h}$. First, as $h_1 \geq \underline{h}$, the economy is copying the technological frontier. Second, the relevant function for the law of motion for knowledge accumulation [equation (27b)], is below the 45-degree line. Consequently, the economy loses knowledge in relative terms until it reaches knowledge threshold $\underline{h}$. From that point on, the stock of absolute knowledge grows in line with the technological frontier.

3. DISCUSSION

To exploit the behavior of a developing economy through its path of development, we present some numerical exercises in order to study comparative statics and dynamics.

3.1. Numerical Exercises

In order to obtain plausible and meaningful results, we calibrate the model with parameters obtained from data and the previous literature. We need values for nine parameters, which are reported in Table 1.

Based on Jorgenson et al. (1987), we assign a value of 0.5 to the material share of output ϵ. The productivity parameter of noninnovating firms η_F can be associated with the inverse of markups. We also obtain this value from Norrbin (1993).[21] We assign a value of 2.2% to the world innovation rate and the growth rate of the technology frontier g. This value corresponds to the average per capita growth rate in the United States for the years 1960–2006 based on Maddison (2009). We set the long-run riskless interest rate at 4%, which corresponds approximately to the average value of this rate for the United States for post-1950 data.

As we are studying the effects of the different knowledge and R&D investment intensities on relative productivity, we run the model for $\alpha + \gamma$ in the range [0, 0.9]. This range accounts for all relevant cases.

As for the education parameter β, equation (14), it is a function of the education quality and quantity and the spillover parameter ξ. We assume that skills are linear in the economy's knowledge stock and assume for the numerical exercises

TABLE 1. Parameters

	Definition	Value	Source
ϵ	Material share	0.490	Jorgenson et. al. (1987)
η	Inverse of markup	0.910	Norrbin (1993)
g	Growth rate of the technology frontier	0.022	Author's calculation/ Maddison (2009)
r	Riskless interest rate	0.040	Post-1950 US data
μ	Fraction of time devoted to educational activities	0.139	Author's calculation/Barro and Lee (2013)
β	Education parameter, equation (14)	0.626	Author's calculation, see main text
α, γ	R&D and skill intensities	[0, 0.9]	
λ	Adoption barriers	[0,1]	

that $\xi = 1$. Thus, β depends only on the expression $\theta\gamma$, which depends on the fraction of time spent at school. From equations (10) and (14), we get that $\beta = \mu^{\mu/(1-\mu)}(1-\mu)$, which has a counterpart in the data. To calculate μ, we obtain data for the average years of schooling from the Barro and Lee dataset. For all countries listed in this dataset, the average years of schooling is 8.2 in year 2010.[22] If we assume that people work until age 65 and enter primary school at six years, then the average time of education corresponds to approximately 13% of the total work years. Consequently, $\beta = 0.626$. Finally, we assume that both the unit cost of adopting a technology κ and the education quality are equal to one.[23]

We focus on two determinants of the technology target and long-run growth. The first one is a political variable, λ, which corresponds to adoption barriers that make it more difficult and costlier for countries to adopt better technologies. These barriers depend to a large extent on domestic policies and can be modified by the government. The second variable is a technological one, γ, and corresponds to the intensity in which the adoption activity needs skills to produce a technological change. This variable does not depend on domestic factors, but rather depends on the type and characteristics of the production of technological innovations. Thus, with these two variables, we can analyze the implications of changing domestic and foreign conditions on long-run growth.

To put our results in context, we will use as a proxy of barriers to technology adoption the Information and Communication Technology Index constructed by the World Bank. This variable does not directly measure government policies, but rather the results of these policies. This index is the simple average of the following variables for 2007: (i) telephones per 1,000 people, (ii) computers per 1,000 people, and (iii) internet users per 1,000 people. The index tries to measure each country's information infrastructure. The easier the communication, dissemination, and processing of information is, the more effective the diffusion of new technologies.

TABLE 2. Adoption barriers: $\lambda = 1$—No barriers; $\lambda = 0$—High barriers. Means and standard deviations.[1] GDP per worker relative to the United States.

	Adoption barriers		Relative GDP per worker	
Type of economy	Mean	Std. deviation	Mean	Std. deviation
High-income countries	0.89	0.05	0.79	0.10
High-to-middle income countries	0.66	0.14	0.34	0.13
Middle-income countries	0.38	0.11	0.13	0.04
Low-income countries	0.22	0.07	0.05	0.01
Average	0.59	0.26	0.36	0.29
Percentile 100—Sweden	0.96	–	0.76	–
Percentile 75—Spain	0.84	–	0.63	–
Percentile 50—Bulgaria	0.58	–	0.32	–
Percentile 25—Algeria	0.36	–	0.17	–

Source: Author's calculations. [1]Mean: simple average of country groups; std. deviation: simple average of standard deviations of country groups.

The index ranges from zero to one, denoting zero highest barriers and one, lowest barriers.

We measure these variables for a sample of eighty-six countries. The country sample is presented in Table D.1 in Appendix D. We classify each country by its income level according to the World Bank's income classification. Table 2 presents the results. The upper panel shows the average index by income group. High-income countries have almost four times more information infrastructure than low-income countries. To get an idea of the countries in this distribution, the lower panel of Table 2 identifies the countries that constitute the 25, 50, 75, and 100 percentiles of the distribution. Sweden is the country with lowest barriers according to this measure and Bulgaria is the median country.

3.2. The Role of the Optimal Target: The Steady State

In this section, we focus our analysis on the value of adoption targets in relative and absolute terms. The first measure enables us to analyze the effects on converging economies while the second measure allows us to analyze the behavior of nonconverging ones.

Figure 4 shows the technology target relative to the technology frontier for different combinations of adoption barriers and skill intensities. A value of the relative technology target equal to one implies that the economy is adopting frontier technologies. A value of zero implies that the economy is copying more advanced technologies in the long run, but these technologies are increasingly

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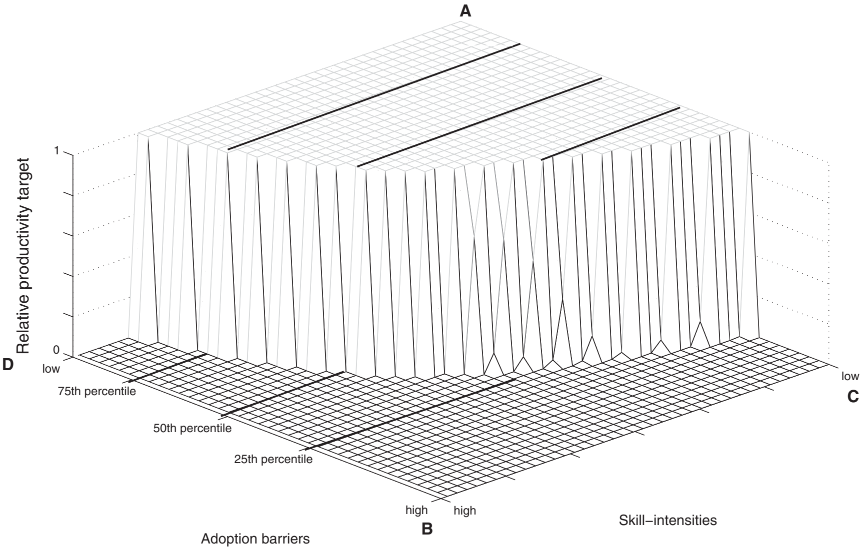

FIGURE 4. Relative technology target in steady state: Technology target relative to the technology frontier.

laggard relative to the technology frontier. The figure shows that a country like Spain (75th percentile) can afford a larger increase in skill intensity compared to a country like Algeria (25th percentile).

In the steady state and in relative terms, there are two possible outcomes. Either, the economy is copying frontier technologies in the long run (or a constant fraction of them) or it is copying backward technologies. When both adoption barriers and skill intensity are low, the optimal target is the technology frontier. Moreover, when skill intensity is low, the economy can afford medium to high barriers and still copy frontier technologies in the long run (moving from point A to point C). However, if technologies become more skill intensive (moving from point A to point D or point C to point B), it is increasingly difficult to copy state-of-the-art technology in the long run.

There is a one-to-one correspondence between copying frontier technologies and growing at a high rate in steady state. As shown by Results 1–3, a developing economy can achieve two types of long-run growth equilibria: In the high-growth equilibrium ($a_{(ss)} > 0$), the economy grows at the rate of the technology frontier, which equals the growth rate of leading economies. In the low-growth equilibrium, the economy grows at a rate given by its domestic R&D conditions. Figure 5 presents the long-run productivity steady state in relation to both variables. The vertical axis measures the average steady-state technology level (productivity) of a country relative to the technology frontier.

This figure encompasses both steady-state technology levels and long-run growth rates.[24] All countries that are targeting the technology frontier in the

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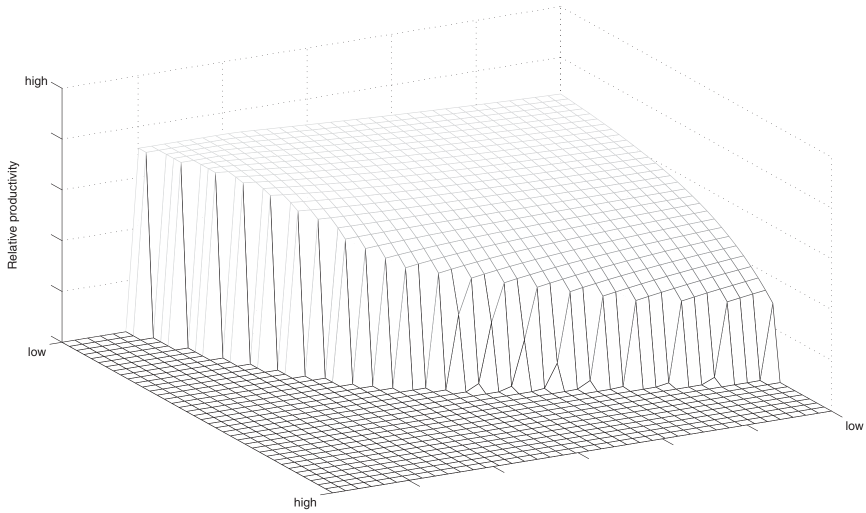
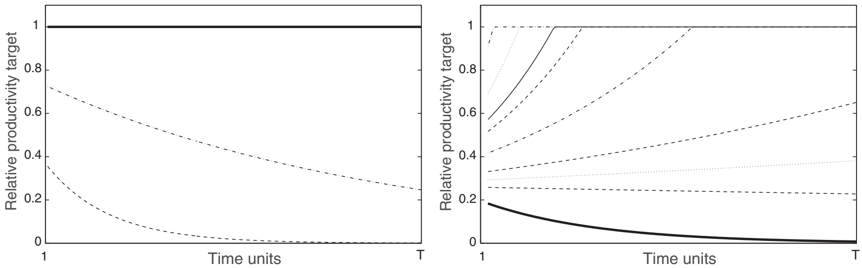


FIGURE 5. Different steady states.

long run ($a^T \to 1$ in Figure 4) are also achieving a high-growth equilibrium in steady state. Note that this group of countries can embrace economies with different economic structures (e.g., education system, barriers). However, all of these economies generate at least the minimal additional skill flow that allows for copying frontier technologies through time. What happens with economies exhibiting a relative technology target converging to zero in Figure 4? First, these are economies that are not copying frontier technologies in steady state and technologies are increasingly laggard relative to the frontier ones. Note that this case does not correspond to cases in which an economy is copying a constant fraction of the technology frontier and thus maintain a constant TFP ratio relative to it (the latter corresponds to the cases discussed in Result 3).

Another interesting question is to analyze whether both types of economies (convergent and nonconvergent) are affected in a similar way by a change in the world innovation rate. There are different responses depending on the economy's technological absorptive capacity. Suppose that the world innovation rate increases from g to g'. According to equations (26a) and (26b) and equations (27a) and (27b), economies with $q[\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\beta\lambda w]^{1/\gamma} \geq g'$ increase their growth rate in steady state. The reason is that these economies had an excessive capacity to copy frontier technologies. In fact, they would copy even more advanced technologies if available. As a consequence, these economies increase their growth rate. However, countries that do not satisfy this condition will maintain their (lower) growth rate in steady state and will become (further) polarized; these economies will not benefit



(a) Low skill intensity, γ=0.15 (b) High skill intensity, γ=0.45

FIGURE 6. The technology target path, high and low skill intensity, different adoption barriers. (a) Low skill intensity, $\gamma = 0.15$. (b) High skill intensity, $\gamma = 0.45$.

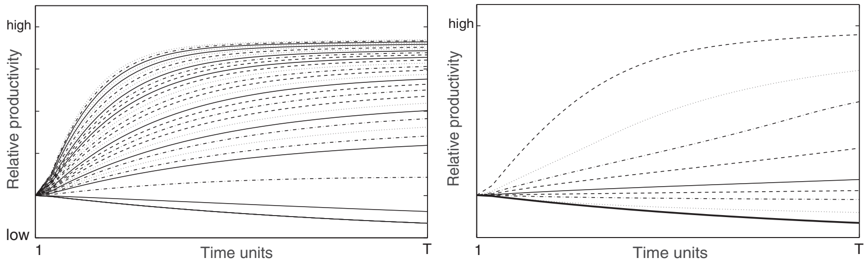
from a higher innovation rate as they were not able to copy frontier technologies in the first place.

3.3. The Role of the Optimal Target: The Dynamics

Turning to the dynamics of the target choice, Figure 6 shows the evolution of the ratio of the technology target [described in equation (32)] relative to the technology frontier. Again, when the target is the technology frontier, the ratio takes the value of one. The horizontal axis denotes periods of time, and T corresponds to a large number of periods. Every line denotes the evolution of the technology target for a given parameter value of the adoption barrier. We simulate the technology target trajectory for 38 different values. For a given period, the lower the line, the higher the implied adoption barrier. Panel (a) shows the technology target evolution when the skill intensity parameter γ is low and panel (b) shows when it is high.

Overall, when the economy has a small stock of knowledge (for instance, at the beginning of the development process at t=1), R&D firms choose a low-technology target. This target rises as the technology absorptive capacity of the economy increases. If the adoption skill intensity parameter is low, shown in Figure 6(a), R&D firms target the technology frontier for almost all values of the adoption barriers (the many values are represented by the black thick line). Only when adoption barriers are too high (the other two lines in the graph), is it optimal for the R&D firm to copy a less-advanced technology. These two values correspond to adoption barriers with values below the 15th percentile of the distribution. If adoption does not need domestic skills to be performed, that is $\gamma = 0$, then the optimal technology target is the technology frontier for all adoption barrier values.

What happens if technologies involve more skills in their implementation, i.e., if γ increases? Figure 6(b) presents how the target changes for the same values of the adoption barriers. First, for all simulated adoption barriers, R&D firms start by choosing technologies below the frontier. For low-adoption barriers



(a) Low skill intensity, γ=0.15

(b) High skill intensity, γ=0.45

FIGURE 7. Relative productivity path, high and low skill intensity, different adoption barriers. (a) Low skill intensity, $\gamma = 0.15$. (b) High skill intensity, γ=0.45.

(higher functions in the figure) the economy builds technology absorptive capacity and eventually targets the technology frontier. However, for medium- and high-adoption barriers, the less-developed economy is not able to copy the technology frontier (the 30 highest values for adoption barriers are represented by the black thick line). The diverging countries correspond to those below the 65th percentile of the distribution.

The counterpart of the relative technology target is the relative average technology achieved by the economy, which is presented in Figure 7. For low-knowledge-intensity technologies, panel (a) illustrates that economies copying the technology frontier [the thick line in Figure 6(a)] show different paths for relative productivity if their economic structures differ. On the other hand, for diverging trajectories of the relative technology target [all cases described by functions with negative slope in Figure 6(b)], we observe diverging trajectories for the average relative technology [functions with negative slope in Figure 7(b)].

3.4. Implications of Assuming that the Technology Target is the Technology Frontier

In this section, we analyze the implications for income of endogenously determining the optimal level of technology to adopt compared with the case in which R&D firms always copy the technology frontier (i.e., no choice of technology adoption target).

RESULT 4. *The assumption of copying the technology frontier (independently from the economic structure) is more significant, the adoption barriers are higher, the education system provides less skills, and more intense skills are required to implement new technologies.*

The intuition is straightforward. As skills become more relevant for adoption or as the economy provides fewer incentives to perform R&D, the optimal adoption target no longer remains the technology frontier. Thus, efforts to continuously implement frontier technology reduce the probability of being successful in adopting

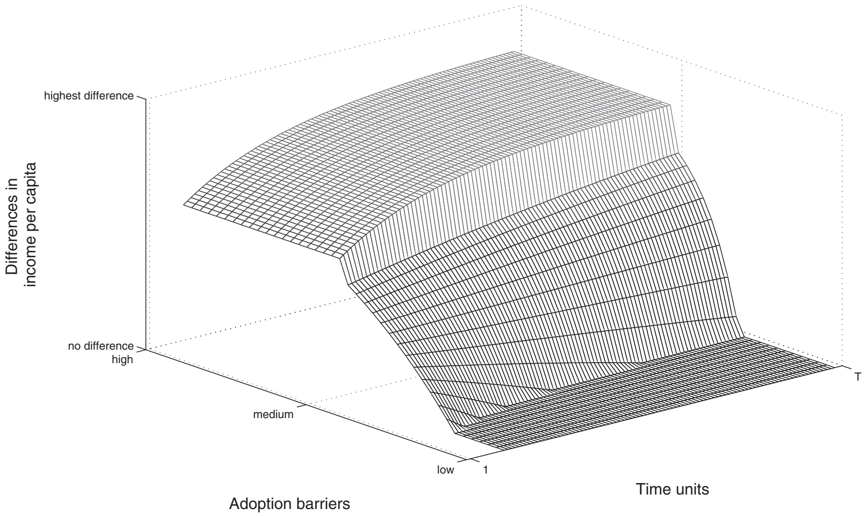


FIGURE 8. Transitional dynamics: Differences in income. Path resulting from using a high skill intensity parameter.

technology, increase adoption costs, and also lower expected technological change. The effect is a decrease in aggregate income.

What are the implications for relative income? To answer this, we measure relative income as GDP relative to the technology frontier, net the costs of R&D activities. Relative net income is given by

$$y_t^{net} = a_t x^{\epsilon} - i_t - \kappa a_t^T. \tag{34}$$

Again, when skill intensities are low, the economy targets the technology frontier throughout the development process and for almost all adoption barrier levels. Consequently, relaxing the assumption of copying frontier technologies does not provide significant new insights for the path of development. However, as skill intensities increase and it becomes optimal to choose a lower target, nontrivial differences in income arise during the transitional dynamics.

Figure 8 presents the implied income differences of allowing an optimal technology choice versus imposing the copying of frontier technologies during the path of development for a medium skill intensity.

Positive differences imply that choosing an optimal target produces a net benefit and zero differences imply that there are not any long-run gains of choosing an optimal target during the development process. The gains of optimally choosing the technology frontier emerge during the transitional dynamics and in steady state. For high-adoption barriers, these differences arise in the transition and in steady state, as high-adoption barriers constitute a permanent distortion. In contrast, for medium-adoption barriers, these differences arise only during the transition as having a low stock of knowledge is common for economies that are beginning to develop.

Income gains of choosing an adoption target are larger if higher skills are needed for copying new technologies and for higher adoption barriers. In order to better understand this reasoning, it is helpful to consider that when R&D firms are required to copy the technology frontier when they optimally would have chosen a lower one, the following happens: if successful, the R&D firm implements a better technology (positive effect), however, the probability of success decreases (negative effect) and the R&D firm has to pay a higher adoption cost ($\kappa A_{\max,t} - \kappa A_t^T$, additional negative effect). At the aggregate level, these results mean that the economy spends too much for copying frontier technologies, producing reduced income in the steady state.

In summary, relaxing the common assumption of copying frontier technologies results in resources being used more efficiently. This would indicate that overly ambitious R&D policies may be a poor choice for developing countries.

4. CONCLUDING REMARKS

A novel aspect of this paper is permitting the R&D firm to choose its technology target. Related studies usually assume that R&D firms will adopt the most advanced technologies available. However, is it always optimal to target the technology frontier? Copying the technology frontier may put a heavy burden on the R&D firms adoption capacity, as it requires a high stock of knowledge for successful implementation. This can considerably hinder being able to benefit from and grow via adoption as the potential technology improvement can drastically fall. However, even though absolute knowledge may be scant for acquiring the technology frontier, it may be sufficient for copying less-advanced technologies. In fact, the lower the technology that the R&D firm is copying, the more productive the stock of knowledge is and the more likely the technology will be successfully implemented.

Skills needed to absorb foreign technology depend on the technology copied. Within this framework, two situations arise. If the skill intensity parameter in the adoption activity is high, then the economy may fall into a low-growth equilibrium if its economic structure is poor. In this case, the adoption capacity can follow a declining path, becoming an impediment for growth. Reducing adoption barriers may help the economy escape this low-growth equilibrium, as a better R&D environment can compensate for the scarcity of knowledge in the early stages of development. However, the model shows that a complementary way to avoid the low-growth equilibrium is to copy technology not at the frontier, as these less-advanced technologies require less skills and less R&D resources to be implemented. The model suggests that countries will adopt laggard technologies when they possess a low stock of knowledge and will target more advanced technologies as they develop. In such a scenario, the economy may improve its growth prospects and eventually be able to copy frontier technologies to sustain high growth. If the skill intensity parameters increase through time, the process of catching up becomes more difficult and the prospects of achieving a high-growth equilibrium become less likely.

If skill intensity in the adoption activity is low, then a low stock of relative knowledge does not decisively impact the economys adoption capacity. In this case, it is always optimal to target frontier technologies. This strategy maximizes growth during the transition. Moreover, in this case all countries achieve a high-growth equilibrium in the long run.

NOTES

1. OECD (2010). Perspectives on Global Development 2010. Shifting Wealth. OECD Publishing. The index is constructed by assigning different values (1–5) to different types of technologies embedded in export manufacturing. These values correspond to: 1 for primary products, 2 for resource-based manufacturing, 3 to low-technology manufacturing, and 4 and 5 to medium- and high-technology manufacturing, respectively. The percentage of exports in each category is then multiplied by the corresponding value, and these are added together and divided by 100.

2. Knowledge refers to the stock of experiences and learning gained through R&D activities. Technology, on the other hand, refers to blueprints, guidelines, and procedures that determine a certain productivity level of intermediate inputs. We assume that the complexity of a technology has a one-to-one correspondence with the productivity that it generates so that both terms, technology and productivity, can be used interchangeably.

3. As in these papers, all economies grow at the same rate in the long run, these studies focus on explaining the differences in per capita income levels, usually stressing economic, legal, or technological barriers that prevent technology transfers.

4. This result complements an extensive body of literature that studies growth traps. These models introduce an economic friction or externality that impedes the accumulation of a productive factor, such as physical capital or knowledge. These factors directly enter the production function or are inputs of the technology production function. See, for example, Becker et al. (1990), Galor and Weil (1996), Becker and Barro (1989), Azariadis and Drazen (1990), Durlauf (1993), Benabou (1996), Galor and Zeira (1993), Murphy et al. (1989), Galor (2005), and McDermott (2002). Feyrer (2008) contrasts stylized facts with the implications of several of these models. Empirical studies consistent with this framework are Barro and Sala-i-Martin (1992), Mankiw et al. (1992), and Evans (1996), among others.

5. A few exceptions are Howitt (2000), Howitt and Mayer-Foulkes (2005), Acemoglu and Zilibotti (2001), and Basu and Weil (1998). Pritchett (1997) and Feyrer (2008) give empirical support for this approach.

6. Relatedly, Acemoglu et al. (2003) explore the relationship between an economy's distance to the technological frontier and the internal organization of the firm (integration/outsourcing decisions). Integration creates managerial overload, thereby, discouraging innovation activities while outsourcing relaxes time constraints, allowing the firm to focus on innovation activities. Based on the same ideas of Acemoglu et al. (2006), they show that there will be a nonconvergence trap if the economy is very uncompetitive, impeding the switch from an integration (imitation based) to an outsourcing (innovation-based) strategy.

7. This assumption ensures that at the equilibrium prices and quantities, the successful R&D firm supplies the whole market demand for input *i*.

8. This overlapping generation setting has been used in different Schumpeterian growth extensions [see examples in Aghion and Howitt (2008)].

9. The key assumption is that $\kappa > 0$. Linearity in A_{it}^T was chosen to obtain an analytical solution.

10. The assumption of temporary monopolies has been used in different settings in the growth literature, for example, in the literature on innovation cycles. Growth papers working on this topic include, for example, Matsuyama (1999) or Furukawa (2015).

11. Some models associate these skills with human capital. Both variables tend to capture the idea that adoption needs some domestic input to be performed. The inclusion of this variable is also supported by the empirical literature both macro [for example, Barro and Sala-i-Martin (1995)] and micro [for example, Bartel and Lichtenberg (1987)]. Moreover, the complementarity between R&D and educational attainment is directly addressed by Benhabib and Spiegel (1994) and supported by the data.

12. In the early stages of development, the country could target less skill-intensive technologies if available (lower γ). In fact, this strategy could help countries that are transitioning to a low-growth equilibrium to increase its adoption capacity and elude this equilibrium. In this paper, we do not focus on this mechanism and assume that the skill intensity in the adoption activity does not change across countries or time.

13. This assumption is also important for technical reasons. According to equations (6) and (9), expected profits are proportional to the factor $A_t^{T(1-\alpha-\gamma)}$. If the probability function had constant returns to scale (CRA) to the accumulative factors (i.e., $\alpha+\gamma=1$), profits would be independent of the technology level. As there is a positive cost of buying a new blueprint (κ), the optimal outcome would be to not buy new technologies. Even if we assume that the cost of acquiring a new technology is zero, then the assumption of CRS would lead that the technology target would be indeterminate for an R&D firm.

14. For example, access to internet and to communication systems, economic and legal regulations, adoption-related policies (e.g., opportunities to attend seminars and congresses), and all other variables that affect the overall efficiency of the adoption activity.

15. In this setting, we are not interested in exploiting sectoral heterogeneities. However, if sectors face different adoption costs or financial conditions, the former condition would be sector-dependent.

16. This variable captures all factors that determine the social knowledge of an economy which are not tradable. It mainly depends on present and past institutions, the education system, and experiences. Having said that, skills acquisition by entrepreneurs, which is the relevant variable for producing technology adoption, can still be affected by other education variables if the quality of the education system or time devoted to learning activities change.

17. Equations (16)–(18) if $h_t < \underline{h}_t$; and $a_t^T=1$ and equations (20) and (21) if $h_t \geq \underline{h}_t$.

18. Period T is given by $T = \log \underline{h} - \log(h_1) - \log\left(q[\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\beta\lambda w]^{1/\gamma}\right) + \log(1+g)+1$.

19. We obtain the steady-state values for this case by imposing the steady-state conditions $a_{t+1} = a_t = a_{ss}$ and $h_{t+1} = h_t = h_{ss}$ in equations (26b) and (27b), respectively, and by combining the resulting equations to obtain the steady-state values described in equations (28) and (29).

20. We obtain the steady-state value for this case by imposing the steady-state condition $a_{t+1} = a_t = a_{ss}$ in equation (24), and by combining the resulting equation with the technology target in relative terms and the rate of creative destruction described in equations (32) and (19), respectively.

21. There are several papers discussing the value of both parameters, for example, Basu and Fernald (1997) and Hall (1990), among others.

22. Barro and Lee (2013). Average years of schooling for the population 15 years old and older in year 2010. The sample includes 146 countries.

23. Changing the total working years or the value of κ does not change the model's implications.

24. As the average steady-state technology is measured relative to the technology frontier, all values strictly positive ($a_{(ss)} > 0$) imply that the average technology level is growing at the growth rate of the technology frontier (high-growth equilibrium). A relative steady-state technology equal to zero ($a_{(ss)} = 0$) implies that the economy is growing at a lower rate than the technology frontier in the long run.

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APPENDIX A

PROPOSITION 1. *When surpassing the minimum H_t-threshold for copying frontier technologies, the R&D firm starts investing a larger amount of R&D in terms of the technological frontier compared with the case in which it is copying nonfrontier technologies.*

Proof. Variables without subscript i denote the corresponding variable at the aggregate level, i.e., $X = \int_0^1 X_i di$. Variables in lowercases define variables in terms of the technology frontier, i.e., $x_t \equiv X_t / A_{\max,t}$. According to equations (15) and (20), R&D investment in

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terms of the technology frontier is as follows:

$$i_t = a_{it}^T \left(\frac{\alpha\kappa}{1-\alpha-\gamma} \right) \qquad \text{if } h_t \leq \underline{h} \tag{15}$$

$$i_t = h_t \left[\left(\frac{1-\alpha-\gamma}{\kappa} \right)^{1-\alpha-\gamma} \alpha^{\alpha+\gamma} \beta\lambda w \right]^{1/\gamma} \qquad \text{if } h_t \geq \underline{h} \tag{20}$$

with $a_{it}^T < 1$ when $h_t < \underline{h}$ and $a_{it}^T = 1$ when $h_t \geq \underline{h}$. Let us denote the period t in which $h_t = \underline{h}$ as T^*. By evaluating equations (15) and (20) at $t = T^*$, we obtain R&D investment at time T^*, which coincides under both rules and is given by

$$i_{T^*} = \left(\frac{\alpha\kappa}{1-\alpha-\gamma} \right). \tag{A.1}$$

For $t \geq T^*$, equation (15) is constant and equal to (A.1) and the investment rule given by equation (20) depends on the relative stock of knowledge.

Let us define $h_t = \underline{h}(1+\chi)$, with $\chi > 0$, $\forall h_t > \underline{h}$. Then, equation (20) can be written as

$$i_t = \underline{h}(1+\chi) \left[\left(\frac{1-\alpha-\gamma}{\kappa} \right)^{1-\alpha-\gamma} \alpha^{\alpha+\gamma} \beta\lambda w \right]^{1/\gamma} \qquad \text{if } h_t \geq \underline{h}. \tag{A.2}$$

Replacing $\underline{h}$ in equation (A.2) yields

$$i_t = \left(\frac{\alpha\kappa}{1-\alpha-\gamma} \right)(1+\chi) \tag{A.3}$$

$$= i_{T^*}(1+\chi) \tag{A.4}$$

$$> i_{T^*}. \tag{A.5}$$

■

APPENDIX B

PROOF OF RESULTS 1–3

The proof consists of characterizing the function h_{t+1} in three dimensions: (i) h_{t+1} is continuous at the threshold of relative knowledge, (ii) h_{t+1}^- is a linear function of h_t and h_{t+1}^+ is a concave function of h_t, and (iii) the slope of h_{t+1}^+ is lower than the slope of h_{t+1}^- and less than one when h_t tends to infinity.

Proposition *i*. *The function h_{t+1} described in equations (27a) and (27b) is continuous at $\underline{h}_t$.*

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Proof. Dividing H_t [equation (17)] with $A_{max,t}$, we obtain the following expression for the knowledge threshold relative to the technological frontier:

$$\underline{h} = \frac{\kappa^{\frac{1-\alpha}{\gamma}}}{\left[(1-\alpha-\gamma)^{1-\alpha} w\lambda\beta\alpha^{\alpha}\right]^{\frac{1}{\gamma}}}. \tag{B.1}$$

Replacing the knowledge threshold value in both functions h^{-}_{t+1} and h^{+}_{t+1}, we obtain

$$\begin{aligned} h^{-}_{t+1} &= \frac{1}{1+g}\left(\frac{\kappa^{\frac{1-\alpha}{\gamma}}}{\left[(1-\alpha-\gamma)^{1-\alpha} w\lambda\beta\alpha^{\alpha}\right]^{\frac{1}{\gamma}}}\right. \\ &\quad \left.\times\left\{1+q\left[\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\beta\lambda w\right]^{1/\gamma}\right\}\right) \\ &= \frac{1}{1+g}\left\{\frac{\kappa^{\frac{1-\alpha}{\gamma}}}{\left[(1-\alpha-\gamma)^{1-\alpha} w\lambda\beta\alpha^{\alpha}\right]^{\frac{1}{\gamma}}} + q\left(\frac{\kappa\alpha}{1-\alpha-\gamma}\right)\right\} \end{aligned} \tag{B.2}$$

$$\begin{aligned} h^{+}_{t+1} &= \frac{1}{1+g}\left(\frac{\kappa^{\frac{1-\alpha}{\gamma}}}{\left[(1-\alpha-\gamma)^{1-\alpha} w\lambda\beta\alpha^{\alpha}\right]^{\frac{1}{\gamma}}}\right. \\ &\quad \left.+q\left\{\frac{\kappa^{1-\alpha}}{\left[(1-\alpha-\gamma)^{1-\alpha} w\lambda\beta\alpha^{\alpha}\right]}\alpha w\lambda\beta\right\}^{\frac{1}{1-\alpha}}\right) \\ &= \frac{1}{1+g}\left\{\frac{\kappa^{\frac{1-\alpha}{\gamma}}}{\left[(1-\alpha-\gamma)^{1-\alpha} w\lambda\beta\alpha^{\alpha}\right]^{\frac{1}{\gamma}}} + q\left(\frac{\kappa\alpha}{1-\alpha-\gamma}\right)\right\}. \end{aligned} \tag{B.3}$$

Thus, $h^{-}_{t+1} = h^{+}_{t+1}$, and the function is continuous at $\underline{h}$. ■

Proposition *ii*. *h^{-}_{t+1} is a linear function of h_t and h^{+}_{t+1} is a concave function of h_t.*
Proof. $\frac{\partial^2 h^{-}_{t+1}}{\partial h_t^2} = 0$ *and* $\frac{\partial^2 h^{+}_{t+1}}{\partial h_t^2} = -\frac{1}{1+g}\left[\frac{\gamma}{1-\alpha-\gamma}\frac{\gamma}{1-\alpha}q\,(\alpha w\lambda\beta)^{\frac{1}{1-\alpha}}\,h^{\frac{-2(1-\alpha)+\gamma}{1-\alpha}}\right] < 0.$ ■

Proposition *iii*. *The slope of h^{+}_{t+1} is lower than the slope of h^{-}_{t+1} on the relevant domain and less than one when h_t tends to infinity.*
Proof.

$$\frac{\partial h^{-}_{t+1}}{\partial h_t} = \frac{1+q\left[\alpha^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma}\beta\lambda w\right]^{1/\gamma}}{1+g} \tag{B.4}$$

$$\frac{\partial h^{+}_{t+1}}{\partial h_t} = \frac{1+q\frac{\gamma}{1-\alpha}\left(h_t^{\gamma+\alpha-1}\alpha w\lambda\beta\right)^{\frac{1}{1-\alpha}}}{1+g}. \tag{B.5}$$

On the domain $[\underline{h},\infty)$, the slope of h_{t+1} is maximal at $\underline{h}$. Replacing $\underline{h}$ in (B.5) and after some calculations, we obtain that $\frac{\partial h^{-}_{t+1}}{\partial h_t} > \frac{\partial h^{+}_{t+1}}{\partial h_t}$ iff $\alpha+\gamma < 1$, which is true given the model assumptions. Then, the slope of h^{+}_{t+1} is lower than the slope of h^{-}_{t+1} on the relevant domain.

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Finally, the slope of $h^+_{t+1} < 1$ *when* h_t *tends to infinity.*

$$\lim_{h_t \to \infty} \frac{\partial h^+_{t+1}}{\partial h_t} = \frac{1}{1+g} < 1. \tag{B.6}$$

■

APPENDIX C

PROPERTIES OF THE LOW-GROWTH LONG-RUN EQUILIBRIUM

PROPOSITION 2. *If* $[q^\gamma \alpha^{\alpha+\gamma} \left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha-\gamma} \beta\lambda w]^{\frac{1}{\gamma}} < g$, *then the economy's growth rate equals the technological frontier's growth rate, which equals the stock of knowledge's growth rate. We reproduce here the growth rate presented in equation (31).*

$$\frac{A_{t+1}-A_t}{A_t} = \frac{A^T_{t+1}-A^T_t}{A^T_t} = q\left[\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{(1-\alpha-\gamma)} \alpha^{(\alpha+\gamma)}\beta\alpha w\right]^{\frac{1}{\gamma}} = \frac{H_{t+1}-H_t}{H_t}. \tag{31}$$

Proof. Let us define a variable $\widetilde{x_t} \equiv X_t/A^T_t$ and the following growth rates:

$$g_A = \frac{A_{t+1}}{A_t} - 1\,; \quad g_H = \frac{H_{t+1}}{H_t} - 1\,; \quad g^T_A = \frac{A^T_{t+1}}{A^T_t} - 1. \tag{C.1}$$

We rewrite the law of motion for relative productivity $\widetilde{a_t} \equiv A_t/A^T_t$ and for relative knowledge $\widetilde{h_t} \equiv H_t/A^T_t$ as

$$\widetilde{a_{t+1}} = \frac{\widetilde{a_t} + \phi_t\,(1-\widetilde{a_t})}{1+g^T_A} \tag{C.2}$$

$$\widetilde{h_{t+1}} = \frac{\widetilde{h_t} + q\widetilde{i_t}}{1+g^T_A}. \tag{C.3}$$

Accordingly, steady-state values correspond to

$$\widetilde{a_{ss}} = \frac{\phi_{ss}}{g^T_A + \phi_{ss}} = \frac{\kappa}{g^T_A(1-\alpha-\gamma)w_t + \kappa} \tag{C.4}$$

$$\widetilde{h_{ss}} = \frac{q\widetilde{i_{ss}}}{g^T_A} = \frac{q\alpha\kappa}{g^T_A(1-\alpha-\gamma)}, \tag{C.5}$$

where the probability of success has been replaced by its expression presented in equation (19) in equation (C.4), and relative R&D investment has been replaced by its expression from equation (15) in equation (C.5). In steady state, both absolute productivity and absolute knowledge are growing at the same rate as the technology target.

Clearing g^T_A from equation (C.5) and replacing the (constant) values for $\widetilde{i}$ and $\widetilde{h}$ from equations (15) and (16) when $A^T_t < A_{\max,t}$, we obtain the growth rate of the technology target in steady state.

$$\widetilde{g_A^T} = q\left(\frac{\alpha\kappa}{1-\alpha-\gamma}\right)\left[\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{1-\alpha}\alpha^{\alpha}\beta\lambda w_t\right]^{-1/\gamma}$$
$$= q\left[\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{(1-\alpha-\gamma)}\alpha^{(\alpha+\gamma)}\beta\alpha w\right]^{\frac{1}{\gamma}}. \quad \textbf{(C.6)}$$

Finally, when calculating the growth rate for A_T from equation (16), we obtain

$$g_A^T = g_H. \quad \textbf{(C.7)}$$

From equations (C.6) and (C.7), we get

$$g_A = g_A^T = g_H = q\left[\left(\frac{1-\alpha-\gamma}{\kappa}\right)^{(1-\alpha-\gamma)}\alpha^{(\alpha+\gamma)}\beta\alpha w\right]^{\frac{1}{\gamma}}. \quad \textbf{(C.8)}$$

■

APPENDIX D

Table D.1. Country sample, by income group

High-income countries	Middle-to high-income countries	Middle-income countries	Low-income countries
Australia	Algeria	Albania	Bangladesh
Austria	Argentina	Bolivia	Cambodia
Belgium	Brazil	Cameroon	Ghana
Canada	Bulgaria	China	Haiti
Denmark	Chile	El Salvador	Kenya
Finland	Colombia	Fiji	Lao PDR
France	Costa Rica	Guyana	Nepal
Germany	Croatia	Honduras	Senegal
Hong Kong	Czech Republic	India	Sierra Leone
Ireland	Egypt	Indonesia	Sudan
Italy	Greece	Jamaica	Uganda
Japan	Guatemala	Morocco	Vietnam
Netherlands	Hungary	Nicaragua	Zimbabwe
New Zealand	Iran	Pakistan	
Norway	Israel	Paraguay	
Singapore	Jordan	Peru	
Spain	Korea	Philippines	
Sweden	Latvia	Sri Lanka	
Switzerland	Lithuania	Syria	
United Kingdom	Malaysia	Yemen	
United States	Mexico		
	Panama		
	Poland		
	Portugal		
	Romania		
	Slovak Republic		
	South Africa		
	Taiwan		
	Thailand		
	Turkey		
	Ukraine		
	Uruguay		
Number of observations: $N = 86$			
21	32	20	13

Source: Author's classification based on data of The World Bank.

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