and $$\theta = \arctan x/a \tag{8}$$

where $\theta$ is the angle between PX and PB.

6. CORRECTION TO ESTIMATE OF SHIP'S AVERAGE SPEED, $V_1$. Since the favourable current is not dead astern on the P to X course, nor is a foul current dead ahead on this leg, a small correction is desirable to achieve a reasonable degree of precision in the calculation. This correction is given by the formulae:

$$V_1 = V+(D_a \sin\theta) \tag{9}$$

The problem can now be reworked using the new value of $V_1$. Due to the simplicity of the formula, this should not be unduly laborious when using a calculator. When using a programmable calculator, only the second values of $x$ and need be displayed.

It is now only necessary to subtract $\theta$ from 90 degrees to obtain the difference between ship's heading and the current axis. The course to make good can then be found by adding or subtracting this difference to the current axis and the course to steer solved by diagram or other simple method of determining compensation for leeway.

# 'Trans-oceanic Passages by Rhumbline Sailing'

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The recent paper[1] by Captain Ivica Tijardović describing trans-oceanic navigation, composed of a rhumbline and a latitude parallel sailing was very interesting. I would like to point out that the problem can be solved without meridional parts assuming a spherical Earth and taking the angles in radians.

According to formula (26) in my own recent paper[2] the combined distance is:

$$D = \frac{\phi_2-\phi_1}{\cos\theta}+\{\lambda_2-\lambda_1-[\operatorname{ar\,sinh}(\tan\phi_2)-\operatorname{ar\,sinh}(\tan\phi_1)]\tan\theta\}\cos\phi_2. \tag{1}$$

This is a minimum if $\theta$ is the solution of:

$$\sin\theta = \frac{[\operatorname{ar\,sinh}(\tan\phi_2)-\operatorname{ar\,sinh}(\tan\phi_1)]\cos\phi_2}{\phi_2-\phi_1}. \tag{2}$$

The inverse hyperbolic sine is:

$$\operatorname{ar\,sinh} z = \ln(z+\sqrt{(z^2+1)}). \tag{3}$$

There are given in the example given by Tijardović:

| | | |
|---|---|---|
| Position 1 | $\phi_1 = 35°$ N | $\lambda_1 = 140° 30'$ E. |
| Position 2 | $\phi_2 = 46° 20'$ N | $\lambda_2 = 124° 30'$ W. |

The corresponding great circle distance is:

$$D_G = 4113{\cdot}3 \text{ n.m.}$$

with the initial heading of:

$$H_I = 47{\cdot}6°$$

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while the rhumbline distance is:

$$D_R = 4359{\cdot}4 \text{ n.m.}$$

with the course:

$$C = 81{\cdot}0°.$$

The method with meridional parts proposed by Tijardović gives the results:

$$D = 1644{\cdot}0 + 2572{\cdot}9 = 4216{\cdot}9 \text{ n.m.}$$

$$\theta = 65° \; 34' = 65{\cdot}6°.$$

The formulae (1) and (2) give the results:

$$D = 1675{\cdot}9 + 2535{\cdot}6 = 4211{\cdot}5 \text{ n.m.}$$

$$\theta = 66° \; 03{\cdot}7' = 66{\cdot}1°.$$

As can be seen the distance by these formulae is 5·4 n.m. shorter than that by meridional parts. The difference is insignificant.

The method presented by Tijardović is worth remembering when planning trans-oceanic navigation.

## REFERENCES

[1] Tijardović, I. (1990). Trans-oceanic passages by rhumbline sailing. This *Journal*, **43**, 292.

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KEY WORDS

1. Marine Navigation. 2. Voyage Planning.

# Measuring True Distance on a Mercator Chart

Captain Ivica Tijardović

A Mercator chart has the great advantage that rhumblines are straight lines and, since the projection is conformal, the course between any two points may be precisely measured. However, the expanding latitude scale means that distances may only be measured approximately. It is now suggested that, by the addition of a constant scale of latitude, distances may also be measured precisely through the simple use of a pair of dividers.

Figure 1 shows two triangles superimposed on one another. The larger triangle, $P_1 P_2 G$ is that corresponding to the rhumbline joining typical positions $P_1$ and $P_2$ as plotted on a Mercator chart and such that G is the intersection of the meridian through $P_1$ and the parallel through $P_2$. We have the relationship:

$$\text{Tan}\,\theta = P_2 G / P_1 G, \tag{1}$$

where $\theta$ is the rhumbline course.