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Fully Distributed Region-Reaching Control with Collision Avoidance for Multi-robot Systems

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SUMMARY
This paper proposes a fully distributed continuous region-reaching controller for multi-robot systems which can effectively eliminate the chattering issues and the negative effects caused by discontinuities. The adaptive control gain technique is employed to solve the distributed region-reaching control problem. By performing Lyapunov function-based stability analysis, it is shown that all the robots can move cohesively within the desired region under the proposed distributed control algorithm. In addition, collision avoidance and velocity matching within the moving region can be guaranteed under properly designed control gains. Simulation examples are given to verify the capabilities of the proposed control method.

KEYWORDS: Distributed control; Multi-robot systems; Adaptive gain techniques; Region control; Collision avoidance.

1. Introduction

Collective behaviors of multi-robot systems, including consensus,[1–3] formation motion,[4–6] and flocking,[7,8,10] have recently received an increasing attention in the control community. The research interest was originated from the broad applications of coordinated behavior in modern industry, since the coordinated multi-robot systems offer advantages in scalability, reliability, flexibility, and manipulability over a team of individually operating robots. A key issue for the coordinated control of multi-robot systems is to design a control algorithm such that the robots can simultaneously reach a common interest. While much of the recent work on the coordinated control of multi-robot systems was focused on the centralized controller design,[11–13] the collision potentially occurring in motion process was not considered in the control design.[14–16] This paper aims to develop a distributed region-reaching control with collision avoidance for multi-robot systems. The research motivation of the present work mainly comes from three application domains: (i) A centralized control strategy is not suitable for controlling a large group of robots due to the limitations of the communication and computing resources required. (ii) The discontinuous algorithm may cause the chattering issues for a large network of robots or even lead to the damage to the whole controlled system with higher speed

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of motion. (iii) The coordinated control of the multi-robot systems enables the robots to accomplish complex tasks in the same workspace, in which avoidance of collision between any robots is necessary and cannot be ignored in a practical real-world environment. A distributed continuous control algorithm with collision avoidance would be the most promising approach for multi-robot systems in terms of its ability to scale with the size of networked robotic systems.

It is well known that the specification of a desired region rather than a point is relatively flexible in implementing control tasks. A large region instead of a precise point for multi-robot systems can provide more freedoms to perform complex tasks.[14,15] On the other hand, when a large number of robots are controlled within the objective region, various geometric shapes of the objective region for robots can be formed by choosing the the appropriate potential energy functions.[16,17] This kind of collective behaviors can also be regarded as the formation control problem of multi-robot systems. At present, various formation control strategies for multi-robot systems have been developed from different perspectives, which can be roughly divided into three categories, namely, leader-following approach,[18,19] virtual structure approach,[20,21] and behavior-based approach.[25,26] These approaches have their own advantages and disadvantages. In particular, the leader-following approach heavily depends on the leader. If the leader is out of action, then the whole system will be paralyzed. The virtual structure approach has its limitations on the flexibility of the relative positions, since the relative positions of the robots are strictly fixed in the whole system during their movement, which is an unfavorable situation toward the obstacle and the alteration of the formation. Moreover, the rigid geometric constraints among robots will become more and more complicated as the number of robots increases. Although the behavior-based approach can solve the collision avoidance problems, it is difficult to achieve the specific formation for robots. A region-based formation control approach was proposed in ref. [11] to make all the robots move into a dynamic region while achieving the velocity matching and maintaining a minimum distance between any robots. However, the algorithm designed in ref. [11] is a centralized approach that each robot has to communicate with the objective region, which is not suitable for controlling a large group of robots because of the potential challenge in communication bandwidth as the number of robots in the group increases. Furthermore, collision among the moving robots in dynamically varying environment could not be completely avoided. Thus, it is desirable to design a distributed control algorithm with collision avoidance for multi-robot systems implemented in practical applications.

Motivated by the above discussions, this paper aims to develop a distributed region-reaching control algorithm with collision avoidance, for a team of robots to reach the objective region in coordinated motion. First, by introducing the adaptive control gain techniques and a distributed discontinuous acceleration estimator, we present a different methodology for analyzing region-reaching control problem for multi-robot systems. Then, we develop a simple yet generic criterion for solving region-reaching control problem. Compared with the results in the existing literature, the main contributions of this work can be stated in three aspects.

(i) The control algorithm to be designed for multi-robot systems is fully distributed in comparison with the work.[11] In the designed control strategy, each robot only needs to communicate with its neighbors. The state information (e.g., the relative position and velocity measurement) of the region is required to be available only for several robots (at least one), not for the entire community. Compared with the recent work,[8,9] the proposed control strategy is continuous and thus does not cause the chattering issues.

(ii) This paper considers the collision avoidance problems between robots during the movement. This advances the existing studies,[5,6,11,21,22] where the collision among robots was not considered. The developed control algorithm is easy to be implemented. Specifically, the control law is designed using potential energy functions and the control component for collision avoidance is activated only in the bounded sensing areas of each robot. The robots do not need to interact with other robots outside the sensing areas. This will save some control efforts.

(iii) A novel controller is developed for the region-reaching control of multi-robot systems by utilizing the adaptive gain techniques and a distributed continuous acceleration estimator. In contrast to the existing research work,[9,14,16,23–25,27–29] a unified combination of region-reaching, collision avoidance, and fully distributed control algorithm is firstly considered for the networked robotic systems under undirected communication network. Moreover, the developed control methodology possesses the advantages in scalability, reliability, flexibility, and manipulability. The developed control strategy can be extended to deal with multiple physical constraint problems for the coordination control

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of multi-robot systems, since the proposed algorithm can effectively eliminate the negative effects caused by discontinuities such as the chattering issues.

The remainder of this paper is organized as follows. In Section 2, the mathematical preliminaries and the main results are provided in detail. Examples and simulation results are presented to validate the correctness of the developed region-reaching control algorithm in Section 3. Finally, the conclusion is given in Section 4.

2. Distributed Region-Reaching Control

2.1. Preliminaries

Consider a network composed of N robots, whose dynamics can be represented by ref. [11]:

$$M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i)\dot{x}_i + g_i(x_i) = \tau_i,\ i = 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^p$ is the generalized coordinate vector, $M_i(x_i) \in \mathbb{R}^{p\times p}$ denotes the positive definite inertia matrix, $C_i(x_i, \dot{x}_i) \in \mathbb{R}^{p\times p}$ is the Coriolis and centrifugal matrix, $D_i(x_i)\dot{x}_i \in \mathbb{R}^p$ represents the damping force in which $D_i(x_i) \in \mathbb{R}^{p\times p}$ is a positive definite matrix, $g_i(x_i) \in \mathbb{R}^p$ is the vector of gravitational force, and $\tau_i \in \mathbb{R}^p$ stands for the generalized control force exerting on the i-th robot. Furthermore, each robot enjoys the following three properties, namely, the skew symmetry property: $\dot{M}_i(x_i) = C_i(x_i, \dot{x}_i) + C_i(x_i, \dot{x}_i)^T$; the bounded property: $m_{ir} I_p \leq M_i(x_i) \leq m_{iR} I_p$, where both m_{ir} and m_{iR} are positive constants; and the dynamic parameter linearization property: $M_i(x_i)\dot{y}_i + C_i(x_i, \dot{x}_i)y_i + D_i(x_i)y_i + g_i(x_i) = Y_i(x_i, \dot{x}_i, \dot{y}_i, y_i)\theta_i$, where $y_i \in \mathbb{R}^p$, $Y_i(x_i, \dot{x}_i, \dot{y}_i, y_i)$ is the regressor matrix.

A distributed region-reaching controller for multi-robot systems will be designed in this subsection. First, a weighted undirected graph $\mathcal{G}$ will be defined for the communication structure of robots based on the graph theory. This can be viewed as a communication mode among N robots, which is also a key tool to design the distributed algorithm for multi-agent systems. Second, an actual dynamic region and a virtual region are, respectively, defined for each robot to stay inside. The actual dynamic region can be regarded as a global objective for all the robots, while the virtual region can be viewed as a local objective for each robot. The control method in this paper is to make the virtual region converge to the actual dynamic region using distributed algorithms. Third, collision avoidance between robots is considered. This can be viewed as a local objective for each robot. The repulsive force to maintain a minimum distance between any two robots is active only within the bounded sensing area of each robot. This is very important to design the distributed algorithm for multi-robot systems, since an individual robot does not need to sense the distances to any other robots.

A weighted undirected graph $\mathcal{G}$ is employed to model the communication interaction among the N robots. The graph $\mathcal{G}$ consists of a node set $\mathcal{V} = \{1, \ldots, N\}$, an unordered edge set $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. An edge of an undirected graph is denoted by $(i, j) \in \mathcal{E}$ or $(j, i) \in \mathcal{E}$. The matrix $\mathcal{A}$ is defined as $a_{ij} \neq 0$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ if $(i, j) \notin \mathcal{E}$. Here, an undirected edge $(i, j) \in \mathcal{E}$ in graph $\mathcal{G}$ means robot i can receive the information from robot j, and vice versa. An undirected graph is connected if there is a path between any distinct pair of nodes. A Laplician matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ associated with the adjacency matrix $\mathcal{A}$ is defined as $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. $\Lambda = diag\{a_{10}, a_{20}, \ldots, a_{N0}\}$ represents a diagonal matrix with the pivots denoting the connection between the region and robots. Specifically, $a_{i0} > 0$ indicates that the i-th robot can obtain the region's information, and $a_{i0} = 0$ otherwise. Without loss of generality, it is assumed in this paper that the robots interact each other by communication with their neighbors under an undirected connected graph.

An actual global objective region Ω for all robots is defined as:[11]

$$f_\Omega(\Delta x_i) = \left[f_{\Omega 1}(\Delta x_{io1}), \ldots, f_{\Omega N_0}\left(\Delta x_{ioN_0}\right)\right]^T \leq 0, \tag{2}$$

where $\Delta x_i = x_i - x_0$, x_o is the reference point of the objective region, $\Delta x_{iok} = x_i - x_{ok}$, and x_{ok} is the reference point of the k-th desired region, $k = 1, \ldots, N_0$. Here, N_0 represents the number of the global actual objective functions. It is assumed that $f_{\Omega k}(\Delta x_{iok})$ is a continuous and first-order derivative scalar function, and $\dot{x}_{ok} = \dot{x}_0$, where $\dot{x}_0$ is the velocity of the objective region. In addition, both $\dot{x}_0$ and $\ddot{x}_0$ are bounded under the assumption $\| 1_N \otimes \ddot{x}_0 \| < \delta_0$, where δ_0 is a positive constant.

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Accordingly, the potential energy function of the global objective region Ω for each robot can be defined as:

$$P_{\Omega i} = \sum_{k=1}^{N_0} \frac{l_k}{2} \left[\max(0,\ f_{\Omega k}(\Delta x_{iok}))\right]^2, \tag{3}$$

where l_k, $k = 1, \ldots, N_0$, are positive gain constants.

Performing partial differentiation to Eq. (3) with respect to Δx_{iok} yields

$$\left(\frac{\partial P_{\Omega_i}(\Delta x_{iok})}{\partial \Delta x_{iok}}\right)^T = \sum_{k=1}^{N_0} l_k \max\bigl(0,\ f_{\Omega_k}(\Delta x_{iok})\bigr) \times \left(\frac{\partial f_{\Omega_{iok}}(\Delta x_{iok})}{\partial \Delta x_{iok}}\right)^T \stackrel{\text{def}}{=} \Delta \xi_i. \tag{4}$$

It is easy to notice that if robot i is outside the objective region, the attractive force $\Delta\xi_i$ will be active to move the robot i toward the objective region until it is inside the objective region and $\Delta\xi_i = 0$.

Similarly, a virtual objective region $\hat{\Omega}$ for each robot can be described as:

$$f_{\hat{\Omega}}(\Delta\hat{x}_i) = \left[f_{\hat{\Omega}1}(\Delta\hat{x}_{io1}), \ldots, f_{\Omega N_0}(\Delta\hat{x}_{ioN_0})\right]^T \leq 0, \tag{5}$$

where $\Delta\hat{x}_i = x_i - \hat{x}_o^i$, $\hat{x}_o^i$ is the estimation of the objective region for the i-th robot, $\Delta\hat{x}_{iok} = x_i - \hat{x}_{ok}$, and $\hat{x}_{ok}$ is the estimation of the k-th desired region.

Subsequently, the estimated potential energy function for each robot can be constructed as:

$$P_{\hat{\Omega} i} = \sum_{k=1}^{N_0} \frac{l_k}{2} \left[\max\left(0,\ f_{\hat{\Omega} k}\left(\Delta\hat{x}_{iok}\right)\right)\right]^2. \tag{6}$$

The partial derivative of Eq. (6) with respect to $\Delta\hat{x}_{iok}$ is then given by:

$$\left(\frac{\partial P_{\hat{\Omega}_i}\left(\Delta\hat{x}_{iok}\right)}{\partial \Delta\hat{x}_{iok}}\right)^T = \sum_{k=1}^{N_0} l_k \max\left(0,\ f_{\hat{\Omega}_k}\left(\Delta\hat{x}_{iok}\right)\right) \times \left(\frac{\partial f_{\hat{\Omega}_{iok}}\left(\Delta\hat{x}_{iok}\right)}{\partial \Delta\hat{x}_{iok}}\right)^T \stackrel{\text{def}}{=} \Delta\hat{\xi}_i. \tag{7}$$

From the definition of the virtual region, it is easy to notice that the i-th robot can reach inside the objective region Ω if and only if $\Delta\hat{\xi}_i$ is convergent to $\Delta\xi_i$, namely, $\hat{x}_0^i$ converges to x_0, and $\dot{\hat{x}}_0^i$ converges to $\dot{x}_0$. In other words, $\Delta\xi_i = 0$ can be guaranteed if and only if $\Delta\hat{\xi}_i = 0$.

Then, the collision avoidance energy function between robots i and j is introduced as:

$$V_{ij}(x_i, x_j) = V_{ji} = \left(\min\left\{0, \frac{\| x_i - x_j \|^2 - R^2}{\| x_i - x_j \|^2 - r^2}\right\}\right)^2, \tag{8}$$

where R is the maximal sensing radius representing that the i-th robot can detect the presence of the other robots in the area. r is the avoidance radius which specifies the minimum safe distance among robots.

Partially differentiating Eq. (8) with respect to x_i gives rise to:

$$\frac{\partial V_{ij}(x_i, x_j)}{\partial x_i} = \frac{4\left(R^2 - r^2\right)\left(\| x_i - x_j \|^2 - R^2\right)}{\left(\| x_i - x_j \|^2 - r^2\right)^3} \times \left(x_i - x_j\right), \tag{9}$$

where $r < \| x_i - x_j \| < R$, and $\frac{\partial V_{ij}(x_i, x_j)}{\partial x_i} = 0$ if $0 < \| x_i - x_j \|$. For the sake of simplicity, we define $\frac{\partial V_{ij}(x_i, x_j)}{\partial x_i} \stackrel{\text{def}}{=} \Delta\omega_i^{ij}$.

From the definition of the collision avoidance energy function $V_{ij}(x_i, x_j)$, it is easy to observe that the collision between robots i and j will never happen as long as the initial states of robots i and j satisfy that $\| x_i(0) - x_j(0) \| > 0$, that is $\lim_{\| x_i - x_j \| \to r^+} \Delta\omega_i^{ij} = +\infty$. It should be noticed that each robot only detects the presence of the other robots in the perception area, not in the entire community, which is vital for the distributed architecture design.

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2.2. Controller design

A variable reference velocity $\dot{x}_{ri} \in \mathbb{R}^p$ is defined as:

$$\dot{x}_{ri} = \dot{\hat{x}}_0^i - \alpha_i \sum_{j=1}^{N} a_{ij} \left(\hat{x}_0^i - \hat{x}_0^j \right) - \alpha_i a_{i0} \left(\hat{x}_0^i - x_0 \right) - \beta_i \Delta \hat{\xi}_i , \tag{10}$$

where $a_{i0} > 0$ if robot i has access to the desired objective region Ω, and $a_{i0} = 0$ otherwise. $\hat{x}_0^i$ is the estimation of x_0 with regard to robot i. $\Delta \hat{\xi}_i$ is the gradient estimation of the desired objective region Ω. It is obvious that $\hat{x}_0^0 = x_0$, that is, the region does not require estimating its own state information. α_i and β_i are both differentiable time-varying coupling weights (or the adaptive gain coefficients), which will be determined later.

Differentiating the reference velocity $\dot{x}_{ri}$ with respect to time yields the reference acceleration:

$$\ddot{x}_{ri} = \ddot{\hat{x}}_0^i - \alpha_i \sum_{j=0}^{N} a_{ij} \left(\dot{\hat{x}}_0^i - \dot{\hat{x}}_0^j \right) - \beta_i \Delta \dot{\hat{\xi}}_i - \dot{\alpha}_i \sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - \hat{x}_0^j \right) - \dot{\beta}_i \Delta \hat{\xi}_i . \tag{11}$$

Furthermore, a sliding vector $s_i \in \mathbb{R}^p$ for robot i is defined as:

$$s_i = \dot{x}_i - \dot{x}_{ri} . \tag{12}$$

Differentiating Eq. (12) with respect to time results in:

$$\dot{s}_i = \ddot{x}_i - \ddot{x}_{ri} . \tag{13}$$

Then the region-reaching controller for robot i is designed as:

$$\tau_i = Y_i (x_i, \dot{x}_i, \ddot{x}_{ri}, \dot{x}_{ri}) \hat{\theta}_i - K_i s_i - \sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - \hat{x}_0^j \right) - \Delta \hat{\xi}_i - \sum_{j=1}^{N} \Delta \omega_j^{ij} , \tag{14}$$

where K_i is a symmetric positive definite matrix and $\hat{\theta}_i$ is the estimate of θ_i.

The estimated parameter $\hat{\theta}_i$ is updated by:

$$\dot{\hat{\theta}}_i = -\Gamma_i Y_i^{\mathrm{T}} (x_i, \dot{x}_i, \ddot{x}_{ri}, \dot{x}_{ri}) s_i , \tag{15}$$

where Γ_i is a symmetric positive definite matrix.

The estimated acceleration $\ddot{\hat{x}}_0^i$ is given by:

$$\ddot{\hat{x}}_0^i = -\gamma \sum_{j=0}^{N} a_{ij} \left(\dot{\hat{x}}_0^i - \dot{\hat{x}}_0^j \right) - \frac{1}{\rho} \sum_{j=1}^{N} \Delta \omega_j^{ij} , \tag{16}$$

where γ is a positive gain constant, $\rho \in (0, 1]$ is a positive control parameter which can be selected as $\rho = \exp(- \sum_{j=1}^{N} a_{ij} \eta \int_0^t \parallel \dot{\hat{x}}_0^i - \dot{\hat{x}}_0^j \parallel d\tau)$, in which η is a positive control gain to be determined later in the proof of Theorem 1.

The adaptive gains α_i and β_i are, respectively, updated by:

$$\dot{\alpha}_i = -\alpha_i \left(\sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - \hat{x}_0^j \right) \right)^T \times \left(\sum_{j=1}^{N} \Delta \omega_j^{ij} + \Delta \hat{\xi}_i + \left(\dot{x}_i - \dot{\hat{x}}_0^i \right) \right) , \tag{17}$$

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and

$$\dot{\beta}_i = -\beta_i \left(\sum_{j=1}^{N} \Delta\omega_i^{ij} + \sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - \hat{x}_0^j \right) \right)^T \Delta\hat{\xi}_i. \tag{18}$$

By substituting Eq. (14) and applying the parameter linearization property to Eq. (1), the closed-loop system (1) can be rewritten as:

$$\begin{aligned} M_i(x_i)\dot{s}_i = &-C_i(x_i, \dot{x}_i)s_i - Y_i\tilde{\theta}_i - D_i(x_i)s_i - \sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - \hat{x}_0^j \right) \\ &-\Delta\hat{\xi}_i - \sum_{j=1}^{N} \Delta\omega_j^{ij} - K_i s_i, \end{aligned} \tag{19}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ is the parameter estimation error.

It should be noted that the estimated acceleration $\ddot{\hat{x}}_0^i$ is continuous in Eq. (16). Compared with the discontinuous strategy in refs. [8,9], the algorithm proposed in this paper can effectively eliminate the negative effects created by discontinuities and does not cause the chattering issues. By introducing the distributed estimator, the robots do not require accurate position and velocity measurements of the objective region but only need to estimate the state information related to their neighbors. In addition, the estimated velocity $\dot{\hat{x}}_0^i$ and position $\hat{x}_0^i$ in Eq. (10) can be easily calculated from the estimated acceleration through integral operation.

From Eqs. (17) and (18), it is not difficult to find that the adaptive control gains are of exponential types, that is, the control gains α_i and β_i are all positive definite if and only if the initial values $\alpha_i(0) > 0$ and $\beta_i(0) > 0$ hold. The adaptive control gains will be always changeable until the robots reach inside the objective region and realize velocity matching within the region and collision avoidance between each other. Therefore, the adaptive control gains are the semantics of robot motion states. Furthermore, both α_i and β_i play an important role in the stability analysis of the overall closed-loop system.

The control objective consists of three essential parts: region-reaching, collision avoidance, and distributed architecture design. It should be emphasized that the anticipated objective cannot be achieved using the existing controllers for multi-robot systems reported in refs. [11,14,21,30]. The developed distributed control algorithm (14) has five components. The first term is an estimate of the robot dynamics, the second term is the sliding feedback, the third term is used for the distributed region matching between the local virtual region and the overall objective region, the fourth term is utilized to provide the attractive force to make robots reach inside the virtual region, and the fifth term is employed to generate the repulsive force to make robots avoid collisions. It is easy to observe that the proposed control algorithm is designed by considering the constraints imposed by the control objective and by simply utilizing the adaptive technique to perform the collision avoidance, distributed architecture, and region-reaching tasks. Thus, the presented controller can be conveniently designed by this methodology for multi-robot systems.

Now the main results are readily given below.

Theorem 1. *For a group of N robots with Lagrangian dynamics* (1)*, by using the fully distributed adaptive controller* (14)*, parameter estimation law* (15)*, the estimated acceleration* (16)*, and the adaptive control gain laws* (17) *and* (18)*, the multi-robot system can reach into the objective region with collision avoidance as long as the initial states of the robots satisfy that* $\| x_i(0) - x_j(0) \| > r$, $i, j = 1, 2, \ldots, N$, $(i \neq j)$, *and* $\eta \geq \frac{\delta_0}{\lambda_{\min}(L+\Lambda)}$.

Proof. First, the Lyapunov function V_1 for the multi-robot systems is introduced to deal with the parametric uncertainty of the controlled system as:

$$V_1(t) = \frac{1}{2} \sum_{i=1}^{N} s_i^T M_i(q_i) s_i + \frac{1}{2} \sum_{i=1}^{N} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i. \tag{20}$$

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Differentiating $V_1(t)$ with respect to time yields

$$\begin{aligned}\dot{V}_1(t) = &-\sum_{i=1}^{N} s_i^T D_i s_i - \sum_{i=1}^{N} s_i^T K_i s_i - \sum_{i=1}^{N} s_i^T \left(\sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - \hat{x}_0^j \right) \right) \\ &- \sum_{i=1}^{N} s_i^T \Delta\hat{\xi}_i - \sum_{i=1}^{N} s_i^T \left(\sum_{j=1}^{N} \Delta\omega_i^{ij} \right).\end{aligned} \tag{21}$$

To avoid collisions among robots, the potential function V_2 is constructed as:

$$V_2(t) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} V_{ij} + \sum_{i=1}^{N} V_{i0}. \tag{22}$$

And its derivative is given by:

$$\begin{aligned}\dot{V}_2(t) =& \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\dot{x}_i^T \Delta\omega_i + \dot{x}_j^T \Delta\omega_j \right) + \sum_{i=1}^{N} \left(\dot{x}_i^T \Delta\omega_i^{i0} + \dot{x}_0^T \Delta\omega_0^{i0} \right) \\ =& \sum_{i=1}^{N} \left(\dot{x}_i^T \Delta\omega_i^{i0} - \dot{x}_0^T \Delta\omega_i^{i0} \right) + \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\dot{x}_i^T \Delta\omega_i^{ij} \right) \\ =& \sum_{i=1}^{N} \sum_{j=0}^{N} \left(\left(\dot{x}_i^T - \dot{x}_0^T \right) \Delta\omega_i^{ij} \right).\end{aligned} \tag{23}$$

To attract robots into the objective region, the potential energy function V_3 is introduced as:

$$V_3(t) = \sum_{i=1}^{N} P_{\hat{\Omega}i}. \tag{24}$$

Differentiating $V_3(t)$ with respect to time leads to:

$$\dot{V}_3(t) = \sum_{i=1}^{N} \left(\dot{x}_i^T - \dot{\hat{x}}_0^i \right) \Delta\hat{\xi}_i. \tag{25}$$

Then, the adaptive gain potential function is constructed as:

$$V_4(t) = \sum_{i=1}^{N} (\alpha_i + \beta_i) + \rho. \tag{26}$$

Differentiating $V_4(t)$ with respect to time and exploiting Eqs. (17) and (18) results in:

$$\begin{aligned}\dot{V}_4(t) = &-\alpha_i \sum_{i=1}^{N} \left(\sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - x_0^j \right) \right)^T \times \left(\sum_{j=1}^{N} \Delta\omega_j^{ij} + \Delta\hat{\xi}_i + \left(\dot{x}_i - \dot{\hat{x}}_0^i \right) \right) \\ &- \beta_i \sum_{i=1}^{N} \left(\sum_{j=1}^{N} \Delta\omega_i^{ij} + \sum_{j=0}^{N} a_{ij} \left(\hat{x}_0^i - x_0^j \right) \right)^T \Delta\hat{\xi}_i - \rho\eta \left(\sum_{j=0}^{N} a_{ij} \left(\| \dot{\hat{x}}_0^i - \dot{\hat{x}}_0^j \| \right) \right).\end{aligned} \tag{27}$$

To match the velocity of the objective region Ω, the Lyapunov function candidate $V_5(t)$ is formulated as:

$$V_5(t) = \frac{1}{2} \sum_{i=1}^{N} \rho \left(\dot{\hat{x}}_0^i - \dot{x}_0 \right)^T \left(\dot{\hat{x}}_0^i - \dot{x}_0 \right). \tag{28}$$

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Differentiating $V_5(t)$ with respect to time yields

$$\dot{V}_5(t)=\sum_{i=1}^{N}\rho\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)^T\left(-\gamma\sum_{j=0}^{N}a_{ij}\left(\dot{\hat{x}}_0^i-\dot{\hat{x}}_0^j\right)\right)+\frac{1}{2}\sum_{i=1}^{N}\dot{\rho}\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)^T\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)$$
$$+\sum_{i=1}^{N}\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)^T\left(-\sum_{j=1}^{N}\Delta\omega_i^{ij}\right)-\sum_{i=1}^{N}\rho\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)^T\ddot{x}_0. \tag{29}$$

Accordingly, an overall Lyapunov function for the multi-robot system is given by:

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t). \tag{30}$$

The derivative of $V(t)$ can be expressed as:

$$\begin{aligned}
\dot{V}(t)=&-\sum_{i=1}^{N}s_i^TD_is_i-\sum_{i=1}^{N}s_i^TK_is_i-\beta_i\sum_{i=1}^{N}\Delta\hat{\xi}_i^T\Delta\hat{\xi}_i\\
&-\alpha_i\sum_{i=1}^{N}\left(\sum_{j=0}^{N}a_{ij}\left(\hat{x}_0^i-\hat{x}_0^j\right)\right)^T\left(\sum_{j=0}^{N}a_{ij}\left(\hat{x}_0^i-\hat{x}_0^j\right)\right)\\
&-\gamma\sum_{i=1}^{N}\rho\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)^T\left(\sum_{j=0}^{N}a_{ij}\left(\dot{\hat{x}}_0^i-\dot{\hat{x}}_0^j\right)\right)\\
&+\frac{1}{2}\sum_{i=1}^{N}\dot{\rho}\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)^T\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)-\sum_{i=1}^{N}\rho\left(\dot{\hat{x}}_0^i-\dot{x}_0\right)^T\ddot{x}_0\\
&-\rho\eta\left(\sum_{j=0}^{N}a_{ij}\left(\left\|\dot{\hat{x}}_0^i-\dot{\hat{x}}_0^j\right\|\right)\right)\\
\le&-\sum_{i=1}^{N}\left(\lambda_{\min}(D_i)+\lambda_{\min}(K_i)\right)\left\|s_i\right\|^2\\
&-\alpha_i\lambda_{\min}\left((\mathcal{L}+\Lambda)^T(\mathcal{L}+\Lambda)\right)\sum_{i=1}^{N}\left\|\tilde{x}_0^i\right\|^2\\
&-\beta_i\sum_{i=1}^{N}\left\|\Delta\hat{\xi}_i\right\|^2-\rho\eta\left\|(L+\Lambda)\otimes I_p\right\|\left\|\dot{\tilde{x}}_0^i\right\|\\
&-\gamma\rho\left\|(L+\Lambda)\otimes I_p\right\|\left\|\dot{\tilde{x}}_0^i\right\|^2+\rho\delta_0\left\|\dot{\tilde{x}}_0^i\right\|\\
\le&-\sum_{i=1}^{N}\left(\lambda_{\min}(D_i)+\lambda_{\min}(K_i)\right)\left\|s_i\right\|^2\\
&-\gamma\rho\lambda_{\min}(\mathcal{L}+\Lambda)^T\left\|\dot{\tilde{x}}_0^i\right\|^2-\beta_i\sum_{i=1}^{N}\left\|\Delta\hat{\xi}_i\right\|^2\\
&-\alpha_i\lambda_{\min}\left((\mathcal{L}+\Lambda)^T(\mathcal{L}+\Lambda)\right)\sum_{i=1}^{N}\left\|\tilde{x}_0^i\right\|^2\\
&-\left(\eta\lambda_{\min}(L+\Lambda)-\delta_0\right)\rho\left\|\dot{\tilde{x}}_0^i\right\|,
\end{aligned} \tag{31}$$

where $\tilde{x}_0^i=\hat{x}_0^i-x_0$ and $\dot{\tilde{x}}_0^i=\dot{\hat{x}}_0^i-\dot{x}_0$.

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From Eq. (31), it is easy to know $\dot{V}(t) \leq 0$ if $\eta \geq \frac{\delta_0}{\lambda_{\min}(L+\Lambda)}$ holds. Then, we can obtain that s_i, $\Delta\hat{\xi}_i$, $\tilde{x}_0^i$ and $\dot{\tilde{x}}_0^i \in L^2$, $\tilde{\theta}_i$, α_i, and β_i are all bounded. From Eq. (10), we can say that $\dot{x}_{ri}$ is bounded. By differentiating Eq. (7), we can state $\Delta\dot{\hat{\xi}}_i$ is bounded since $\dot{f}_{\hat{\Omega}}(\Delta\hat{x}_i)$ is bounded. From Eq. (9), it is easy to see that $\Delta\omega_i^{ij}$ is bounded. This means that the collision between any two robots can be avoided during the movement. Furthermore, from Eq. (16), we have $\ddot{\hat{x}}_0^i$ is bounded. Then from Eq. (11), we can obtain $\ddot{x}_{ri}$ is bounded because $\dot{\alpha}_i$ and $\dot{\beta}_i$ are both bounded. Thus, we can confirm that $Y_i(x_i, \dot{x}_i, \ddot{x}_{ri}, \dot{x}_{ri})\tilde{\theta}_i$ is bounded. Hence, we can conclude from the closed-loop system (19) that $\dot{s}_i$ is bounded. It follows that $\ddot{V}(t)$ is bounded which implies that $\dot{V}(t)$ is uniformly continuous. By applying the Barbalat's Lemma,[31] we can conclude that $\dot{V}(t) \to 0$ as $t \to \infty$. Following Eq. (31), we have $s_i \to 0$, $\dot{\tilde{x}}_0^i \to 0$, $\tilde{x}_0^i \to 0$, and $\Delta\hat{\xi}_i \to 0$ as $t \to \infty$. Consequently, we have $\Delta\xi_i \to 0$ due to $\Delta\hat{\xi}_i \to \Delta\xi_i$ as $t \to \infty$. This means that the robots can move inside the objective region. Finally, from Eq. (12), we have that $\dot{x}_i \to \dot{x}_0$ as $t \to \infty$ since $\dot{x}_i \to \dot{\hat{x}}_0^i$ and $\dot{\hat{x}}_0^i \to \dot{x}_0$. This means that the velocities of the robots are synchronized with the dynamic region. Therefore, all the robots can move inside the objective region with collision avoidance in a steady state. This completes the proof of the Theorem. □

In order to design the fully distributed controllers satisfying these performance requirements, the distributed acceleration estimator and the exponential adaptive gains have been introduced in this paper, which are quite different from the formation controllers in the developed control methods.[6,13,21,30] The presented control algorithm can be simply designed using the adaptive technique to compensate for the uncertainties which are completely unknown. Therefore, the proposed control scheme is easy to be implemented in practical engineering applications.

3. Simulation Results

Simulation examples of eight robots are given in this section to demonstrate the effectiveness of the developed control algorithm. The dynamic equation of motion of each robot is described by ref. [31]:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_{ix} \\ \ddot{q}_{iy} \end{bmatrix} + \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_{ix} \\ \dot{q}_{iy} \end{bmatrix} = \begin{bmatrix} \tau_{ix} \\ \tau_{iy} \end{bmatrix},$$

where $M_{11} = a_1 + 2a_3\cos(q_{iy}) + 2a_4\sin(q_{iy})$, $M_{12} = M_{21} = a_2 + a_3\cos(q_{iy}) + a_4\sin(q_{iy})$, $M_{22} = a_2$, $N_{11} = -b\dot{q}_{iy}$, $N_{12} = -b(\dot{q}_{ix} + \dot{q}_{iy})$, $N_{21} = b\dot{q}_{ix}$, $N_{22} = 0$, $b = a_3\sin(q_{iy}) - a_4\cos(q_{iy})$, $a_1 = 7$, $a_2 = a_3 = a_4 = 2$.

The values of the other parameters used in the simulation are given by $r = 0.1$, $R = 0.5$, $\Gamma_i = \mathrm{diag}\{1, 1\}$, $K_i = \mathrm{diag}\{3, 3\}$, and $\gamma = 15$. The initial values of control gains are chosen as $\alpha_i(0) = 1$ and $\beta_i(0) = 1$. The initial estimated parameters $\hat{x}_0^i$, $\dot{\hat{x}}_0^i$, $\ddot{\hat{x}}_0^i$ are all set as $(0, 0)$. Furthermore, the communication network among robots is set as a ring network with the weight of 1, and only one robot can maintain the connectivity with the desired region, that is, $a_{10} = 1$, and $a_{i0} = 0$, $i \neq 1$. The moving circular region is formed by $(x_{i1} - x_{01})^2 + (x_{i2} - x_{02})^2 = 1$, where $x_{01} = t$, $x_{02} = \sin(t)$.

The initial positions of the eight robots are selected as $[-1, 1]^T$, $[-1, -1]^T$, $[1, 1]^T$, $[1, -1]^T$, $[-2, 2]^T$, $[-2, -2]^T$, $[2, 2]^T$, $[2, -2]^T$, respectively. And the initial velocities of the robots are set as $[0, -2]^T$, $[0, 2]^T$, $[0, -2]^T$, $[0, 2]^T$, $[0, -2]^T$, $[0, 2]^T$, $[0, -2]^T$, $[0, 2]^T$ respectively. The other parameters are specified as $l_1 = 0.1$ and $\eta = 10$. Figure 1 shows the simulation results of the eight robots with the moving target region (the circular region). In particular, Fig. 1(a) displays the positions of the robots at different time, and it is shown that the robots can eventually reach inside the desired region. Figure 1(b) depicts the evolution of distances between robots during the time period of 0–30 s. It is shown that the distance between robots is greater than 0.1 in the process of movement and the distance between robots is greater than 0.5 when they finally stabilize in the region. Figure 1(c) and (d) illustrate the corresponding velocity variations in two directions. The robots can reach velocity matching with the objective region. It can be observed from Fig. 1 that the eight robots are able to move inside the dynamic region without collision and reach velocity matching with the desired moving objective region.

The region-reaching concept was firstly proposed in ref. [17]. Although subsequent research work has been done in refs. [11, 18–20], the collision avoidance between robots during the movement was not ensured. On the contrary, the control algorithm developed in this paper guarantees the robots can

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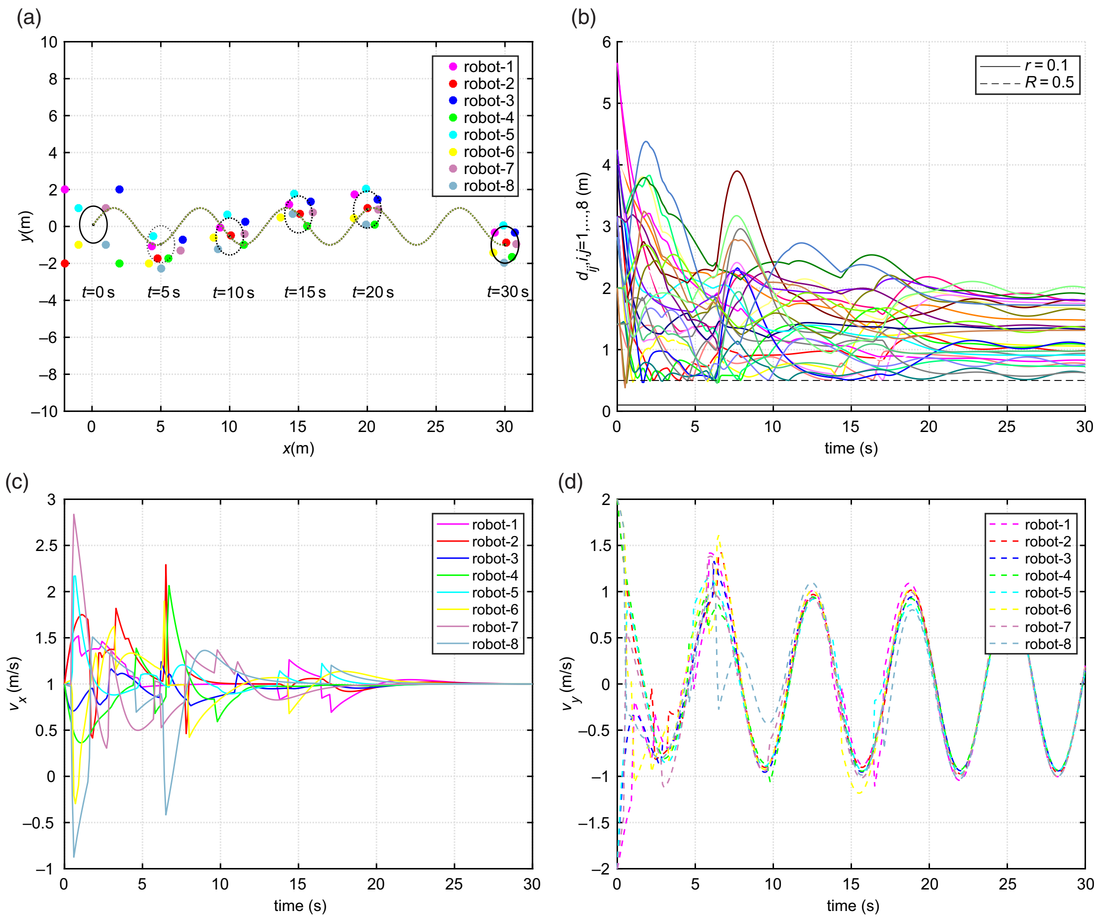

Fig. 1. A team of robots (eight robots) moving together along a sine wave path in a circular shape: (a) position states of eight robots at different time, (b) evolution of distances with time, (c) velocity states along x-axis of eight robots with time, and (d) velocity states along y-axis of eight robots with time.

always avoid collisions as long as the initial positions of the robots are out of the avoidance radius. On the other hand, compared with the recent work,[29] the proposed control scheme is fully distributed to overcome the communication limitations in a network for the predetermined performance design (only one robot is required to receive the information of the region).

Theoretically, the fully distributed control algorithm proposed in this paper can make a large number of robots reach into the objective region. It should be noted that the number of robots depends on the shape and size of the objective region as well as the small distances between robots required for collision avoidance. Due to the constraints of different target regions and the collision avoidance constraints between robots, it would be very hard to find the maximum number of robots that can reach into different target regions without performing mathematical optimization.

4. Conclusion

This paper has studied the distributed region-reaching control with collision avoidance problem of multi-robot systems under undirected network. A fully distributed continuous acceleration estimator and adaptive gain design have been presented to develop a local adaptive controller with collision avoidance and region-reaching ability. Furthermore, a simple yet generic control criterion for solving region-reaching control problems has been obtained using the Lyapunov stability theorem.

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Conflict of Interest

The authors declare that there is no conflict of interest to this work.

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