Laser and Particle Beams (2015), *33*, 123–133.

doi:10.1017/S0263034615000142

Coaxial propagation of Laguerre–Gaussian (LG) and Gaussian beams in a plasma

SHIKHA MISRA,[1] SANJAY K. MISHRA,[2] AND P. BRIJESH[3,4]

[1]Centre for Energy Studies (CES), Indian Institute of Technology Delhi (IITD), New Delhi, India
[2]Institute for Plasma Research (IPR), Gandhinagar, India
[3]Tata Institute of Fundamental Research (TIFR), Mumbai, India
[4]UM-DAE-CBS, Mumbai, India

(Received 20 September 2014; Accepted 22 January 2015)

Abstract

This paper investigates the non-linear coaxial (or coupled mode) propagation of Laguerre–Gaussian (LG) (in particular L_0^1 mode) and *Gaussian* electromagnetic (em) beams in a homogeneous plasma characterized by ponderomotive and relativistic non-linearities. The formulation is based on numerical solution of non-linear Schrödinger wave equation under Jeffreys–Wentzel–Kramers–Brillouin approximation, followed by paraxial approach applicable in the vicinity of intensity maximum of the beams. A set of coupled differential equations for spot size (beam width) and phase evolution with space corresponding to coupled mode has been derived and numerically solved to determine the propagation dynamics. Using focusing equation a critical condition describing the self-trapped (i.e., spatial soliton) mode of laser beam propagation in the plasma has been discussed; as a consequence oscillatory focusing/defocusing of the beams in coupled mode propagation have been analyzed and presented graphically. As an important outcome, significant enhancement in the intensity of LG beam is noticed when it is coupled with the Gaussian mode.

Keywords: Coaxial propagation; Paraxial & paraxial-like approach; Self-focusing

1. INTRODUCTION

In general, the lasers are characterized by non-uniform radial intensity profile and the non-linear effects invoked by the propagation of such electromagnetic (em) beams in the plasmas are highly sensitive to the irradiance distribution along the wave front of the beam (Sodha *et al.*, 1976). As the beam propagates in the plasma, the electrons/ions redistribute itself under the influence of non-uniform irradiance profile and this self-consistent redistribution cause non-linearities in the plasma, characterized by dielectric function. Among many of the non-linear phenomena associated with interplay between em beam and plasma, self-focusing/defocusing is of considerable interest (Sodha *et al.*, 1976; Hora, 1991; Sprangle & Esarey, 1991; Berge, 1998; Saini & Gill, 2006; Yu *et al.*, 2007) on account of its relevance to promising applications in inertial confinement fusion (ICF) (Tabak *et al.*, 1994; Deutsh *et al.*, 1996), charged particle acceleration (Sprangle *et al.*, 1988; Umstadter *et al.*, 1996), *X*-ray generation (Eder *et al.*, 1994), and ionospheric modification (Gurevich, 1978). The investigations exploring the laser propagation dynamics in plasmas frequently take account of Gaussian nature of the irradiance profile of the laser where the intensity peaks at the central axis and falls off radially. However, in the last few years significant interest has been gained by optical beams with zero central intensity (Sodha *et al.*, 1974; 2009*a*; 2009*b*; Gupta *et al.*, 2011*a*; 2009*b*; Khamedi & Bahrampour, 2013; Sharma *et al.*, 2013; Misra *et al.*, 2014) because of its numerous applications in modern atomic optics and plasma physics. In particular, the off-axis dipole-like potential associated with the central shadow beams can be utilized to guide and trap the atoms; this effect has drawn significant attention to the central shadow beam dynamics (O'Neil *et al.*, 2002). The theoretical and analytical investigations describing the propagation dynamics of central shadow (in particular hollow Gaussian) beams in plasma, predicts weak divergence of hollow beams than that for Gaussian profile; such beams thus can be utilized to achieve large-energy transport in the plasma (Misra *et al.*, 2014). Apart from the hollow Gaussian beams (HGBs), another class of central shadow beams is described by the Laguerre–Gaussian (LG) (Sueda *et al.*, 2004) profile; in particular such beams besides a non-*Gaussian*

Address correspondence and reprint requests to: Shikha Misra, Centre for Energy Studies (CES), Indian Institute of Technology Delhi (IITD), New Delhi, India 110016. E-mail: shikhamish@gmail.com

intensity modulation comprises an inherent orbital angular momentum (l). The general algebraic form of LG profile can be expressed as

$$A(r, z, \theta) = (E_0/f)(r/r_0 f)^l \exp[-(r^2/2r_0^2 f^2)] \\ L_p^l(r^2/r_0^2 f^2)\exp(-il\theta), \tag{1}$$

where p refers the radial index, $r_0 f$ refers to the half-waist width, f is the beam width parameter, θ is the azimuthal angle, and E_0 is the amplitude of the electric field.

This can readily be seen that the LG beam displays an off-axis ($r \neq 0$) intensity maxima. As an em beam propagates in the plasma, it may trigger the non-linear effects depending on the laser intensity and the beam can undergo continuous focusing/defocusing. The situation when the LG beam in the self-focusing regime, the axial trapping efficiency of the optical tweezers gets significantly enhanced. The propagation dynamics of such LG beams in the non--linear (in particular the Kerr dielectric) medium has recently been investigated by Thakur and Berakdar (2010) and Khamedi and Bahrampour (2013); however, these analyses are based on paraxial approximation whose validity is limited to the finite region near the central axis (i.e., $r = 0$). It is necessary to point out here that paraxial approximation in general is applicable to Gaussian beams where the non-linear effects are pronounced in the vicinity of its intensity maximum, that is, at the central axis. Hence, the use of paraxial approximation in analyzing the propagation dynamics of central shadow LG (off-axis) beams is inconsistent and the formulation should be modified. The applicability of paraxial approach in the vicinity of off-axis ($r \neq 0$) intensity maxima in particular for the HGBs has been examined in recent studies (Misra & Mishra, 2008; 2009*a*; 2009*b*; Sodha *et al.*, 2009*a*; 2009*b*).

Apart from individual beam dynamics, the phenomenon of mutual interaction of multiple beams has numerous applications in filamentation, optics, electron acceleration, and trapping of atoms (Sprangle *et al.*, 1988; Sprangle & Esarey, 1991; Umstadter *et al.*, 1996; Scheller *et al.*, 2014). The focusing/de-focusing of two or more Gaussian em-coaxial beams in the plasma have been extensively studied (Sodha *et al.*, 1976; 1979; 2008; Konar & Jana, 2005; Gupta *et al.*, 2011*a*; 2011*b*) and the influence of basic saturating non-linearities, for example, ponderomotive, collisional, and relativistic on the beam dynamics has been explored. In such cases, the plasma density redistribution and hence the dielectric function gets modified by the combined effect of intensities of both the em beams. In coaxial propagation, the propagation dynamics (i.e., focusing/defocusing) of one beam is influenced by the other beam and hence one beam can be utilized as the controlling tool for the other beam (Sodha *et al.*, 2008). The cross-focusing of two HGBs in the relativistic regime has been analyzed by Gupta *et al.* (2011*a*; 2011*b*); however, the mutual influence of the beams on one another and dielectric function is not very clear from their analysis. In this study, we aim to demonstrate that how the coaxial propagation of LG (in particular the L_0^1 mode) and Gaussian beams mutually influences each other dynamics in different non-linear plasma regimes. The existence of such beams has been experimentally verified by Brijesh *et al.* (2007) who were able to generate the horseshoe-shaped longitudinal beam; it is primarily a coaxial combination of LG and Gaussian modes. Another motivation comes from a recent work (Scheller *et al.*, 2014) where the length of the laser-induced filaments in air is enhanced significantly when an annular beam is coupled with the Gaussian mode; such enhanced filamentation has promising applications in remote sensing (Luo *et al.*, 2006), atto-second physics (Stibenz *et al.*, 2006), channeling microwaves (Ren *et al.*, 2013), and lightning protection (Kasparian *et al.*, 2008).

In this analysis, a formalism describing the non-linear coaxial propagation dynamics of finite size intense coherent LG (L_0^1 mode) and Gaussian beams, in a plasma characterized by ponderomotive and relativistic non-linearities has been developed. The formulation is based on paraxial and a modified paraxial-like approach, applicable to the Gaussian and central shadow LG beams have been utilized to investigate the space evolution and consequent transverse focusing/defocusing of the coupled mode; as discussed before the plasma modification in such case is influenced by the intensity profiles of both the beams. The details of the paraxial-like approach can be seen from the recent literature (Misra & Mishra, 2008; Sodha *et al.*, 2009*a*; Sharma *et al.*, 2013) where it is applied to study the non-linear propagation of dark hollow Gaussian beams (DHGBs) in a plasma. The propagation of an *em* beam is characterized by non-linear Schrödinger wave equation (NLSE). Using the paraxial and paraxial-like approaches for the two beams *viz.* Gaussian and LG (L_0^1 mode) beams, the NLSE is solved and a set of non-linear coupled differential equation describing the space evolution of beam width parameters (f, i.e., electric field) and phase have been derived; this space evolution describes the coaxial propagation of the Gaussian and LG beams as in advances in plasmas. The deviation of the propagation characteristic of the coupled mode (Gaussian plus L_0^1 mode) from that in case of individual beam has been demonstrated graphically. The critical curves characterizing the region of focusing/defocusing in radius-intensity space and consequent transverse self-compression of the coaxial mode of propagation in plasma characterized by different non-linearities *viz.* ponderomotive and relativistic regimes have been examined in this paper. In the next section, the focusing equation for Gaussian and LG profiles has been established and the critical condition for the propagation of beams in the self-trapped mode is discussed. In Section 3, the effective dielectric function of the plasma under the influence of the coaxial beams corresponding to ponderomotive and relativistic non-linearities has been evaluated. Section 4 includes the discussion on the numerical results based on analysis, whereas a summary of the outcome in Section 5 concludes the paper.

2. ANALYSIS

2.1. Propagation

Consider the coaxial propagation of coherent circularly polarized LG and Gaussian beams in a homogeneous plasma with its electric vector polarized along the z-axis; the laser beams are considered to be of same frequency (ω). It is convenient to express the electric field vectors $\mathbf{E}_l$ and $\mathbf{E}_g$ associated with these beams in cylindrical coordinate system with azimuthal symmetry and can be expressed as

$$\mathbf{E}_l(r, z, \theta_l) = A_l(r, z)(\hat{x} + i\hat{y}) \exp[i(kz - \omega t - l\theta_l)] \tag{2}$$

and

$$\mathbf{E}_g(r, z) = A_g(r, z)(\hat{x} + i\hat{y}) \exp[i(kz - \omega t)], \tag{3}$$

where $A_l(r, z) = (E_{0l}/f_l)(r/r_{0l}f_l)^l \exp[-(r^2/2r_{0l}^2 f_l^2)] L_p^l(r^2/r_{0l}^2 f_l^2) \exp(-i\varphi_l)$, $A_g(r, z) = (E_{0g}/f_g)\exp[-(r^2/2r_{0g}^2 f_g^2)]$, E_{0l} and E_{0g} refers to the maximum amplitude of the LG and Gaussian beams of initial width r_{0l} & r_{0g} (in space), L_p^l is the associated Laguerre polynomial, l & p refer the binomial coefficients characterizing intensity modulation on the wavefront and ϕ_l refers the initial phase difference between the electric field vectors between Gaussian ($\mathbf{E}_g$) and LG ($\mathbf{E}_l$) beams in coaxial propagation. The dispersion relation characterize the em field in a plasma and can be expressed as $\omega^2 = (c^2k^2 + \omega_p^2)$, where ω_p is the plasma frequency, $k[=(\omega/c)\varepsilon_{0j}^{1/2}]$ refers to the wave number associated with the *em* beam, ε_{0j} is the dielectric function corresponding to the axis of maximum electric field on the wavefront of the beam and c refers to the speed of light in vacuum; here j stands for LG (L_0^1 mode) and Gaussian profiles of the beam.

The NLSE) describing the propagation of an em beam in a plasma (Sodha *et al.*, 1976); following the Jeffreys–Wentzel–Kramers–Brillouin (JWKB) approximation (Ghatak & Loknathan, 2004) for a slowly varying amplitude of the wave with propagation distance z (i.e., the term $\partial^2 A_j/\partial z^2$ is neglected), the wave equation characterizing the electric field vector takes the form

$$2ik\frac{\partial A_j}{\partial z} + \frac{\partial k}{\partial z} = \left(\frac{\partial^2 A_j}{\partial r^2} + \frac{1}{r}\frac{\partial A_j}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 A_j}{\partial \theta_j^2} + \frac{\omega^2}{c^2}(\varepsilon_j - \varepsilon_{0j}), \tag{4}$$

where ε_j refers the dielectric function of the plasma and j stands for Gaussian and LG beams.

The first two terms in the right-hand side of the above equation refer to the contribution in field evolution due to diffraction and phase variation; the manifestation of these terms with plasma non-linear effects (last term) causes the transverse focusing/defocusing of the *em* beam as it propagates in the plasma. The non-linearity in plasma primarily arises on account of electron density modification due to non-uniformity in the irradiance profile of the beam. The solution for Eq. (4) can be written as (Sodha *et al.*, 1976)

$$A_j(r, z, \theta_j) = A_{0j}(r, z) \exp[-i(kS_j(r, z) + l\theta_j)], \tag{5}$$

where $S_j(r, z)$ refers to the eikonal associated with the LG and Gaussian beam propagations.

Substituting for $A_j(r, z, \theta_j)$ in Eq. (4) and comparing the real and imaginary terms one gets (Thakur & Berakdar, 2010; Misra *et al.*, 2014)

$$\begin{aligned} &\frac{2S_j}{k}\frac{\partial k}{\partial z} + 2\frac{\partial S_j}{\partial z} + \left(\frac{\partial S_j}{\partial r}\right)^2 = \\ &\frac{1}{k^2 A_{0j}}\left(\frac{\partial^2 A_{0j}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{0j}}{\partial r}\right) - \frac{l^2 A_{0j}}{r^2} + \frac{\omega^2}{c^2}(\varepsilon_j - \varepsilon_{0j}) \end{aligned} \tag{6a}$$

and

$$\frac{\partial A_{0j}^2}{\partial z} + A_{0j}^2\left(\frac{\partial^2 S_j}{\partial r^2} + \frac{1}{r}\frac{\partial S_j}{\partial r}\right) + \frac{\partial A_{0j}^2}{\partial r}\frac{\partial S_j}{\partial r} + \frac{A_{0j}^2}{k}\frac{\partial k}{\partial z} = 0. \tag{6b}$$

It is noticed that Gaussian and LG beams have different intensity profiles and characterized by on-axis and off-axis maxima, respectively; thus it is instructive that the dynamics of each beam should be analyzed in the vicinity of its intensity maxima.

2.1.1. LG Beam: L_0^1 Mode

As discussed earlier, LG beam displays an off-axis intensity profile and the above set of equations [Eq. (5)] should be transferred to the axis ($r \neq 0$) in the vicinity of intensity maximum. For the sake of simplicity in the analysis, the propagation dynamics of a specific LG profile corresponding to L_0^1 mode has been considered herein. For this particular case (i.e., L_0^1 mode) the intensity maximum occurs at $r = r_{0l}f_l$. To proceed further a paraxial-like approach (Misra & Mishra, 2008; Sodha *et al.*, 2009*a*) analogous to that of paraxial approximation is adopted where the coordinate system is transformed from (r, z) to (η_l, z) space such that

$$\eta_l = [(r/r_{0l}f_l) - 1], \tag{7}$$

where $r_{0l}f_l(z)$ is the width of the beam and $r = r_{0l}f_l$ represents the position of the maximum irradiance on the wavefront as it advances in the plasma. In the vicinity of off-axis maxima, it is justified to expand other relevant parameters around intensity maximum, that is, $\eta_l = 0$. Using this relation the set of

https://doi.org/10.1017/S0263034615000142 Published online by Cambridge University Press

Eqs (6) thus transformed as (Misra & Mishra, 2008)

$$\frac{2S_l}{k}\frac{\partial k}{\partial z}+2\frac{\partial S_l}{\partial z}+\frac{1}{r_{0l}^2 f^2}\left(\frac{\partial S_l}{\partial \eta_l}\right)^2=$$
$$\frac{1}{(r_{0l}^2 f_l^2)(k^2 A_{0l})}\left(\frac{\partial^2 A_{0l}}{\partial \eta_l^2}+\frac{1}{(1+\eta_l)}\frac{\partial A_{0l}}{\partial \eta_l}-\frac{l^2 A_{0l}}{(\eta_l+1)^2}\right)-\left(\frac{\eta_l^2\omega^2}{k^2c^2}\right)\varepsilon_{2l} \tag{8a}$$

and

$$\frac{\partial A_{0l}^2}{\partial z}+\frac{A_{0l}^2}{(r_{0l}^2 f_l^2)}\left(\frac{\partial^2 S_l}{\partial \eta_l^2}+\frac{1}{(1+\eta_l)}\frac{\partial S_l}{\partial \eta_l}\right)+\frac{1}{r_{0l}^2 f_l^2}\frac{\partial A_{0l}^2}{\partial \eta_l}\frac{\partial S_l}{\partial \eta_l}+\frac{A_{0l}^2}{k}\frac{\partial k}{\partial z}=0, \tag{8b}$$

where $\varepsilon_l\ (\eta_l,\ z)=\varepsilon_{0l}\ (z)-\eta_l^2\ \varepsilon_{2l}\ (z)$ and like the paraxial theory the present analysis is strictly applicable when $\eta_l \ll 1$. In paraxial regime, the solution of Eqs. (8) can be written as

$$E_l E_l{*}=A_{0l}^2(r,z)=(2E_{0l}^2/f_l^2)(\eta_l+1)^2\exp[-(\eta_l+1)^2] \tag{9a}$$

and

$$S_l=(1/2)(\eta_l+1)^2\beta_l(z)+\Theta_l(z), \tag{9b}$$

where $\beta_l=r_{0l}^2 f_l\ (\partial f_l\ /\partial z)$, $\Theta_l(z)$ is the arbitrary phase function describing the departure of the curvature from spherical nature and f_l refers to the beam width parameter that characterizes the irradiance profile as it propagates in the plasma.

The solutions for A_{0l}^2 and S_l is consistent with Eq. (8b) and essentially characterize the maintenance of the shape of the beam as it advances through the plasma. Substituting for A_{0l}^2, S_l, and ε_l in Eq. (8a) and equating the coefficients of η_l^0 and η_l^2 on both sides of the resulting equation one obtains

$$\left[\varepsilon_{0l}\left(\rho_{0l}^2 f_l\frac{\partial^2 f_l}{\partial \xi^2}+2\frac{\partial \Phi_l}{\partial \xi}\right)+\frac{3}{\rho_{0l}^2 f_l^2}\right]+\left(\Phi_l/2+\rho_{0l}^2 f_l\frac{\partial f_l}{\partial \xi}\right)\frac{\partial \varepsilon_{0l}}{\partial \xi}=0 \tag{10a}$$

and

$$\varepsilon_{0l}\frac{\partial^2 f_l}{\partial \xi^2}=\frac{1}{\rho_{0l}^2 f_l}\left(\frac{1}{\rho_{0l}^2 f_l^2}-\varepsilon_{2l}\right)-\frac{\partial f_l}{\partial \xi}\frac{\partial \varepsilon_{0l}}{\partial \xi}, \tag{10b}$$

where $\rho_{0l}=(r_{0l}\omega/c)$, $\Phi_l=(\Theta_l\omega/c)$, and $\xi=(z\omega/c)$.

The set of equations [Eq. (10)] characterizes the spatial evolution of the electric field envelop of LG (L_0^1 mode) beam as it propagates in the plasma; in this course the transverse focusing/defocusing of the beam occurs. It should be noted here that the wave equations [Eqs. (7)–(10)] strictly correspond to the L_0^1 mode of LG beam; however, the similar analysis based on the paraxial-like approach can be performed for various orders (i.e., arbitrary *l* & *p*) of off-axis LG intensity profiles.

2.1.2. Gaussian Beam

In case of *Gaussian* beam, the intensity maximum occurs at $r=0$. Following earlier analyses (Sharma *et al.*, 2009) based on paraxial approximation, the coupled differential equations describing the beam width parameter and phase in corresponding to Gaussian wavefront in the plasma can be written as

$$\varepsilon_{0g}\rho_g^2(d\Phi_g/d\xi)+(1/f_g^2)+\Phi_g(d\varepsilon_{0g}/d\xi)=0 \tag{11a}$$

and

$$\varepsilon_{0g}\frac{d^2 f_g}{d\xi^2}=\frac{1}{\rho_g^2}\left(\frac{1}{\rho_g^2 f_g^3}-f_g\varepsilon_{2g}\right)-(1/2)\left(\frac{\partial f_l}{\partial \xi}\frac{\partial \varepsilon_{0l}}{\partial \xi}\right), \tag{11b}$$

where $\rho_{0g}=(r_{0g}\omega/c)$ and $\Phi_g=(\Theta_g\omega/c)$. The above equations [Eqs (11)] characterizing f_g and Φ_g are consistent with the irradiance profile

$$A_{0g}^2=(E_{0g}^2/f_g^2)\exp(-\eta_g^2/f_g^2) \tag{12}$$

with $\varepsilon_g\ (r,z)=\varepsilon_{0g}\ (z)-\eta_g^2\ \varepsilon_{2g}\ (z)$, $\eta_g=(r/r_{0g})$.

It is interesting to note here that the algebraic form of the focusing equation [i.e., Eqs. (10b)–(11b)] is similar to that obtained in references (Nasalski, 1995; 1996; Thakur & Berkdar, 2010), where laser propagation dynamics in Kerr dielectric medium corresponding to Gaussian and LG profiles has been explored. In contrast to quadratic dependence of the dielectric function on the laser field in Kerr medium, the plasma exhibits saturating nature of non-linearity; this causes the oscillatory focusing/defocusing as the laser beam propagates in the plasma.

The set of Eqs. (10) and (11) is coupled through the dielectric function and characterize the coaxial propagation of the LG (L_0^1 mode) and Gaussian beams. Using appropriate expressions for ε corresponding to plasma with dominant ponderomotive and relativistic non-linearities in addition to initial boundary conditions (corresponding to plane wavefront of the pulse at $z=0$) *viz.* $\Phi_l(0)=\Phi_g(0)=0$, $f_l(0)=f_g(0)=1$ and $f_l'\ (0)=f_g'\ (0)=0$, the equations can numerically be solved to evaluate the beam width parameter and phase dependence on the propagation distance ξ; for our computations Mathematica software is used. The knowledge of f_j and Φ_j leads to the information about spatial evolution of intensity profile as it propagates in the plasma.

2.2. Critical Condition for Focusing: Critical Curve

For an initial plane wavefront (i.e., $df_j/d\xi=0$) of the beam, $(d^2f_j/d\xi^2)_{\xi=0}=0$ refers to $f_j(\xi)=1$ and the beam can propagate without convergence or divergence in plasma; this refers the critical condition for focusing of the *em* beam. Thus by substituting $(d^2f_j/d\xi^2)_{\xi=0}=0$ in Eqs. (10b) and (11b), one obtains a relation between dimensionless initial width of the beam $\rho_j(=r_j\omega/c)$ and initial irradiance EE^*, as the critical curve that ensures the propagation of the beam in the self-trapped mode. The critical condition is thus given by Sharma *et al.* (2003)

$$\rho_j^2 = (1/\varepsilon_{2j}). \tag{13}$$

The beam displays self-focusing for the condition $(d^2f_j/d\xi^2) < 0$, whereas in case of $(d^2f_j/d\xi^2)>0$ the beam undergoes either oscillatory or steady divergence.

3. DIELECTRIC FUNCTION OF THE PLASMA

The non-linear propagation of any beam in the plasma is characterized by the non-linear dielectric function that is in the present analysis, modified by the coupling of fields (intensities) of both the beams. The spatial profile of the dielectric function as a consequence of the combined intensity profile (derived later) drives the non-linear effects in the plasma and hence propagation dynamics of the beams. Following Sodha *et al.* (1974), the effective dielectric function of the plasma can be expressed as

$$\varepsilon(\mathrm{r, z}) = 1 - \Omega^2(\mathrm{n_e/n_{e0}}), \tag{14}$$

where $\Omega = (\omega_{\mathrm{pe}}/\omega)$, $\omega_{\mathrm{pe}} = (4\pi\ n_{\mathrm{e}0}e^2/m_{\mathrm{e}})^{1/2}$ is the electron plasma frequency, n_e is the plasma density in the presence of *em* field, $n_{\mathrm{e}0}$ refers the undisturbed plasma density, m_{e} is the mass of the electron, and e is the electronic charge.

As mentioned before, the plasma density redistribution and hence the effective dielectric function is determined by combined intensities of LG and Gaussian beams. The effective irradiance generated by LG (L_0^1 mode) and Gaussian beams can be written as

$$\begin{aligned} EE^* &= (E_l + E_g).(E_l + E_g)^* = E_l^2 + E_g^2 + 2Re(E_l.E_g^*) \\ &= E_{g0}^2[(\gamma^2/f_l^2)(r^2/\alpha^2 r_{0g}^2 f_l^2)\exp(-r^2/\alpha^2 r_{0g}^2 f_l^2) \\ &\quad + (1/f_g^2)\exp(-r^2/r_{0g}^2 f_g^2) \\ &\quad + 2(\gamma/f_g f_l)(r/\alpha r_g f_l)\exp[-(r^2/2r_{0g}^2)(1/\alpha^2 f_l^2 + 1/f_g^2)\cos\varphi_l] \end{aligned} \tag{15}$$

with $\alpha(=r_{0l}/r_{0g})$ and $\gamma = E_{l0}/E_{g0}$.

In the paraxial regime, the effective irradiance [EE^*, Eq. (15)] can be expanded around its intensity maximum. Thus for Gaussian beam ($r = 0$), one gets

$$EE^* = a_g + b_g\eta_g + c_g\eta_g^2, \tag{16a}$$

where $a_g = (E_{g0}^2/f_g^2)$, $b_g = E_{g0}^2(2\gamma/\alpha f_g f_l^2)\ \cos\varphi_l$, and $c_g = E_{g0}{}^2[(\gamma^2/\alpha^2 f_l^4) - (1/f_g^4)]$.

Similarly in the case of L_0^1 mode of the LG beam, EE^* in the vicinity of irradiance maximum (i.e., $\eta_l = 0$) can be written as

$$EE^* = a_l + b_l\eta_l + c_l\eta_l^2, \tag{16b}$$

where $a_l = E_{g0}^2[(\gamma^2/f_l^2)\exp(-1) + (1/f_g^2)\exp(-\alpha^2 f_{lg}^2) + (2\gamma/f_g f_l)\exp[-(1+\alpha^2 f_{lg}^2)/2]\cos\varphi_l]$, $b_l = -E_{g0}^2(\alpha^2 f_{lg}^2)[(2/f_g^2)\exp(-\alpha^2 f_{lg}^2) + (2\gamma/f_g f_l)\exp[-(1+\alpha^2 f_{lg}^2)/2]\cos\varphi_l]$, $c_l = -E_{g0}^2[(2\gamma^2/f_l^2)\exp(-1) - (1/f_g^2)(\alpha^2 f_{lg}^2)\exp(-\alpha^2 f_{lg}^2) + (2\gamma/f_g f_l)\exp[-(1+\alpha^2 f_{lg}^2)/2]\cos\varphi_l]$, and $f_{lg} = (f_l/f_g)$.

3.1. Evaluation of Dielectric Function

3.1.1. *Ponderomotive Non-Linearity*

In collisionless plasmas under influence of *an em* radiation, the redistribution of the electron density is determined by the balance of ponderomotive force with electron gas pressure gradient and the space charge electric field; the magnitude of the ponderomotive force is proportional to the gradient of beam irradiance. Such non-linearity sets in a period of the order of ω_{pi}^{-1}. For a collisionless plasma at moderate power of the beam (when the quiver speed of the electron is much smaller than the speed of light in vacuum), the modified electron density function n_{e} is given by (Akhmanov *et al.*, 1968; Sodha *et al.*, 1976),

$$n_{\mathrm{e}} = n_{\mathrm{e}0}\exp(-\beta EE^*), \tag{17}$$

where $\beta = (e^2/8k_{\mathrm{b}}T_0 m_{\mathrm{e}}\omega^2)$, k_{b} is the Boltzmann constant, and T_0 is the temperature of the atoms/ions. By substituting Eq. (17) into (14), one obtains

$$\begin{aligned} \varepsilon_j(r, z) &= \varepsilon_{0j}(z) - \eta_j^2\varepsilon_{2j}(z) = 1 - \Omega^2\exp(-\beta EE^*) \\ &\approx [1 - \Omega^2\exp(-\beta a_j)] - \eta_j^2[\Omega^2(\beta c_j - \beta^2 b_j^2/2)\exp(-\beta a_j)]. \end{aligned} \tag{18}$$

Here $\varepsilon_j(r, z)$ refer to the dielectric function for LG (L_0^1 mode) and Gaussian beams in the vicinity of their intensity maximum.

3.1.2. *Relativistic Non-Linearity*

In the presence of high-intensity em radiation, the electrons may gain the quiver speed equivalent to the light speed in vacuum. This causes relativistic variation in the electron mass and consequent change in plasma frequency leads to the redistribution of electrons and triggers relativistic non-linearity (Hora, 1975). This non-linearity sets in a period of the order of ω_{pe}^{-1}. The dielectric function in the case of circularly polarized beam can be expressed as (Hora, 1991)

$$\varepsilon_j(r, z) = 1 - \Omega^2(1 + \varsigma EE^*)^{-1/2}, \tag{19}$$

where $\varsigma = (e^2/m_e^2\omega^2 c^2)$. The dielectric function in the vicinity of intensity maxima can be written as

$$\begin{aligned}\varepsilon_j(r,z) &= \varepsilon_{0j}(z) - \eta_j^2\varepsilon_{2j}(z) = 1 - \Omega^2(1+\varsigma EE^*)^{-1/2}\\ &\approx [1-\Omega^2(1+\varsigma a_j)^{-1/2}] - \eta_j^2(\Omega^2/2)(1+\varsigma a_j)^{-3/2}\\ &\quad [\varsigma c_j - (3/4)\varsigma^2 b_j^2(1+\varsigma a_j)^{-1}].\end{aligned} \tag{20}$$

As can be seen from the final expressions for dielectric function, it is influenced by the irradiance profile of both the beams that are coupled through the phase difference between them; manifesting the appropriate expressions for the dielectric function [Eqs. (18) and (20)] with the beam dynamics equations [Eqs. (10) and (11)], coaxial (or coupled mode) propagation of LG (L_0^1 mode) and Gaussian beams has been analyzed and discussed in the next section.

4. NUMERICAL RESULTS AND DISCUSSION

For a numerical appreciation of the analytical formulation and physics understanding of the coaxial propagation dynamics of central shadow LG (L_0^1 mode in particular) and Gaussian em beams, critical curve, and non-linear space evolution are computed for an arbitrarily set of laser–plasma parameters and different kind of basic non-linearities; the estimates have been illustrated graphically. The propagation of the beams in the plasma is primarily characterized by dielectric function that becomes a complex function of electric fields of both the beams in coupled mode. The critical curves *viz.* the relation between initial irradiance $g_j(=\beta\ E_{0j}^2 \text{or} \varsigma\ E_{0j}^2)$ and initial beam width parameter (ρ_j) has been obtained using Eq. (13) and the appropriate expression for the transverse component of dielectric function and non-linearities. The non-linear space evolution of the beam width parameters (f_j) and eikonal phase (Θ_j) has been obtained by simultaneous numerical integration of Eqs. (10) and (11) along with the suitable laser plasma parameters and appropriate boundary conditions (as stated in the last paragraph of Section 2.1.2). The effect of varying initial laser parameters of both beams *viz.* initial width (ρ_j), initial irradiance (g_j), and initial phase difference (ϕ_l) on non-linear coaxial propagation dynamics and transverse compression have been evaluated and presented in the form of curves. The computations have been performed for the following standard set of parameters *viz.* $\beta\ E_{0g}^2(=\varsigma\ E_{0g}^2) = 5$, $\beta\ E_{0l}^2(=\varsigma\ E_{0l}^2) = 2$, $\gamma^2 = 2/5$, $\alpha = 1$, $\Omega^2 = 0.8$, and $\varphi_l = \pi/4$; the choice of these normalized parameters refers to laser propagation in the weakly relativistic plasma regime having uniform background plasma density $n_{e0}{\sim}10^{18}$ cm^{-3}, laser wavelength $\lambda{\sim}10$ μm, and intensity $I_0 \approx 10^{17}$ W/cm^2, respectively. The effect of various laser–plasma parameters on the critical curves and beam parameters has been studied by varying one and keeping others the same.

The critical curves (*cc's*) corresponding to dominant ponderomotive non-linearities have been illustrated in the set of Figure 1. The critical curves describe the self-trapping mode of the propagation of the *em* beams and characterize the em beam propagation in (ρ_j,g_j) space. It can readily be seen from Eq. (13) that the initial beam width (i.e., ρ_j^{-2}) follows the behavior similar to that of transverse (azimuthal) component of the dielectric function ε_{2j}. The critical curve divides the (ρ_j,g_j) space primarily in two regions where the propagation of em beam is characterized by self (oscillatory)-focusing

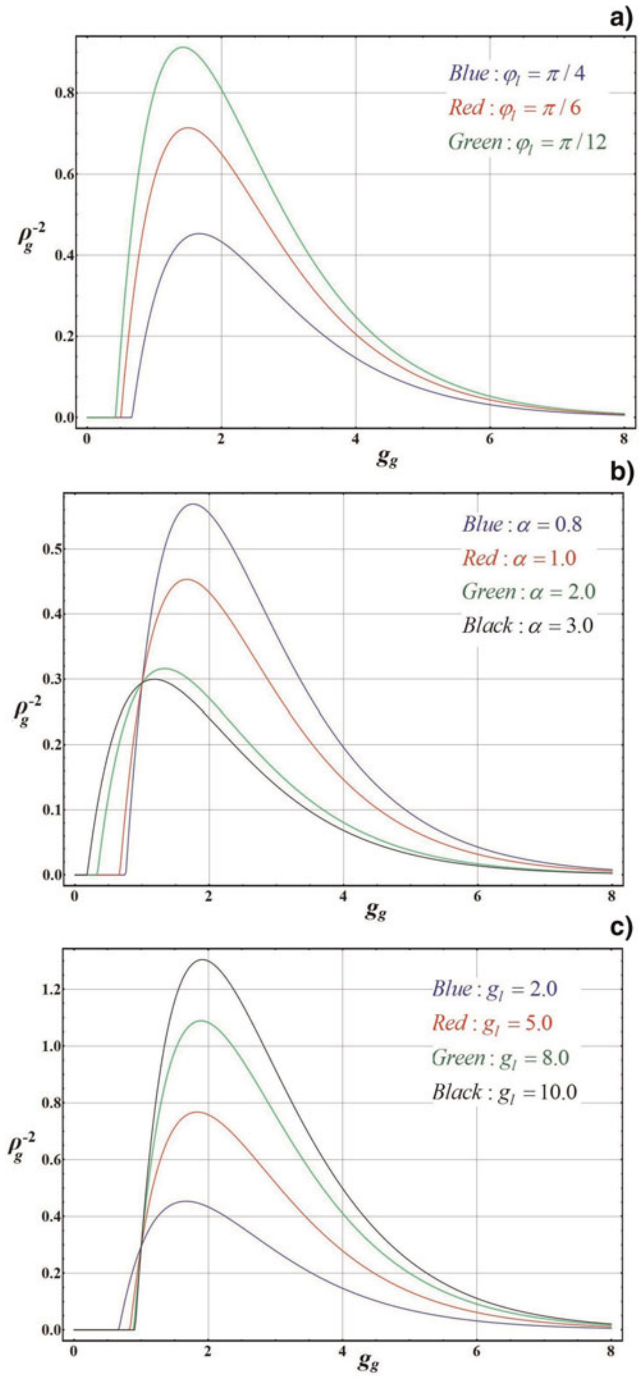

Fig. 1. Dependence of *cc's* ($\rho_g^{-2} - g_g$) on (a) phase ϕ_l associated with LG beams, (b) parameter α and (c) irradiance of LG beams g_l corresponding to ponderomotive non-linearity for the standard set of parameters stated in the text.

https://doi.org/10.1017/S0263034615000142 Published online by Cambridge University Press

(region below cc) and oscillatory defocusing (region above in proximity of cc), or steady defocusing (region far away from cc). The figures also indicate the fact that the beam having large ρ_j and large irradiance in the self-focusing region, gives rise to larger non-linear effects. The parameters corresponding to an initial point (ρ_j,g_j) lying on critical curves correspond to $(d^2f/d\ \xi^2)_{\xi=0} = 0$ and for an initially plane wavefront $(df/d\ \xi) = 0$, f remains constant and the beam propagates without convergence or divergence in plasma; such self-trapped motion of the beam is termed as stationary spatial soliton propagation. As stated before, the phase difference ϕ_l describes the coupling of electric field vectors associated with Gaussian and LG (L_0^1 mode) beams and effectively characterize the intensity pattern and propagation of the coupled mode. The effect of varying ϕ_l on critical curves ($\rho_g^{-2} - g_g$) corresponding to ponderomotive non-linearity for the given values of parameters α & g_l ($=\beta E_{0l}^2$), has been illustrated in Figure 1a. The figure indicates that the self-focusing region decreases with increasing ϕ_l; this nature can be attributed to large coupling between the coaxial beams with decreasing ϕ_l which enhances the effective irradiance of the beam and therefore more pronounced non-linear effects. It is seen that the self-focusing region decreases with the increasing value of the parameter α; this behavior has been displayed in Figure 1b and can be understood in terms of decrease in effective intensity [*via* Eqs. (16) and (18)] with increasing α. The self-focusing region is seen to increase with increasing parameter g_l; this nature has been displayed in Figure 1c. Similarly the critical curves can also be obtained in terms of parameters associated with LG beams, that is ($\rho_l^{-2} - g_l$) for given values of α and g_g; however, these curves are another way of presentation of cc's and carry the same information as described in the case of Figure 1. The dependence of critical curves corresponding to dominant relativistic non-linearity has been displayed in the set of Figure 2; the nature of the curves is similar to that obtained in the case of ponderomotive non-linearity (Fig. 1) and physically interpretable in the similar fashion. These critical curves characterize the regions for self-focusing and oscillatory/steady divergence and valid throughout the propagation dynamics.

The dependence of beam width parameters associated with coaxial propagation of LG (f_l) and Gaussian (f_g) beams in the plasma having dominant ponderomotive non-linearity has been illustrated in Figure 3a (solid lines). It is seen that during coaxial propagation each beam mutually influence the dynamics of the other beam; in order to illustrate this fact the independent propagation of individual beams has been shown by broken lines in the figure. It can be seen from the figure that the additional influence of LG beam in coupled mode propagation causes the focusing of the Gaussian beam as it advances through the plasma. The effect of coaxial propagation on space evolution of the eikonal phase associated with LG (Φ_l) and Gaussian (Φ_g) beams (as in Fig. 3a) has been displayed in Figure 3b. The behavior of the curves corresponding to coaxial propagation is in well conformance with the critical curves as shown in Figure 1. The effect of phase difference (i.e., ϕ_l) on coaxial propagation of the beams corresponding to $\phi_l = \pi/4$ & $\pi/8$ has been displayed in Figure 4a. The figure reflects the strong coupling in case of $\phi_l = \pi/4$ as the focusing curves approach each other; this nature is well appreciated with critical curves in Figure 1a. The co-axial beam is seen to exhibit larger

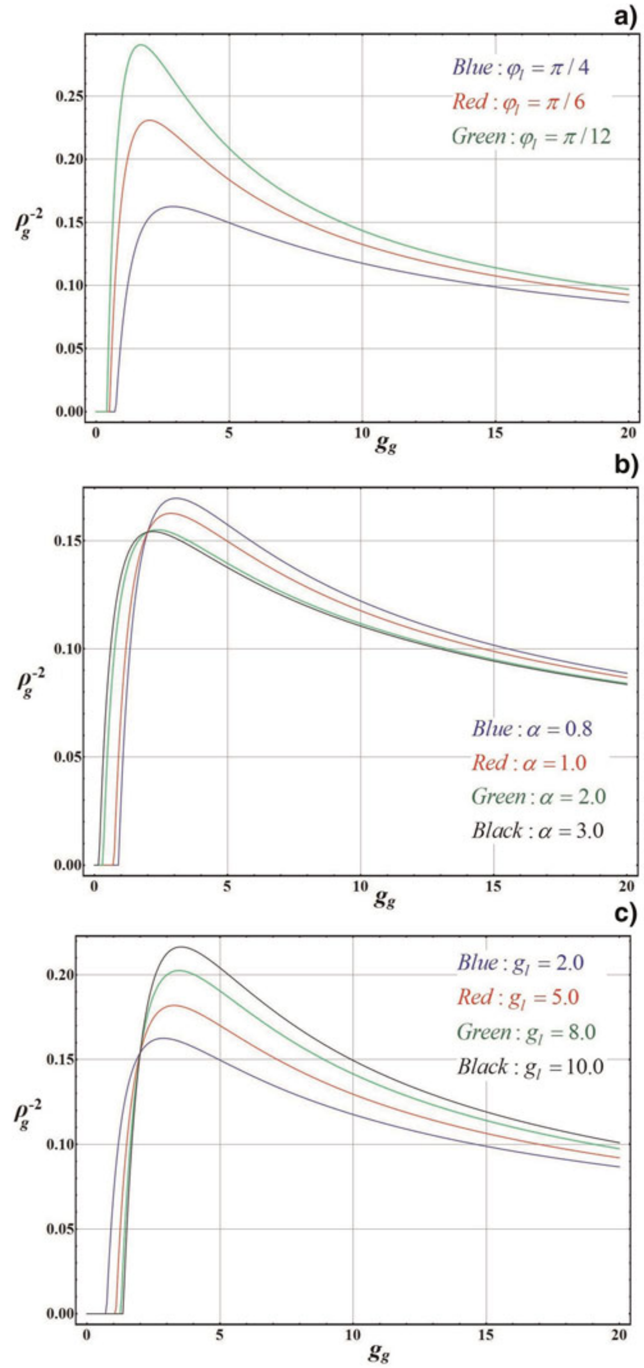

Fig. 2. Dependence of cc's ($\rho_g^{-2} - g_g$) on (a) phase ϕ_l associated with LG beams, (b) parameter α and (c) irradiance of LG beams g_l corresponding to relativistic non-linearity for the standard set of parameters stated in the text.

https://doi.org/10.1017/S0263034615000142 Published online by Cambridge University Press

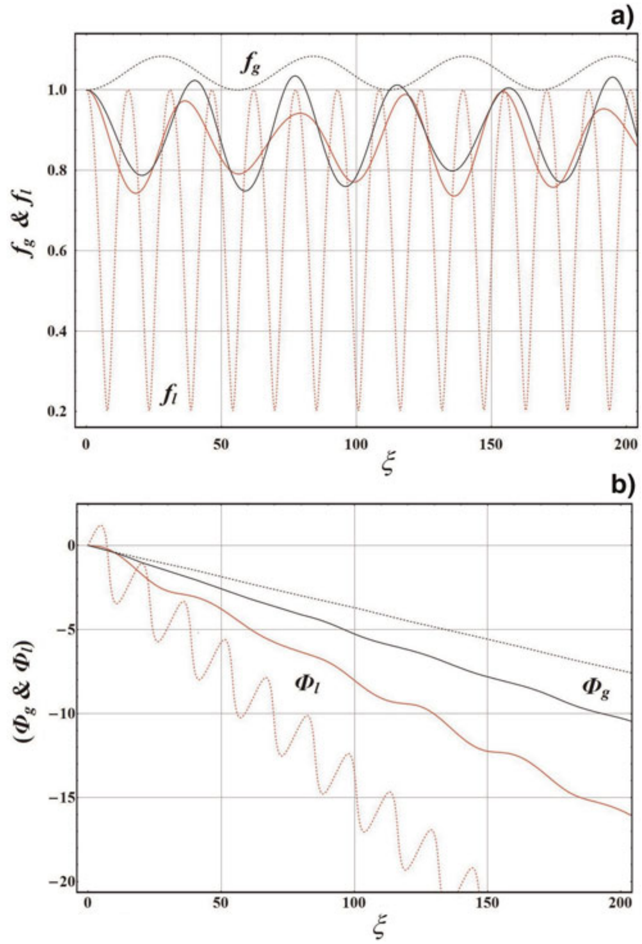

Fig. 3. Space evolution of the (a) beam width parameters (f_j) and (b) phase (Φ_j) corresponding to ponderomotive non-linearity for the standard set of parameters stated in the text with $\phi_l = \pi/6$; solid lines refer to coaxial propagation, while the broken curves correspond to separate propagation of Gaussian (f_g, black lines) and LG (f_l, red lines) beams.

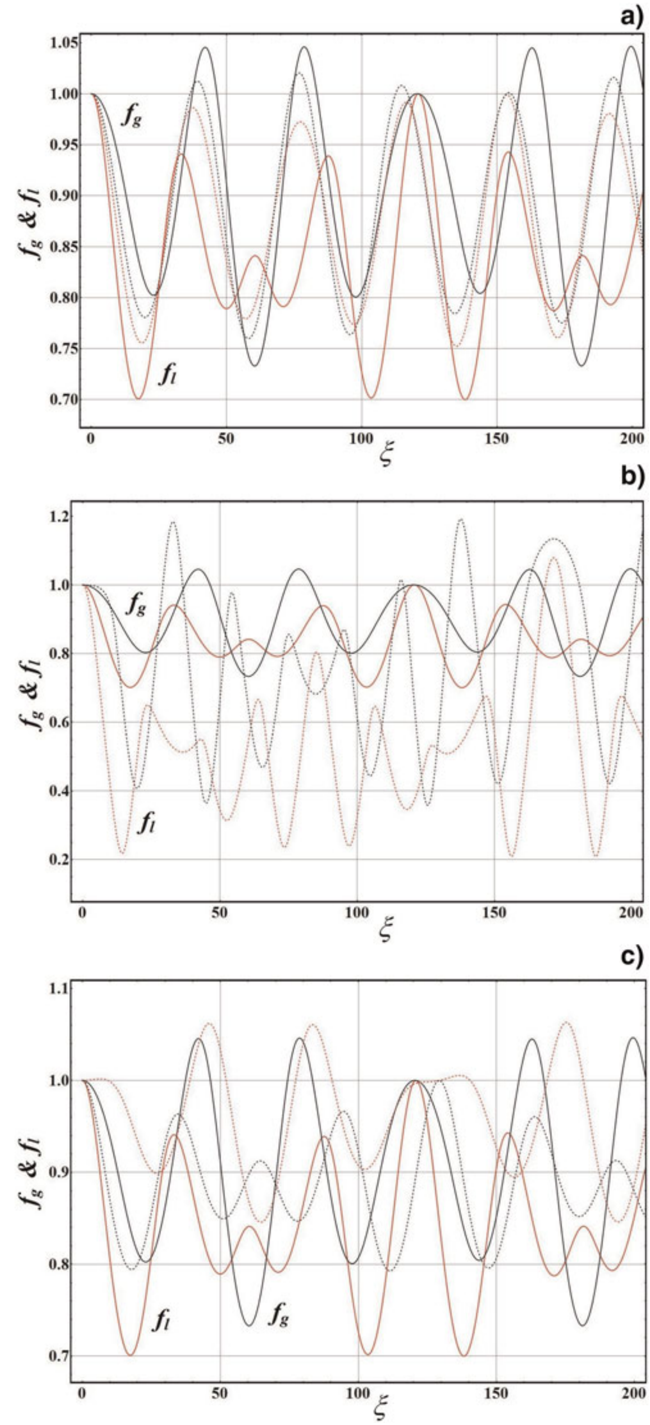

Fig. 4. Space evolution of the beam width parameters (f_j) corresponding to ponderomotive non-linearity for the standard set of parameters stated in the text. Panel (a) corresponds to varying phase $\phi_l(=\pi/4$, solid) and $\phi_l(=\pi/8$, broken), panel (b) refers to varying parameter $\alpha(=1$, solid) and $\alpha(=1.5$, broken), panel (c) refers to varying irradiance of LG beam $g_l(=2$, solid) and $g_l(=5$, broken); red and black color lines refer to f_l and f_g, respectively.

focusing with increasing parameter α and is shown in Figure 4b. The increase in α effectively refers to large initial beam width (ρ_1) associated with the LG mode (for constant ρ_g) that enhances non-linearity and hence the focusing. Further the influence of the initial irradiance of the LG mode on the non-linear propagation dynamics of Gaussian beam coaxially has been displayed in Figure 4c; the strong non-linear effects in the case of $g_1 = 5$ can be understood as a consequence of critical curves displayed in Figure 1c. The figures corresponding to the coupled mode propagation suggest that the dynamics of one beam can be controlled up to significant extent by varying the parameters of the other beam.

The beam width parameters *f s* certainly describe the evolution of electric field or intensity envelop (i.e., EE^*) of the *em* beam and hence the power (energy) transfer as it advances through the plasma. In order to have an idea of the radial distribution of the intensity during coaxial propagation of LG–Gaussian modes, the space evolution of the effective irradiance profile in non-linear regime of the plasma characterized by dominant ponderomotive non-linearity has been displayed in Figure 5; the distribution is shown at different ξ values for the parameters $g_g = 10$, $g_1 = 5$, $\alpha = 3$, and $\phi_1 = \pi/6$. It is shown that during coaxial propagation, the

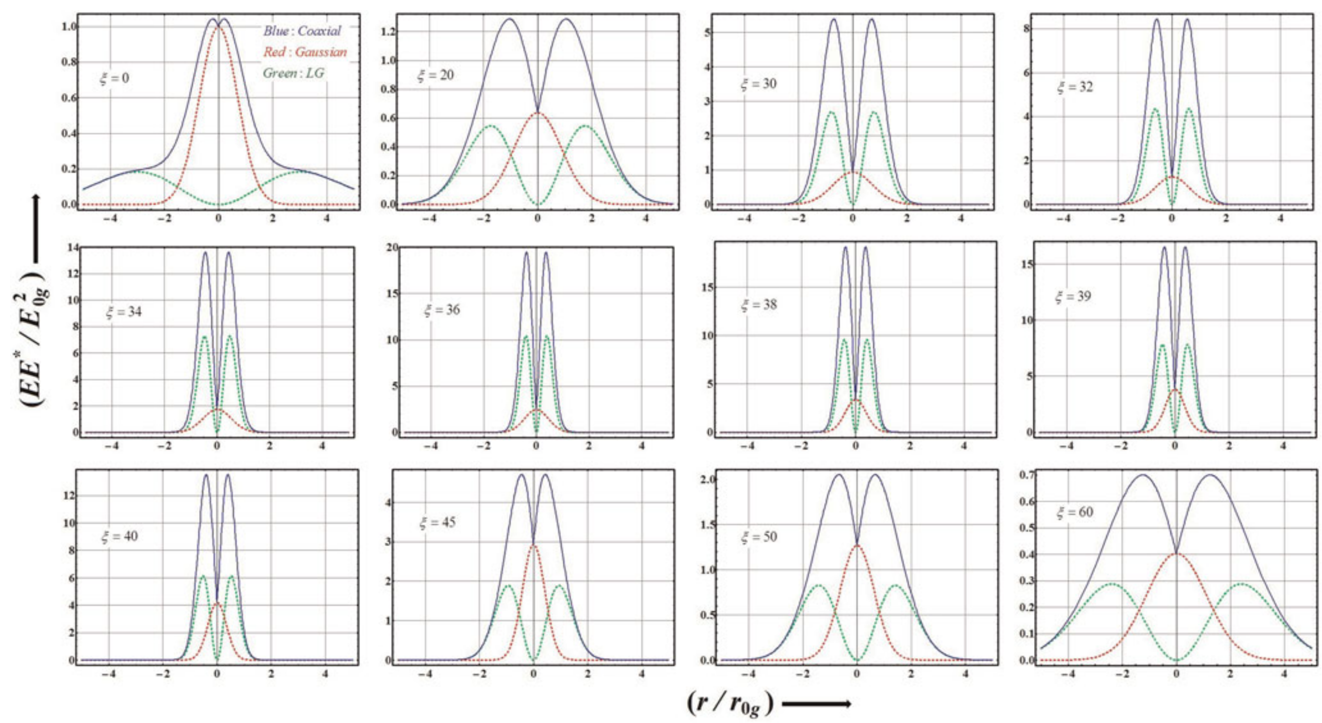

Fig. 5. Space evolution of the radial distribution of the beam irradiance (EE^*/E_{0g}^2) as a function of parameters (ξ); the figure corresponds to dominant ponderomotive non-linearity for the parameters $g_g = 10$, $g_l = 5$, $\alpha = 3$, and& $\phi_l = \pi/6$.

beam dynamics mutually influence each other and gives rise to large non-linear effects; as a consequence the transverse laser field itself gets modified as it advances through the plasma. It can be seen from the figure that on account of mutual focusing/defocusing the radial distribution of intensity displays compression/rarefaction of the beams; the intensity distribution of LG and Gaussian modes independently has been shown by the broken curves. It is also noticed that the coupling of Gaussian mode with annular (L_0^1 mode) field profile leads to significant enhancement (e.g., ξ = 36) in its irradiance as beam propagates through the plasma; this nature is qualitatively similar as obtained in one of the recent experimental investigation (Scheller *et al.*, 2014). The spatial evolution of such profiles (characterized by potential dipoles) can also efficiently be utilized for trapping of plasma particles. Furthermore, it is also necessary to point out that a specific case, that is, propagation dynamics of LG em beam corresponding to L_0^1 mode has been considered herein the analysis however the methodology of paraxial-like approach can be extended to investigate the propagation dynamics of higher order LG modes.

5. SUMMARY

A formalism describing the non-linear coaxial propagation dynamics of finite size intense LG (L_0^1 mode) and Gaussian beams in a plasma characterized by ponderomotive and relativistic non-linearities has been established. In order to analyze the off-axis contribution of annular LG beams the formulation takes account of paraxial-like approach, while usual paraxial approximation is utilized to analyze the dynamics of Gaussian mode propagation. The dynamics of coaxial propagation is coupled through the dielectric function which is considered to be a function of combined electric field of both the propagating modes. The coaxial propagation dynamics is described by NLSE which governs the spatial evolution of the coupled mode as it advances through the plasma. Based on this analysis, the critical curves and space evolution of beam width parameter (f_j) and phase (Φ_j) of coupled mode have been computed and presented graphically. The critical curves predict the regimes of oscillatory (self) focusing/defocusing and steady divergence of the coupled mode propagation; this characteristic feature is a consequence of competing phenomenon of diffraction with non-linear effects. It is shown that the coupling of Gaussian profile with L_0^1 mode significantly enhances its intensity as the coupled mode advances through the plasma. The focusing dynamics of such profiles (i.e., existence of sharp potential dipoles) is of relevance to the particle/atomic trapping and efficient energy transport which has significantly enhanced due to self-focusing of the coaxial beams.

ACKNOWLEDGEMENTS

One of the authors (SM) is grateful to the Department of Science and Technology (DST) for the financial support for the work.

REFERENCES

Akhmanov, S.A., Sukhorukov, A.P. & Khokhlov, R.V. (1968). Self-focusing and diffraction of light in a nonlinear medium. *Sov. Phys. Usp*. **10**, 609–636.

Berge, L. (1998). *Wave Collapse In Physics: Principles And Applications To Light And Plasma Waves*. Phys. Rep. **303**, 259–370.

Brijesh, P., Kessler, T., Zuegel, D. & Meyerhofer, D. (2007). *Demonstration of a horseshoe-shaped longitudinal focal profile*. J. Opt Soc. Am. B **24**, 1030–1038.

Deutsh, C., Furukaw, H., Mima, K., Murakami, M. & Nishihara, K. (1996). *Interaction Physics of the Fast Ignitor Concept*. Phys. Rev. Lett. **77**, 2483–2486.

Eder, D.C., Amendt, P., Da Silva, L.B., London, R.A., Mac Gowan, B.J., Mathews, D.L., Penetrante, B.M., Rosen, M.D., Silks, S.C., Donnelly, T.D., Falcone, R.W & Strobel, G.L., (1994). *Tabletop x-ray lasers*. Phys. Plasmas **1**, 1744–1752.

Ghatak, A. & Loknathan, S. (2004). *Quantum Mechanics: Theory and Applications*. New Delhi: Springer Science and Business Media.

Gupta, R., Rafat, M. & Sharma, R.P. (2011a). *Effect of relativistic self- focusing on plasma wave excitation by a hollow Gaussian beam*. J. Plasma Phys. **77**, 777–784.

Gupta, R., Sharma, P., Rafat, M. & Sharma, R.P. (2011b). *Cross-focusing of two hollow Gaussian laser beams in plasmas*. Laser Part. Beams **29**, 227–230.

Gurevich, A.V. (1978). *Nonlinear Phenomena in the Ionosphere*. Berlin: Springer.

Hora, H. (1975). *Theory of relativistic self-focusing of laser radiation in plasmas*. J. Opt. Soc. Am. **65**, 882–886.

Hora, H. (1991). *Plasmas at High Temperature and Density*. Heidelberg: Springer.

Kasparian, J., Ackermann, R., Andre, Y. B., Mechain, G., Mejean, G., Prade, B., Rohwetter, P., Salmon, E., Stelmaszczkyk, K., YU, J., Mysyrowicz, A., Sauerbrey, R. Woste, L. & Wolf, J., (2008). *Electric events synchronized with laser filaments in thunderclouds*. Opt. Express **16**, 5757–5763.

Khamedi, M. & Bahrampour, A.R. (2013). *Analysis of twisted laser beam focusing and defocusing in plasma*, Phys. Scr. **88**, 035503–035506.

Konar, S. & Jana, S. (2005). *Nonlinear Propagation of a Mixture of TEM00 and TEM01 Modes of a Laser Beam in a Cubic Quintic Medium*, Phys. Scr. **71**, 198–203.

Luo, Q., Xu, H.L., Hosseini, S.A., Daigle, J.F., Theberge, F., Sharifi, M. & Chin, S.L., (2006). *Remote sensing of pollutants using femtosecond laser pulse fluorescence spectroscopy*. Appl. Phys. B **82**, 105–109.

Misra, S. & Mishra, S.K. (2008). *On focusing of a ring ripple on a Gaussian electromagnetic beam in a plasma*. Phys. Plasmas **15**, 0923071–8.

Misra, S. & Mishra, S.K. (2009a). *Focusing of a ring ripple on a Gaussian electromagnetic beam in a magnetoplasma*. J. Plasma Phys. **75**, 545–561.

Misra, S. & Mishra, S.K. (2009b). *Focusing of dark hollow Gaussian electromagnetic beam in a plasma with relativistic ponderomotive Regime*. PIER B **16**, 291–309.

Misra, S., Mishra, S.K., Sodha, M.S. & Tripathi, V.K. (2014). *Effect of Electron-Ion Recombination on Self-focusing/ defocusing of Laser Pulse in Tunnel Ionized Plasmas*. Laser Part. Beams **32**, 21–31.

Nasalski, W. (1995). *Complex ray tracing of nonlinear propagation*. Opt. Commun. **119**, 218–226.

Nasalski, W. (1996). *Aberrationless effects of nonlinear propagation*. J. Opt. Soc. Am. B **13**, 1736–1747.

O'Neil, A.T., Macvicar, I., Allen, L. & Padgett, M.J. (2002). *Intrinsic and extrinsic nature of the orbital angular momentum of a light beam*. Phys. Rev. Lett. **88**, 0536011–4.

Ren, Y., Alshershby, M., Qin, J., Hao, Z. & Lin, J. (2013). *Microwave guiding in air along single femtosecond laser filament*. J. Appl. Phys. **113**, 094904-1-5.

Saini, N.S. & Gill, T.S. (2006). *Self-focusing and self-phase modulation of an elliptic Gaussian laser beam in collisionless magnetoplasma*. Laser Part. Beams **24**, 447–453.

Scheller, M., Mills, M.S., Miri, M.A., Cheng, W., Moloney, J.V., Kolesik, M., Polynkin, P. & Christodoulides, D.N., (2014). *Externally refueled optical filaments*. Nat. Photonics **8**, 297–301.

Sharma, A., Borhanian, J. & Kourakis, I. (2009). *Electromagnetic beam profile dynamics in collisional plasmas*. J. Phys. A: Math. Theor. **42**, 465501.

Sharma, A., Prakash, G., Verma, M.P. & Sodha, M.S. (2003). *Three regimes of intense laser propagation in plasmas*, Phys. Plasmas **10**, 4079–4084.

Sharma, A., Sodha, M.S., Misra, S. & Mishra, S.K. (2013). *Thermal de-focusing of intense dark hollow Gaussian laser beams in atmosphere*. Laser Part. Beams **31**, 403–410.

Sodha, M.S., Ghatak, A.K. & Tripathi, V.K. (1974). *Self-Focusing of Laser Beams in Dielectrics, Semiconductors and Plasmas*. Delhi: Tata-McGraw-Hill.

Sodha, M.S., Govind, & Sharma, R.P. (1979). *Cross-focusing of two co-axial Gaussian electromagnetic beams in a magnetoplasma and plasma wave generation*. Plasma Phys. **21**, 13–26.

Sodha, M.S., Mishra, S.K. & Misra, S. (2009a). *Focusing of dark hollow Gaussian beams in a plasma*. Laser Part. Beams **27**, 57–68.

Sodha, M.S., Mishra, S.K. & Misra, S. (2009b). *Focusing of dark hollow Gaussian electromagnetic beam in a magnetoplasma*. J. Plasma Phys. **75**, 731–748.

Sodha, M.S., Sharma, A. & Agarwal, S. (2008). *A condition for simultaneous propagation of coaxial Gaussian electromagnetic beams in a plasma, without convergence or divergence*. J. Plasma Phys. **74**, 293–299.

Sodha, M.S., Ghatak, A.K. & Tripathi, V.K. (1976). *Self-focusing of laser beams in plasmas in plasmas and semiconductors*. Prog. Opt. **13**, 169–265.

Sprangle, P. & Esarey, E. (1991). *Stimulated back scattered harmonic generation from intense laser interaction with beams and plasmas*. Phys. Rev. Lett. **67**, 2021–2024.

Sprangle, P., Esarey, E., Ting, A. & Joyce, G. (1988). *Laser wakefield acceleration and relativistic optical guiding*. Appl. Phys. Lett. **53**, 2146–2148.

Stibenz, G., Zhavoronkov, N. & Steinmeyer, G. (2006). *Self-compression of millijoule pulses to 7.8 fs duration in a white-light filament*. Opt. Lett. **31**, 274–276.

Sueda, K., Miyaji, G., Miyanaga, N. & Nakatsuka, M. (2004). *Laguerre-Gaussian beam generated with a multilevel spiral phase plate for high intensity laser pulses*. Opt. Express **12**, 3548–3553.

Tabak, M., Hammer, J., Glinisky, M.E., Kruer, W.L., Wilks, S.C., Woodworth, J., Campbell, E.M., Perry, M.D. & Mason, R.J. (1994). *Ignition and high gain with ultra-powerful lasers*. Phys. Plasmas **1**, 1626–1634.

Thakur, A. & Berakdar, J. (2010). *Self-focusing and defocusing of twisted light in non-linear media*. Opt. Express **18**, 27691–27696.

Umstadter, D., Chen, S.Y., Maksimchuk, A., Mourou, G. & Wagner, R. (1996). *Nonlinear optics in relativistic plasmas and laser wake field acceleration of electrons*. Science **273**, 472–475.

Yu, W., Yu, M.Y., Xu, H., Tian, Y.W., Chen, J. & Wong, A.Y. (2007). *Intense local plasma heating by stopping of ultrashort ultraintense laser pulse in dense plasma*. Laser Part. Beams **25**, 631–638.