

One of the features of the text is an elaborate code which is used to refer to certain axioms, definitions, and theorems. For example, T_Ir is the Theorem on Irrational Numbers, which runs as follows: "If a non-zero rational number r is combined with an irrational number p by any one of the four operations of arithmetic, the result produced is an irrational number; in symbols, $r + p$, $r - p$, $p - r$, rp , r/p , p/r are irrational numbers." According to the author's preface, "Experience in classroom teaching shows that the students use the code with alacrity and effectiveness in making full and concise proofs."

This reviewer feels that the book under review is a worthy addition to the literature; but on the whole he found the exposition somewhat clumsy. In a few places terms are used before they are explained (e.g. "empty set," page 99) and in some places no explanation is offered where one is clearly required. (e.g. $0!$ is used, but never defined. Since $3!$ is defined, the reviewer presumes that no knowledge of factorials is assumed.) Functions are never mentioned, even though the use of functions could have simplified the treatment considerably. These objections, however, may possibly be regarded as minor. Finally, the exercises in the book are many in number and generally non-computational in nature.

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An Introduction to Functional Analysis, by Angus E. Taylor. John Wiley and Sons, New York, 1958. 423 pages. \$12.50.

This book is intended primarily as a text for graduate students. The majority of it is taken up with development of the basic abstract theory of linear spaces and operators. This material is interspersed with many examples, and each chapter begins with an introduction which gives some motivation and points out the most important results. The numerous exercises are a valuable feature.

The first chapter is concerned with the purely algebraic aspects of vector spaces and linear operators. There is a concise review of the finite dimensional case and several examples of infinite dimensional spaces and operators on them. It is pointed out how classical problems such as the existence and uniqueness of solutions for linear integral or differential equations can be formulated in terms of linear operators. The algebraic dual of a space and transpose of an operator are discussed, as well as algebraic (Hamel) bases.

The second chapter is a concise introduction to general topology, with concentration on what is needed for linear analysis. For example, completion of a metric space and Baire

category are covered in detail, while Tychonoff's theorem on the compactness of product spaces is stated without proof.

The next two chapters "Topological Linear Spaces" and "General Theorems on Linear Operators" cover thoroughly the basic definitions and theorems of abstract linear analysis. There is enough discussion of semi-norms, local convexity, and the like, to serve as a useful introduction to the more extensive treatment of Bourbaki, but the main emphasis is on normed linear spaces. (For instance the closed graph theorem is proved for operators on complete metric spaces, but the space of continuous linear functionals and adjoint operators are considered for normed spaces only.) One interesting feature is the exhaustive listing of the "states" possible for a bounded operator and its conjugate (adjoint). The range of an operator from one normed linear space, X , to another, Y , may be all of Y , a proper dense subset of Y , or such that its closure is a proper subset of Y . The operator may have a continuous inverse, an unbounded inverse, or no inverse. These various possibilities allow one to classify operators into nine "states". The state of an operator and the state of its conjugate are not independent; the situation is fully set forth in a diagram showing which combinations are impossible, and which are impossible under the added conditions that X be complete, that Y be complete, or that X be norm-reflexive. (Incidentally, it is this diagram which forms the theme of the jacket design.)

Chapter five is devoted to the spectral theory of arbitrary closed operators on a complex Banach space. The treatment is quite full, and makes this chapter a useful reference. The main topics are the theory of compact operators (the application to integral equations is pointed out) and the use of contour integration to obtain an operational calculus. The classical spectral theory of normal and self-adjoint operators on Hilbert space occupies the next chapter. A distinctive point is that compact symmetric operators on an inner-product space are considered without the assumption that the space is complete. As a result, for example, the theory applies directly to suitable integral operators on the space of continuous functions on the unit interval, without completing the space to L^2 , and yields the conclusion that there is a complete orthonormal set of continuous eigenfunctions.

There is a final chapter which describes in detail the relation between linear functionals and integrals. Almost no details of proofs are given.

The book is not at all self-contained. The author very properly economizes on space by simply giving references for the Minkowski and Hölder inequalities and various basic results in topology and measure-theory. Very few of the examples of applications of the theory are carried through in full detail.

It is hard to call this a defect since the author always gives explicit references and clearly states or describes the propositions whose proofs are omitted. Nevertheless I feel there is some danger that a student reading on his own might not appreciate what is "concealed" in the references. As a result he might come to think that once a problem is formulated in abstract terms it is as good as solved, or, if he is more sceptical, that while many problems of classical analysis can be formulated in an abstract way, the general theory is not much help in solving them. Either impression would be unfortunate.

Despite this criticism, Professor Taylor's book deserves careful consideration as a text for almost any introductory course in functional analysis. The basic theory is presented with great clarity and the numerous problems, while they should be in the grasp of a student who has mastered the text, are by no means routine exercises. Doubtless no two functional analysts have the same idea of what an introductory course should be. In any case here is a good foundation for a course, and an instructor who wishes to supplement it (by topics from Riesz and Sz. -Nagy's "Leçons d'Analyse Fonctionnelle", for example) should find it easy to do so.

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Problems in Euclidean Space (Applications of Convexity), by H.G. Eggleston. Pergamon Press, New York, 1957. 165 pages. \$6.50.

This 165 page book is a collection of problems, most of them being reprints of the author's papers.

There are four groups of problems. The first one opens with two beautiful examples: 1) when is an open set an intersection of a descending sequence of open connected sets? and 2) can a homeomorphism of E_2 onto itself be approximated by a finite succession of homeomorphisms of the form $x_1 = f(x, y)$, $y_1 = y$ or $x_1 = x$, $y_1 = g(x, y)$? Complete answers are given, but the connection with convexity is negligible.

The next group consists of a single problem which is a special case of Borsuk's conjecture on the possibility of covering a set in E_n by $n + 1$ sets of smaller diameter. The author obtains a complicated proof of this for $n = 3$. A far superior proof of B. Gruenbaum is relegated to the limbo of a footnote as being 'a lucky fluke' (sic!). Gruenbaum not only proves the same in less than a quarter of the space but he also shows that a set A in E_3 is a union of four sets of diameter less than .9886 times the diameter of A .