On 'A pretty series revisited': Graham Jameson writes: In [1], A succession of three substitutions is used to equate the integral

$$\int_0^1 \frac{u^{-1/3}(1-u)^{1/3}}{1-4a^2u(1-u)}\,du$$

to formula (5) on p. 453. This can be achieved by the single substitution $u = t/(1+t)$: then $1-u = 1/(1+t)$, so $u/(1-u) = t$ and the integral becomes

$$\int_0^\infty \frac{t^{-1/3}}{1-4a^2\frac{t}{(1+t)^2}}\frac{1}{(1+t)^2}\,dt = \int_0^\infty \frac{t^{-1/3}}{(1+t)^2-4a^2t}\,dt$$
$$= \int_0^\infty \frac{t^{-1/3}}{1+(2-4a^2)t+t^2}\,dt,$$

as seen in formula (5). For the case $2a^2 = 1$, the required equivalent form $\frac{1}{2}\int_0^\infty \frac{1}{v^{2/3}(1+v)}dv$ now follows by the substitution $t = v^{1/2}$.

Reference

1. Sean M. Stewart, A pretty series revisited, *Math. Gaz.* **105** (November 2021) pp. 450-457.

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On 'Correct answer – dodgy method': Lars Lund-Hansen writes: In [1], Des MacHale gives a fine list of "dodgy proofs" with correct results. Most of them are fine or even funny as is the crazy trigonometric derivation in example 8. But in his example 2 he deems the standard procedure to transform an infinite periodic decimal number to a quotient "not valid". The reason should be that infinite decimals are undefined objects when this proof is taught and hence the rule of moving the decimal point by two when multiplying with 100 is unjustified. At a purely technical level I don't disagree, but I don't think that should make the proof "not valid".

When mathematics is taught or studied it is impossible to start from the most fundamental concepts and then step by step work the way up. For example, one must meet and learn to work with the real numbers long before one can appreciate that they are undefined until they are established on top of the rationals in a rather complicated development. And before that the rationals must be constructed on top of the integers and before that... (as Des would says, what does this ... mean?). Hence most of the proofs one encounters before that should - according to Des - be "not valid" until the full machinery has been spelled out. The undefined infinite decimals are similar. Just like the real numbers most pupils can use them and think they know what they are, regardless that (some of) their teachers know better.

However, I think that if we are going to call a proof 'dubious' when it deals with objects which are not yet defined, this must depend on how it fares when definitions become sharper later on. Some proofs then fall apart or require a lot of work to save, but the "infinite decimal" example is not one of them. If one later learns what an infinite decimal really is and why the rule of multiplication by a power of ten is that simple, the proof may be understood at a deeper level, but it is precisely the same proof and nothing wrong has been taught.

Reference

1. Des MacHale, Correct item – dodgy method. *Math. Gaz.* **105** (November 2021) pp. 507-510.

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On 105.28: Alan Beardon writes: In this Note Clive Johnson studies the iterates g^n of a Möbius map, say $g(z) = (az + b)/(cz + d)$, with particular reference to the case in which the iterates are periodic (that is when g^n is the identity map). He also remarks that "This is probably reproducing known results but I have not been able to find references for them". These results are indeed well documented. Möbius maps arise in hyperbolic geometry (real Möbius maps are the isometries of the hyperbolic plane; complex Möbius maps are the isometries of three-dimensional hyperbolic space), in number theory (continued fractions, diophantine approximation and quadratic forms), and in complex analysis where they are intimately connected to Riemann surfaces and the uniformization theorem. At a more elementary level, we classify Möbius maps as follows. If g is a Möbius map then there is a Möbius map f such that $fgf^{-1}(z)$ is a map of one of the forms $z \to z + 1$ or $z \to kz$, and the periodic case is precisely when $k^n = 1$. As $\text{trace}(g) = \text{trace}\left(fgf^{-1}\right)$, this classification is determined entirely in terms of $\text{trace}(g)$. Finally, the finite Möbius groups are (abstractly) the cyclic groups, the dihedral groups, and the symmetry groups of the five Platonic solids.

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In the Pipeline for November 2022

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