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# PUBLIC DEBT IN A POLITICAL ECONOMY

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This paper analyzes the determination of public debt in a dynamic politico-economic model with overlapping generations. Sizeable levels of public debt can be rationalized in this model. The elasticity of substitution between public and private consumption determines the size of public debt and could explain differences of debt across countries. I compare the optimal policies under commitment and in a "political equilibrium" without commitment. Public debt can be higher or lower when commitment is absent, depending on the elasticity of substitution between public and private consumption. Consequently, under certain conditions, the no-commitment debt level can be closer to a normative benchmark with higher weight for future generations.

**Keywords:** Government Debt, Fiscal Policy, Political Economy, Markov Equilibrium

## 1. INTRODUCTION

The level of public debt varies substantially across countries. Normative theories that assume infinite planning horizons and full commitment to future policies have a hard time explaining this variation. In the benchmark model of Lucas and Stokey (1983), state-contingent public debt is indeterminate, and Aiyagari et al. (2002) show that without state-contingent public debt it is optimal to accumulate a buffer stock of public assets instead of debt. Although the absence of a commitment technology resolves the indeterminacy of public debt, Debortoli and Nunes (2013) show that sizeable public debt levels that differ between countries are still hard to rationalize in this model, because the economy often converges to a steady state with no debt accumulation at all.

In this paper, I aim to contribute to the theoretical understanding of how public debt is determined by introducing an intergenerational conflict into the benchmark model by Lucas and Stokey (1983). More precisely, I build an infinite-horizon two-period overlapping-generations model without capital, where agents work in the first period and retire in the second period to live off their savings. The

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government levies lump-sum taxes from the young working population, issues debt, and with those revenues provides public consumption and pays back the debt from the previous period. Public debt (or government bonds) is used as a savings instrument by the agents in the first period to save for retirement. Lack of commitment in the sense of this paper refers to the inability of a government to commit to the whole path of future policies. However, I abstract from government default as an additional source of lack of commitment to focus on this particular "political" source of lack of commitment arising in modern democracies from the periodical reelection of governments. I find that in the context of such a model the elasticity of substitution between public and private consumption is an important factor in the determination of public debt.

The contribution of this paper is to show that it is possible to obtain a positive steady state debt level in a model à la Lucas and Stokey (1983) where the debt is determined by the resolution, through repeated voting, of the intergenerational conflict between young and old agents. The model presented is closely related to Song et al. (2012) (abbreviated SSZ in the following). However, they focus on the stationary equilibrium set of small open economies in which the world interest rate is endogenous, but "exogenous" from the standpoint of each individual government. In contrast, I study a closed economy where the fiscal policy has an effect on the equilibrium interest rate.

Although financial markets are internationally open in the OECD countries, a large fraction of all government bonds are still held domestically in most countries.[1] In reality most economies are somewhere in between a completely closed and an open economy, as suggested by empirical findings of a substantial home bias in asset markets [see Fidora et al. (2007)]. Furthermore, there is evidence that the interest rate indeed reacts to changes in national supply of bonds [see Laubach (2009)]. It is thus interesting to analyze the closed economy model as a complementary analysis to SSZ.

For simplicity, I abstract in the analysis from physical capital, so that government bonds are the sole asset in the economy. Including private capital or other means of savings would not change the qualitative results as long as there was some crowding out of private by public bonds, although it could matter quantitatively. In this way, I am able to derive clear-cut analytical results with respect to the determination of public debt when utility is of the constant-elasticity-of-substitution (CES) functional form.

To derive the analytical results I proceed as follows. First, as a benchmark of comparison, I analyze the case where the first generation of agents can commit to all future policies; i.e., they choose taxes, public consumption, and the new issuance of public debt in the current and every future period (called "commitment" in the following). Because the government bonds are bought by young agents to be used for old-age consumption in the next period, the choice of new debt issuance today determines the relative share of resources allocated to private or public consumption by the old in the next period. Second, I analyze a "political equilibrium," where each generation of agents decides only on the contemporaneous fiscal policy, i.e., the contemporaneous taxes, public consumption, and new issuance of public debt. In such a political equilibrium, a strategic debt bias arises because of strategic interactions between consecutive generations of agents. Intuitively, one generation of young agents forces the next generation of young agents to levy higher taxes. Because only young agents are working and pay taxes, such an increase in taxation leads to an increase in the total resources (public and private consumption) of those (previously young, now old) agents.

I find three main results. First, the higher the elasticity of substitution between public and private consumption, the higher is the level of public debt at equilibrium (both under commitment and in the political equilibrium). This result is due to the assumption that public consumption has a smaller weight in the utility function of agents than private consumption, a standard assumption in the literature (for instance, SSZ show that this is the case in their calibrated economy). Thus, the higher the substitutability between the two sorts of consumption, the more private consumption the agents will prefer in their old age. Because public debt from the last period constitutes the savings of the agents and is thus equal to old-age private consumption, it will rise with higher substitutability. Second, the higher the elasticity of substitution between public and private consumption, the higher is the level of public debt in the political equilibrium relative to the commitment case. Intuitively, the more substitutable the two sorts of consumption, the stronger is the (positive) income effect arising from more total resources for the old agents relative to the (negative) substitution effect of having to substitute public for private consumption in old age.

Finally, I also analyze a normative benchmark where a benevolent planner is also committed to future policies (similar to the "commitment" case), but puts a geometrically decaying weight on consecutive generations [see also Farhi and Werning (2007)]. I find that when the elasticity of substitution is relatively small (smaller than 1 in absolute value), the level of public debt in the political equilibrium is closer to the level of debt implied by the normative benchmark than debt under commitment. Intuitively, the commitment case can be viewed as the upper bound of this normative benchmark.

This paper contributes to the literature on the politico-economic determinants of government debt [recent examples include but are not restricted to Battaglini and Coate (2008), Debortoli and Nunes (2010), and Yared (2010)]. Like SSZ, but differently than most other papers in the literature, I focus on intergenerational conflicts as a driving force for the determination of public debt. Another notable exception is the paper by Cukierman and Meltzer (1989), who analyze an overlapping-generations model with majority voting and bequests. Despite the presence of bequests, Ricardian equivalence does not hold, because of heterogeneity in abilities and bequests. Similarly to the present paper, they thus show how positive levels of public debt can arise out of intergenerational conflicts. The present paper is complementary by adopting a very different focus. Whereas Cukierman and Meltzer (1989) analyze the joint effect of absence of commitment and heterogeneity [in the spirit of the static redistribution model of Meltzer and

Richard (1981)], the present paper focuses on the effect of the absence of commitment alone. Furthermore, the paper is also related to another strand of literature studying fiscal policy in politico-economic setups abstracting from explicit debt [see for example Mateos-Planas (2010) for an overlapping-generations model or Klein et al. (2008) for an infinite-horizon model].

The paper is structured as follows. Sections 2-4 present a general two-period OLG model and provide intuition for the differences between the commitment solution, the solution under a benevolent planner, and the solution in the political equilibrium. Section 5 derives closed-form solutions for the case of CES-utility and analyzes the role of the elasticity of substitution between public and private consumption for the different allocations. Section 6 provides a summary and a discussion of the results. All derivations and proofs are relegated to the Appendix.

## 2. AN OVERLAPPING-GENERATIONS MODEL OF PUBLIC DEBT AND FISCAL POLICY

In this section, I introduce the overlapping-generations model with public debt and fiscal policy, which will be used to analyze the determination of public debt. The model economy is an endowment economy consisting of a household sector and a government sector. The overlapping-generations structure considered here is of the following form: Agents in the model live for two periods only. In the first period they work and in the second period they retire and live off their savings. The population size is constant.

### 2.1. Endowments and Feasibility

Each young agent receives an endowment, $y$, at the beginning of life. For simplicity the endowment is assumed to be constant over time. In each period the endowment can be consumed by a young agent, $c_t$, by an old agent, $b_t$, or by both agents at the same time in the form of a public good, $g_t$. Because the economy is closed, an aggregate feasibility constraint has to hold:

$$y = c_t + g_t + b_t. \tag{1}$$

### 2.2. Preferences and Constraints of the Agents

Agents are assumed to be altruistic toward their children. The lifetime utility of a young agent in period $t$ can be summarized as follows:

$$v_t = u(c_t) + \theta u(g_t) + \beta[u(b_{t+1}) + \theta u(g_{t+1}) + \lambda v_{t+1}], \tag{2}$$

where $c_t$ is private consumption of the agent in youth in period $t$, $g_t$ is public consumption in youth, $b_{t+1}$ is private consumption in old age in period $t+1$, $g_{t+1}$ is public consumption in old age, $v_{t+1}$ is the lifetime utility of the child, $\beta$ is the discount factor, $\theta$ is the preference for public consumption relative to private

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consumption, and $\lambda$ is the degree of altruism. Note that $\lambda = 0$ nests a pure OLG model where everyone only cares about his own utility, and $\lambda = 1$ nests the case of perfect altruism or dynasties (infinitely lived agents).

Note that I assume public and private goods to benefit young and old agents equally. In reality, some public goods, for example general public expenditures or safety expenditures, arguably have this feature, whereas other expenditure categories such as public transport, museums, or parks might be more beneficial for the old than for the young. I abstract here from these differences for simplicity.

I assume that agents can save for their retirement by buying one-period bonds from their own national government. $b_{t+1}$ is thus the number of bonds that an agent buys at price $p_t$ in period $t$ to yield one unit of consumption each in period $t+1$. The government levies a lump sum tax $\tau_t$ on the endowment in each period. The lifetime budget constraint of each agent is thus given by

$$c_t + p_t b_{t+1} = (1 - \tau_t)y.$$

## 2.3. The Government

The government provides public consumption goods, $g_t$, which enter the utility function of the agents; levies a tax on the endowment, $\tau_t$; and issues government bonds, $b_{t+1}$. The government revenues thus consist of new bonds that are issued and tax revenues: $p_{t+1}b_{t+1} + \tau_t y$. The government expenditures consist of the debt to repay (in units of consumption), which is given from the previous period, and the government expenditure for provision of the public good: $g_t + b_t$. Therefore the budget constraint of the government is given by

$$p_t b_{t+1} = g_t + b_t - \tau_t y.$$

Note that the maximum debt limit is equal to the total endowment, $y$, in this model.

It should be noted that I abstract from the possibility of transfers (such as gifts or bequests) between parents and children. The conceptual difference between transfers and government bonds is that the transfers are given voluntarily by the private agents. However, even if transfers were allowed, the agents would not use them in this model. The old would not want to leave bequests, as they are imperfectly altruistic. The young would not want to transfer, because I abstract from reverse altruism. To rule out negative bequests is more restrictive, but this is a common assumption in the literature, because they are not often observed.

# 3. THE COMMITMENT SOLUTION

Consider first the "commitment solution," i.e., an allocation such that the whole path of fiscal policy is set to maximize the utility of the first generation of young agents. $b_t$ is the state variable because it is the stock of debt that the government has to honor and has to be known before it can decide how to set the other fiscal policy variables. Because taxes are lump-sum, the competitive equilibrium corresponds to the (Pareto-efficient) planner solution. Thus, one can simply maximize the utility of the first generation directly with respect to the allocations $c_t$, $g_t$, and $b_{t+1}$ subject to the aggregate feasibility constraint, equation (1).[2]

The commitment problem can be formulated recursively in two stages:

DEFINITION 1. *The commitment allocation is the solution to the following maximization problem:*

$$\{c_0, g_0, b_1\} = \arg\max \left\{u(c_0) + \theta u(g_0) + \beta V^{\mathrm{CO}}(b_1)\right\},$$
$$V^{\mathrm{CO}}(b) = \max_{c,g,b'} \left\{u(b) + \lambda u(c) + (1+\lambda)\theta u(g) + \lambda\beta V^{\mathrm{CO}}(b')\right\} \text{ for } t > 0$$
$$\text{s.t.}(1).$$

As one can see from the two-stage formulation, the commitment problem is time-inconsistent. Indeed, the first generation set taxes, public consumption, and public debt in the interest of their children and grandchildren (and thus in the same way as future generations would) from the second period onward. But for the initial period they choose a solution with lower taxes or, equivalently, higher youth private consumption, $c_t$. Intuitively, there is more weight on private consumption relative to public consumption in the first period, because the altruism is only one-sided, so that the old in the first period are not considered.

To gain some intuition about the determination of public debt under commitment in this model, it is useful to consider the following equation, which can be derived from the first-order conditions of the problem defined in the preceding:

$$\frac{\partial u(b')}{\partial b'} = \theta(1+\lambda)\frac{\partial u(g')}{\partial g'}. \tag{3}$$

Equation (3) shows that public debt will be chosen to solve the tradeoff between public and private consumption in old age optimally. The higher $\theta$, the higher the preference for public consumption relative to private consumption. With higher altruism, $\lambda$, public consumption is valued more as well, because both young and old agents enjoy the public good. It also becomes intuitively clear that the elasticity of substitution is an important parameter for the determination of public debt in this model, because it determines how well private consumption can substitute for public consumption and vice versa. Intuitively, regardless of $\theta$ and $\lambda$, the ratio of public to private consumption will optimally be close to one if those two sorts of consumption are very complementary. The more substitutable the two kinds of consumption are, the more important their different weights in the utility function become. Under the (quite plausible) parameter condition $\theta(1+\lambda) < 1$, meaning that the weight on private consumption is higher than that on public consumption despite altruism, the level of public debt increases with the degree of substitutability.

### 3.1. The Benevolent Planner Allocation

Although the commitment allocation is often chosen as a natural benchmark for comparison, it is not entirely satisfactory as a normative criterion. Under commitment, only the lifetime utility of the first generation directly enters the objective function, whereas the lifetime utilities of other generations enter only indirectly through altruism of the first generation. It is thus useful to consider an alternative allocation obtained by maximizing a weighted sum of the lifetime utilities of all the generations, i.e., a welfare criterion, where future generations enter with a higher weight. A difficult question arising in this context is how exactly to weight the different generations. Here I focus on a welfare criterion with a geometrically decaying weight, $0 < \tilde{\beta} < 1$ [see Farhi and Werning (2007)]:

$$\sum_{t=0}^{T} \tilde{\beta}^t v_t.$$

More precisely, the allocation under a benevolent planner of the sort described previously can be defined as follows:

DEFINITION 2. *The benevolent planner allocation is the solution to the following maximization problem:*

$$\{c_t, g_t, b_{t+1}\}_{t=0}^{\infty} = \arg\max \left\{ \sum_{t=0}^{T} \left[ \sum_{j=0}^{t} \tilde{\beta}^{t-j} (\lambda\beta)^j \right] [u(c_t) + \theta u(g_t) + \beta [u(b_{t+1}) + \theta u(g_{t+1})]] \right\} \quad \textbf{(4)}$$

s.t.(1),

*where* $0 < \tilde{\beta} < 1$.

Note that for $\tilde{\beta} = 0$ we obtain the commitment allocation, where the first generation has all the weight. Furthermore, it is interesting to consider another boundary case, where $\tilde{\beta} = 1$ and the criterion is defined slightly differently as follows:

$$\lim_{T\to\infty} \inf \frac{1}{T} \sum_{t=0}^{T} v_t.$$

This means that all generations are weighted equally and each particular generation has a weight that approaches zero, so that effectively only the steady state counts. Intuitively, future generations have all the weight in this case. The allocation under this *long-run* planner can be defined as follows:

DEFINITION 3. *The long-run planner allocation is the solution to the following maximization problem:*

$$\{c_t, g_t, b_{t+1}\}_{t=0}^{\infty} = \arg\max \left\{ \lim_{T\to\infty} \inf \frac{1}{T} \sum_{t=0}^{T} \left[ \sum_{j=0}^{t} (\lambda\beta)^j \right] \right.$$

$$\left. [u\,(c_t) + \theta u\,(g_t) + \beta\,[u(b_{t+1}) + \theta u(g_{t+1})]] \right\} \quad \textbf{(5)}$$

s.t.(1).

## 4. THE POLITICAL EQUILIBRIUM

The commitment solution outlined in the previous section is a useful benchmark. However, it is not a realistic positive description, because in democratic countries elections are held repeatedly and a government cannot tie the hands of future governments. Therefore, in this section, I consider the political equilibrium under repeated voting, which I consider to be a more realistic environment. I focus on a majority voting environment in which the fiscal policy is set by the young only.[3] The model can be extended to a probabilistic voting model à la Lindbeck and Weibull (1987). The political objective function can then be characterized by an average of the objective functions of group of voters weighted by the political power of the group,

$$U(\{c_t, g_t, b_t\}_{t=0,\ldots,\infty}) = \omega v_t + (1-\omega)[u(b_t) + \theta u(g_t) + \lambda v_t],$$

where $\omega$ is the political power of the old and $v_t$ is lifetime utility as defined in equation (2). The majority voting model considered here is nested in this specification (setting $\omega = 0$), because then the utility of the young only is maximized by the political candidates who want to win the election. I simplify the presentation by not including the old voters. This is not crucial for the results (all propositions can be shown for $\omega = 0$ and $\omega > 0$). In Section B.1 of the Appendix, I generalize to the case where the old generation participates in the political decision as well.[4]

Fiscal policy is determined by a dynamic game between successive generations of voters. I focus on Markov perfect equilibria of this game conditional on the only payoff-relevant state variable (public debt, $b_t$). Although it is not possible for a present government to commit a future government to any policies directly, the level of public debt, as a decision variable affecting the future, can be used by the present government to influence actions of a future government *indirectly*.

More specifically, the "political equilibrium" is defined as follows:

DEFINITION 4. *The (Markov perfect) political equilibrium is a 3-tuple of functions* $\{C, G, B\}$*, where* $C : [0, y] \to [0, y]$ *is private consumption of young agents as a function of the previous period's debt,* $c = C(b)$*;* $G : [0, y] \to [0, y]$

*is public consumption as a function of the previous period's debt,* $g = G(b)$*; and* $B : [0, y] \to [0, y]$ *is the new issuance of debt as a function of the previous period's debt,* $b' = B(b)$*, such that*

$$\{(b), G(b), B(b)\} = \arg\max_{c,g,b'} \left\{u\,(c) + \theta u\,(g) + \beta V^{PE}(b')\right\}, \tag{6}$$

$$V^{PE}(b') = u\left(b'\right) + \lambda u[C(b')] + (1+\lambda)\theta u[G(b')] + \lambda\beta V^{PE}[B(b')], \tag{7}$$

$$\text{s.t. } G(b) = y - C(b) - b. \tag{8}$$

Again, to gain intuition, consider the following equation for the tradeoff between public and private old age consumption in the political equilibrium that can be derived from the first-order conditions:

$$\frac{\partial u(b')}{\partial b'} + \lambda \frac{\partial C(b')}{\partial b'} \frac{\partial u[C(b')]}{\partial C} = \theta(1+\lambda)\left[1 + \frac{\partial C(b')}{\partial b'}\right] \frac{\partial u(g')}{\partial g'}. \tag{9}$$

Compared with equation (3), the corresponding equation under commitment, equation (9), besides including a similar tradeoff between public and private consumption in old age, additionally contains two terms involving the reaction of the next generation to the level of public debt. Intuitively, increasing public debt leads to a tighter government budget in the next period and thus to relatively lower public consumption. This incites the next generation to decrease its own private consumption (allow higher taxes) to sustain higher public consumption. Therefore, today's generation of young can force the next generation of young to restrain their private consumption by setting a higher debt level, $b'$. This "strategic" effect of public debt is represented by the derivative of the reaction function, $\frac{\partial C(b')}{\partial b'} < 0$, in equation (9). More precisely, there are two additional effects on the tradeoff of today's generation between public and private consumption compared to the commitment outcome: (1) Public consumption is relatively less attractive compared to private consumption, because by substituting away from public toward private consumption an agent can use the strategic effect and induce higher taxes on the next generation. This is represented by the term $\frac{\partial C(b')}{\partial b'} < 0$ on the right-hand side of equation (9). (2) A negative side-effect of the strategic behavior (insofar as agents are altruistic) is that the "child" has less private consumption. This is represented by the additional term $\lambda \frac{\partial C(b')}{\partial b'} \frac{\partial u[C(b')]}{\partial C}$ on the left-hand side, which matters only when $\lambda > 0$.

Whether public debt is higher or lower under commitment or in the political equilibrium is generally ambiguous. Intuitively, one could think that because of the strategic effect, public debt would be higher in the political equilibrium as the current generation tried to exert a strategic influence on the next generation. However, given that in the political equilibrium, consumption of the future generations of young agents will be relatively higher than under commitment (lower taxation), which also implies a lower public good provision, it cannot be generally derived from comparing equation (9) and equation (3), under which institutional setting public debt will be higher. In Section 5, I will analyze the special case of CES-utility and derive results for the level of public debt under commitment and in the political equilibrium as functions of the elasticity of substitution. The intuition for why the elasticity of substitution would matter is as follows: Consider very complementary public and private consumption. Clearly, it would be much more difficult for the old to deviate from the level of public debt that resolves the trade-off between public and private consumption optimally. Overall, because public consumption is lower in the political equilibrium (because of higher consumption of the young), public debt could thus be even lower than under commitment.

In the next section I consider a CES-utility function. With this functional form of utility, it is possible to derive analytical closed form solutions for the allocations. Furthermore, it will be shown that the elasticity of substitution between public and private consumption plays a key role.

## 5. CLOSED FORM RESULTS WITH CES-UTILITY

Suppose the different sorts of consumption (public and private in youth and in old age) are all valued according to the following CES-utility function:

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma} \text{ with } x \in [c, b, g]. \tag{10}$$

(Note that in this functional form for the utility function $\frac{1}{\sigma}$ is the elasticity of substitution between public and private consumption.) With this functional form of utility one can derive closed form solutions for the debt level under commitment, in the political equilibrium, and under a benevolent planner by using the first-order conditions of the maximization problems defined previously.

### 5.1. Allocations

Under commitment the allocations are given by

$$b_{t+1}^{CO} = \alpha_{b,t}^{CO} y,\ c_t^{CO} = \alpha_{c,t}^{CO}(y - b_t),\ g_t^{CO} = \alpha_{g,t}^{CO}(y - b_t), \tag{11}$$

where

$$\alpha_{b,t}^{CO} \equiv \frac{1}{\{[\theta(1+\lambda)]^{1/\sigma} + 1 + \lambda^{1/\sigma}\}} y \text{ for all } t,$$

$$\alpha_{c,t}^{CO} \equiv \begin{cases} \frac{1}{\theta^{1/\sigma}+1} & \text{for } t = 0 \\ \frac{\lambda^{1/\sigma}}{\{[\theta(1+\lambda)]^{1/\sigma}+\lambda^{1/\sigma}\}} & \text{for } t > 0, \end{cases}$$

$$\alpha_{g,t}^{CO} \equiv \begin{cases} \frac{\theta^{1/\sigma}}{\theta^{1/\sigma}+1} & \text{for } t = 0 \\ \frac{(\theta(1+\lambda))^{1/\sigma}}{\{[\theta(1+\lambda)]^{1/\sigma}+\lambda^{1/\sigma}\}} & \text{for } t > 0. \end{cases}$$

The commitment problem is time-inconsistent in the sense that the first generation ($t = 0$) would choose a different $c_t$ and $g_t$ for itself than for other generations given the same $b_t$. However, the level of public debt is always the same from period $t > 0$ onward. The reason is that it is chosen one period in advance, so that there is no time inconsistency here, given the other policies.

For period $t > 0$ onward (in the steady state), consumption of the old is optimally a constant share $\alpha^{CO}_{b,t>0}$ of the endowment, consumption of the young a constant share $\alpha^{CO}_{c,t>0}(1-\alpha^{CO}_{b,t>0})$, and public consumption a constant share $\alpha^{CO}_{g,t>0}(1-\alpha^{CO}_{b,t>0})$. In contrast, in period $t = 0$, consumption of the old is influenced by the initial level of public consumption, $b_0$, and private consumption is a relatively higher share of the rest, $y - b_0$. (This is easy to see by comparing the two multipliers of $(y - b)$ in the reaction functions: $\alpha^{CO}_{c,0} > \alpha^{CO}_{c,t>0}$.) The reason for this higher relative weight on private than on public consumption in period 0 compared to later periods is intuitively based on the assumption that altruism is one-sided: in the first period only the utility of public consumption for the young themselves is included without considering the utility of public consumption for the parents in the objective function, whereas in later periods utility from public consumption for both children and parents is equally present.

Under the benevolent planner (BP) discussed in Section 3.1, the allocations will be

$$b^{BP}_{t+1} = \alpha^{BP}_{b,t} y,\ c^{BP}_t = \alpha^{BP}_{c,t}(y - b_t),\ g^{BP}_t = \alpha^{BP}_{g,t}(y - b_t),$$

where

$$\alpha^{BP}_{b,t} \equiv \frac{\left[\hat{\beta}(t)\beta\right]^{1/\sigma}}{\theta^{1/\sigma}\left[\hat{\beta}(t+1) + \hat{\beta}(t)\beta\right]^{1/\sigma} + \hat{\beta}(t+1)^{1/\sigma} + \left[\hat{\beta}(t)\beta\right]^{1/\sigma}}$$

$$\alpha^{BP}_{c,t} \equiv \frac{\left[\hat{\beta}(t)\right]^{1/\sigma}}{\theta^{1/\sigma}\left[\hat{\beta}(t) + \hat{\beta}(t-1)\beta\right]^{1/\sigma} + \hat{\beta}(t)^{1/\sigma}}$$

$$\alpha^{BP}_{g,t} \equiv \frac{\theta^{1/\sigma}\left[\hat{\beta}(t) + \hat{\beta}(t-1)\beta\right]^{1/\sigma}}{\theta^{1/\sigma}\left[\hat{\beta}(t) + \hat{\beta}(t-1)\beta\right]^{1/\sigma} + \hat{\beta}(t)^{1/\sigma}}$$

and

$$\hat{\beta}(t) \equiv \sum_{j=0}^{t} \tilde{\beta}^{t-j}\,(\lambda\beta)^j\,.$$

Similarly to the commitment allocation, the allocations under this benevolent planner are a fraction of the available income $y$. However, here the weights are time-varying and the economy does not jump instantly into the steady state. Under the boundary case of a long-run planner (LRP) we obtain the following (constant) solutions for the reaction functions:

$$\alpha_{b,t}^{\text{LRP}} = \frac{(\beta)^{1/\sigma}}{\theta^{1/\sigma}[1+\beta]^{1/\sigma} + 1 + \beta^{1/\sigma}}$$

$$\alpha_{c,t}^{\text{LRP}} = \frac{1}{\theta^{1/\sigma}[1+\beta]^{1/\sigma} + 1}$$

$$\alpha_{g,t}^{\text{LRP}} = \frac{\theta^{1/\sigma}[1+\beta]^{1/\sigma}}{\theta^{1/\sigma}[1+\beta]^{1/\sigma} + 1}.$$

For this planner, all generations enter with the same weight. Consumption in youth is in the proportion $1 : \beta^{1/\sigma}$ to consumption in old age at equilibrium because the latter is discounted by $\beta$. Consumption in youth is in the proportion $1 : [\theta^{1/\sigma}(1+\beta)^{1/\sigma}]$ to public consumption. This proportion quite intuitively depends on $\theta$, the relative preference for public consumption, and includes a weight $(1+\beta)$ for the public good, as it can be consumed by both old and young agents.

In the political equilibrium, the allocations are given by

$$b_{t+1}^{\text{PE}} = \alpha_b^{\text{PE}} y,\ c_t^{\text{PE}} = \alpha_c^{\text{PE}}(y - b_t),\ g_t^{\text{PE}} = \alpha_g^{\text{PE}}(y - b_t),$$

where

$$\alpha_b^{\text{PE}} \equiv \frac{\xi}{\xi + 1 + \theta^{1/\sigma}} \text{ with } \xi \equiv \frac{(1+\theta^{1/\sigma})^{1/\sigma}}{[(1+\lambda)\theta^{1/\sigma} + \lambda]^{1/\sigma}},$$

$$\alpha_c^{\text{PE}} \equiv \frac{1}{\theta^{1/\sigma} + 1},$$

$$\alpha_g^{\text{PE}} \equiv \frac{\theta^{1/\sigma}}{\theta^{1/\sigma} + 1}.$$

Note that in the political equilibrium, differently from the commitment allocation, because it is time consistent, the relative shares of private youth and public consumption are always the same. There is no difference between the first period and the subsequent periods (except for the fact that public debt is given in the first period). From period $t > 0$ onward (in the steady state) the model predicts that consumption of the old will be a constant share $\alpha_b^{\text{PE}}$ of the endowment, consumption of the young a constant share $\alpha_c^{\text{PE}}(1 - \alpha_b^{\text{PE}})$, and public consumption a constant share $\alpha_g^{\text{PE}}(1 - \alpha_b^{\text{PE}})$.

### 5.2. The Role of the Elasticity of Substitution Between Public and Private Consumption

A first important result about the role of the elasticity of substitution is that public debt increases with higher substitutability, as summarized by the following proposition:

PROPOSITION 1. *Denote by $b^{CO}$ the steady state debt level under commitment according to Definition 1 and by $b^{PE}$ the steady state debt level in the political equilibrium according to Definition 4. Assume a CES-utility function [as defined in equation (10)]. Then*

1. *If $\theta(1+\lambda) < 1$, then a higher elasticity of substitution, $\frac{1}{\sigma}$, leads to a higher level of public debt under commitment, $b^{CO}$.*
2. *If $\lambda < 0.5$, then a higher elasticity of substitution, $\frac{1}{\sigma}$, leads to a higher level of public debt in the political equilibrium, $b^{PE}$.*

The proposition shows that the model implies a positive relationship between the elasticity of substitution and the level of public debt under certain parameter conditions.[5] The higher the elasticity of substitution between public and private consumption, the higher the level of public debt. Intuitively, as $\theta(1+\lambda) < 1$ is assumed (and likely to hold in reality), public consumption has less weight in the objective function than private consumption, although it can be used by both young and old. Therefore the more substitutable the two sorts of consumption are, the more voters will prefer private to public consumption in their old age and the higher is the level of public debt.

Another interesting question is whether public debt is higher or lower under commitment than in political equilibrium. Interestingly, for the case of CES-utility, it can be shown that the elasticity of substitution between public and private consumption, $\frac{1}{\sigma}$, plays a key role in how the level of public debt under commitment compares to the political equilibrium. The following proposition summarizes this result:

PROPOSITION 2. *Denote by $b^{CO}$ the steady state debt level under commitment according to Definition 1 and by $b^{PE}$ the steady state debt level in political equilibrium according to Definition 4; then under a CES-utility function [as defined in equation (10)], one can differentiate between three cases:*

1. *Under log-utility, i.e., if $\frac{1}{\sigma} = 1$, $b^{CO} = b^{PE}$.*
2. *If public and private consumption are relatively complementary, i.e., if $\frac{1}{\sigma} < 1$, then $b^{PE} < b^{CO}$.*
3. *If public and private consumption are relatively substitutable, i.e., if $\frac{1}{\sigma} > 1$, then $b^{CO} < b^{PE}$.*

The proposition shows that the level of debt in political equilibrium can be higher or lower than under commitment, depending on the elasticity of substitution between public and private consumption. Indeed, the elasticity of substitution between public and private consumption determines the strength of the strategic effect (in political equilibrium) as well as the tradeoff between public and private consumption (under commitment and in political equilibrium). To use public debt strategically, today's generation has to deviate from the level of public debt that optimally solves their tradeoff between public and private consumption tomorrow. The higher the elasticity of substitution, the less costly it is to deviate in that way.

Remember that under commitment, as the future tax is higher (and therefore more public consumption is provided), the level of public debt resolving the tradeoff between public and private consumption optimally is higher than in political equilibrium. According to Proposition 2, if $\frac{1}{\sigma} = 1$, the debt level under commitment is exactly equal to the debt level in political equilibrium. Intuitively, under log-utility, the effect of lower taxation on the tradeoff between public and private consumption and the effect of using public debt strategically cancel each other out, so that public debt is equal under commitment and in political equilibrium.

The proposition thus shows that it is not necessarily the case that a commitment device leads to lower debt levels than democratic voting, as one might have guessed initially. If public and private consumption are substitutes, public debt is higher in political equilibrium than under commitment. In this case, a commitment device would help to avoid high debt levels. If public and private consumption are complements, public debt is lower in political equilibrium than under commitment. In this situation, a commitment device that was decided upon by the first generation of voters would lead to a higher debt level.

Given that two different systems were analyzed with their implications for public debt (commitment or repeated elections), a salient question is what public debt should be, according to a normative benchmark. Proposition 3 addresses this question:

PROPOSITION 3. *Denote by $b^{\text{CO}}$ the steady state debt level under commitment according to Definition 1 and by $b^{\text{BP}}$ the steady state debt level under a benevolent planner according to Definition 2. Assume a CES-utility function [as defined in equation (10). Then $b^{\text{BP}} \leq b^{\text{CO}}$.*

This proposition is fairly intuitive. For any $\tilde{\beta} > 0$, the benevolent planner puts more weight on future generations than the objective under commitment. Thus public debt will be smaller, because it is the first generation that profits most from it, as it does not have to pay the taxes used to pay back the debt. Concerning the level of public debt in political equilibrium, Proposition 2 and 3 together imply that if public and private consumption are relatively substitutable ($\frac{1}{\sigma} > 1$), then public debt in political equilibrium is definitely farther away from the normative benchmark than the commitment level of debt no matter what the weight of the planner is.

Proposition 4 describes the relationship between the level of debt in political equilibrium and the one in the steady state under a long-run planner:

PROPOSITION 4. *Denote by $b^{\text{PE}}$ the steady state debt level in political equilibrium according to Definition 4 and by $b^{\text{LRP}}$ the steady state debt level under a long-run planner according to Definition 3. Assume a CES-utility function [as defined in equation (10)] and assume that altruism and the discount factor are not too high, or more precisely $\beta(1 + \lambda) < 1$. Then $b^{\text{LRP}} < b^{\text{PE}}$.*

The long-run planner allocation is the lower bound and the commitment allocation is the upper bound for allocations of public debt under a benevolent planner with a geometrically decaying weight. Public debt in political equilibrium always lies above the lower bound. It now depends on the exact weight or patience of the planner under which setup, political equilibrium or commitment, the level of public debt is closer to the level of debt under the benevolent planner. In the next section, I present a numerical example for different possible weights of the planner.

### 5.3. A Numerical Example

In this section, I compute concrete policy functions and the effect of the elasticity of substitution between public and private consumption on public debt levels in the steady state by looking at a numerical example. The parameter values are set in a way similar to that in SSZ (see Table 1 in their paper), where a similar model was calibrated for OECD countries. One model period is defined as 30 years. The discount factor $\beta$ is set to obtain an annual discount factor of 0.97. The preference for public consumption, $\theta$, is set to 0.38, because SSZ find that cross-country values of $\theta$ are between 0.37 and 0.39. The altruism parameter is set to 0.45, which approximately corresponds to the inverse of the higher preference for public consumption of old agents compared to young agents (2.2) in SSZ. The idea is to yield similar relative weights for public and private consumption, as in SSZ.

For illustrative purposes, consider the policy functions for $\frac{1}{\sigma} = 2$ shown in Figure 1. Figure 1a shows the policy functions for public debt as a function of the previous period's debt under commitment and in political equilibrium. Public debt does not depend on the previous period's debt level, but instantly jumps to the steady state. The reason is that the new debt issued concerns only the allocation between public and old-age private consumption in the next period, so that it is irrelevant to the present period. Note also that the steady state debt levels are different in political equilibrium and under commitment. In line with Proposition 2, the debt level is higher in political equilibrium for $\frac{1}{\sigma} = 2 > 1$.

Figure 1b shows the policy function for public consumption. Public consumption is lower in political equilibrium, where the young generation decide because they prefer a higher private consumption of their own. Consequently, the young have higher private consumption (as shown in Figure 1c) and the taxes are lower (as shown in Figure 1d) in political equilibrium than under commitment. Note that the tax rate could in general also be negative for very low values of public debt, meaning that in this case the voters would prefer the funds from newly issued bonds to be used to finance a consumption subsidy. However, at equilibrium, income taxes are positive.

Figure 2 illustrates the effect of the elasticity of substition between public and private consumption, $\frac{1}{\sigma}$, on the level of public debt in the steady state, where the initial level of debt does not matter anymore.

The figure illustrates the result of Proposition 1 that public debt rises with the elasticity of substitution both under commitment and in political equilibrium. Note

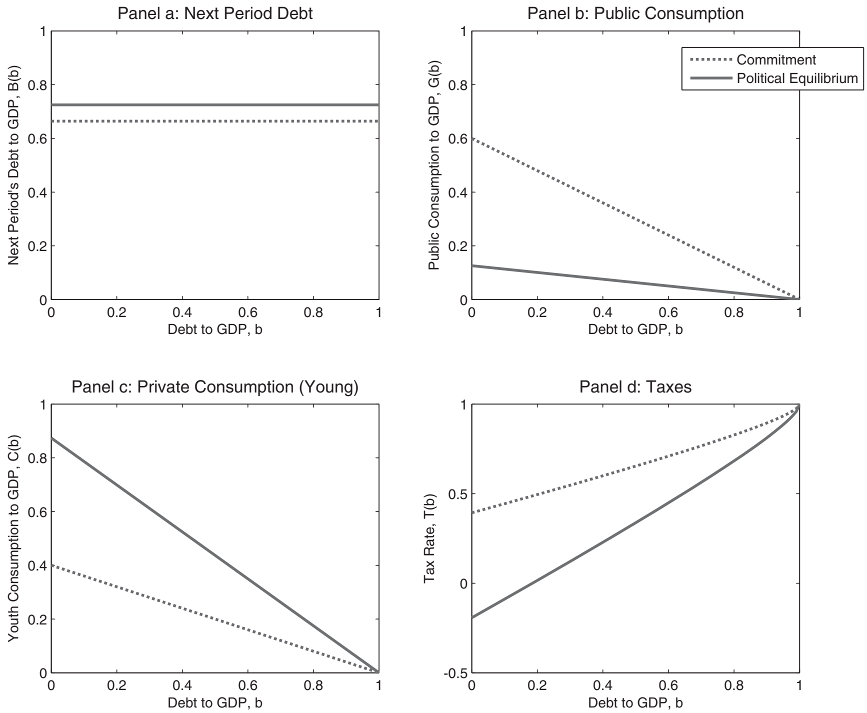


**FIGURE 1.** Equilibrium policy functions. Panels a–c plot the equilibrium policy functions for debt next period, public consumption, and private consumption. Panel d plots the implied policy function for the tax rate. The commitment case is represented by the dotted line and the political equilibrium by the solid line.

also that the figure again illustrates the result of Proposition 2, which relates to how public debt under commitment compares with that in political equilibrium: it is higher for low elasticities (complementary consumption) and lower for high elasticities (substitutable consumption), the threshold being at $\frac{1}{\sigma} = 1$ (the knife-edge case where debt is equal under commitment and in political equilibrium). We can view the commitment case as the upper bound (see also Proposition 3) and the debt level under the long-run planner as a lower bound for the debt level under a benevolent planner with some intermediate patience ($0 < \tilde{\beta} < 1$). The figure illustrates the result of Proposition 4 that the level of public debt in political equilibrium is always higher than the lower bound.

## 6. SUMMARY AND DISCUSSION

To summarize, in this paper, I presented an overlapping-generations model where public debt constitutes the savings for retirement. Under commitment, public debt

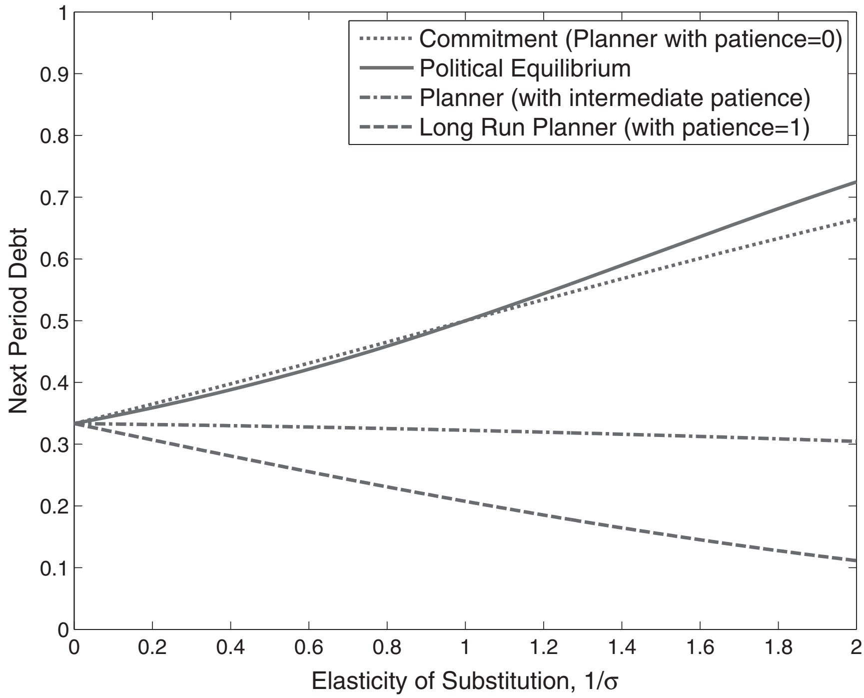


**FIGURE 2.** The effect of the elasticity of substitution on the level of public debt. The elasticity of substitution, $\frac{1}{\sigma}$, is on the $x$-axis and the level of public debt (in the steady state) on the $y$-axis. The dotted line shows the commitment case, the dashed–dotted line the case of the benevolent planner with intermediate patience ($\tilde{\beta} = 0.5$), the dashed line the case of the long-run planner, and the solid line the political equilibrium.

solves the tradeoff between public and private consumption in old age. In the absence of commitment, there is a strategic role for public debt.

The elasticity of substitution between public and private consumption is crucial for the determination of public debt in this model. Three main results were shown: I find that the higher the elasticity of substitution between public and private consumption, (1) the higher is the level of public debt at equilibrium (both under commitment and in "political equilibrium") and (2) the higher is the level of public debt in "political equilibrium" relative to the commitment case. Moreover, (3) when the elasticity of substitution is lower than unity, the level of public debt in "political equilibrium" can be closer to a normative benchmark that puts a geometrically decaying weight on consecutive generations than the commitment level of debt.

The first two results are of a positive nature and yield interesting implications that could be tested empirically. This paper mainly focused on theoretical results. For future research it would thus be interesting to investigate the empirical implications of the model more closely. The third result, in contrast, is normative. If private and public consumption are relative substitutes in a given country, commitment results in a public debt level closer to the optimal benchmark than absence of commitment.

However, there are some important caveats to the model presented in this paper and therefore one should be cautious when interpreting the results in terms of policy implications. For instance, taxation was assumed to be nondistortionary, whereas in reality important distortions might be present. Under distortionary taxation, there are additional intertemporal effects through the need to smooth taxation over time. Moreover, if one introduced aggregate shocks, those might yield an additonal motive for tax smoothing. In this paper, I took a first step by analyzing the basic model à la Lucas and Stokey (1983), including intergenerational conflicts. This approach has the advantage that one can derive closed form solutions under the assumption of CES-utility. In this way, I aimed at clarifying the basic mechanism underlying the determination of public debt in this class of models. Extending the model to include additional motives for debt to change dynamically, such as tax distortions or aggregate shocks, and analyzing the interactions with the strategic effect found in this paper could be a fruitful avenue for future research.

## NOTES

1. For example, a study from Reinhart and Rogoff (2011) shows that the median share of domestic debt in total debt has been rising over time, from 33% in developing countries and 68% in advanced countries from 1900–1940 to 49% in developing countries and 93% in advanced economies from 1991–2010.

2. It is straightforward to show that those allocations can be implemented using the fiscal policy instruments of the government. $g_t$ and $b_{t+1}$ are already set directly by the government and taxes, and $\tau_t$ can be set to implement $c_t$. Given $b_{t+1}$ one simply has to set it to satisfy the first - conditions of the household: $y(1-\tau_t) = c_t + \beta \frac{u'(b_{t+1})}{u'(c_t)} b_{t+1}$.

3. Because I assume no population growth and I assume the same lengths for the working period and the retirement period, the population share of the young implied by the model is 50%, which does not conform withto actual data. It would be easy, however, to extend the model to incorporate a growing population. For simplicity, I abstract from it for the purpose of this paper.

4. The Appendix contains derivations of the proofs of the propositions for the general case with $\omega > 0$, where one can set $\omega = 0$ to see the proof of the proposition presented in the main text.

5. The condition $\lambda < 0.5$ might seem restrictive. In Section E of the Appendix, I show that in a more general model (including old voters) the condition for the political equilibrium is even less restrictive and we can still find this positive relationship. If the conditions on parameters are not fulfilled, the relationship is in general ambiguous, but can still be positive for certain parameter constellations. In Section 3.3, I will discuss a numerical example with realistic parameter values and show that the positive relationship is also given.

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# APPENDIX A: THE COMMITMENT ALLOCATION

## A.1. RECURSIVE FORMULATION OF THE COMMITMENT PROBLEM

Because taxes are lump-sum, the competitive equilibrium corresponds to the (Pareto-efficient) planner solution

$$\max_{\{b_{t+1}\}_{t=0}^{\infty},\{c_t\}_{t=0}^{\infty},\{g_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\lambda\beta)^t \left\{u(c_t) + \theta u(g_t) + \beta[u(b_{t+1}) + \theta u(g_{t+1})]\right\}$$
$$\text{s.t. } g_t + c_t + b_t = y.$$

For the period $t > 0$, one can reformulate the problem recursively as a Bellman equation, taking government debt as the state variable:

$$V^{\mathrm{CO}}(b) = \max_{c,g,b'} \left\{u(b) + \lambda u(c) + (1+\lambda)\theta u(g) + \lambda\beta V^{\mathrm{CO}}(b')\right\} \tag{A.1}$$
$$\text{s.t. } g = y - c - b.$$

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### A.2. FIRST-ORDER CONDITIONS UNDER COMMITMENT

Use the feasibility constraint (1) to substitute $g$ into (A.1) and differentiate with respect to $c$ and $b'$:

$$\lambda u'(c) - \theta(1+\lambda)u'(y-c-b) = 0,$$
$$u'(b') - (1+\lambda)\theta u'(y-c'-b') + \frac{\partial V^{\mathrm{CO}}[B(b')]}{\partial B(b')}\frac{\partial B(b')}{\partial b'} = 0, \quad \textbf{(A.2)}$$

where $B(b) = b'$ denotes the optimal policy function for government debt. By an envelope theorem (stating that $\frac{\partial V^{\mathrm{CO}}[B(b')]}{\partial B(b')} = 0$), we can simplify (A.2) to

$$u'(b') - (1+\lambda)\theta u'(y-c'-b') = 0. \quad \textbf{(A.3)}$$

To obtain (3), reinsert (1) into (A.3) and rearrange.

# APPENDIX B: THE BENEVOLENT PLANNER ALLOCATION

### B.1. OBJECTIVE OF THE BENEVOLENT PLANNER

The benevolent planner adds up all lifetime utilities of all generations of agents, weighting them with a geometrically decaying weight, $\tilde{\beta}$:

$$\begin{aligned}\sum_{t=0}^{T}\tilde{\beta}^t v_t = \sum_{t=0}^{T}\tilde{\beta}^t \{&u(c_t) + \theta u(g_t) + \beta[u(b_{t+1}) + \theta u(g_{t+1})]\\ &+(\lambda\beta)\{u(c_{t+1}) + \theta u(g_{t+1}) + \beta[u(b_{t+2}) + \theta u(g_{t+2})]\}\\ &+(\lambda\beta)^2\{u(c_{t+2}) + \theta u(g_{t+2}) + \beta[u(b_{t+3}) + \theta u(g_{t+3})]\} + \ldots\}.\end{aligned}$$

Writing out the infinite sum,

$$\begin{aligned}\sum_{t=0}^{T}\tilde{\beta}^t v_t &= \left\{v_0 + \tilde{\beta}v_1 + \tilde{\beta}^2 v_2 + \ldots\right\}\\ &= \left\{\underbrace{u(c_0) + \theta u(g_0) + \beta[u(b_1) + \theta u(g_1)] + (\lambda\beta)[u(c_1) + \ldots] + \ldots}_{=v_0}\right.\\ &\quad + \tilde{\beta}\underbrace{\{u(c_1) + \theta u(g_1) + \beta[u(b_2) + \theta u(g_2)] + (\lambda\beta)[u(c_2) + \ldots] + \ldots\}}_{=v_1}\\ &\quad \left. + \tilde{\beta}^2\underbrace{\{u(c_2) + \theta u(g_2) + \beta[u(b_3) + \theta u(g_3)] + \ldots\}}_{=v_2}\right\}.\end{aligned}$$

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Reordering the terms according to the time periods,

$$\sum_{t=0}^{T} \tilde{\beta}^t v_t = \Big\{ u(c_0) + \theta u(g_0) + \beta[u(b_1) + \theta u(g_1)] + (\tilde{\beta} + \lambda\beta)\{u(c_1) + \theta u(g_1) + \beta[u(b_2) + \theta u(g_2)]\} + [\tilde{\beta}^2 + \tilde{\beta}(\lambda\beta) + (\lambda\beta)^2][u(c_2) + ...] + ... \Big\}.$$

Writing this more compactly yields the objective function in (4):

$$\sum_{t=0}^{T} \tilde{\beta}^t v_t = \sum_{t=0}^{T} \left[ \sum_{j=0}^{t} \tilde{\beta}^{t-j} (\lambda\beta)^j \right] \{u(c_t) + \theta u(g_t) + \beta[u(b_{t+1}) + \theta u(g_{t+1})]\}.$$

## B.2. OBJECTIVE OF THE LONG RUN PLANNER

The long-run planner adds up all lifetime utilities of all generations of agents, weighting them equally ($\tilde{\beta} = 1$). To include the (equal) weights formally I divide by $T$ and take the limit of the infimum, when $T$ goes to infinity:

$$\lim_{T\to\infty} \inf \frac{1}{T} \sum_{t=0}^{T} v_t = \lim_{T\to\infty} \inf \frac{1}{T} \sum_{t=0}^{T} \{u(c_t) + \theta u(g_t) + \beta[u(b_{t+1}) + \theta u(g_{t+1})] + (\lambda\beta)\{u(c_{t+1}) + \theta u(g_{t+1}) + \beta[u(b_{t+2}) + \theta u(g_{t+2})]\} + (\lambda\beta)^2\{u(c_{t+2}) + \theta u(g_{t+2}) + \beta[u(b_{t+3}) + ...]\} + ... \Big\}.$$

Writing this more compactly yields the objective function in (5):

$$\lim_{T\to\infty} \inf \frac{1}{T} \sum_{t=0}^{T} v_t = \lim_{T\to\infty} \inf \sum_{t=0}^{T} \frac{1}{T} \left[ \sum_{j=0}^{t} (\lambda\beta)^j \right] \times \{u(c_t) + \theta u(g_t) + \beta[u(b_{t+1}) + \theta u(g_{t+1})]\}.$$

## B.3. FIRST-ORDER CONDITIONS UNDER THE BENEVOLENT PLANNER

Use the feasibility constraint (1) to substitute $g$ into (A.1) and differentiate with respect to $c_t$ and $b_{t+1}$:

$$\hat{\beta}(t) \left[u'(c_t) - \theta u'(y - c_t - b_t)\right] - \beta\hat{\beta}(t-1)\theta u'(y - c_t - b_t) = 0$$

$$-\hat{\beta}(t+1)\theta u'(y - c_{t+1} - b_{t+1}) + \beta\hat{\beta}(t) \left[u'(b_{t+1}) - \theta u'(y - c_{t+1} - b_{t+1})\right] = 0,$$

where $\hat{\beta}(t) \equiv \sum_{j=0}^{t} \tilde{\beta}^{t-j} (\lambda\beta)^j$.
Simplify:

$$\hat{\beta}(t)u'(c_t) - \left[\hat{\beta}(t) + \beta\hat{\beta}(t-1)\right] \theta u'(y - c_t - b_t) = 0,$$

$$\beta\hat{\beta}(t)u'(b_{t+1}) - \theta \left[\hat{\beta}(t+1) + \beta\hat{\beta}(t)\right] u'(y - c_{t+1} - b_{t+1}) = 0.$$

The first-order conditions under the long-run planner are very similar.

# APPENDIX C: THE POLITICAL EQUILIBRIUM

## C.1. MICROFOUNDATION OF THE POLITICAL EQUILIBRIUM: A PROBABILISTIC VOTING MODEL

Assume the following probabilistic voting model. Candidates or parties can extract some exogenous rent if they are elected. Before the election, each candidate proposes a policy platform characterized by the policy variables $g_t$, $\tau_t$, and $b_{t+1}$. Candidates can also differ in some other dimension unrelated to policy, usually referred to as "ideology" in the literature. Voters differ in their valuation of this other dimension. Lindbeck and Weibull (1987) have shown that under such conditions, the political choice is equivalent to maximizing a weighted objective of the indirect utilities, where the weights represent the fraction of the population with these particular preferences. In the context of the present model, the equilibrium of such a probabilistic voting model can thus be represented as the choice of $g_t$, $\tau_t$, and $b_{t+1}$, maximizing a political objective function that is a weighted average of young and old households, given the state variable $b_t$. Similarly to the commitment problem, we can maximize subject to the allocations directly, and the feasibility constraint is the only relevant constraint:

$$\max_{b_{t+1},c_t,g_t} (1-\omega)\sum_{i=0}^{\infty}(\lambda\beta)^i\left\{u(c_{t+i})+\theta u(g_{t+i})+\beta[u(b_{t+i+1})+\theta u(g_{t+i+1})]\right\}$$

$$+\,\omega\sum_{i=0}^{\infty}(\lambda\beta)^i\left[u(b_{t+i})+\lambda u(c_{t+i})+\theta(1+\lambda)u(g_{t+i})\right] \qquad \textbf{(C.1)}$$

s.t. (1),

where $\omega$ represents the population share (and political power) of the old voters (and correspondingly $(1-\omega)$ represents the young voters). Note that in the paper, for simplicity, I omitted the old population in the political decision problem. Because their problem is time-consistent, omitting them does not lead to any qualitative difference. However, for completeness, I include them in the derivations in the Appendices. To obtain the corresponding results from the main paper, set $\omega = 0$.

## C.2. RECURSIVE FORMULATION IN THE ABSENCE OF COMMITMENT

First, write out the infinite sums in (C.1) and denote them by $\tilde{v}_t$:

$$\tilde{v}_t = (1-\omega)\left\{u(c_t)+\theta u(g_t)+\beta[u(b_{t+1})+\theta u(g_{t+1})]+(\lambda\beta)\left\{u(c_{t+1})\right.\right.$$
$$\left.\left.+\;\theta u(g_{t+1})+\beta[u(b_{t+2})+\theta u(g_{t+2})]\right\}+\dots\right\}$$
$$+\,\omega\left\{[u(b_t)+\lambda u(c_t)+\theta(1+\lambda)u(g_t)]+(\lambda\beta)[u(b_{t+1})+\dots]+\dots\right\}.$$

Reorder the terms to simplify:

$$\tilde{v}_t = \omega u(b_t)+\tilde{\omega}u(c_t)+(1+\omega\lambda)\theta u(g_t)$$
$$+\,\tilde{\omega}\beta\left[u(b_{t+1})+\lambda u(c_{t+1})+(1+\lambda)\theta u(g_{t+1})\right]$$
$$+\,\tilde{\omega}\beta(\lambda\beta)\left[u(b_{t+2})+\lambda u(c_{t+2})+(1+\lambda)\theta u(g_{t+2})\right]$$
$$+\,\tilde{\omega}\beta(\lambda\beta)^2\left[u(b_{t+3})+\lambda u(c_{t+3})+(1+\lambda)\theta u(g_{t+3})\right]+\dots,$$

where $\tilde{\omega} \equiv (1 - \omega + \lambda\omega)$. Because every government will face the same optimization problem given the state variable (government debt), we can define the optimal policy functions for public goods, $G(b)$, for private consumption, $C(b)$, and for government debt next period, $B(b)$, and we can write recursively in two stages:

$$\tilde{v} = \omega u(b) + \tilde{\omega} u(c) + (1 + \lambda\omega)\theta u(g) + \tilde{\omega}\beta V^{\mathrm{PE}}(b'), \tag{C.2}$$

$$V^{\mathrm{PE}}(b') = u(b') + \theta(1 + \lambda)u[G(b')] + \lambda u[C(b')] + \lambda\beta V^{\mathrm{PE}}[B(b')]. \tag{C.3}$$

The optimal policy functions will result from maximizing (C.2) given (C.3) subject to (1). To obtain (6)–(8), set $\omega = 0$.

### C.3. FIRST-ORDER CONDITIONS IN THE ABSENCE OF COMMITMENT

Substitute (1) into (C.2) and (C.3) and differentiate with respect to $c$ and $b'$:

$$\tilde{\omega} u'(c) - (1 + \lambda\omega)\theta u'(y - c - b) = 0, \tag{C.4}$$

$$u'(b') + \lambda u'[C(b')]\frac{\partial C(b')}{\partial b'} - (1 + \lambda)\theta u'[y - C(b') - b'] \times \left[\frac{\partial C(b')}{\partial b'} + 1\right] + \lambda\beta \frac{\partial V^{\mathrm{PE}}[B(b')]}{\partial B(b')}\frac{\partial B(b')}{\partial b'} = 0. \tag{C.5}$$

Use the envelope theorem (which states that $\frac{\partial V^{\mathrm{PE}}[B(b')]}{\partial B(b')} = 0$) to simplify (C.5):

$$u'(b') + \lambda u'[C(b')]\frac{\partial C(b')}{\partial b'} - (1 + \lambda)\theta u'[y - C(b') - b'] \times \left[\frac{\partial C(b')}{\partial b'} + 1\right] = 0. \tag{C.6}$$

To obtain (9), reinsert (1) into (C.6) and reorder.

# APPENDIX D: ALLOCATIONS UNDER CES-UTILITY

In this Appendix, I show how to derive the allocations under the CES-utility function that was defined in (10) of the main paper. I show the derivation under commitment only. The derivations in the other cases follow the same logic. First, derive the first-order conditions under CES-utility:

$$c_0^{-\sigma} - \theta(y - c_0 - b_0)^{-\sigma} = 0 \text{ if } t = 0, \tag{D.1}$$

$$\lambda c^{-\sigma} - \theta(1 + \lambda)(y - c - b)^{-\sigma} = 0 \text{ if } t > 0, \tag{D.2}$$

$$b'^{-\sigma} - (1 + \lambda)\theta(y - c' - b')^{-\sigma} = 0 \text{ for all } t. \tag{D.3}$$

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Solve (D.1) and (D.2) for consumption of the young to obtain the policy function for $c$ conditional on the state variable $b$:

$$C(b) = \begin{cases} \frac{1}{1+\theta^{1/\sigma}}(y - b_0) & \text{if } t = 0 \\ \frac{\lambda^{1/\sigma}}{\lambda^{1/\sigma}+[\theta(1+\lambda)]^{1/\sigma}}(y - b) & \text{if } t > 0. \end{cases} \tag{D.4}$$

Insert (D.4) into (D.3):

$$[(1+\lambda)\theta]^{1/\sigma} b' - \left(1 - \frac{\lambda^{1/\sigma}}{\lambda^{1/\sigma}+[\theta(1+\lambda)]^{1/\sigma}}\right)(y - b') = 0 \text{ for all } t. \tag{D.5}$$

Solve (D.5) for $b'$, and use the result in (D.4) and (1) to obtain the optimal allocations given in (11).

# APPENDIX E: PROOF OF PROPOSITION 1

## E.1. PART 1: $b^{CO}$ INCREASES WITH $\frac{1}{\sigma}$

The first part of the proposition says that given $\theta(1+\lambda) < 1$, the level of government debt under commitment increases with the elasticity of substitution. The level of government debt under commitment can be conceived as a function, $h(.)$, of the elasticity of substitution $\frac{1}{\sigma}$:

$$b^{CO} = h\left(\frac{1}{\sigma}\right) \equiv \frac{1}{1+\lambda^{1/\sigma}+[\theta(1+\lambda)]^{1/\sigma}} y.$$

Differentiating this function with respect to $\frac{1}{\sigma}$ yields

$$\frac{dh\left(\frac{1}{\sigma}\right)}{d\frac{1}{\sigma}} = -\left(\frac{1}{1+\lambda^{1/\sigma}+[\theta(1+\lambda)]^{1/\sigma}}\right)^2 \left\{\ln(\lambda)\lambda^{1/\sigma} + \ln[\theta(1+\lambda)][\theta(1+\lambda)]^{1/\sigma}\right\} y$$
$$> 0.$$

The derivative is positive because the inside of the square brackets is negative. This stems from the fact that $\ln(\lambda) < 0$ and $\ln[\theta(1+\lambda)] < 0$, because $\lambda < 1$ and $\theta(1+\lambda) < 1$ by assumption. This also shows that the assumption that $\theta(1+\lambda) < 1$ is crucial.

## E.2. PART 2: $b^{PE}$ INCREASES WITH $\frac{1}{\sigma}$

The second part of the proposition says that if $\lambda < 0.5$ and there are only young voters ($\omega = 0$), the level of government debt in the political equilibrium increases with the elasticity of substitution. In this proof, I will show an even more general case where old voters are also included. For this purpose, I have to assume two parameter conditions instead of only one (because I have one parameter more, $\omega$, the power of the old, which

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could be different from 0):

$$\frac{1+\lambda}{1+\omega\lambda}(1-\omega+\omega\lambda)+\lambda<2, \tag{E.1}$$

$$\theta\frac{1+\omega\lambda}{\tilde{\omega}}<1. \tag{E.2}$$

The two parameter conditions impose restrictions on $\lambda$, $\theta$, and $\omega$ at the same time. Note that the first parameter condition is the equivalent to $\lambda < 0.5$, if we insert $\omega = 0$. The second parameter condition collapses to $\theta < 1$, if we insert $\omega = 0$. The following proof will show why those conditions are needed to unambiguously show that government debt increases with substitutability. In other cases it is ambiguous, but it can still be the case. To obtain the equivalent proof for the case discussed in the main paper, simply set $\omega = 0$.

Suppose the level of government debt in the political equilibrium is given by a function $\tilde{h}(.)$:

$$b^{PE}=\tilde{h}\left(\frac{1}{\sigma}\right)\equiv\frac{1}{1+\frac{f_1\left(\frac{1}{\sigma}\right)f_2\left(\frac{1}{\sigma}\right)}{f_3\left(\frac{1}{\sigma}\right)}}y,$$

$$\text{where}\quad f_1\left(\frac{1}{\sigma}\right)\equiv\left(1+\eta^{1/\sigma}\right),$$

$$f_2\left(\frac{1}{\sigma}\right)\equiv\left[\theta(1+\lambda)\eta^{1/\sigma-1}+\lambda\right]^{1/\sigma},$$

$$f_3\left(\frac{1}{\sigma}\right)\equiv\left(1+\eta^{1/\sigma}\right)^{1/\sigma},$$

$$\eta=\frac{\theta(1+\omega\lambda)}{1-\omega+\omega\lambda}.$$

Differentiating this function with respect to $\frac{1}{\sigma}$ yields, where for notational simplicity I leave out the arguments of the functions,

$$\frac{d\tilde{h}\left(\frac{1}{\sigma}\right)}{d\frac{1}{\sigma}}=-\left(\frac{1}{1+\frac{f_1f_2}{f_3}}\right)^2\frac{(f_1'f_2+f_2'f_1)f_3-f_1f_2f_3'}{f_3^2}y,$$

where $f_1$, $f_2$, $f_3$ as before

and $f_1'\equiv\ln(\eta)\eta^{1/\sigma}$,

$$f_2'\equiv\ln\left[\frac{\theta(1+\lambda)}{\eta}\eta^{1/\sigma}+\lambda\right]f_2+\frac{1}{\sigma}\left[\frac{\theta(1+\lambda)}{\eta}\eta^{1/\sigma}+\lambda\right]^{-1}$$

$$\cdot f_2\frac{\theta(1+\lambda)}{\eta}\ln(\eta)\eta^{1/\sigma},$$

$$f_3'\equiv\ln\left(1+\eta^{1/\sigma}\right)\left(1+\eta^{1/\sigma}\right)^{1/\sigma}+\frac{1}{\sigma}\left(1+\eta^{1/\sigma}\right)^{1/\sigma-1}\ln(\eta)\eta^{1/\sigma}.$$

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By reshuffling this expression, I obtain another expression, where it is easier to analyze the sign of the derivative:

$$\frac{d\tilde{h}\left(\frac{1}{\sigma}\right)}{d\frac{1}{\sigma}} = -(...)^2 \left\{ \ln(\eta)\eta^{1/\sigma} \left[\frac{q_1(x) f_2}{f_3}\right] + \frac{q_2(x) f_1 f_2}{f_3} \right\} y,$$

$$\text{where } q_1(x) \equiv 1 + \frac{1}{\sigma}\frac{\theta(1+\lambda)}{\eta}\left[\frac{\theta(1+\lambda)}{\eta}x + \lambda\right]^{-1}(1+x) - \frac{1}{\sigma},$$

$$q_2(x) \equiv \ln\left[\frac{\theta(1+\lambda)}{\eta}x + \lambda\right] - \ln(1+x),$$

$$x \equiv \eta^{1/\sigma}.$$

Under the third condition, stated in equation (E.2), it follows that $\ln(\eta) < 0$. Now the question is, which sign do $q_1(x)$ and $q_2(x)$ have? In the following, I will first show that $q_1(x) > 0$ under the conditions assumed, and then I will show that $q_2(x) < 0$ under the conditions assumed. With those signs, we obtain a debt level in the political equilibrium increasing with the elasticity of substitution, as can easily be seen by introspection.

Step 1: Show that $q_1(x) > 0$. To see which sign $q_1(x)$ has, I first show that $q_1(x)$ is nonincreasing in $x$. When $\eta < 1$, the highest possible value of $x = \eta^{1/\sigma}$ is 1 and the lowest possible value of $x = \eta^{1/\sigma}$ is 0. Because $q_1(x)$ is nonincreasing in $x$, it suffices then to show that $q_1(1) > 0$, and we know that $q_1(x) > 0$ must hold for all $x \in [0, 1]$. To show that $q_1(x)$ is nonincreasing in $x$, differentiate $q_1(x)$:

$$q_1'(x) = -\frac{1}{\sigma}\left[\frac{\theta(1+\lambda)}{\eta}x + \lambda\right]^{-2}\left[\frac{\theta(1+\lambda)}{\eta}\right]^2(1+x)$$

$$+\frac{1}{\sigma}\left[\frac{\theta(1+\lambda)}{\eta}x + \lambda\right]^{-1}\left[\frac{\theta(1+\lambda)}{\eta}\right] \leq 0$$

$$\Longleftrightarrow \frac{\theta(1+\lambda)}{\eta\lambda} \geq 1.$$

Note that (E.3) must always be true, because even at the maximum of $\eta$ (in terms of $\omega$), $\eta_{\max} \equiv \frac{\theta(1+\lambda)}{\lambda}$, it is true. As explained earlier, it is now sufficient to check whether $q_1(x_{\max}) = q_1(1) > 0$:

$$q_1(1) = 1 + \frac{1}{\sigma}\left\{\left[\frac{\frac{\theta(1+\lambda)}{\eta}}{\frac{\theta(1+\lambda)}{\eta} + \lambda}\right]2 - 1\right\}$$

$$= 1 + \frac{1}{\sigma}\left\{\left[\frac{\theta(1+\lambda)}{\theta(1+\lambda) + \lambda\eta}\right]2 - 1\right\} > 0.$$

$q_1(1)$ is clearly monotonically decreasing in $\eta$. By inserting the maximum of $\eta$, $\eta_{\max} = \frac{\theta(1+\lambda)}{\lambda}$, we can check that it must always be positive:

$$1 + \frac{1}{\sigma}\left\{\left[\frac{\theta(1+\lambda)}{\theta(1+\lambda)+\lambda\eta_{\max}}\right]2 - 1\right\} > 0$$
$$\iff 1 + \frac{1}{\sigma}[1-1] = 1 > 0.$$

(E.3) is always true. Thus we have shown that $q_1(x) > 0$ for all $x \in [0, 1]$.

Step 2: Show that $q_2(x) < 0$. Differentiate $q_2(x)$ with respect to $x$:

$$q_2'(x) = \frac{\frac{\theta(1+\lambda)}{\eta}}{\frac{\theta(1+\lambda)}{\eta}x + \lambda} - \frac{1}{1+x} \geq 0$$
$$\iff \frac{\theta(1+\lambda)}{\eta\lambda} \geq 1.$$

Because (E.3) has to hold, by assumption $q_2(x)$ is an increasing function. Thus, similarly to the preceding, it is now enough to check whether $q_2(x) < 0$ at $x = 1$:

$$q_2(1) = \ln\left[\frac{\theta(1+\lambda)}{\eta} + \lambda\right] - \ln(2).$$

This expression is negative if $\frac{\theta(1+\lambda)}{\eta} + \lambda = \frac{1+\lambda}{1+\omega\lambda}(1-\omega+\omega\lambda) + \lambda < 2$, which holds by (E.1). ■

# APPENDIX F: PROOF OF PROPOSITION 2

Consider the following formulation for the level of government debt in political equilibrium (PE) and under commitment (CO):

$$b^{\mathrm{CO}} = \frac{1}{\left[\frac{\theta(1+\lambda)\hat{\xi}^{1-\sigma}+\lambda}{(1+\hat{\xi})^{1-\sigma}}\right]^{1/\sigma} + 1},$$
$$b^{\mathrm{PE}} = \frac{1}{\left[\frac{\theta(1+\lambda)\xi^{1-\sigma}+\lambda}{(1+\xi)^{1-\sigma}}\right]^{1/\sigma} + 1},$$

where $\xi \equiv \left[\frac{(1+\omega\lambda)\theta}{1-\omega+\omega\lambda}\right]^{1/\sigma}$ and $\hat{\xi} \equiv \left[\frac{(1+\lambda)\theta}{\lambda}\right]^{1/\sigma}$. Note that now the two debt levels look very similar, which has been achieved by setting $\hat{\xi}$ in a certain way. Now define the following function:

$$f(x) \equiv \left[\frac{\theta(1+\lambda)x^{1-\sigma}+\lambda}{(1+x)^{1-\sigma}}\right]^{1/\sigma}.$$

Note that one can write the debt levels in terms of this function $f$ at different realizations for the function argument, $x = \xi$ for the political equilibrium and $x = \hat{\xi}$ for the commitment case:

$$b^{\text{CO}} = \frac{1}{f(\hat{\xi})+1},$$
$$b^{\text{PE}} = \frac{1}{f(\xi)+1}.$$

The proof now consists of two parts. First, it can be shown that in the range $x \le \hat{\xi}$, $f'(x) < 0$ for $\frac{1}{\sigma} < 1$ and $f'(x) > 0$ for $\frac{1}{\sigma} > 1$. To see this, differentiate the function $f(.)$ with respect to its argument:

$$\frac{df(x)}{dx} = \left(\frac{1}{\sigma} - 1\right)\left[\frac{\theta(1+\lambda)x^{1-\sigma}+\lambda}{(1+x)^{1-\sigma}}\right]^{1/\sigma-1} \times \frac{\left\{\theta(1+\lambda)x^{-\sigma} - [\theta(1+\lambda)x^{1-\sigma}+\lambda](1+x)^{-1}\right\}}{(1+x)^{1-\sigma}}.$$

Now we can show that the term in braces must be non-negative for $x \le \hat{\xi}$. To show this, first simplify:

$$\theta(1+\lambda)x^{-\sigma} - [\theta(1+\lambda)x^{1-\sigma}+\lambda](1+x)^{-1} \ge 0$$
$$\iff \theta(1+\lambda)x^{-\sigma} - \lambda \ge 0.$$

Then realize that for $x = \hat{\xi}$ this holds with equality. Therefore because $x^{-\sigma}$ is a decreasing function in $x$, the term inbraces will be non-negative for all values $x \le \hat{\xi}$. Thus, in this range for $x$, the derivative $\frac{df(x)}{dx}$ is negative in the case of $\frac{1}{\sigma} < 1$ and positive in the case of $\frac{1}{\sigma} > 1$ for all $0 < \omega < 1, \theta, \lambda > 0$. For the knife-edge case of $\frac{1}{\sigma} = 1$, the derivative is zero. This means that in this case the value of $x$ is irrelevant, as $f(x)$ is constant in $x$ (which can also be seen by introspection). Thus, for $\frac{1}{\sigma} = 1$, the commitment solution is equal to the political equilibrium.

Second, it can be shown that $\xi < \hat{\xi}$ for $\omega < 1$ and $\lambda \le 1$:

$$\hat{\xi} = \left[\frac{(1+\lambda)\theta}{\lambda}\right]^{1/\sigma} > \left[\frac{(1+\omega\lambda)\theta}{(1-\omega+\omega\lambda}\right]^{1/\sigma} = \xi$$
$$\iff (1+\lambda)(1-\omega+\omega\lambda) > \lambda(1+\omega\lambda). \quad \textbf{(F.1)}$$

(F.1) must be true because $(1+\lambda) > (1+\omega\lambda)$ and $(1-\omega)+\omega\lambda \ge \lambda$. For $\omega = 1$, we obtain $\hat{\xi} = \xi$ for all $\lambda, \theta$. Thus, it has been shown that for all interesting parameter constellations $\xi \le \hat{\xi}$. Combined with the previous finding that in the range $x \le \hat{\xi}$, $f'(x) < 0$ for $\frac{1}{\sigma} < 1$ and $f'(x) > 0$ for $\frac{1}{\sigma} > 1$, it is easy to see that the following must hold:

$$b^{\text{CO}} = \frac{1}{f(\hat{\xi})+1} < b^{\text{PE}} = \frac{1}{f(\xi)+1} \text{ for } \frac{1}{\sigma} > 1,$$
$$b^{\text{CO}} = \frac{1}{f(\hat{\xi})+1} > b^{\text{PE}} = \frac{1}{f(\xi)+1} \text{ for } \frac{1}{\sigma} < 1.$$

This completes the proof. ∎

# APPENDIX G: PROOF OF PROPOSITION 3

We want to show that $b^{\mathrm{BP}} \leq b^{\mathrm{CO}}$ in the steady state. In the steady state the debt levels are equal to

$$b^{\mathrm{CO}} = \frac{1}{1+\lambda^{1/\sigma}+[\theta(1+\lambda)]^{1/\sigma}},$$

$$b^{\mathrm{BP}} = \frac{\beta^{1/\sigma}}{\beta^{1/\sigma}+\max(\tilde{\beta},\lambda\beta)^{1/\sigma}+[\theta(\max(\tilde{\beta},\lambda\beta)+\beta)]^{1/\sigma}}.$$

Differentiate two cases. In the first case, $\tilde{\beta} > \lambda\beta$,

$$b^{\mathrm{CO}} = \frac{1}{1+\lambda^{1/\sigma}+[\theta(1+\lambda)]^{1/\sigma}} > \frac{1}{1+(\tilde{\beta}/\beta)^{(1/\sigma)}+[\theta(\frac{\tilde{\beta}}{\beta}+1)]^{1/\sigma}} = b^{\mathrm{BP}}.$$

In the second case, $\tilde{\beta} < \lambda\beta$,

$$b^{\mathrm{CO}} = \frac{1}{1+\lambda^{1/\sigma}+[\theta(1+\lambda)]^{1/\sigma}} = \frac{\beta^{1/\sigma}}{\beta^{1/\sigma}+(\beta\lambda)^{1/\sigma}+[\theta\beta(1+\lambda)]^{1/\sigma}} = b^{\mathrm{BP}}.$$

# APPENDIX H: PROOF OF PROPOSITION 4

We want to show that $b^{\mathrm{LRP}} < b^{\mathrm{PE}}$ if $\beta(1+\lambda) < 1$. Before starting, note that from the results in Propositions 2 and 3, we know already that for $\frac{1}{\sigma} > 1$ the debt level of the benevolent planner must be below that in political equilibrium. (We can deduce this from $b^{\mathrm{LRP}} < b^{\mathrm{BP}} < b^{\mathrm{CO}}$, which was shown in the previous section, and $b^{\mathrm{CO}} < b^{\mathrm{PE}}$, which holds for $\frac{1}{\sigma} > 1$ according to Proposition 2.) Thus we now only have to show that $b^{\mathrm{LRP}} < b^{\mathrm{PE}}$ for the case $\frac{1}{\sigma} < 1$ when the parameter condition $\beta(1+\lambda) < 1$ holds. First, define two functions with similar shapes that constitute an important term in the expression for the debt levels:

$$b^{\mathrm{PE}} = \frac{1}{f(\xi;\lambda)+1}, \text{ where } f(\xi;\lambda) \equiv \left[\frac{\theta(1+\lambda)\xi^{1-\sigma}+\lambda}{(1+\xi)^{1-\sigma}}\right]^{1/\sigma};$$

$$b^{\mathrm{LRP}} = \frac{1}{f(\tilde{\xi};\frac{1}{\beta})+1}, \text{ where } f(\tilde{\xi};\frac{1}{\beta}) \equiv \left[\frac{\theta(1+\frac{1}{\beta})\tilde{\xi}^{1-\sigma}+\frac{1}{\beta}}{(1+\tilde{\xi})^{1-\sigma}}\right]^{1/\sigma}.$$

Second, notice that the minimal possible value for both $\xi$ and $\tilde{\xi}$ is $\theta^{1/\sigma}$. Third, notice that for the case $\frac{1}{\sigma} < 1$ both $\xi$ and $\tilde{\xi}$ are always smaller than $\hat{\xi} \equiv \left[\frac{(1+\lambda)\theta}{\lambda}\right]^{1/\sigma}$. Thus, we will analyze those functions on the interval $[\theta^{1/\sigma}, \hat{\xi}]$. Fourth, from the definition of both functions it is clear by inspection that if we evaluated them at the same point, $\xi = \tilde{\xi}$, the function $f(.;\frac{1}{\beta})$ would always lie above $f(.;\lambda)$, because $\frac{1}{\beta} > 1 > \lambda$. However, it is still not clear that $f(\tilde{\xi};\frac{1}{\beta})$ must lie above $f(\xi;\lambda)$ if $\xi \neq \tilde{\xi}$. To show that, one can use the fact shown in the proof for Proposition 1 that $f(.;\lambda)$ is monotonically increasing on the interval under

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consideration and thus must have its maximum at $\theta^{1/\sigma}$. Furthermore, it can be shown that $f(.; \frac{1}{\beta})$ has a minimum at $\tilde{\xi}$:

$$\frac{df(x)}{dx} = \left(\frac{1}{\sigma} - 1\right)\left[\frac{\theta(1+\frac{1}{\beta})x^{1-\sigma} + \frac{1}{\beta}}{(1+x)^{1-\sigma}}\right]^{1/\sigma - 1} \frac{\left\{\theta(1+\frac{1}{\beta})x^{-\sigma} - [\theta(1+\frac{1}{\beta})x^{1-\sigma} + \frac{1}{\beta}](1+x)^{-1}\right\}}{(1+x)^{1-\sigma}} = 0.$$

Simplifying,

$$\Leftrightarrow \theta\left(1+\frac{1}{\beta}\right)x^{-\sigma} - \frac{1}{\beta} = 0.$$

Solving this equation for $x$ yields

$$x = \left[\frac{\theta(1+\frac{1}{\beta})}{\frac{1}{\beta}}\right]^{1/\sigma} = \tilde{\xi}.$$

It is a minimum and not a maximum because the first derivative is negative at $\theta^{1/\sigma}$ and positive at $\hat{\xi}$ and the point $\tilde{\xi}$ lies in this interval, as shown previously. That the first derivative is negative at $\theta^{1/\sigma}$ can be seen by considering

$$\theta\left(1+\frac{1}{\beta}\right)\left(\theta^{1/\sigma}\right)^{-\sigma} - \frac{1}{\beta} > 0.$$

Then the curly brackets are positive, and because $\frac{1}{\sigma} < 1$, the derivative is negative in this interval. That the first derivative is positive at $\hat{\xi}$ can be seen by considering

$$\begin{aligned}
&\theta(1+\frac{1}{\beta})\left\{\left[\frac{\theta(1+\lambda)}{\lambda}\right]^{1/\sigma}\right\}^{-\sigma} - \frac{1}{\beta} < 0 \\
&\Longleftrightarrow \lambda\left(1+\frac{1}{\beta}\right) - \frac{1}{\beta}(1+\lambda) < 0 \\
&\Longleftrightarrow \lambda - \frac{1}{\beta} < 0.
\end{aligned}$$

To show that $f(.; \frac{1}{\beta})$ must lie above $f(.; \lambda)$, it is sufficient to show that the minimum of $f(.; \frac{1}{\beta})$ lies above the maximum of $f(.; \lambda)$ on the interval under consideration. More

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precisely, we must show that

$$f(\theta^{1/\sigma};\lambda) = \left[\frac{\theta(1+\lambda)\theta^{(1-\sigma)/\sigma}+\lambda}{(1+\theta^{1/\sigma})^{1-\sigma}}\right]^{1/\sigma}$$

$$< \; f(\tilde{\xi};\frac{1}{\beta}) = \left\{\frac{\theta(1+\frac{1}{\beta})\left[\frac{\theta(1+\frac{1}{\beta})}{\frac{1}{\beta}}\right]^{(1-\sigma)/\sigma}+\frac{1}{\beta}}{\left\{1+\left[\frac{\theta(1+\frac{1}{\beta})}{\frac{1}{\beta}}\right]^{1/\sigma}\right\}^{1-\sigma}}\right\}^{1/\sigma}.$$

Simplifying the right-hand side,

$$\left[\frac{\theta^{1/\sigma}+\lambda\theta^{1/\sigma}+\lambda}{(1+\theta^{1/\sigma})^{1-\sigma}}\right]^{1/\sigma} < \left(\frac{1}{\beta}\right)^{1/\sigma}\left\{1+\left[\frac{\theta(1+\frac{1}{\beta})}{\frac{1}{\beta}}\right]^{1/\sigma}\right\}.$$

To see that this must be the case, realize that there is an expression that must lie in between the two values:

$$\left[\frac{\theta^{1/\sigma}+\lambda\theta^{1/\sigma}+\lambda}{(1+\theta^{1/\sigma})^{1-\sigma}}\right]^{1/\sigma} < \left[\frac{\theta^{1/\sigma}+\lambda\theta^{1/\sigma}+\lambda+1}{(1+\theta^{1/\sigma})^{1-\sigma}}\right]^{1/\sigma} < \left(\frac{1}{\beta}\right)^{1/\sigma}+\left[\theta(1+\frac{1}{\beta})\right]^{1/\sigma},$$

$$\left[\frac{\theta^{1/\sigma}+\lambda\theta^{1/\sigma}+\lambda}{(1+\theta^{1/\sigma})^{1-\sigma}}\right]^{1/\sigma} < (1+\lambda)^{1/\sigma}+\theta^{1/\sigma}(1+\lambda)^{1/\sigma} < \left(\frac{1}{\beta}\right)^{1/\sigma}+\left[\theta(1+\frac{1}{\beta})\right]^{1/\sigma}.$$

To see that the last inequality must be true, multiply both sides by $\left(\frac{1}{\beta}\right)^{1/\sigma}$:

$$[\beta\,(1+\lambda)]^{1/\sigma}+\theta^{1/\sigma}(\beta+\beta\lambda)^{1/\sigma} < 1+[\theta(1+\beta)]^{1/\sigma}.$$

This must be true under the parameter condition $\beta(1+\lambda) < 1$. This completes the proof. ■

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