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ON SOME QUESTIONS RELATED TO KOETHE'S NIL IDEAL PROBLEM

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Abstract We study properties of two-sided and one-sided ideals of A-rings, i.e. rings that are sums of their nil left ideals. We show that the question as to whether one-sided ideals of A-rings are again A-rings is equivalent to the famous Koethe problem. We also obtain some results on another related open problem that asks whether annihilators of elements of non-zero A-rings are non-zero.

Keywords: Koethe problem; nil rings; ideals

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1. Introduction

All rings considered in this paper are associative but do not necessarily have identities. We denote by R^* the ring (or algebra if we consider algebras) obtained by adjoining an identity to a ring (or algebra) R if R has no identity, and set $R^* = R$ otherwise. To denote that I is a two-sided ideal (left ideal, right ideal) of a ring R we write $I \lhd R$ ($I <_l R$, $I <_r R$, respectively). Obviously, if L is a left ideal of R, then the two-sided ideal of R generated by L is equal to LR^*.

For a given ring R we set $A(R) = \sum\{L <_l R \mid L \text{ nil}\}$. If $R = A(R)$, then we say that R is an *A-ring*. It is well known and not hard to check (see [**8**]) that $A(R) = \sum\{K <_r R \mid K \text{ nil}\}$ and consequently $A(R) \lhd R$. Moreover, if R is an algebra over a field F, it suffices to take the sum of all nil left (equivalently, right) F-ideals of R. Consequently, $A(R)$ is an F-ideal in this case.

The ideal $A(R)$ was introduced by Andrunakievich [**1**] and we call it the *Andrunakievich ideal*. Koethe's problem [**6**] asks whether the nil radical $\mathrm{Nil}(R)$ of any ring R contains all the nil left (equivalently, right [**9**]) ideals of R, that is, $\mathrm{Nil}(R) = A(R)$.

For an arbitrary ring R, define $\overline{\mathrm{Nil}}(R) = \bigcap\{I \lhd R \mid A(R/I) = 0\}$. It is not hard to see that $\overline{\mathrm{Nil}}$ is the smallest radical (in the Kurosh–Amitsur sense [**5**]) such that $\overline{\mathrm{Nil}}(R)$ contains all the nil one-sided ideals of R for every ring R. Koethe's problem is obviously also equivalent to the question as to whether the radicals $\overline{\mathrm{Nil}}$ and Nil coincide.

Andrunakievich posed the following question, which was open for a long time.

Problem 1.1 (Andrunakievich [1]). *Does $A(R/A(R)) = 0$ hold for every ring R?*

It is clear that this problem asks whether $\overline{\mathrm{Nil}}(R) = A(R)$ for every ring R. It turned out [**2**, **8**] that this problem is equivalent to Koethe's problem.

Not knowing whether the radicals Nil and $\overline{\mathrm{Nil}}$ coincide, one can ask whether $\overline{\mathrm{Nil}}$ at least enjoys the most important properties of the nil radical. Positive results of that sort would give approximations of a positive solution of Koethe's problem, whereas if any of these questions turned out to be equivalent to Koethe's problem, it would show some extra properties that a potential counterexample to Koethe's problem would have to satisfy. One such question was raised already by Andrunakievich in [**1**].

Problem 1.2 (Andrunakievich [1]). *Can every ring R with $A(R) = 0$ be mapped homomorphically onto a prime ring R' with $A(R') = 0$?*

This problem is still open. It can also be formulated as whether $\overline{\mathrm{Nil}}$-semi-simple rings are subdirect products of prime $\overline{\mathrm{Nil}}$-semi-simple rings. It is well known that nil-semi-simple rings are subdirect products of prime nil-semi-simple rings.

One of the most obvious properties of nil rings is that their subrings are also nil rings. However, it is not hard to check [**8**] that the counterpart to this property for the $\overline{\mathrm{Nil}}$ radical is already equivalent to Koethe's problem. On the other hand, it is known [**8**] that one-sided ideals of $\overline{\mathrm{Nil}}$-radical rings are again $\overline{\mathrm{Nil}}$-radical. In that context it is natural to ask whether the ideals of A-rings are also A-rings. It does not look evident and Sands asked the following question.

Problem 1.3 (Sands [8]). *Is Koethe's problem equivalent to showing that the ideals of A-rings are A-rings?*

One can also ask another question.

Problem 1.4 (Chebotar *et al.* [3]). *Is Koethe's problem equivalent to showing that the left ideals of A-rings are A-rings?*

In this paper we will prove that Problem 1.4 has a positive answer. We will also obtain some results related to the following open problem [**8**].

For $a \in R$ we denote by $l_R(a)$ the left annihilator of a in R, that is, $l_R(a) = \{x \in R \mid xa = 0\}$. The right annihilator $r_R(a)$ of a in R is defined in the dual way.

Problem 1.5. *Let L be a nil left ideal of a non-zero ring R.*

(i) *Is it true that $l_R(a) \neq 0$ for all $a \in LR^*$?*

(ii) *Is it true that $r_R(a) \neq 0$ for all $a \in LR^*$?*

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It is also not known whether Problems 1.5 (i) and (ii) are equivalent.

We will need the following well-known result proved by Krempa.

Theorem 1.6 (Krempa [7] and Chebotar *et al.* [2, Lemma 4.1]). *Koethe's problem has a negative solution if and only if there exists a non-zero nil-semi-simple A-algebra over a field.*

2. Properties of the Andrunakievich ideal

We start by describing some relations between $A(R)$ and $A(I)$, where I is an ideal or a left ideal of R.

Obviously, $A(A(R)) = A(R)$ for an arbitrary ring R, so $A(R)$ is an A-ring.

For a subring S of a ring R, we define $A_R(S) = \sum\{L \mid L <_l R,\ L \subseteq S,\ L \text{ nil}\}$. Just like $A(R)$, if R is an algebra over a field F and S is an F-subalgebra, it suffices to take the sum of all nil left (right) F-ideals of R contained in S.

It is clear that $A_R(S) \subseteq A(S)$ for every subring S of a ring R and $A_R(S) \subseteq A_R(T)$ for subrings S and T of R with $S \subseteq T$.

Proposition 2.1. *If $L <_l R$, then*

(1) $LA(L) \subseteq A_R(L) \subseteq A(L)$,

(2) $A(R)L \subseteq A_R(L)$ *and*

(3) *if $R = A(R)$, then $A(L) <_l R$ and $(L/A(L))^2 = 0$.*

Proof. (1) Suppose that $K <_l L$ and that K is nil. Obviously, $LK <_l R$ and $LK \subseteq K$. Thus, LK is a nil left ideal of R contained in L, so $LK \subseteq A_R(L)$. Consequently, $LA(L) \subseteq A_R(L)$.

(2) Suppose that $K <_l R$ and that K is nil. Take $l \in L$. Then Kl is a nil left ideal of R contained in L, so $Kl \subseteq A_R(L)$. Consequently, $A(R)L \subseteq A_R(L)$.

(3) If $R = A(R)$, then $RL = A(R)L \subseteq A_R(L) \subseteq A(L)$ by (2), so $A(L) <_l R$. Moreover, $L^2 \subseteq RL \subseteq A(L)$ and so $(L/A(L))^2 = 0$. □

Proposition 2.2. *Let $I \lhd R$. Then the following hold.*

(1) *$A(I) = A_R(I) \subseteq A(R)$. In particular, if $I = A(I)$, then $I \subseteq A(R)$.*

(2) *$A(I) \lhd R$.*

(3) *If $A(R) \subseteq I$, then $A(I) = A(R)$.*

Proof. (1) Suppose that $K <_l I$ and that K is nil. Obviously, $R^*K <_l R$ and $R^*K \subseteq I$. Moreover, R^*K is nil since $(R^*K)^2 \subseteq IK \subseteq K$. This shows that $K \subseteq A_R(I)$. Consequently, $A(I) \subseteq A_R(I)$. The rest is clear.

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(2) From (1) we get $A(I) = A_R(I) <_l R$. As was mentioned in the introduction, $A(I) = \sum\{K \mid K <_r I,\ K \text{ nil}\}$. Applying arguments dual to those in the proof of (1), one gets that $\sum\{K \mid K <_r I,\ K \text{ nil}\} = \sum\{K \mid K <_r R,\ K \subseteq I,\ K \text{ nil}\}$. Hence, $A(I) <_r R$ and $A(I) \lhd R$.

(3) If $A(R) \subseteq I$, then $A(R) \lhd I$ and $I \lhd R$, so it follows from (1) that $A(R) = A(A(R)) \subseteq A(I) \subseteq A(R)$, and hence $A(I) = A(R)$. □

Remark 2.3. Note that the one-sided variant of Proposition 2.2 does not hold. For instance, let $R = M_2(F)$ be the ring of 2×2 matrices over a field F and let $L = \left(\begin{smallmatrix} F & 0 \\ F & 0 \end{smallmatrix}\right) <_l R$. Since every nilpotent element in L is of the form $\left(\begin{smallmatrix} 0 & 0 \\ a & 0 \end{smallmatrix}\right)$ for some $a \in F$, it follows that $A(L) = \left(\begin{smallmatrix} 0 & 0 \\ F & 0 \end{smallmatrix}\right)$, which is not a left ideal of R. Moreover, $A(R) = 0$, so $A(L) \not\subseteq A(R)$.

From Proposition 2.2 (1) it follows that Problem 1.3 can be expressed as asking whether Koethe's problem is equivalent to the statement that $I = A_R(I)$ for every ideal I of an arbitrary A-ring R. However, the remark above shows that it is not evident whether Problem 1.4 is equivalent to the following one.

Problem 2.4. *Is Koethe's problem equivalent to whether $A_R(L) = L$ for every left ideal L of an A-ring R?*

The second part of the following result is dual in a sense to Proposition 2.1 (3).

Proposition 2.5. *For an arbitrary $L <_l R$:*

(1) *if L is nil, then $A(LR^*) = LR^*$;*

(2) *if $A(L) = L$, then $(LR^*/A(LR^*))^2 = 0$.*

Proof. (1) Since L is nil, $L \subseteq A(LR^*)$. By Proposition 2.2, $A(LR^*) \lhd R$. These imply that $LR^* \subseteq A(LR^*)$ and so $A(LR^*) = LR^*$.

(2) Applying Proposition 2.1, we obtain $L^2 = LA(L) \subseteq A_R(L) \subseteq A_R(LR^*) \subseteq A(LR^*)$. Hence, $L^2R^* \subseteq A(LR^*)$ since $A(LR^*)$ is an ideal of R by Proposition 2.2 (2). Now, $(LR^*)^2 \subseteq L^2R^* \subseteq A(LR^*)$, so $(LR^*/A(LR^*))^2 = 0$. □

It is natural to ask the following question.

Problem 2.6. *Is Koethe's problem equivalent to whether $A(LR^*) = LR^*$ for every $L <_l R$ with $A(L) = L$?*

3. Main results

First we answer Problems 1.4 and 2.4 in the affirmative.

Theorem 3.1. *The following conditions are equivalent:*

(i) *Koethe's problem has a positive solution;*

(ii) *every left ideal of an A-ring is itself an A-ring;*

(iii) *$A_R(L) = L$ for every A-ring R and $L <_l R$.*

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Proof. It is evident that (i) implies (ii).

Assume that (ii) is satisfied and suppose that $L <_l R = A(R)$. Since $A(R) = R$, we have $A(M_2(R)) = M_2(R)$, by [**2**, Corollary 3.2]. Now, $T = \left(\begin{smallmatrix} R & L \\ R & L \end{smallmatrix}\right) <_l M_2(R)$, so $A(T) = T$ by the assumption. Hence, $T = \sum V_i$, where all the V_i are nil left ideals of T. Let V be one of V_i and let $U = U_i$ be the set of right-upper entries of matrices from V_i. Clearly, $U \subseteq L$. Let $u \in U$. Then there are $x, y \in R$ and $z \in L$ such that $\left(\begin{smallmatrix} x & u \\ y & z \end{smallmatrix}\right) \in V$. For any $r \in R$, we have

$$\begin{pmatrix} rx & ru \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x & u \\ y & z \end{pmatrix} \in V$$

and so $ru \in U$. Consequently, $U <_l R$. Since

$$\begin{pmatrix} 0 & 0 \\ ux & u^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ u & 0 \end{pmatrix} \begin{pmatrix} x & u \\ y & z \end{pmatrix} \in V,$$

it is a nilpotent matrix of T and so u^2 is a nilpotent element of R. Hence U is nil. Thus, all the U_i are nil left ideals of R contained in L. From $T = \sum V_i$ it follows that $L = \sum U_i$. Therefore, $L = A_R(L)$ and we get (iii).

Assume now that Koethe's problem has a negative solution. Then there is a nil-semi-simple A-algebra R over a field F by Theorem 1.6. Suppose that $A_R(L) = L$ for every left ideal L of R. Let r be a non-nilpotent element of R and take $L = R^*r$, a left F-ideal of R. Since $L = A_R(L)$, there are nil left F-ideals $L_1, \dots, L_n$ of R contained in L such that $r = l_1 + \cdots + l_n$, where $l_i \in L_i$. Now, each $l_i = (\alpha_i + a_i)r$ for some $\alpha_i \in F$ and $a_i \in R$. Note that R is a Jacobson radical algebra since $R = A(R)$ by assumption. Hence, if $\alpha_i \neq 0$ for some i, then there are $\beta_i \in F$ and $b_i \in R$ such that $(\beta_i + b_i)(\alpha_i + a_i) = 1$. Consequently, $r = (\beta_i + b_i)l_i \in L_i$, which is impossible as r is non-nilpotent. Thus, all $\alpha_i = 0$ and so $(1 - a_1 - \cdots - a_n)r = 0$. Since R is a Jacobson radical algebra, we get that $r = 0$, a contradiction. This proves that (iii) does not hold. The proof is now complete. □

Now we will show that Problem 2.6 has a positive answer.

Theorem 3.2. *Koethe's problem has a positive solution if and only if $A(LR^*) = LR^*$ for arbitrary $L <_l R$ with $A(L) = L$.*

Proof. The 'only if' part is clear. Thus, suppose that Koethe's problem has a negative solution. Then, by Theorem 1.6, there exists a non-nil A-algebra S over a field F. Let R be the subalgebra $\left(\begin{smallmatrix} S & S^* \\ S & S \end{smallmatrix}\right)$ of $M_2(S^*)$ over F. In the proof of [**2**, Theorem 4.6] it was shown that $A(R) = M_2(S)$ for such an S. Note that $L = \left(\begin{smallmatrix} 0 & S^* \\ 0 & S \end{smallmatrix}\right) <_l R$ and $I = \left(\begin{smallmatrix} 0 & S^* \\ 0 & 0 \end{smallmatrix}\right) \lhd L$. Since $L/I \simeq S$, $A(S) = S$ and $I^2 = 0$, we have that $A(L) = L$. Note that $LR^* = \left(\begin{smallmatrix} S & S^* \\ S^2 & S \end{smallmatrix}\right)$. By Proposition 2.2 (1), we have $A(LR^*) \subseteq A(R) = M_2(S)$, so $A(LR^*) \neq LR^*$. □

We now obtain some results related to Problem 1.5.

Proposition 3.3. *Let R be a non-zero Jacobson radical algebra over a field and let $r \in R$. If r is a sum of nilpotent elements in R^*r, then $l_R(r) \neq 0$.*

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Proof. If $r = 0$, then $l_R(r) = R \neq 0$. So assume that $r \neq 0$. Since R is a Jacobson radical algebra and $r \neq 0$, we have $r \notin Rr$. Hence, there are $a_1, \dots, a_n \in R^*$, not all in R, such that $r = a_1 r + \cdots + a_n r$ and each $a_i r$ is a non-zero nilpotent element. Suppose that $a_1 \notin R$ and set $b = a_2 + \cdots + a_n$. Then, $(1 - a_1 - b)r = 0$. Since R is a Jacobson radical algebra over a field, all elements in $R^* \setminus R$ are invertible in R^*. Hence, if $1 - a_1 - b \notin R$, then it is invertible in R^* and so $r = 0$, a contradiction. Thus, $1 - a_1 - b \in R$. However, $a_1 \notin R$, so $1 - b \notin R$ either, and hence $1 - b$ is invertible in R^*. Since $(1 - b)r = a_1 r$ is a non-zero nilpotent element, there exists an integer $m > 1$ such that $((1 - b)r)^m = 0$ but $((1-b)r)^{m-1} \neq 0$. Hence, $t = ((1-b)r)^{m-1}(1-b) \neq 0$. However, $tr = 0$, so $l_R(r) \neq 0$. □

We shall need the following lemma, which slightly generalizes [**4**, Lemma 3.11] and can be obtained by applying essentially the same arguments.

Lemma 3.4. *Let R be a ring, $r \in R$ and $L <_l R$. Then:*

(1) *$rl_R(r) \lhd rR$, $r_R(r)r \lhd Rr$ and $(rl_R(r))^2 = (r_R(r)r)^2 = 0$;*

(2) *the map $rx + rl_R(r) \to xr + r_R(r)r$ for $x \in L$ is an isomorphism of $(rL + rl_R(r))/rl_R(r)$ onto $(Lr + r_R(r)r)/r_R(r)r$.*

It is evident that if L is a nil left ideal of a ring R, then all elements in LR^* are sums of nilpotent elements. The following theorem shows, in particular, that the answer to Problem 1.5 is positive if all left ideals of LR^* have this property.

Theorem 3.5. *Let R be a non-zero Jacobson radical ring. Suppose that, for every $L <_l R$, each element of L is a sum of nilpotent elements in L. Then, $l_R(r) \neq 0$ and $r_R(r) \neq 0$ for each $r \in R$.*

Proof. Suppose first that R is an algebra over a field F and take any $r \in R$. By assumption, r is a sum of nilpotent elements in $R^* r$. Hence, $l_R(r) \neq 0$ by Proposition 3.3. By Lemma 3.4 we see that $rR^*/rl_{R^*}(r) \simeq R^* r/r_{R^*}(r)r$ and $(rl_{R^*}(r))^2 = (r_{R^*}(r)r)^2 = 0$. Hence, r is also a sum of nilpotent elements in rR^*. Thus, R^{op}, the ring opposite to R, and r satisfy the assumptions of Proposition 3.3. Consequently, $r_R(r) = l_{R^{\mathrm{op}}}(r) \neq 0$.

Suppose now that R is a ring and $r \in R$. We will show that both $l_R(r)$ and $r_R(r)$ are non-zero for $r \neq 0$. Assume first that $nr = 0$ for an integer $n > 1$ and n is the smallest possible. Obviously, $nRr = rnR = 0$. Hence, we are done if $nR \neq 0$. Thus, assume that $nR = 0$. If n is a prime number, then R is an algebra over a field and we are done. Thus, assume that there is a prime p such that $n = pk$ for an integer $k > 1$. Let $I = \{x \in R \mid kx = 0\}$. Obviously, $I \lhd R$ and $r \notin I$, by the minimality of n. Write $\bar{R} = R/I$ and $\bar{x} = x + I$ for $x \in R$. Then $p\bar{R} = 0$, so $\bar{R}$ is a non-zero algebra over a field. It is also clear that $\bar{R}$ satisfies the assumption of the theorem. Consequently, $l_{\bar{R}}(\bar{r}) \neq 0$ and $r_{\bar{R}}(\bar{r}) \neq 0$. That is, there are $a, b \in R \setminus I$ such that $ar \in I$ and $rb \in I$. Hence, $ka \neq 0$ and $kb \neq 0$ but $kar = krb = 0$. Thus, $0 \neq ka \in l_R(r)$ and $0 \neq kb \in r_R(r)$, so we are done. Assume next that $nr \neq 0$ for every positive integer n. Let $T = \{x \in R \mid nx = 0 \text{ for some positive integer } n\}$. It is clear that $T \lhd R$, $r \notin T$ and $nx \neq 0$ for every non-zero integer n and every non-zero element $x \in R/T$. It is not hard

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to see that the localization S of R/T at the set of non-zero integers is an algebra over the field F of rational numbers and every left F-ideal of S satisfies the assumption of the theorem. Hence, by the preceding paragraph, there are $a, b \in R \setminus T$ such that $ar \in T$ and $rb \in T$. Now, there is a non-zero integer k such that $kar = krb = 0$. Obviously, $0 \neq ka \in l_R(r)$ and $0 \neq kb \in r_R(r)$. The result follows. □

We know from Theorem 3.1 that if all left ideals of A-rings were A-rings, then Koethe's problem would have a positive solution. However, even Koethe's problem has a negative solution; there are rings all of whose left ideals are A-rings. From Theorem 3.5 we obtain immediately that for such rings we have the following corollary.

Corollary 3.6. *Let R be a non-zero ring such that $L = A(L)$ for every left ideal L of R. Then $l_R(r) \neq 0$ and $r_R(r) \neq 0$ for each $r \in R$.*

In the context of the above theorem and corollary, the following questions arise.

Problem 3.7.

(a) *Suppose that $L <_l R = A(R)$. Is every element in L a sum of nilpotent elements in L?*

(b) *Suppose that $R = \overline{\mathrm{Nil}}(R)$. Is every element in R a sum of nilpotent elements in R?*

(c) *Is $l_R(a) \neq 0$ for every non-zero $a \in \overline{\mathrm{Nil}}(R)$?*

Obviously, Problem 3.7 (c) is more general than Problem 1.5. Since left ideals of $\overline{\mathrm{Nil}}$-rings are $\overline{\mathrm{Nil}}$-rings, Problem 3.7 (b) is more general than Problem 3.7 (a) and from Theorem 3.5 it follows that Problem 3.7 (b) is more general than Problem 3.7 (c).

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