Laser and Particle Beams (2008), *26*, 37–40. Printed in the USA.

doi: 10.1017/S0263034608000050

Ion heating due to ionization and recombination

R.G. EVANS
Plasma Physics Group, Physics Department, Imperial College, London United Kingdom and CLF Rutherford Appleton Laboratory, Chilton, Didcot, United Kingdom

(Received 20 September 2007; Accepted 23 January 2008)

Abstract

The charge on plasma ions may fluctuate due to random ionization and recombination processes. The resulting motion has a time dependent Hamiltonian and in simple cases, the ions steadily gain energy. Quantitative rates are estimated for some cases involving high Z plasmas.

Keywords: Fluctuations; Ion heating; Ionization

1. INTRODUCTION

It is very common in plasma physics, particularly in plasmas of moderately high atomic number, for the average charge state $<Z> = n_e/n_i$ to be somewhat less than the nuclear charge Z_{nuc}. For instance, Neon is only fully ionized for temperatures of more than 500 eV or so, and Xenon is only fully ionized above around 50 keV. It is usual to distinguish between Z_{nuc} and $<Z>$ in the description of the plasma, but much less common (Gitomer, 1986; Epperlein *et al.*, 1994) to recognize that plasmas simultaneously contain ions of many different charge states, which each have different dynamics and collision frequencies.

In reality, any one ion will not have a constant charge state, but its charge will fluctuate because of ionization and recombination events, and each charge state will have slightly different dynamics in the plasma electromagnetic fields. In many ways, the charge fluctuations are analogous to the electron-ion collisions that result in inverse bremsstrahlung absorption of electromagnetic wave energy by electrons. If the electrons are initially hotter than the ions or there is an external source driving plasma waves, then it is possible for the ions to steadily gain energy because of the random changes of charge state.

Somewhat similar effects occur in dusty plasmas (Jana *et al.*, 1993). Where the electric charge on dust particles fluctuates, energy is not conserved since the Hamiltonian

$$H = \frac{p^2}{2M} + Ze\Phi,$$

is time dependent through the fluctuations of Z.

2. MOTION IN AN ION-ACOUSTIC WAVE

We consider an idealized case where the plasma ions of mass M may exist in two charge states Z_1 and Z_2 ($Z_2 > Z_1$) and for simplicity, we consider that the ionization rate ν_{ion} from Z_1 to Z_2 is the same as the recombination rate ν_{rec} from Z_2 to Z_1, so that the equilibrium densities of Z_1 and Z_2 are equal. We also define $\nu = \nu_{\text{ion}}/2 = \nu_{\text{rec}}/2$ which is the rate for a complete cycle of ionization and recombination.

Consider the motion of the ion in charge state Z_1, in a monochromatic ion acoustic wave described by its longitudinal electric field $E = E_0 \cos(\omega t - kx)$. Since the wave is longitudinal and assumed to be small amplitude then the density fluctuation is

$$\frac{\delta n}{n} = \frac{v_o}{v_{ph}},$$

where

$$v_o = \frac{Z_1 e E_0}{M\omega},$$

Address correspondence and reprint request to Roger G. Evans, Plasma Physics Group, Physics Department, Imperial College, Prince Consort Road, London SW7 2AZ. E-mail: r.g.evans@physics.org

is the oscillation velocity and

$$v_{ph} = \frac{\omega}{k} = \left(\frac{Z_1 k_B T_e}{M}\right)^{1/2},$$

is the ion acoustic speed, k_B being Boltzmann's constant. Initially the ion oscillation is around the point $x = 0$ and has no net drift so that

$$v = (Z_1 e E_0 / M\omega)\sin(\omega t).$$

We now consider what happens when the ion of charge state Z_1 is ionized to charge state Z_2 at some phase Φ_1 of the electric field. Ionization generally occurs *via* electron impact or by photoionization, in either case; there is little change in the ion momentum. So immediately after the ionization event, we may write the ion velocity as

$$v = \frac{Z_2 e E_0}{M\omega}\sin\omega t + \frac{(Z_1 - Z_2)eE_0}{M\omega}\sin\Phi_1.$$

The first term corresponds to the oscillatory motion in the new charge state and the second to an average drift velocity. If the ion of charge Z_2 now recombines with an electron at phase Φ_2 to produce an ion of charge Z_1, again with little change in the ion momentum then the drift velocity after one ionization-recombination cycle is

$$v_D = (Z_1 - Z_2)\frac{eE_0}{M\omega}\sin\Phi_1 + (Z_2 - Z_1)\frac{eE_0}{M\omega}\sin\Phi_2.$$

The velocity increments do not cancel out since they occur at different phases of the wave. We rewrite this last equation as

$$v_D = 2\left(\frac{Z_1 - Z_2}{Z_1}\right) v_o \cos(\Phi)\sin\left(\frac{\delta\Phi}{2}\right), \quad (1)$$

where

$$\Phi = (\Phi_1 + \Phi_2)/2 \ \text{and} \ \delta\Phi = \Phi_1 - \Phi_2.$$

After many uncorrelated cycles of this process, the drift velocity will perform a random walk so that the expectation value of the energy increases linearly with time.

$$\langle v_D^2\rangle = 4\nu t\left(\frac{Z_2 - Z_1}{Z_1}\right)^2 v_o^2 \langle\cos^2\Phi\rangle\left\langle \sin^2\left(\frac{\delta\Phi}{2}\right)\right\rangle.$$

The absolute phase Φ is random so $<\cos^2\Phi> = 1/2$ and we evaluate the term in $\delta\Phi = \omega\tau$ by taking the time delay τ between ionization and recombination to be exponentially distributed so that we need to evaluate

$$\int_0^\infty \nu\exp(-\nu\tau)\sin^2\left(\frac{\omega\tau}{2}\right)d\tau = \frac{1}{2}\left(\frac{\omega^2}{\nu^2 + \omega^2}\right),$$

giving our final result for the rate of increase of ion energy

$$\left\langle \frac{1}{2}M v_D^2\right\rangle = \nu t\left(\frac{Z_1 - Z_2}{Z_1}\right)^2 \frac{1}{2}M v_o^2\frac{\omega^2}{\nu^2 + \omega^2}. \quad (2)$$

For small values of ν/ω, the heating rate is proportional to ν while for $\nu >> \omega$ the heating rate falls as ν^{-1}, since the successive ionization and recombination events are strongly correlated. This calculation is strictly valid only for small ion wave amplitudes $(v_o/v_{ph}) = (\delta n/n) << 1$, since we assume that the ionization and recombination rates are independent of the phase of the waves (McWhirter & Wilson, 1974).

3. MOTION IN A STATIC POTENTIAL WELL

We consider the motion of an ion initially of charge Z_1 in a potential well $\Phi = ax^2$ with $x = x_1\sin\omega t$ and initial energy $E_1 = Z_1 a x_1^2$. If the ion changes its charge state to Z_2 at phase Φ_1 with no change in momentum, then the new energy is

$$E_2 = E_1 + (Z_2 - Z_1)a x_1^2\sin^2\Phi_1.$$

The frequency and phase of the oscillation in the new charge state are different to their previous values and the exact algebra becomes lengthy. If we restrict the analysis to ionization and recombination rates smaller than the oscillation period, we can assume all phases to be randomly distributed, and neglect the phase change at ionization and recombination. If the ion now returns to charge Z_1 at phase Φ_2 of its new oscillation and its new energy is E_3, then we find

$$\frac{E_3}{E_1} = \cos^2\Phi_1\cos^2\Phi_2 + \sin^2\Phi_1\sin^2\Phi_2 + \frac{Z_2}{Z_1}\sin^2\Phi_1\cos^2\Phi_2 + \frac{Z_1}{Z_2}\cos^2\Phi_1\sin^2\Phi_2,$$

so that with randomly distributed Φ_1 and Φ_2:

$$\left\langle\frac{E_3}{E_1}\right\rangle = \frac{1}{2} + \frac{1}{4}\left(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right) > 1.$$

This is an exponential increase of the mean energy with each ionization recombination cycle. It is possible because the motion of a particle with time dependent charge corresponds to a time dependent Hamiltonian and energy is in general not conserved. Truly electrostatic structures in plasmas are rare, but the late stage evolution of many plasma instabilities such as the two-stream instability gives rise to long-lived density structures. In most cases, the density structures correspond to the expulsion of electrons by the ponderomotive force associated with wave energy, and so do not provide confining potentials for the ions. Quasi-resonant particles in a wave see an almost static potential and will experience this exponential growth in energy until they are shifted away from resonance.

4. LASER DRIVEN STIMULATED BRILLOUIN SCATTERING

Intense lasers propagating through moderately uniform plasmas are subject to stimulated scattering from both electron oscillations (Raman scattering) and ion-acoustic oscillations (Brillouin scattering). Both instabilities are potentially damaging to laser driven inertial confinement fusion (ICF), the Raman scattering generates high energy electrons and the Brillouin scattering can lead to high reflectivity and low absorption of the laser light.

Typically Brillouin scattering requires the condition $ZT_e >> T_i$ so that the phase velocity of the ion-acoustic waves $c_s = (Zk_BT_e/M_i)^{1/2}$ is well above the ion thermal velocity in order to minimize ion Landau damping.

A recent experimental investigation (Stevenson, 2004) of Brillouin scattering in gas mixtures observed that even small amounts of high Z gases such as Xenon could dramatically reduce the amount of Brillouin backscatter. Here we give a plausible argument that ionization heating of the Xenon ions could contribute to this observation.

The scattering experiment used 0.53 μm laser light and quotes a density around 0.25 of critical density ($n_e \sim 10^{21}$ cm^{-3}) and a temperature around 1.0 keV. The Brillouin backscatter ion acoustic wave has a wavelength of half the laser wavelength and a frequency $\nu_{ia} \sim 1.6 \times 10^{10}\ s^{-1}$.

Simple estimates of ionization equilibrium based on Saha's equation show that Xenon would be around 30 times ionized and have a spread of a few adjacent charge states so that $(Z_2 - Z_1)/Z_1$ in our simple model would be around 0.1.

For the case of equal populations in two adjacent charge states, we can estimate the ionization and recombination rates to be equal, and a simple formula (McWhirter, 1965) for the radiative recombination rate α (which dominates three body recombination at these densities) gives $\alpha \sim 2.7 \times 10^{-13}\ Z^2\ T_e^{-1/2}\ cm^3 s^{-1}$. Evaluating this for $Z = 30$, $n_e = 10^{21} cm^{-3}$ and $T_e = 1.0$ keV gives recombination (and therefore ionization) rates $\nu \sim 10^{10} s^{-1}$.

We see from Eq. (2) that if we assume an ion-acoustic wave amplitude of $(\delta n/n) = (v_o/v_{ph}) = 0.1$, then over the 1 ns duration of the laser pulse, there will be significant ion heating ($v_{th} \sim v_{osc} \sim 0.1\ v_{ph}$). The atomic rates used to calculate this effect are approximate, but are likely to be an underestimate since we have not included significant rates such as di-electronic recombination, which are not very amenable to simple generalized formulae. Raising the above rates or increasing the assumed ion acoustic wave amplitude increases the ion heating to the point where ion Landau damping becomes important ($v_{th} \sim 0.3\ v_{ph}$). A more precise calculation of the atomic rates would be needed to make definitive statements concerning the importance of ionization heating in this particular experiment.

5. ELECTRON ION EQUILIBRATION

Normally electron ion temperature equilibration proceeds *via* the mechanism of screened binary collisions giving the Spitzer rate for $T_e >> T_i$

$$\nu_{ie} = 3.2 \times 10^{-9} Z^2 \ln \Lambda n_e T_e^{-3/2} \mu^{-1},$$

where $\mu = M/m_p$ and m_p is the proton mass. The process is slower than other collision rates because the large electron ion mass ratio gives only a small energy transfer in each collision.

If $ZT_e >> T_i$, the thermal plasma will have an equilibrium excitation of ion-acoustic waves with dispersion relation $\omega_{ia} = kc_s$. The energy in the ion-acoustic modes is calculated by a procedure analogous to the simple derivation of the ratio of kinetic to electrostatic energy in the Langmuir oscillation of electrons.

If the plasma is considered to be in a box of side L then modes exist for $k = 2\pi N/L$ and the maximum value of k is given by the Landau damping limit $kc_s = \omega_{pi}$ where $\omega_{pi}^2 = 4\pi n_i Z e^2/M$. The energy driving the ion waves is due to the electron thermal energy so we now ascribe an energy of k_BT_e to each ion mode giving after a little manipulation exactly the same result as for the Langmuir modes: (Electron Kinetic Energy/Ion Acoustic wave energy) = $n_e\lambda_D^3$ where λ_D is the electron Debye length.

It is only when plasmas become non-ideal (few particles per Debye sphere) that there is significant energy in the electrostatic modes. To use this result with Eq. (2), we need an expression for the average oscillatory velocity of the ion-acoustic modes, and we note that for a mode of wave number k, $v_{osc}^2 \sim k^{-2}$ while the density of modes $\sim k^2$ so that the mean square oscillation velocity is

$$M\langle v_{osc}^2\rangle = \frac{Zk_BT_e}{n_e\lambda_D^3} \text{ or } \frac{v_{osc}^2}{c_s^2} = \frac{1}{n_e\lambda_D^3}.$$

In strongly coupled plasmas of high Z, around solid density and at temperatures of hundreds of eV the ionization heating can be important but it is in a regime where the Spitzer theory is already of limited applicability.

6. CONCLUSION

The ion dynamics driven by stochastic changes of ion charge lead to a novel ion heating mechanism, which may need to be included in the analysis of some high Z plasma experiments.

The simple model presented here would need to be extended for very large amplitude waves where ionization and recombination rates may change with the density (and in some cases temperature) changes associated with the waves and where more than two ionization stages are involved.

REFERENCES

Epperlein, E.M., Short, R.W. & Simon, A. (1994). Transport and sound waves in plasmas with light and heavy ions. *Phys. Rev. E* **49**, 2480–2483.

Gitomer, S.J., Jones, R.D., Begay, F., Ehler, A.W., Kephart, J.F. & Kristal, R. (1986). Fast ions and hot electrons in the laser–plasma interaction. *Phys. Fluids* **29**, 2679–2688.

Jana, M.R., Sen, A. & Kaw, P.K. (1993). Collective effects due to charge-fluctuation dynamics in a dusty plasma. *Phys. Rev. E* **48**, 3930–3933.

McWhirter, R.W.P. & Wilson, R. (1974). The spectroscopy of plasmas carrying sound waves. *J. Phys. B Atom. Mol. Phys*. **7**, 1588–1601.

McWhirter, R.W.P. (1965). Spectral Intensities. In *Plasma Diagnostic Techniques* (Huddlestone, R.H. & Leonard, S.L., eds.). New York: Academic Press.

Stevenson, R.M., Suter, L.J., Oades, K., Kruer, W., Slark, G.E., Fournier, K.B., Meezan, N., Kauffman, R., Miller, M., Glenzer, S., Niemann, C., Grun, J., Davis, J., Back, C. & Thomas, B. (2004). Effects of plasma composition on backscatter, hot electron production, and propagation in underdense plasmas *Phys. Plasmas* **11**, 2709–2715.

https://doi.org/10.1017/S0263034608000050 Published online by Cambridge University Press