

DETECTING SCAPEGOAT EFFECTS IN THE RELATIONSHIP BETWEEN EXCHANGE RATES AND MACROECONOMIC FUNDAMENTALS: A NEW APPROACH

LORENZO POZZI

Erasmus University Rotterdam & Tinbergen Institute

BARBARA SADABA

Bank of Canada

This paper presents a new testing method for the scapegoat model of exchange rates. A number of steps are implemented to determine whether macro-fundamentals are scapegoats for the evolution of exchange rates. Estimation is conducted using a Bayesian Gibbs sampling approach applied to eight countries (five developed and three emerging) versus the USA over the period 2002Q1–2014Q4. The macro-fundamentals that we consider are real GDP growth, the inflation rate, the long-run nominal interest rate, and the current account to GDP ratio. We calculate the posterior probabilities that these macro-fundamentals are scapegoats. For the inflation rate, these probabilities are considerably higher than the imposed prior probabilities of $\frac{1}{2}$ in five out of eight countries (in particular, the Anglo-Saxon economies).

Keywords: Exchange Rate, Scapegoat Model, Unobserved Component, State Space, Bayesian

1. INTRODUCTION

One of the major puzzles in international macroeconomics is the difficulty to link exchange rates to macroeconomic fundamentals such as money supplies, interest rates, and outputs, that is, the so-called “disconnect puzzle” of exchange rates [see Obstfeld and Rogoff (2000)]. The “disconnect puzzle” manifests itself in a variety of ways, among which the lack of out-of-sample predictability of exchange rates and the instability of the ex-post relationship between exchange rates and fundamentals are probably the most striking. The latter manifestation of

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the “disconnect puzzle” constitutes the focus of this paper. The instability of structural parameters has been linked to the poor performance of exchange rate models both in and out of sample [see Meese and Rogoff (1983a,b, 1988), Bacchetta et al. (2010), and Rossi (2006, 2013)].¹ With respect to the reasons for instability, Cheung and Chinn (2001) argue, based on a survey of US foreign exchange traders, that the instability of the impact of macro-fundamentals on exchange rates is driven by the fact that traders regularly change the weight that they attach to macro-fundamentals. Sarno and Valente (2009) conduct an exchange rate model selection procedure that allows them to select the best model in every period out of all possible combinations of fundamentals. They report frequent changes in the optimal model implying frequent shifts in parameters.

The scapegoat theory of exchange rates [see Bacchetta and van Wincoop (2004, 2009, 2012, 2013)] provides a formal theoretical framework for many of these ideas and hence provides a potential explanation for the weak link between macro-fundamentals and exchange rates.² In a scapegoat model, economic agents form rational expectations but are assumed to have incomplete information. Economic agents do not observe some economic variables in the economy (e.g. money demand shifts, real exchange rate shocks,...) and do not know the structural parameters on the macro-fundamentals that drive the exchange rate. Therefore, they form expectations about these structural parameters based on an observed “signal” which typically depends on the observed level of the exchange rate, the observed interest rate differential and the discount factor of the agents (i.e. the discount rate used by investors to discount future observed and unobserved variables). Because of imperfect information, they can be rationally confused and can, as a result, rationally attribute changes in the exchange rate to changes in the observed macro-fundamentals, while in fact these changes are caused by the unobserved variables. Therefore, economic agents may erroneously give too much weight to certain observed macro-fundamentals in the determination of the exchange rate. In this model, it is expectations of the structural parameters on macro-fundamentals that are driving exchange rates rather than the structural parameters themselves. As agents are assumed to frequently update their expectations about the impact of macro-fundamentals on exchange rates, the theory can potentially explain the highly unstable observed relationship between macroeconomic fundamentals and exchange rates. The model has been recently tested empirically by Fratzscher et al. (2015) using survey scores reported biannually by Consensus Economics. These reflect the weight investors attach to certain macro-fundamentals in the determination of the exchange rate in a given period. They regress changes in the exchange rate of 12 currencies on fundamentals and on fundamentals interacted with these scores. They find that the interaction terms have a significant impact on exchange rates, hence providing evidence in favor of the scapegoat model.

In this paper, we propose an alternative empirical testing strategy for the scapegoat theory of exchange rates. More specifically, the contribution of the paper

is to test for scapegoat effects using the exact structural exchange rate equation implied by the scapegoat model instead of estimating ad-hoc empirical specification of the type considered by Fratzscher et al. (2015). This approach should contribute to tighten the link between the theory on scapegoats and the empirical testing of this theory because we explicitly test for scapegoat effects under the restrictions imposed on the data by the theoretical model. These restrictions are twofold. First, in line with existing empirical evidence that suggests that—for instance because of carry trade strategies which are very common in currency markets—there is important time variation in the deviation from uncovered interest rate parity (UIRP) [see e.g. Carriero (2006) and Byrne and Nagayasu (2012)], we estimate the time-varying exchange risk premium or deviation from the UIRP condition that is present in the model. Second, we explicitly estimate the signal extraction problem that is central to the scapegoat model using values for the discount factor reported previously in the literature.³

The scapegoat model that we consider follows the model presented by Bacchetta and van Wincoop (2013), while allowing for time-varying structural parameters (i.e. random walks) on the macro-fundamentals.⁴ We further incorporate to the model a time-varying deviation from the UIRP condition in the derived exchange rate equation.⁵ The model leads to an exchange rate equation that consists of four terms. First, a term that captures the standard impact of the macro-fundamentals on the exchange rate; this term consists of the macro-fundamentals interacted with time-varying structural parameters. Second, a term that captures the impact of macro-fundamentals as scapegoats. This term consists of the macro-fundamentals interacted with the expectations about the time-varying structural parameters. Third, a term related to the unobserved component, which reflects unobserved relative money demand shocks and/or real exchange rate shocks. Fourth, a term related to the time-varying deviation from the UIRP condition or the exchange rate risk premium.

The exchange rate equation is sufficiently complex that an estimation approach in different steps is required. First, the time-varying deviation from the UIRP condition or exchange rate risk premium is estimated using a state-space approach applied to the observed difference between the change in the exchange rate and the interest rate differential. Second, the unobserved time-varying structural parameters on the macro-fundamentals and the unobserved component of the model are estimated using a state-space system applied to the observed “signal” in the model which depends on the level of the exchange rate, on the interest rate differential, and on the discount factor. Third, the scapegoat terms in the model’s exchange rate equation, that is, the expectations of the structural parameters interacted with the macro-fundamentals, are estimated using a regression analysis where the estimation is conditional on the exchange rate risk premium, the structural parameters on the macro-fundamentals, and the unobserved component estimated in the previous steps. Following Fratzscher et al. (2015), we use survey data to proxy the parameter expectations that enter the scapegoat terms.

The estimation in different steps is carried out through a Bayesian Gibbs sampling approach for eight countries versus the USA over the period 2002Q1–2014Q4. We consider five developed economies (Australia, Canada, the euro area, Japan, and the UK) and three emerging countries (Singapore, South Korea, and South Africa). Our choice of macro-fundamentals is based on the availability of corresponding survey data for these fundamentals. More specifically, we incorporate four macro-fundamentals in the estimations that can potentially be scapegoats, that is, the real GDP growth rate (relative to the US), the inflation rate (relative to the US), the long-run nominal interest rate (relative to the USA), and the current account balance to GDP ratio. The applied Gibbs approach is advantageous because the full posterior distributions of parameters and states are calculated in every step and are conditioned upon in the next steps so that both parameter and state uncertainty can fully be taken into account in the estimation of scapegoat effects. Additionally, the Bayesian approach allows for model selection when considering which fundamentals are scapegoats. In particular, we assign binary indicators to each of the potential scapegoat terms in the exchange rate regression [see, e.g. George and McCulloch (1993) and Frühwirth-Schnatter and Wagner (2010)]. These are equal to one if a particular fundamental can be considered a scapegoat and equal to zero if the fundamental does not enter the regression equation as a scapegoat. We sample these binary indicators together with the other parameters using the Gibbs sampler. From the sampled indicators, we compute the posterior probabilities that the included fundamentals are scapegoats.

The results suggest, first, that there is a persistent but stationary time-varying deviation from the UIRP condition or exchange risk premium in all countries considered. Second, we identify a persistent but stationary unobserved component from the “signal” in the model which potentially reflects unobserved quantities such as money demand shocks or real exchange rate shocks. Third, we find that over the sample period the structural parameters on the macro-fundamentals are constant and often close to zero. Fourth, as far as the scapegoat terms in the exchange rate equation are concerned, we calculate posterior probabilities that these macro-fundamentals are scapegoats, and we find, for the inflation rate in five out of eight countries, probabilities that are considerably higher than the imposed prior probabilities of $\frac{1}{2}$. These countries are the three Anglo-Saxon economies (Australia, Canada, and the UK) and South Korea and South Africa. We find little evidence to suggest that the other macro-fundamentals we consider are scapegoats, however.

The paper is structured as follows. Section 2 presents the scapegoat model and derives a testable exchange rate equation from the model. Section 3 shows how to implement the estimation of this equation in a number of steps. Section 4 discusses the choice of macro-fundamentals, the data used, and the Bayesian estimation method (i.e. the outline of the Gibbs sampler and the imposed parameter priors). Section 5 presents and discusses the estimation results of the various steps. Section 6 concludes.

2. THEORY

We consider an interest rate parity condition between a local and a benchmark economy and an equation that contains determinants of the interest rate differential between these economies:

$$E_t(s_{t+1}) - s_t = \bar{i}_t + z_t, \tag{1}$$

$$\bar{i}_t = \mu[s_t - f_t\beta_t - x_t], \tag{2}$$

where s_t is the log nominal exchange rate (expressed as the amount of local currency per unit of benchmark currency), E_t denotes the rational expectations operator conditional on time t information, \bar{i}_t is the short-term nominal interest rate differential between the local country and the benchmark country, z_t is the exchange risk premium or deviation from UIRP, f_t is a $1 \times K$ vector of observed macroeconomic fundamentals with β_t the $K \times 1$ vector of corresponding time-varying parameters, and x_t is an unobserved fundamental or component. As noted by Engel and West (2005), equation (2) may represent the interest rate differential as obtained from the differential in Taylor rules between the local and the benchmark economies. Alternatively, it may represent the interest rate differential as obtained from the reduced-form monetary model of exchange rates, that is, obtained by combining a Purchasing Power Parity condition with money market equilibrium in both countries. In the Taylor rule model, the unobserved component represents a relative shock to the Taylor rules and/or potentially omitted Taylor rule terms. In the monetary model, the unobserved component represents an unobserved relative money demand shock, possibly augmented with a real exchange rate shock. We depart from the standard exchange rate framework of Engel and West (2005) by assuming that some parameters in the model are unknown [see, e.g. Bacchetta and van Wincoop (2009, 2013)]. In particular, we assume that the parameter vector β_t is unknown; that is, we have $E_t(\beta_{kt}) \neq \beta_{kt}$ for $k = 1, \dots, K$. With respect to the parameter μ , we follow Bacchetta and van Wincoop (2009, 2013) and assume that μ is known and constant.

The signal y_t is given by

$$y_t = f_t\beta_t + x_t. \tag{3}$$

The agents know this signal as, from equation (2), we have $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$ and agents observe s_t and \bar{i}_t and they know μ . If β_t were known, agents could infer the value of the unobserved component x_t . Given that β_t is not known, y_t gives an imperfect signal of β_t because of the unobserved component x_t .

Combining equations (1)–(3) and solving forward then gives

$$s_t = (1 - \lambda) \left[y_t + \sum_{j=1}^{\infty} \lambda^j E_t(y_{t+j}) \right] - \lambda \left[z_t + \sum_{j=1}^{\infty} \lambda^j E_t(z_{t+j}) \right]$$

or

$$s_t = (1 - \lambda) \left[f_t \beta_t + x_t + \sum_{j=1}^{\infty} \lambda^j E_t (f_{t+j} \beta_{t+j} + x_{t+j}) \right] - \lambda \left[z_t + \sum_{j=1}^{\infty} \lambda^j E_t (z_{t+j}) \right], \tag{4}$$

where the result is obtained by imposing the transversality condition $\lambda^\infty E_t(s_{t+\infty}) = 0$ and where we define the discount factor λ as $\lambda \equiv \frac{1}{1+\mu}$.

We assume that the unobserved variables x_t and z_t follow AR(1) processes, and that the observed macroeconomic fundamentals f_t and corresponding time-varying unknown parameters β_t follow random walk processes, that is,

$$x_t = \rho_x x_{t-1} + \varepsilon_t^x, \tag{5}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \tag{6}$$

$$f_{kt} = f_{k,t-1} + \varepsilon_{kt}^f \quad k = 1, \dots, K, \tag{7}$$

$$\beta_{kt} = \beta_{k,t-1} + \varepsilon_{kt}^\beta \quad k = 1, \dots, K, \tag{8}$$

where all processes are assumed to be mutually independent. Using these processes in equation (4) then gives

$$s_t = (1 - \Psi) f_t \beta_t + \Psi f_t E_t(\beta_t) + (1 - \Psi) x_t + \Phi z_t, \tag{9}$$

where $\Psi \equiv \frac{\lambda(1-\rho_x)}{1-\rho_x\lambda}$ and $\Phi \equiv -\frac{\lambda}{1-\rho_z\lambda}$. We refer to Appendix B for the derivation. The first term, $(1 - \Psi) f_t \beta_t$, captures the standard impact of macro-fundamentals on the exchange rate s_t via the time-varying structural parameters β_t . The second term, $\Psi f_t E_t(\beta_t)$, captures the impact of macro-fundamentals on s_t through the occurrence of scapegoat effects as captured by the expectations about the unknown parameters, that is, through $E_t(\beta_t)$. The third term, $(1 - \Psi) x_t$, captures the role of the unobserved component x_t . The fourth term, Φz_t , captures the impact of UIRP deviations or exchange rate risk premiums z_t . It should be noted that for the scapegoat effects to enter the model, three conditions must be fulfilled. First, the discount factor λ is nonzero as otherwise $\Psi = 0$ and the scapegoat term drops out of the model. Second, the unobserved variable x_t is stationary; otherwise, if $\rho_x = 1$, we have $\Psi = 0$ and the scapegoat term drops out of the model.⁶ Third, the parameters in β_t are unknown; that is, β_t is different from $E_t(\beta_t)$. The first condition is met as the literature finds that λ is positive and typically close to 1 [see, e.g. Engel and West (2005) and Sarno and Sojli (2009)]. Our results show that the second condition is also met as the estimates that we obtain for ρ_x are well below 1. The third condition constitutes the focus of this paper. In the empirical section we test whether proxies used for the expected parameters on macro-fundamentals $E_t(\beta_t)$ have an impact on the exchange rate.

The estimation of equation (9) is conducted in four steps, which are discussed one by one in the next section.

3. EMPIRICAL IMPLEMENTATION

This section explains how the estimation of equation (9) is implemented. In Section 3.1, the estimation of the UIRP deviation z_t using a state-space approach is discussed. In Section 3.2, we explain how to estimate the time-varying structural parameters β_t as well as the unobserved component x_t from the signal $y_t \equiv s_t - \frac{1}{\mu} \bar{i}_t$ also using a state-space approach. In Section 3.3, we discuss how the parameters Ψ and Φ are estimated. Finally, in Section 3.4, we use the estimates obtained for $z_t, x_t, \beta_t, \Psi,$ and Φ in equation (9) and then estimate the scapegoat term $\Psi f_t E_t(\beta_t)$ using survey data to proxy $E_t(\beta_t)$. We note that the Gibbs sampler approach discussed in Section 4 incorporates the parameter uncertainty of the first three steps into the estimation of equation (9) as the scapegoat effects are calculated conditional on the full posterior distributions obtained for $z_t, x_t, \beta_t, \Psi,$ and Φ .

3.1. Estimating the Exchange Risk Premium z_t

To calculate the time-varying deviation from UIRP or exchange rate risk premium z_t , we estimate a state-space system consisting of the following equations:

$$\Delta s_{t+1} - \bar{i}_t = z_t + \varepsilon_{t+1}^s \quad \varepsilon_{t+1}^s \sim iid(0, \sigma_s^2), \tag{10}$$

$$z_{t+1} = \rho_z z_t + \varepsilon_{t+1}^z \quad \varepsilon_{t+1}^z \sim iid(0, \sigma_z^2), z_1 \sim iid\left(0, \frac{\sigma_z^2}{1 - \rho_z^2}\right). \tag{11}$$

The observation equation, equation (10), equals the interest parity condition, equation (1), in the model. This can be seen by taking expectations in period t from both sides of equation (10) and noting that $E_t(\varepsilon_{t+1}^s) = 0$. It relates the observed variable $\Delta s_{t+1} - \bar{i}_t$ to the unobserved variable z_t . The state equation, equation (11), is equation (6) rewritten for period $t + 1$. We refer to Appendix D for the exact specification of the state-space model. Estimation of this system provides estimates of z_t . Given z_t , we can then calculate ρ_z and σ_z^2 from a simple AR(1) regression on z_t and we can calculate σ_s^2 which is the variance of the error term $\varepsilon_{t+1}^s = \Delta s_{t+1} - \bar{i}_t - z_t$. The estimates of z_t and ρ_z are then used in the estimation of equation (9). Note that we implicitly assume $0 < \rho_z < 1$. If $\rho_z = 0$, no distinct identification of z_t versus ε_{t+1}^s is possible. If $\rho_z = 1$, estimation is possible after adjusting the initialization for z_1 . However, this is not necessary as the literature reports that UIRP deviations tend to be stationary [see, e.g. Carriero (2006) and Byrne and Nagayasu (2012)]. When estimating equations (10) and (11), we do indeed find that $0 < \rho_z < 1$, as we report below.

3.2. Estimating the Time-Varying Structural Parameters β_t and the Unobserved Component x_t

We estimate the time-varying structural parameters β_t using equation (3), where the signal $y_t \equiv s_t - \frac{1}{\mu} \bar{i}_t$ can be calculated from s_t, \bar{i}_t and the known parameter μ . Since, from the model discussed in Section 2 we have $\lambda \equiv \frac{1}{1+\mu}$, we can use a value

for μ obtained from estimates reported in the literature for λ . Sarno and Sojli (2009) report an average monthly discount factor of 0.989, which then amounts to setting $\lambda = 0.967$ in quarterly data.⁷ This value for λ implies setting $\mu = 0.034$.

When y_t is calculated, we estimate the following state-space model to obtain estimates for the time-varying structural parameters β_{kt} where $k = 1, \dots, K$,

$$(1 - \rho_x L)y_t = (1 - \rho_x L)f_t \beta_t + \varepsilon_t^x \quad \varepsilon_t^x \sim iid(0, \sigma_x^2), \tag{12}$$

$$\beta_{k,t+1} = \beta_{kt} + \varepsilon_{k,t+1}^\beta \quad \varepsilon_{k,t+1}^\beta \sim iid(0, \sigma_{\beta_k}^2), \beta_{k1} \sim iid(0, 10^6), \tag{13}$$

where equation (12) is the observation equation, which relates the observed signal y_t to the unobserved states β_t . Equation (12) equals equation (3) premultiplied by $(1 - \rho_x L)$ (with L the lag operator), a transformation that guarantees that the observation equation has a noise shock ε_t^x as, from equation (5), we have $(1 - \rho_x L)x_t = \varepsilon_t^x$. The state equation, equation (13), is equation (8) rewritten for period $t + 1$. Since the state β_{kt} follows a random walk, its initialization is diffuse. We refer to Appendix D for the exact specification of the state-space model. Estimation of this system provides estimates of β_t . Given these, we can calculate estimates for the variances $\sigma_{\beta_k}^2$. Estimates for the unobserved component x_t are then obtained by noting that $x_t = y_t - f_t \beta_t$. Given estimates for x_t , we can then calculate ρ_x and σ_x^2 from an AR(1) regression on x_t . Note that estimates for ρ_x should be smaller than 1 because, as noted in Section 2, a non-stationary x_t implies that scapegoat effects drop out of the model. As reported below, we do indeed find that $0 < \rho_x < 1$. The obtained estimates of β_t and x_t are then used in the estimation of equation (9).

3.3. Estimating the Parameters Ψ and Φ

The parameters Ψ and Φ in equation (9) are given by $\Psi \equiv \frac{\lambda(1-\rho_x)}{1-\rho_x \lambda}$ and $\Phi \equiv -\frac{\lambda}{1-\rho_x \lambda}$, respectively. Hence, to estimate Ψ and Φ we need estimates for ρ_z , ρ_x , and λ . Estimates for ρ_z and ρ_x are obtained when estimating z_t and x_t , respectively, as detailed in Sections 3.1 and 3.2. For λ , as noted in Section 3.2, we use the estimates reported by Sarno and Sojli (2009) and set $\lambda = 0.967$. When calculating the posterior distributions of Ψ and Φ , we keep λ fixed so that the posteriors of Ψ and Φ incorporate only the dispersion contained in the posterior distributions of ρ_x and ρ_z , respectively. We find, however, that our results are robust to imposing slightly different values for the discount factor λ .⁸

3.4. Estimating the Scapegoat Effects $E_t(\beta_t)$

Using the estimates obtained for z_t , x_t , β_t , Ψ , and Φ in Sections 3.1–3.3, we rewrite equation (9) as,

$$\tilde{s}_t = \tilde{f}_t E_t(\beta_t), \tag{14}$$

where $\tilde{s}_t \equiv s_t - (1 - \hat{\Psi})f_t\hat{\beta}_t - (1 - \hat{\Psi})\hat{x}_t - \hat{\Phi}\hat{z}_t$ and $\tilde{f}_t \equiv \hat{\Psi}f_t$. Upon noting that \tilde{f}_t and $E_t(\beta_t)'$ are $1 \times K$ vectors, we can write

$$\tilde{s}_t = \sum_{k=1}^K E_t(\beta_{kt})\tilde{f}_{kt}, \tag{15}$$

where $k = 1, \dots, K$. Following Fratzscher et al. (2015), we proxy the scapegoat effects $E_t(\beta_{kt})$ by setting $E_t(\beta_{kt}) = \phi_k \tau_{kt}$ for $k = 1, \dots, K$ where τ_{kt} is a survey outcome denoting the weight attached to fundamental k by investors in period t and where ϕ_k captures the impact of τ_{kt} on the exchange rate.⁹ Assuming that τ_{kt} is a good proxy for the scapegoat effect $E_t(\beta_{kt})$, if the macro-fundamental f_{kt} functions as a scapegoat in the exchange rate determination, we should find a nonzero ϕ_k for this fundamental. Hence, equation (15) becomes

$$\tilde{s}_t = \sum_{k=1}^K \phi_k \tau_{kt} \tilde{f}_{kt}. \tag{16}$$

We then add an intercept and error term to the equation, which gives

$$\tilde{s}_t = c' + \sum_{k=1}^K \phi_k \tau_{kt} \tilde{f}_{kt} + \varepsilon_t, \tag{17}$$

where c' is a constant and ε_t is a zero mean error term. Next, as a model selection device, we add binary indicators δ_k to each of the K scapegoat terms [see George and McCulloch (1993) and Frühwirth-Schnatter and Wagner (2010)]. If fundamental k can be considered a scapegoat, then $\delta_k = 1$. Otherwise, $\delta_k = 0$. From the sampled indicator δ_k (for $k = 1, \dots, K$) we can then calculate the posterior probability that fundamental k is a scapegoat. Our estimable test equation is now given by

$$\tilde{s}_t = c' + \sum_{k=1}^K \delta_k \phi_k \tau_{kt} \tilde{f}_{kt} + \varepsilon_t. \tag{18}$$

Finally, if the model fits the data well, the error term ε_t should be *iid*. Our results suggest, however, that there is residual autocorrelation. This is not surprising since we include only a limited number of fundamentals in the equation, that is, fundamentals k for which we have survey data τ_{kt} available to proxy $E_t(\beta_{kt})$. Calculated autocorrelation and partial autocorrelation functions applied to estimates for ε_t obtained under the assumption that ε_t is *iid* suggest that, for all currencies, there is substantial first-order autocorrelation of the autoregressive form (see Tables C1 and C2 in Appendix C). To deal with this when estimating equation (18), we explicitly model this autocorrelation so that we have

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \varepsilon_t^* \qquad \varepsilon_t^* \sim iid(0, \sigma_\varepsilon^2). \tag{19}$$

Upon multiplying both sides of equation (18) by $(1 - \rho_\varepsilon L)$, we obtain

$$(1 - \rho_\varepsilon L)\tilde{s}_t = c + \sum_{k=1}^K \delta_k \phi_k (1 - \rho_\varepsilon L)\tau_{kt} \tilde{f}_{kt} + \varepsilon_t^*, \tag{20}$$

where $c = (1 - \rho_\varepsilon L)c' = (1 - \rho_\varepsilon)c'$ and where the regression error term ε_t^* is now an *iid* shock so that Bayesian OLS can be applied to estimate this equation. Technical details are provided in Appendix D.

4. ESTIMATION METHOD

In this section, we discuss the macroeconomic fundamentals included in f_t that could potentially be scapegoats. Then, we discuss the data used and its sources. Finally, we elaborate on the Bayesian estimation method, that is, we discuss the Gibbs sampler and the assumed parameter priors.

4.1. Choice of Macroeconomic Fundamentals f_t that can be Scapegoats

We include macroeconomic fundamentals in f_t that can be expected to have an impact on both the interest rate differential as given by equation (2) and the exchange rate as given by equation (9). Additionally, as these variables are the ones that can become scapegoats according to the model—that is, the parameters β_t on f_t are unknown, and therefore we can have $E_t(\beta_t) \neq \beta_t$ —we choose to include macroeconomic fundamentals in f_t for which proxies are available for $E_t(\beta_t)$. As detailed in Section 4.2, following Fratzscher et al. (2015), we proxy $E_t(\beta_t)$ by survey data from Consensus Economics.¹⁰ As such, we use the following fundamentals in f_t for which survey data from Consensus Economics are available. First, the real GDP growth rate differential between the local and benchmark country, \bar{g}_t . Second, the inflation rate differential between the local and benchmark country, $\bar{\pi}_t$. Third, the long-term nominal interest rate differential between the local and benchmark country, \bar{i}_t^L .¹¹ Fourth, the current account balance to GDP ratio of the local country, ca_t (where the latter is not considered in deviation from the benchmark country).^{12,13}

4.2. Data

We use quarterly data over the period 2002Q1–2014Q4. Our sample period is determined by the availability of the survey data used for the variable τ_{kt} in the model. We discuss these surveys in more details below. We conduct estimations using data for five developed or industrialized economies (Australia, Canada, the euro area, Japan, and the UK) and for three emerging economies (Singapore, South Korea, and South Africa). The data for exchange rates and all macro-fundamentals used in the estimations, with the exception of the long-term interest rate, are taken from Oxford Economics via Datastream. Data for the long-run interest rates are taken from national sources.¹⁴

The currencies considered are the Australian dollar (AUD), the Canadian dollar (CAD), the euro (EUR), the Japanese yen (JPY), the UK pound (GBP), the Singapore dollar (SGD), the Korean won (KRW), and the South African rand (ZAR). All exchange rate data are expressed versus the US dollar. In particular, the (log) nominal exchange rate s_t is expressed as (the log of) the amount of local currency that one US dollar is worth.

The rest of the variables are calculated as follows. For \bar{i}_t , we use 3-month interest rates as proxy for the short-term nominal interest rate (relative to the USA).¹⁵ For \bar{g}_t , we use real GDP growth (relative to the USA). For the inflation rate $\bar{\pi}_t$, we use the quarterly change in the consumer price index (relative to the USA). For \bar{l}_t , we use the yield on 10-year government bonds relative to the USA as a proxy for the long-term interest rate differential. For ca_t , we use the ratio of the current account balance to GDP where a positive value of ca_t indicates a surplus.¹⁶

Following Fratzscher et al. (2015), we obtain our measure of the surveys τ_{kt} from a dataset provided by Consensus Economics. These data provide a quantitative measure of the weights attached by financial market participants to macro-fundamentals in the determination of the exchange rate. To collect these data, Consensus Economics conducts a survey at regular 6-month intervals. In these surveys, they ask a group of 40–60 foreign exchange market participants to rank on a quantitative scale the current importance of a number of macro-fundamentals for the determination of exchange rate movements against the US dollar. Whenever possible, the same financial market participants are included in the survey each time. Each macro-fundamental receives a score listed on a scale of 0 (no influence) to 10 (very strong influence). Consensus Economics reports the cross-sectional average, at each point in time, of these experts' scores for each fundamental. These data can be considered representative because of two reasons. First, the views of the group of experts included in the survey are highly representative of overall market views since these views are formed by asset managers and investment managers (from major financial institutions), treasury executives, corporate planners, central bankers, and government officials. The majority of these experts are located in important financial centers. Second, by conducting direct interviews of front-line market participants, accurate real-time expectations of investors are obtained which represent market sentiments in the most direct way.

The survey scores are available biannually over our sample period, for all currencies in our sample and for the macro-fundamentals included in f_t , that is, for \bar{g}_t , $\bar{\pi}_t$, \bar{l}_t , and ca_t . In order to obtain quarterly series to match the frequency of the rest of our data over the period 2002Q1–2014Q4, we use linear interpolation.¹⁷ Figure A1 in Appendix A shows these survey scores for each of the four fundamentals included in our regressions for all eight currencies in our sample. These graphs illustrate the time variation present in the weights attached by investors to the different macro-fundamentals.

While some papers in the literature model exchange rates at the monthly frequency [see, e.g. Bacchetta and van Wincoop (2013) and Fratzscher et al. (2015)],

we prefer to use quarterly data for three reasons. First, this avoids the need to interpolate quarterly macro-data to obtain macro-data at the monthly frequency. Second, since the survey data that are used as proxies for the parameter expectations $E_t(\beta_t)$ are only available on a biannual basis, it makes more sense to consider the model at the quarterly frequency than at the monthly frequency. Third, we can use real GDP—which is available at the quarterly but not at the monthly frequency—to construct \bar{g}_t instead of having to use industrial production.

4.3. Bayesian Estimation

Bayesian methods are used to estimate equation (9). In particular, we use a Gibbs sampling approach, which is a Markov Chain Monte Carlo (MCMC) method used to simulate draws from the intractable joint and marginal posterior distributions of the parameters and unobserved states using only tractable conditional distributions. As described in Section 3, estimation is conducted in different steps with steps 1 and 2 (Sections 3.1 and 3.2) being fully independent, step 3 (Section 3.3) using the results of steps 1 and 2, and step 4 (Section 3.4) using the results obtained in steps 1, 2, and 3. A Bayesian method is advantageous when compared to classical methods like maximum likelihood because the full posterior distributions of parameters and states are calculated in every step and can be used in the next steps. Hence, the parameter uncertainty of the first two steps can be incorporated into step 3, and the parameter and state uncertainty of the first three steps can be incorporated into the estimation of equation (9) in step 4. Additionally, our Bayesian approach allows us to do model selection, that is, compute the posterior probabilities that the macro-fundamentals included in equation (9) are scapegoats. Finally, a Bayesian approach can be conducted without making specific assumptions about the orders of integration of the variables used in the analysis. As a Bayesian analysis relies on sampling posterior distributions rather than on using asymptotic approximations, statistical inference in the presence of non-stationarity variables is less complicated compared to inference conducted in a classical setting.

The general outline of the Gibbs sampler is presented in Section 4.3.1 whereas technical details about the implementation of the Gibbs sampler are relegated to Appendix D. The parameter priors used in the Bayesian estimation are then discussed in Section 4.3.2.

4.3.1. Gibbs sampler. The Gibbs sampling scheme is as follows:

1. Sample the exchange rate risk premium z_t and parameters from the state-space model equations (10) and (11). First, sample the state z_t conditional on the data and the parameters in the system, namely σ_s^2 , ρ_z , and σ_z^2 . To this end, the Bayesian state-space approach with multimove sampling of Carter and Kohn (1994) and Kim and Nelson (1999) is implemented (i.e. the forward filtering, backward sampling approach). Second, sample the parameter σ_s^2 conditional on the data and the state z_t using a Bayesian OLS regression approach applied to equation (10) [see e.g.

- Bauwens et al. (2000)]. Third, sample ρ_z and σ_z^2 conditional on the data and the state z_t using a Bayesian OLS regression approach applied to equation (11).
2. Sample the time-varying structural parameters β_t from the state-space model equations (12) and (13). First, sample the K states β_t conditional on the data and the parameters in the system, namely ρ_x , σ_x^2 , and $\sigma_{\beta_k}^2$, using the Bayesian state-space approach with multimove sampling of Carter and Kohn (1994) and Kim and Nelson (1999). Second, obtain estimates for the unobserved component x_t from $x_t = y_t - f_t \beta_t$. Third, sample the parameter $\sigma_{\beta_k}^2$ conditional on the state β_{kt} (for $k = 1, \dots, K$) using a Bayesian OLS regression approach applied to equation (13). Fourth, sample the parameters ρ_x and σ_x^2 conditional on x_t using a Bayesian OLS regression approach applied to equation (5).
 3. Calculate the parameters $\Psi = \frac{\lambda(1-\rho_x)}{1-\rho_x\lambda}$ and $\Phi = -\frac{\lambda}{1-\rho_x\lambda}$ using the sampled values for ρ_z and ρ_x and the imposed value for λ , that is, $\lambda = 0.967$.
 4. a. Using estimates for z_t , x_t , β_t , Ψ , and Φ , calculate $\tilde{s}_t \equiv s_t - (1 - \hat{\Psi})f_t \hat{\beta}_t - (1 - \hat{\Psi})\hat{x}_t - \hat{\Phi}\hat{z}_t$ and $\tilde{f}_t \equiv \hat{\Psi}f_t$.
 - b. For $k = 1, \dots, K$, sample the binary indicator δ_k using equation (20) while marginalizing over the parameter vector ϕ_k over which model selection is carried out. The approach follows the stochastic variable selection procedure for regressions by George and McCulloch (1993) and Frühwirth-Schnatter and Wagner (2010).
 - c. Jointly sample c , σ_ε^2 , and the slope coefficients ϕ_k for which the corresponding binary indicators δ_k are equal to 1 via a Bayesian OLS regression approach applied to equation (20). Set the slope coefficients ϕ_k to 0 if the corresponding binary indicators δ_k are equal to 0.
 - d. Calculate the estimates of the residuals ε_t from $\varepsilon_t = \tilde{s}_t - c' - \sum_{k=1}^K \delta_k \phi_k \tau_{kt} \tilde{f}_{kt}$. Sample the AR coefficient ρ_ε for given variance σ_ε^2 using Bayesian OLS applied to equation (19).

Sampling from these steps is iterated D times and, after a sufficiently large number of burn-in draws B , the sequence of draws $(B + 1, \dots, D)$ approximates a sample from the posterior distributions of the sampled quantities. The results reported below are based on $D = 50,000$ iterations with the first $B = 20,000$ draws discarded as a burn-in sequence, that is, the reported results are based on posterior distributions constructed from $D - B = 30,000$ draws. A convergence analysis of the Gibbs sampler is presented in Appendix E and shows that these numbers are sufficient for the Markov chain to converge for all reported estimations.

4.3.2. *Parameter priors.* For the regression parameters—that is, ρ_z , ρ_x , c , ϕ , and ρ_ε —we use a Gaussian prior $\mathcal{N}(b_0, V_0)$ defined by setting the prior mean b_0 and prior variance V_0 . For the variance parameters—that is, σ_s^2 , σ_z^2 , σ_x^2 , $\sigma_{\beta_k}^2$ (for $k = 1, \dots, K$), and σ_ε^2 —we use the Inverse Gamma prior $\mathcal{IG}(c_0, C_0)$ where the shape $c_0 = \nu_0 T$ and scale $C_0 = c_0 \sigma_0^2$ parameters are calculated from the prior belief σ_0^2 about the variance parameter and the prior strength ν_0 which is expressed as a fraction of the sample size T . For the binary indicators δ_k (with $k = 1, \dots, K$) in equation (20) that determine which fundamentals are scapegoats, we choose Bernoulli prior distributions where every indicator δ_k has a prior probability p_0 of being equal to 1, that is, $p(\delta_k = 1) = p_0$ (for $k = 1, \dots, K$).

With respect to the parameter priors used when estimating the state-space system given by equations (10) and (11), we set $b_0 = 0$ and $V_0 = 1$ for the AR(1) parameter ρ_z so that the prior distribution covers the full range of possible values for this parameter. For both variance parameters σ_s^2 and σ_z^2 , we set the prior belief equal to half the unconditional variance of the data series $\Delta s_{t+1} - \bar{i}_t$, that is, for σ_s^2 and σ_z^2 we set $\sigma_0^2 = 0.5 \times V(\Delta s_{t+1} - \bar{i}_t)$.¹⁸ The strength is set equal to $\nu_0 = 0.05$ for both variances which amounts to imposing a relatively loose prior.

With respect to the parameter priors used when estimating the state-space system given by equations (12) and (13), we set $b_0 = 0$ and $V_0 = 1$ for the AR(1) parameter ρ_x so that the prior distribution covers the full range of possible values for this parameter. For the variance parameter σ_x^2 , we set the prior belief equal to half the unconditional variance of the signal y_t , that is, for σ_x^2 we set $\sigma_0^2 = 0.5 \times V(y_t)$.¹⁹ For the variances $\sigma_{\beta_k}^2$ (for $k = 1, \dots, K$), we set belief $\sigma_0^2 = 0.01$ which is not too low and not too high to allow for slow structural movement in β_k without imposing that β_k is constant. The strength is set equal to $\nu_0 = 0.05$ for all variances so that the priors are given relatively little weight in the estimation results.

With respect to the parameters of regression equation (20), we set $p_0 = 0.5$ for every binary indicator δ_k (with $k = 1, \dots, K$) which amounts to assuming that there is an a-priori 50% chance that fundamental k is a scapegoat. For the intercept c and the regression slope parameters ϕ_k that are included in the regression (i.e. those for which $\delta_k = 1$), we set $b_0 = 0$ and $V_0 = 10$ which allows for a wide range of possible estimates for c and ϕ_k . For the regression error variance σ_ε^2 , we set belief $\sigma_0^2 = 0.01$ and strength $\nu_0 = 0.05$ which, again, implies a relatively loose prior imposed on a variance, that is, in this case on σ_ε^2 . Finally, with respect to the AR parameter of regression equation (19), we set $b_0 = 0$ and $V_0 = 1$ for ρ_ε so that the prior distribution covers the full range of possible values for this parameter.

5. RESULTS

5.1. Estimates of the Exchange Rate Risk Premium

Table 1 presents the parameter estimates of the state-space system equations (10) and (11) while Figure 1 presents the signal $\Delta s_{t+1} - \bar{i}_t$ and the estimated exchange risk premium z_t for all eight currencies in the sample. From the table, we note that for all currencies the AR(1) parameter lies between 0.15 and 0.84 suggesting that there is a persistent though stationary deviation of the UIPR condition in the model. This result is in line with results reported in the literature [see e.g. Carriero (2006) and Byrne and Nagayasu (2012)]. As the scapegoat model discussed in Section 2 shows that z_t enters the exchange rate equation derived from the model, that is, equation (9), we condition the estimation of this equation on the estimates obtained for z_t (i.e. on the full posterior distribution of z_t).

TABLE 1. Posterior distributions of the parameters of the state-space system equations (10) and (11)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
ρ_z	0.6407 [0.370;0.866]	0.4198 [0.088;0.698]	0.1519 [-0.359;0.593]	0.6009 [0.267;0.862]	0.5018 [0.214;0.756]	0.3468 [-0.035;0.649]	0.6002 [0.298;0.847]	0.8314 [0.648;0.971]
σ_z^2	0.0020 [0.001;0.004]	0.0008 [0.000;0.001]	0.0010 [0.000;0.002]	0.0008 [0.000;0.002]	0.0009 [0.000;0.001]	0.0002 [0.000;0.001]	0.0011 [0.000;0.002]	0.0024 [0.001;0.004]
σ_s^2	0.0017 [0.001;0.003]	0.0007 [0.000;0.001]	0.0014 [0.001;0.003]	0.0008 [0.000;0.001]	0.0007 [0.000;0.001]	0.0002 [0.000;0.001]	0.0010 [0.000;0.002]	0.0026 [0.001;0.004]

Note: Reported are the medians and the 90% highest posterior density (HPD) intervals of the posterior distributions of the AR parameter and variance parameters of state-space system equations (10) and (11).

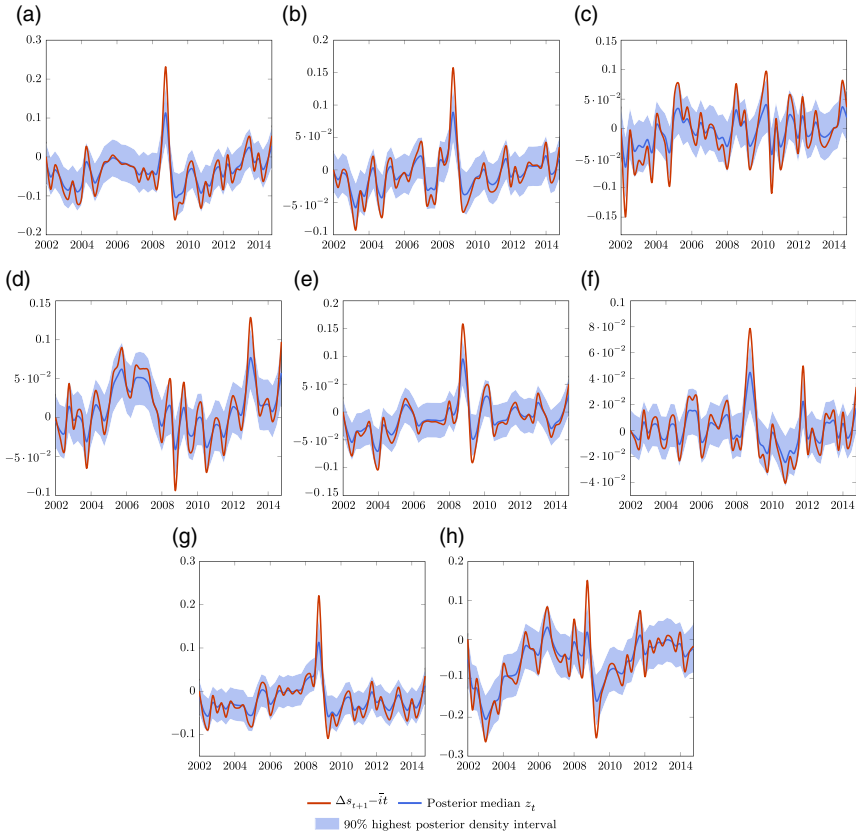


FIGURE 1. The signal $\Delta s_{t+1} - \bar{i}_t$ and the estimated exchange risk premium z_t . (a) Australian dollar (AUD). (b) Canadian dollar (CAD). (c) euro (EUR). (d) Japanese yen (JPY). (e) UK pound (GBP). (f) Singapore dollar (SGD). (g) South Korean won (KRW). (h) South African rand (ZAR).

5.2. Structural Parameters Macro-Fundamentals and Unobserved Component

In this section, we discuss the results of the estimation of the state-space system equations (12) and (13). This estimation provides estimates for the potentially time-varying structural parameters β_t on the macro-fundamentals \bar{g} , $\bar{\pi}$, \bar{i}^L , and ca . These are presented in Figure 3 while the posterior distributions of the estimates of the variances $\sigma_{\beta_k}^2$ (for $k = 1, \dots, K$) of the shocks to the random walks β_t are reported in Table 2. From Figure 3, we note that—while there is some time variation in the structural parameters as the variances reported in Table 2 are positive—the HPD intervals around the β_t 's are rather wide and these parameters can de facto all be considered constant. This means that our theoretical model

TABLE 2. Posterior distributions of the parameters of the state-space system equations (12) and (13)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
ρ_x	0.5191 [0.137;0.817]	0.6990 [0.318;0.912]	0.6703 [0.139;0.958]	0.7164 [0.216;0.944]	0.5073 [0.179;0.786]	0.5215 [0.169;0.804]	0.7063 [0.263;0.936]	0.6559 [0.405;0.903]
σ_x^2	0.0525 [0.028;0.256]	0.0540 [0.017;0.795]	0.1476 [0.031;6.015]	0.9936 [0.269;13.56]	0.0912 [0.049;0.255]	0.0336 [0.018;0.102]	5.7618 [1.912;51.28]	0.2907 [0.113;4.637]
$\sigma_\beta^2(\bar{g})$	0.0116 [0.005;0.043]	0.0112 [0.005;0.041]	0.0115 [0.005;0.043]	0.0111 [0.004;0.038]	0.0113 [0.004;0.043]	0.0113 [0.004;0.042]	0.0114 [0.005;0.041]	0.0115 [0.004;0.044]
$\sigma_\beta^2(\bar{\pi})$	0.0117 [0.004;0.041]	0.0115 [0.004;0.042]	0.0115 [0.004;0.043]	0.0117 [0.005;0.043]	0.0113 [0.004;0.043]	0.0115 [0.004;0.042]	0.0112 [0.004;0.041]	0.0117 [0.005;0.044]
$\sigma_\beta^2(\bar{l}^L)$	0.0111 [0.004;0.041]	0.0116 [0.005;0.041]	0.0112 [0.004;0.041]	0.0114 [0.004;0.042]	0.0117 [0.005;0.044]	0.0114 [0.004;0.039]	0.0114 [0.005;0.041]	0.0118 [0.004;0.042]
$\sigma_\beta^2(ca)$	0.0114 [0.004;0.040]	0.0114 [0.004;0.042]	0.0113 [0.004;0.043]	0.0115 [0.005;0.045]	0.0113 [0.004;0.041]	0.0126 [0.005;0.046]	0.0114 [0.005;0.044]	0.0113 [0.004;0.040]

Note: Reported are the medians and the 90% HPD intervals of the posterior distributions of the AR parameter and variance parameters of the state-space system equations (12) and (13).

essentially collapses to the model considered by Bacchetta and van Wincoop (2013) with constant structural parameters. While this result may be specific to the considered sample period (which contains relatively little observations), the result nonetheless suggests that the instability between exchange rates and macro-fundamentals and the “disconnect puzzle” cannot be explained simply by imposing time-varying structural parameters in the model. This conclusion is corroborated by the argument of Bacchetta and van Wincoop (2013) that the instability of the relationship between exchange rates and macro-fundamentals is hard to detect based solely on data for exchange rates and macro-fundamentals as the econometrician who conducts regressions between these variables is typically less informed than economic agents.²⁰ By providing a role for time-varying and potentially highly volatile *expectations* of parameters, the scapegoat model allows for the addition of more information when estimating the relationship between exchange rates and macroeconomic fundamentals, that is, following Fratzscher et al. (2015) and as discussed in Section 4.2, we use survey data to proxy these expected parameters.

Additionally—and related to the width of the HPD intervals—most structural parameters have HPD intervals that contain the value of zero suggesting that the impact of the macro-fundamentals on the signal $y_t \equiv s_t - \frac{1}{\mu} \bar{i}_t$ is rather limited. Macro-fundamentals that have a clear non-zero structural impact on the exchange rate are the inflation rate $\bar{\pi}$ for Australia and the UK, the long-run interest rate \bar{i}^L for Australia, Japan, the UK, and Singapore, and the current account to GDP ratio ca for Australia and the UK. The inflation differential $\bar{\pi}$ has a positive impact suggesting that higher inflation rates in Australia and the UK versus the USA depreciate the exchange rates of these countries; the long-run interest rate \bar{i}^L has a negative impact suggesting that higher long-run interest rates in Australia, Japan, the UK, and Singapore versus the USA appreciate those countries’ exchange rates. The current account balance ca has a negative impact for Australia and a positive impact for the UK meaning that a higher current account surplus or a lower deficit appreciates the Australian dollar and depreciates the UK pound.

The estimation of equations (12) and (13) then allows us to calculate estimates for the unobserved component x_t from $x_t = y_t - f_t \beta_t$. The estimated AR(1) and variance parameters of this component, that is, ρ_x and σ_x^2 , are reported in Table 2 while Figure 2 presents graphs for the signal y_t and for the posterior median of x_t and its 90% HPD interval. From Figure 2 we note that the difference between y_t and x_t which reflects $f_t \beta_t$ is often rather important suggesting that even though the HPD intervals around the β ’s are wide and often contain the value of 0, the magnitude of the estimated β ’s is non-negligible. From Table 2 we note that, for all currencies, the AR(1) parameter is positive and lies between 0.5 and 0.72 suggesting that there is a persistent though stationary unobserved component in the model which potentially reflects unobserved quantities such as money demand shocks or real exchange rate shocks. The existence of this component is a precondition for the potential presence of scapegoat effects. We investigate the presence of scapegoat effects in the next section.

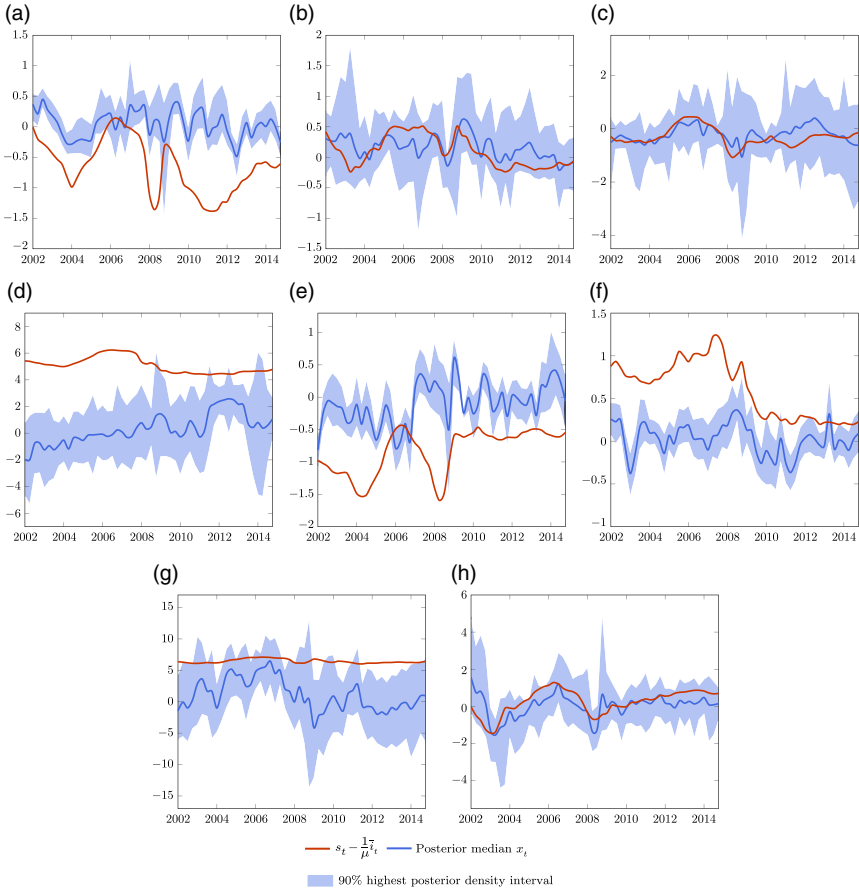


FIGURE 2. The signal $y_t \equiv s_t - \frac{1}{\mu} \bar{i}_t$ and the unobserved component x_t . (a) Australian dollar (AUD). (b) Canadian dollar (CAD). (c) euro (EUR). (d) Japanese yen (JPY). (e) UK pound (GBP). (f) Singapore dollar (SGD). (g) South Korean won (KRW). (h) South African rand (ZAR).

5.3. Scapegoat Effects

We now discuss which fundamentals—if any—can be identified as scapegoats. To this end, we estimate equation (20) using Bayesian OLS. The results are reported in Tables 3 and 4. The posterior inclusion probabilities p that the included macro-fundamentals \bar{g} , $\bar{\pi}$, \bar{i}^L , and ca are scapegoats are shown in Table 3. These probabilities are calculated as the average over the iterations of the Gibbs sampler of the binary indicators δ_k included in equation (20). From the table, we note that only the scapegoat term for the inflation differential $\bar{\pi}$ which is the inflation differential interacted with the Consensus survey score for the inflation differential tends to *systematically* have a posterior inclusion probability that is higher than

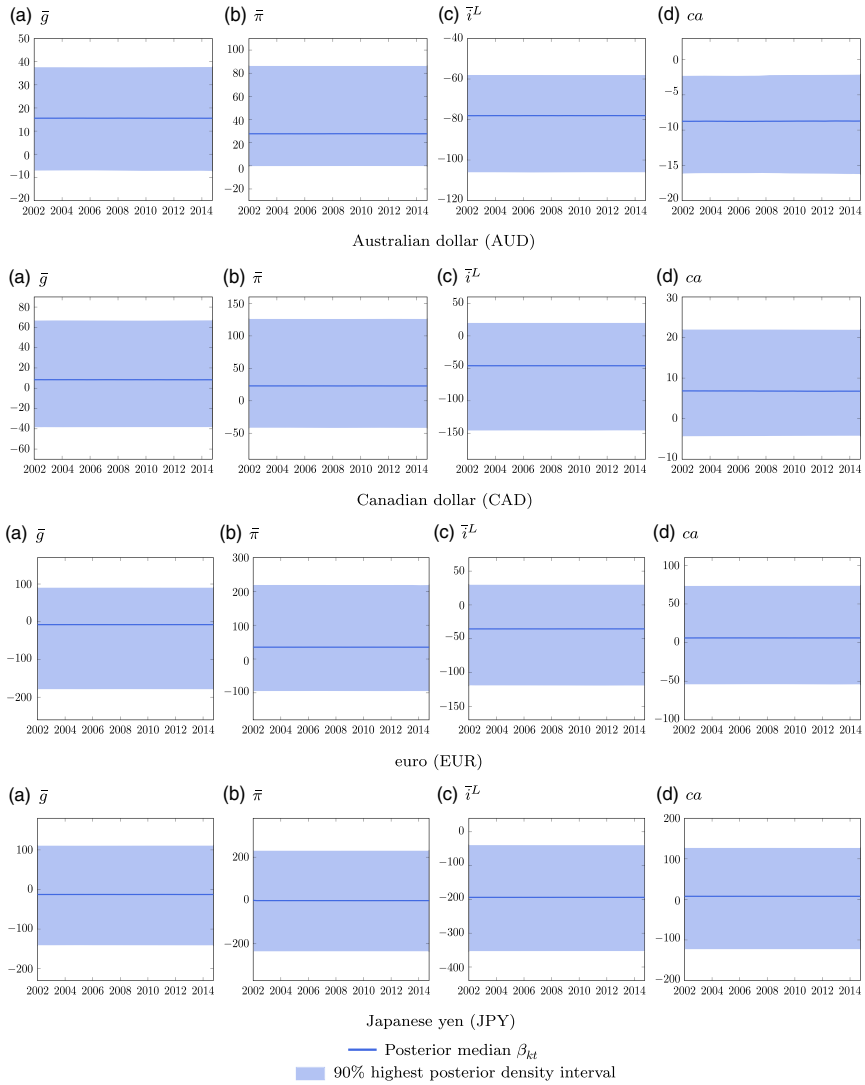


FIGURE 3. Structural parameters β_{kt} .

the imposed prior probability of $p_0 = 0.5$. This is the case for the Anglo-Saxon countries—Australia, Canada, and the UK—where the inclusion probabilities all are close to 0.9 and for South Korea and South Africa where the inclusion probabilities are lower but still well above 0.5. Interestingly, in these five countries, the monetary authority has adopted an inflation targeting regime while in the remaining three economies in our sample, the monetary authority does not adhere explicitly to such a regime [see Hammond (2011)]. This matters because in those

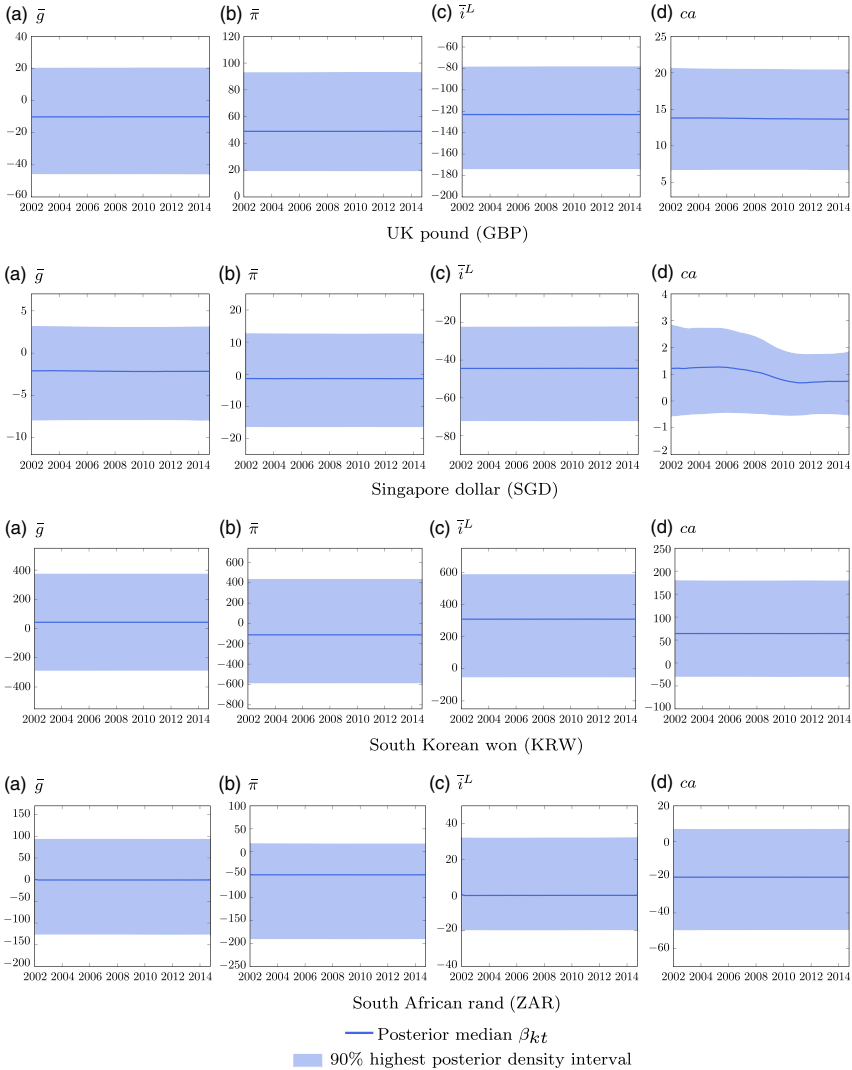


FIGURE 3. (Continued).

countries where the monetary authority is an inflation targeter, it can be expected that investors follow the evolution of the inflation rate more closely.

In Table 4, we then report estimates for the remaining parameters of equation (20). In particular, the coefficients ϕ_k capture the impact that the survey weights τ_{kt} have on the exchange rate. The posterior medians and 90% HPD intervals of the coefficients ϕ_k on the macro-fundamentals \bar{g} , $\bar{\pi}$, \bar{i}^L , and ca interacted with the Consensus survey scores support the results obtained for the posterior probabilities as reported in Table 3. For the Anglo-Saxon economies—Australia, Canada,

TABLE 3. Posterior probabilities that fundamentals are scapegoats

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
\bar{g}	0.5193	0.2972	0.2395	0.1066	0.1353	0.0308	0.1115	0.3854
$\bar{\pi}$	0.8808	0.8953	0.3462	0.2310	0.8971	0.2364	0.6425	0.7418
\bar{i}^L	0.8581	0.4863	0.2466	0.4977	0.3421	0.2036	0.3780	0.4200
ca	0.2903	0.2517	0.1914	0.2974	0.1359	0.0249	0.1026	0.3448

Notes: Reported are the posterior probabilities p that the fundamentals \bar{g} , $\bar{\pi}$, \bar{i}^L , and ca (as defined in Section 4.1) are scapegoats. These probabilities are calculated as the average of the sampled binary indicators δ_k over the iterations of the Gibbs sampler. The prior inclusion probabilities are equal to $p_0 = 0.5$ in all cases.

and the UK—and for South Korea and South Africa, all of which have posterior inclusion probabilities for inflation rate that are higher than 0.5, the impact of the inflation rate interacted with the survey weight is different from zero and positive. It is interesting to note that for the euro area and Singapore, which have posterior inclusion probabilities for the inflation rate that are below 0.5, we do find estimates for ϕ_k that are nonetheless above zero (even though the value of zero is included in the HPD interval). Furthermore, when comparing these results with those of the structural parameters β_k on the inflation rate reported in Figure 3, the β_k for inflation is found to be positive in Australia and in the UK while it is essentially zero in Canada, South Korea, and South Africa. Hence, the scapegoat effects ϕ_k intensify the impact of the structural parameters β_k , a result which supports the predictions of the scapegoat model.

Apart from the inflation rate in the five mentioned countries, only the long-run interest rate in Australia can be considered a scapegoat as the long-run interest rate interacted with its survey weight has a negative impact on the exchange rate, that is, it has a non-zero value for ϕ_k . This result is in line with the posterior probability $p = 0.86$ reported for \bar{i}^L for Australia in Table 3. Since from Figure 3 we note that the structural parameter β_k on the long-run interest rate is also negative, this suggests that for the Australian dollar the long-run interest rate is a scapegoat. For the other countries, no convincing evidence can be found that \bar{i}^L is a scapegoat. Neither is there evidence that the macro-fundamentals \bar{g} and ca are scapegoats.

In Figure 4, we present the posterior medians and 90% HPD intervals of the estimated parameter expectations, that is, $E_t(\beta_{kt}) = \phi_k \tau_{kt}$. These are calculated using the posterior distributions of ϕ_k and the survey data τ_{kt} . In line with the findings reported in both tables, we find that these are strictly larger than zero for the inflation rate in the five aforementioned countries (Australia, Canada, the UK, South Korea, and South Africa) and positive (but with the value of zero contained within the HPD interval) in the euro area and Singapore. The evolution over time of $E_t(\beta_{kt})$ for the inflation differential $\bar{\pi}$ in these countries mirrors the evolution of the survey score τ_{kt} for the inflation differential as can be seen from Figure A1 in Appendix A.²¹ The time variation found in these estimates allows to potentially explain why the relationship between exchange rates and macro-fundamentals—in particular, the inflation rate—can be unstable.

TABLE 4. Posterior distributions of the parameters of regression equation (20)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
c	0.1202 [0.006;0.285]	0.0100 [-0.013;0.039]	-0.0761 [-0.143;-0.015]	0.1644 [0.024;1.228]	-0.1554 [-0.293;-0.023]	0.0085 [-0.009;0.048]	0.3409 [0.040;3.116]	0.3694 [0.003;0.766]
$\phi(\bar{g})$	1.0353 [0.169;2.176]	0.4676 [-0.102;1.146]	-0.4176 [-1.142;0.166]	0.2226 [-0.282;0.834]	-0.0029 [-0.769;0.647]	0.0142 [-0.056;0.090]	-0.2499 [-1.181;0.547]	2.2865 [-0.576;7.545]
$\phi(\bar{\pi})$	2.4186 [0.902;4.687]	1.9017 [0.654;3.510]	0.7408 [-0.592;2.145]	-0.5759 [-2.145;0.580]	1.7664 [0.639;3.356]	0.2490 [-0.070;0.607]	1.8296 [0.357;4.175]	3.1700 [0.802;7.752]
$\phi(\bar{i}^L)$	-3.0915 [-5.904;-0.967]	1.1698 [-0.980;3.373]	0.0902 [-2.337;2.990]	-1.4554 [-4.777;0.246]	-0.4585 [-2.527;1.703]	-0.1093 [-0.767;0.518]	1.5240 [-0.607;7.860]	2.0183 [-0.567;4.280]
$\phi(ca)$	0.8018 [-0.169;2.203]	-0.3729 [-0.924;0.196]	-0.0429 [-1.436;0.988]	0.7452 [-0.230;2.138]	-0.2389 [-0.796;0.260]	0.0065 [-0.066;0.073]	0.2477 [-0.308;1.037]	-1.3626 [-3.804;0.726]
ρ_ε	0.6746 [0.377;0.928]	0.8720 [0.666;0.990]	0.6687 [0.411;0.922]	0.9549 [0.664;0.990]	0.6766 [0.405;0.956]	0.9628 [0.844;0.990]	0.9403 [0.468;0.990]	0.7043 [0.305;0.990]
σ_ε^2	0.0137 [0.007;0.044]	0.0044 [0.002;0.009]	0.0057 [0.003;0.012]	0.0114 [0.005;0.051]	0.0064 [0.004;0.014]	0.0022 [0.001;0.004]	0.0175 [0.006;0.208]	0.0527 [0.022;0.404]

Note: Reported are the medians and the 90% HPD intervals of the posterior distributions of the parameters c , ϕ_k (with corresponding fundamental denoted between brackets), ρ_ε , and σ_ε^2 of regression equation (20).

As we find that the inflation rate is statistically the most important scapegoat variable, we also investigate whether it is economically relevant. To this end, we calculate the variance share of the scapegoat component $\Psi f_i E_t(\beta_i)$ in the exchange rate s_t both for the complete model and for the model estimated under the counterfactual assumption that expectations $E_t(\beta_i)$ for the inflation differential $\bar{\pi}$ do not matter for s_t . From equations (9), (14), and (18), this variance share can be calculated as $\frac{V(\Psi f_i E_t(\beta_i))}{V(s_t)} = \frac{V(\tilde{f}_i E_t(\beta_i))}{V(s_t)} = \frac{V(\sum_{k=1}^K \delta_k \phi_k \tau_{kt} \tilde{f}_{kt})}{V(s_t)}$. To conduct the counterfactual exercise that the inflation survey does not matter for the exchange rate s_t , the model and variance shares are estimated with the binary indicator δ_k for the inflation differential set to zero. The posterior medians and 90% HPD intervals of the variance shares calculated from the model with and without inclusion of the inflation survey are reported in Table 5. From the table, we note that—for the unrestricted model—the scapegoat component explains between 4% (euro area and Singapore) and 32% (Australia) of the variance in the exchange rate. Obviously, the variance shares are higher for countries where scapegoat variables are detected. As noted above, for Australia both the inflation rate $\bar{\pi}$ and the long-run interest rate \tilde{r}^L can be considered scapegoat variables, while for Canada, the UK, South Korea, and South Africa we argue that only the inflation rate $\bar{\pi}$ is a scapegoat variable. In the table, we then report the variance shares obtained when estimating the model without the inclusion of the inflation scapegoat survey, and we find that these variance shares are considerably reduced compared to those estimated from the unrestricted model. The reduction in variance shares is as much as 15 percentage points on average for the Anglo-Saxon economies and about half as much—that is, about 7.5% points on average—for South Korea and South Africa. These are economically relevant numbers. Hence, our results confirm that in those countries where the inflation rate is identified as a scapegoat variable for the exchange rate, it is both a statistically and an economically relevant scapegoat variable.

How do our results compare to the scapegoat results reported by Bacchetta and van Wincoop (2013) and Fratzscher et al. (2015)? Given the considerable differences in set-up and approach of both these papers compared to our paper, the finding of important differences in results is not really surprising. There are however also commonalities in the results obtained. We discuss each paper in turn.

Bacchetta and van Wincoop (2013) conduct a numerical analysis by calibrating their scapegoat model to data on exchange rates, interest rates, and observed fundamentals. Hence, they do not conduct estimations but generate data for parameter expectations from the model where values are imposed for all model parameters. By contrast, we conduct estimations of the scapegoat model and only impose a value for one parameter in the model, that is, the discount factor. Furthermore, the set of fundamentals that they use differs from ours as our choice of fundamentals is motivated by the availability of survey scores from Consensus Economics to proxy for the expected parameters. Common findings of both approaches are that scapegoat effects can account for the instability of the relationship between exchange rates and fundamentals, and that these effects

TABLE 5. Posterior distributions of variance shares of the scapegoat component (with/without inflation survey)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
With infl.surv.	0.32 [0.07;0.61]	0.22 [0.03;0.51]	0.04 [0.00;0.24]	0.08 [0.00;0.30]	0.18 [0.02;0.43]	0.04 [0.00;0.28]	0.13 [0.00;0.40]	0.24 [0.03;0.59]
Without infl.surv.	0.17 [0.02;0.43]	0.08 [0.00;0.35]	0.03 [0.00;0.22]	0.07 [0.00;0.28]	0.02 [0.00;0.17]	0.02 [0.00;0.17]	0.05 [0.00;0.25]	0.17 [0.01;0.56]

Note: Reported are the medians and the 90% HPD intervals of the posterior distributions of the variance shares of the scapegoat component. The variance shares are given by $\frac{V(\Psi_{jt}^* E_t(\beta_{jt}))}{V(s_t)} = \frac{V(\hat{f}_{jt} E_t(\beta_{jt}))}{V(s_t)} = \frac{V(\sum_{k=1}^K \delta_k \phi_k \tau_{kt} \hat{f}_{kt})}{V(s_t)}$. For the case with inflation survey, the variance shares are obtained from estimation of the full model (and correspond to the results reported in the tables and figures in the text). For the case without inflation survey, the variance shares are computed from the estimation of a model without the inflation survey included, that is, the binary indicator δ_k for inflation is set to 0.

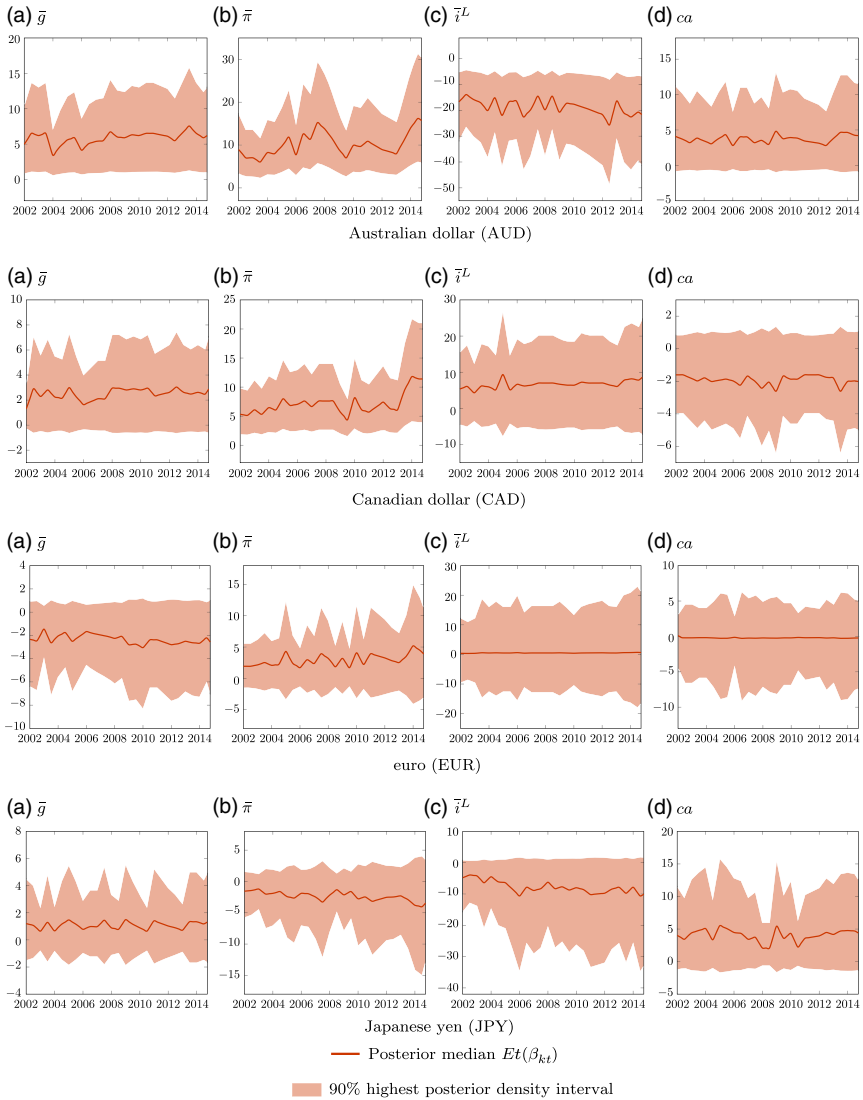


FIGURE 4. Expectations of parameters $E_t(\beta_{kt})$.

increase the predictive power of macro-fundamentals for exchange rates. For our results, this instability can be observed from Figure 4 for the inflation differential for the Anglo-Saxon economies, South Korea, and South Africa, while the increase in predictive power can be observed from Table 5 for the same countries. We note that whereas Bacchetta and van Wincoop (2013) report only modest increases in predictive power, our Table 5 reports more important increases in predictability.

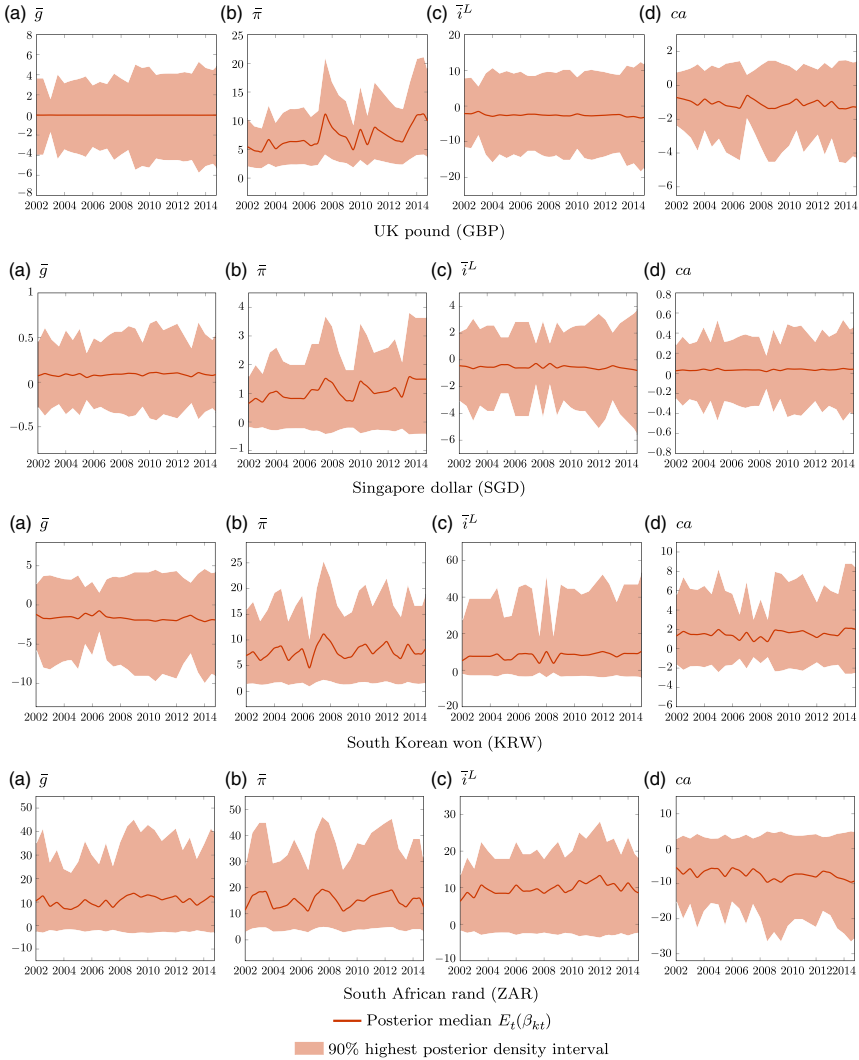


FIGURE 4. (Continued).

Fratzcher et al. (2015) conduct estimations of ad-hoc regressions where the exchange rate is regressed on fundamentals interacted with Consensus Economics survey scores while, as detailed in Section 1, our approach is model-based as it imposes the restrictions implied by the theoretical model on the data. While our results confirm the empirical findings of Fratzscher et al. (2015) as far as the inflation rate is concerned, we find considerably less evidence in favor of scapegoat effects when looking at the other macro-fundamentals. Similar to Fratzscher et al. (2015) who report considerably higher R^2 's when their regression equations

contain interaction terms of fundamentals and surveys, we find an important increase in the predictive power of fundamentals—in our case, the inflation rate—when scapegoat effects are allowed for (i.e. see Table 5).

6. CONCLUSIONS

This paper proposes a new empirical testing strategy for the scapegoat theory of exchange rates that uses the exact structural exchange rate equation implied by a scapegoat model instead of an ad-hoc empirical specification. The approach followed should tighten the link between the theory on scapegoats and the empirical testing of this theory.

From theory, we derive an exchange rate equation that can be estimated in different steps. First, the exchange rate risk premium or time-varying deviation from the UIRP condition is estimated using a state-space approach applied to the observed difference between the change in the exchange rate and the interest rate differential. Second, the unobserved time-varying structural parameters on the macro-fundamentals and the unobserved component of the model are estimated using a state-space system applied to the observed “signal” in the model, which depends on the level of the exchange rate, the interest rate differential, and the discount factor. Third, the scapegoat component in the model’s exchange rate equation is estimated using a regression analysis where the estimation is conditional on the estimates obtained in the previous steps.

The estimation is carried out through a Bayesian Gibbs sampling approach for eight countries versus the USA over the period 2002Q1–2014Q4. We consider five developed economies (Australia, Canada, the euro area, Japan, and the UK) and three emerging countries (Singapore, South Korea, and South Africa), and we incorporate four macro-fundamentals in the estimations that can potentially be scapegoats, that is, the real GDP growth rate (relative to the USA), the inflation rate (relative to the USA), the long-run interest rate (relative to the USA), and the current account balance to GDP ratio. We use survey data from Consensus Economics to proxy the parameter expectations that enter the scapegoat term.

The results suggest, first, that there is a persistent but stationary exchange risk premium or time-varying deviation from the UIRP condition in all countries considered. Second, we identify a persistent but stationary unobserved component from the “signal” in the model which potentially reflects unobserved quantities such as money demand shocks or real exchange rate shocks. Third, we find that, over the sample period, the structural parameters on the macro-fundamentals are constant and often close to zero. Fourth, as far as the scapegoat terms in the exchange rate equation are concerned, we calculate posterior probabilities that these macro-fundamentals are scapegoats, and we find, for the inflation rate in five out of eight countries, probabilities that are considerably higher than the imposed prior probabilities of $\frac{1}{2}$. These countries are the three Anglo-Saxon economies (Australia, Canada, and the UK) and South Korea and South Africa. We find little evidence to suggest that the other macro-fundamentals that we consider are scapegoats.

NOTES

1. We note that Engel and West (2005) and Engel et al. (2007) put this poor performance in perspective, however, by arguing that the low predictability of exchange rates using macro-fundamentals is actually implied by standard present-value models of the exchange rate. When the macro-fundamentals and/or the unobservable shock are non-stationary and the discount factor used to discount expected future fundamentals is high (i.e. close to 1), then the exchange rate will be close to a random walk. In this case, expectations about future fundamentals drive the exchange rate, while current and lagged values are relatively unimportant. A testable implication then is whether exchange rates can predict future fundamentals rather than the other way around [see, e.g. Engel and West (2005) and Sarno and Schmeling (2014)].

2. As noted by Bacchetta and van Wincoop (2013)—see paragraph 6 of their introduction and their citations—other research has considered parameter uncertainty in exchange rate models although these papers have not considered uncertainty about parameters multiplying fundamentals and the role of unobserved fundamentals.

3. By accounting for a non-zero time-varying deviation from the UIRP condition, our paper allows to distinguish between the effects of unobservables such as money demand shocks that affect the unobserved fundamental in the exchange rate equation of the scapegoat model from those of changes in investors risk preferences and perceptions that affect the time-varying deviation from UIRP. Related to this, our approach *estimates* the unobserved fundamental in the exchange rate equation of the scapegoat model and does not use a variable to proxy it [e.g. order flow as used by Fratzscher et al. (2015)]. While there are good arguments to use the latter variable as a proxy [see Fratzscher et al. (2015)], it is nonetheless interesting to use a somewhat broader approach to deal with the unobserved fundamental in the scapegoat model.

4. Bacchetta and van Wincoop (2009) also consider the case of time-varying structural parameters albeit they do not specify them as random walks.

5. A risk premium is included in the calibration exercise of Bacchetta and van Wincoop (2013), however.

6. More specifically, as shown by Bacchetta and van Wincoop (2013), the observed fundamentals f_t and the unobserved component x_t can both follow AR(1) processes (with an AR parameter potentially equal to 1) but the AR parameters of both processes should be different.

7. This is obtained from $\lambda = (0.989)^{\frac{1}{4}}$.

8. These results are unreported but are available from the authors upon request.

9. A constant added to the specification for $E_t(\beta_{it})$ was generally found to be equal to zero.

10. An alternative proxy could be $E_t(\beta_t)$ as estimated from the Kalman filter applied when estimating the state-space system given by equations (12) and (13). However, the Kalman filter output cannot be considered a good proxy for $E_t(\beta_t)$ as it tends to converge to the “smoother” $E_T(\beta_t)$ when filtering nears the end of the sample period where $E_T(\beta_t)$ is basically what is used to estimate the time-varying structural parameter β_t . While the scapegoat model *does* predict that $E_t(\beta_t)$ tends to β_t when the scapegoat effects wear off [see Fratzscher et al. (2015)], there is of course no reason for this convergence to occur only and precisely at the end of the sample period.

11. We do not add the short-term interest rate differential \bar{i}_t in f_t since it appears on the LHS of equation (2).

12. Note that the variables \bar{g}_t , $\bar{\pi}_t$, and \bar{i}_t are, given that \bar{i}_t can be considered a proxy for inflation expectations, in accordance with variables one would include in equation (2) under a Taylor rule differential interpretation of equation (2). The variables \bar{g}_t , $\bar{\pi}_t$, and ca_t are also in accordance with the variables one would use in a reduced-form monetary model of exchange rates [see, e.g. Meese and Rogoff (1983a) and Cheung et al. (2005)]. We note that the expected signs of the coefficients β_t on the fundamentals f_t can vary according to the assumed underlying model.

13. We have also conducted estimations where f_t contains variables for which we have no survey data to proxy $E_t(\beta_t)$, that is, an intercept, the money supply differential between the local and benchmark economies, and the one-period lagged short-term nominal interest rate differential [see Bacchetta and van Wincoop (2009, 2013)]. For these variables, by necessity, we have to assume that $E_t(\beta_t) = \beta_t$ (for all t). The results obtained when these additional fundamentals are included in the regression

equation do not differ much from our reported results. Hence, we do not report them, but they are available from the authors upon request.

14. That is, from the Reserve Bank of Australia, the Bank of Canada, Eurostat, the Japanese Ministry of Finance, the Bank of England, the US Treasury, the Bank of Korea, the South African Reserve Bank, and the Monetary Authority of Singapore.

15. We use the following 3-month rates for each country: deposit rate for Australia, Treasury Bill for Canada, EURIBOR for the euro area, LIBOR for Japan, Sterling Interbank Lending Rate for the UK, money market rate for Singapore, interbank rate for South Korea, and JIBAR rate for South Africa.

16. Seasonally adjusted data are used for GDP, the consumer price index, and the current account.

17. We find very similar results when, instead, we construct a quarterly series by assigning the last available survey score to the quarter in which the survey is not conducted. These results are not reported but are available upon request.

18. This implies that we give equal prior weight to both components z_t and ϵ_{t+1}^s in the decomposition of $\Delta s_{t+1} - \bar{i}_t$ given by equation (10).

19. Given equation (3) this implies that, a priori, we attribute half of the variance of the signal y_t as stemming from the unobserved component x_t .

20. The literature supports this argument as the empirical evidence on time variation in the structural parameters—for both in-sample and out-of-sample analyses—is mixed. Some authors argue that time variation in structural parameters is important to explain exchange rates [see e.g. Engel and Hamilton (1990), Engel (1994), and Cheung and Erlandsson (2005)] while others argue that time variation is not that important [see e.g. Rossi (2006) and Bacchetta et al. (2010)].

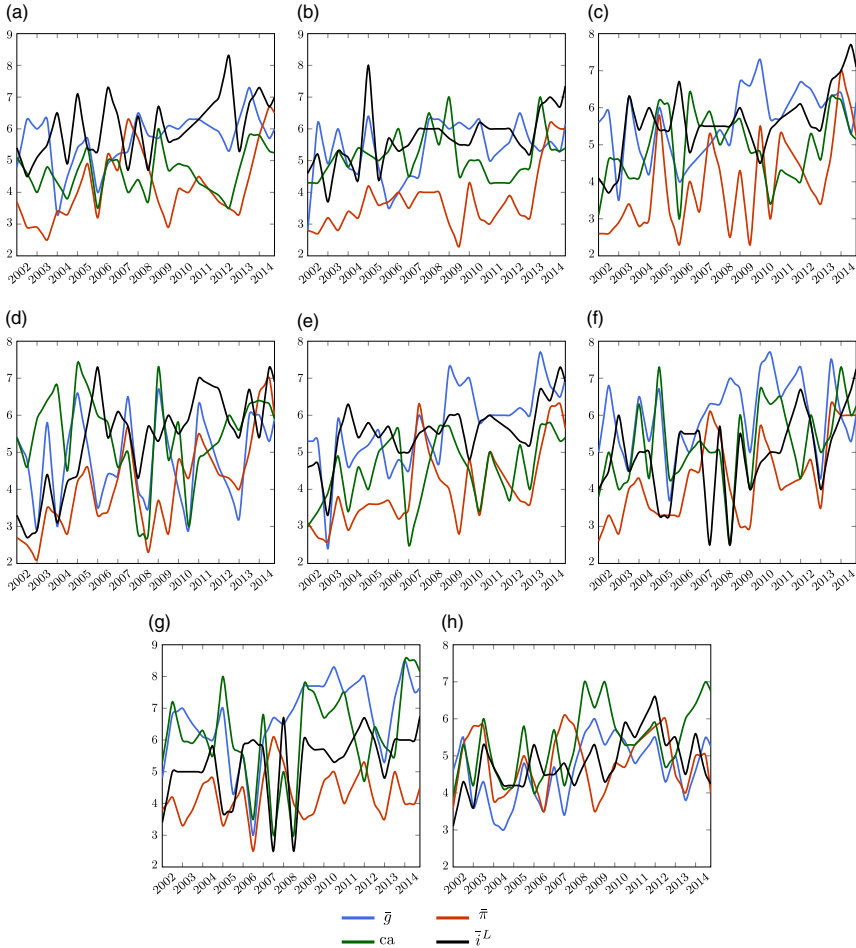
21. From this figure, we also observe that the inflation survey score matters for the exchange rate in terms of its evolution rather than in terms of its magnitude as the average survey score τ_{it} for the inflation differential $\bar{\pi}$ is relatively low for all currencies [see also Fratzscher et al. (2015), Tables I and II in their Internet Appendix B].

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APPENDIX A: SURVEY DATA



Notes: The series τ_{kt} are reported for K macro-fundamentals ($k = 1, \dots, K$ where $K = 4$ with fundamentals \bar{g} , $\bar{\pi}$, \bar{i}^L , and ca). Biannual survey scores for every country in deviation from the USA are from Consensus Economics and are described in Section 4.2. Quarterly series are obtained through linear interpolation.

FIGURE A1. Survey scores τ_{kt} . (a) Australian dollar (AUD). (b) Canadian dollar (CAD). (c) euro (EUR). (d) Japanese yen (JPY). (e) UK pound (GBP). (f) Singapore dollar (SGD). (g) South Korean won (KRW). (h) South African rand (ZAR).

APPENDIX B: DERIVATION OF EQUATION (9)

Note that from equation (5) we can derive $x_{t+j} = \rho_x^j x_t + \sum_{l=0}^{j-1} \rho_x^l \varepsilon_{t+j-l}^x$ and therefore write

$$E_t(x_{t+j}) = \rho_x^j E_t(x_t). \tag{B1}$$

Similarly, from equation (6) we can derive $z_{t+j} = \rho_z^j z_t + \sum_{l=0}^{j-1} \rho_z^l \varepsilon_{t+j-l}^z$ and therefore write

$$E_t(z_{t+j}) = \rho_z^j E_t(z_t). \tag{B2}$$

Given the assumption that the processes f_{kt} and β_{kt} (for $k = 1, \dots, K$) are independent, we can write

$$E_t(f_{k,t+j} \beta_{k,t+j}) = E_t(f_{k,t+j}) E_t(\beta_{k,t+j}) = f_{kt} E_t(\beta_{kt}) \quad k = 1, \dots, K, \tag{B3}$$

where the last step follows from the random walk processes equations (7) and (8) assumed for f_{kt} and β_{kt} .

We note that as the signal $y_t \equiv s_t - \frac{1}{\mu} \bar{t}_t$ is observed, we have $E_t(y_t) = y_t$ or $y_t - E_t(y_t) = 0$ (but, since the parameters β_t are unknown, $E_t(\beta_t) \neq \beta_t$ and $E_t(x_t) \neq x_t$). From equation (3), this implies that $f_t \beta_t + x_t - f_t E_t(\beta_t) - E_t(x_t) = 0$ or

$$E_t(x_t) = x_t + f_t \beta_t - f_t E_t(\beta_t). \tag{B4}$$

For z_t , on the other hand, we have

$$E_t(z_t) = z_t. \tag{B5}$$

This can be seen by taking expectations in period t from both sides of equation (1) in the text to obtain $E_t(s_{t+1}) - s_t = \bar{t}_t + E_t(z_t)$ which can only be equal to equation (1) if equation (B5) holds.

Using equations (B1)–(B5) into equation (4) while noting that $\sum_{j=1}^{\infty} \lambda^j = \sum_{j=0}^{\infty} \lambda^j - 1 = \frac{\lambda}{1-\lambda}$, $\sum_{j=1}^{\infty} (\lambda \rho_x)^j = \sum_{j=0}^{\infty} (\lambda \rho_x)^j - 1 = \frac{\lambda \rho_x}{1-\lambda \rho_x}$ and $\sum_{j=1}^{\infty} (\lambda \rho_z)^j = \sum_{j=0}^{\infty} (\lambda \rho_z)^j - 1 = \frac{\lambda \rho_z}{1-\lambda \rho_z}$, we obtain equation (9) in the text.

APPENDIX C: AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTIONS OF THE ESTIMATED RESIDUALS OF EQUATIONS (18) AND (20)

TABLE C1. Autocorrelations and partial autocorrelations of the residuals ε_t in equation (18) under the *iid* assumption

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
<i>acf</i> (1)	0.5133 [0.267;0.696]	0.4335 [0.211;0.632]	0.6146 [0.408;0.796]	0.6658 [0.457;0.798]	0.4889 [0.276;0.671]	0.6735 [0.470;0.827]	0.3720 [0.123;0.639]	0.3926 [0.171;0.609]
<i>acf</i> (2)	0.3103 [0.005;0.541]	0.2537 [0.065;0.455]	0.5096 [0.263;0.661]	0.4699 [0.181;0.657]	0.2888 [0.050;0.496]	0.4693 [0.177;0.700]	0.1461 [-0.087;0.445]	0.1865 [-0.033;0.424]
<i>acf</i> (3)	0.1465 [-0.179;0.411]	0.1277 [-0.049;0.350]	0.3755 [0.125;0.511]	0.4704 [0.229;0.623]	0.1892 [-0.071;0.402]	0.4442 [0.212;0.643]	0.1140 [-0.114;0.367]	0.1040 [-0.120;0.324]
<i>acf</i> (4)	0.0891 [-0.206;0.334]	0.1465 [-0.042;0.347]	0.2722 [0.039;0.410]	0.3968 [0.177;0.537]	0.1792 [-0.055;0.357]	0.3335 [0.054;0.560]	0.0330 [-0.182;0.294]	0.0385 [-0.168;0.238]
<i>pacf</i> (1)	0.5133 [0.267;0.696]	0.4335 [0.211;0.632]	0.6146 [0.408;0.796]	0.6658 [0.457;0.798]	0.4889 [0.276;0.671]	0.6735 [0.470;0.827]	0.3720 [0.123;0.639]	0.3926 [0.171;0.609]
<i>pacf</i> (2)	0.0442 [-0.194;0.258]	0.0679 [-0.108;0.232]	0.1852 [-0.070;0.333]	0.0394 [-0.145;0.174]	0.0504 [-0.134;0.207]	0.0144 [-0.101;0.133]	-0.0102 [-0.211;0.129]	0.0222 [-0.166;0.202]
<i>pacf</i> (3)	-0.0479 [-0.244;0.139]	-0.0017 [-0.154;0.153]	-0.0135 [-0.147;0.128]	0.2483 [0.062;0.389]	0.0339 [-0.129;0.178]	0.2019 [0.106;0.309]	0.0632 [-0.131;0.227]	0.0153 [-0.176;0.196]
<i>pacf</i> (4)	-0.0013 [-0.183;0.175]	0.0815 [-0.102;0.241]	-0.0166 [-0.163;0.108]	-0.0280 [-0.176;0.123]	0.0654 [-0.086;0.196]	-0.0982 [-0.214;0.028]	-0.0416 [-0.220;0.129]	-0.0296 [-0.201;0.135]

Notes: The *iid* assumption for the residuals in equation (18) implies the assumption that $\rho_\varepsilon = 0$ in equation (19). *acf*(*l*)/*pacf*(*l*) denotes autocorrelation/partial autocorrelation of order *l*. Reported are the medians and 90% HPD intervals of the posterior distributions of the autocorrelations/partial autocorrelations.

TABLE C2. Autocorrelations and partial autocorrelations of the residuals ε_t^* in equation (20)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
<i>acf</i> (1)	-0.0502 [-0.297;0.248]	-0.1381 [-0.353;0.142]	-0.1533 [-0.397;0.381]	-0.0441 [-0.212;0.112]	0.0345 [-0.183;0.277]	-0.0798 [-0.236;0.124]	-0.0042 [-0.213;0.132]	0.0019 [-0.225;0.260]
<i>acf</i> (2)	-0.0815 [-0.283;0.137]	-0.0525 [-0.261;0.186]	0.0657 [-0.138;0.320]	-0.1907 [-0.418;0.022]	-0.1101 [-0.315;0.120]	-0.0909 [-0.256;0.115]	-0.0676 [-0.297;0.076]	-0.0234 [-0.241;0.207]
<i>acf</i> (3)	-0.0901 [-0.294;0.121]	-0.0408 [-0.247;0.170]	-0.0199 [-0.218;0.195]	-0.0148 [-0.164;0.184]	-0.0497 [-0.248;0.146]	-0.0961 [-0.273;0.090]	-0.0234 [-0.175;0.185]	0.0751 [-0.163;0.283]
<i>acf</i> (4)	-0.0345 [-0.227;0.157]	0.0163 [-0.184;0.219]	-0.0547 [-0.218;0.121]	0.1234 [-0.033;0.318]	0.0542 [-0.111;0.195]	-0.1160 [-0.249;-0.056]	-0.0534 [-0.218;0.053]	0.0145 [-0.192;0.213]
<i>pacf</i> (1)	-0.0502 [-0.297;0.248]	-0.1381 [-0.353;0.142]	-0.1533 [-0.397;0.381]	-0.0441 [-0.212;0.112]	0.0345 [-0.183;0.277]	-0.0798 [-0.236;0.124]	-0.0042 [-0.213;0.132]	0.0019 [-0.225;0.260]
<i>pacf</i> (2)	-0.1146 [-0.309;0.098]	-0.0972 [-0.291;0.139]	-0.0018 [-0.216;0.236]	-0.2078 [-0.433;0.005]	-0.1310 [-0.326;0.081]	-0.1109 [-0.267;0.091]	-0.0825 [-0.316;0.065]	-0.0440 [-0.262;0.172]
<i>pacf</i> (3)	-0.1121 [-0.309;0.101]	-0.0687 [-0.255;0.138]	-0.0414 [-0.229;0.144]	-0.0434 [-0.191;0.139]	-0.0474 [-0.260;0.151]	-0.1160 [-0.283;0.075]	-0.0298 [-0.179;0.138]	0.0646 [-0.171;0.273]
<i>pacf</i> (4)	-0.0822 [-0.288;0.127]	-0.0286 [-0.228;0.179]	-0.0957 [-0.278;0.062]	0.0709 [-0.085;0.242]	0.0145 [-0.181;0.171]	-0.1680 [-0.319;0.033]	-0.0710 [-0.265;0.034]	-0.0124 [-0.221;0.181]

Notes: *acf*(*l*)/*pacf*(*l*) denotes autocorrelation/partial autocorrelation of order *l*. Reported are the medians and 90% HPD intervals of the posterior distributions of the autocorrelations/partial autocorrelations.

APPENDIX D: TECHNICAL DETAILS OF GIBBS SAMPLER

This appendix provides technical details on the steps of the Gibbs sampler as outlined in Section 4.3.1 in the main text.

D.1: State-Space Models (Steps 1 and 2)

D.1.1: General approach. The unobserved states are sampled conditional on the parameters using a state-space approach. In particular, we use the forward-filtering backward-sampling approach discussed in detail in Kim and Nelson (1999) to sample the unobserved states. The general form of the state-space model is given by

$$Y_t = Z_t S_t + V_t, \quad V_t \sim iid\mathcal{N}(0, H_t), \tag{D1}$$

$$S_{t+1} = T_t S_t + K_{t+1} E_{t+1}, \quad E_{t+1} \sim iid\mathcal{N}(0, Q_{t+1}), \quad t = 1, \dots, T \tag{D2}$$

$$S_1 \sim iid\mathcal{N}(s_1, P_1), \tag{D3}$$

where Y_t contains observations and S_t is the unobserved $n^s \times 1$ state vector. The matrices Z_t, T_t, K_t, H_t, Q_t , and the mean s_1 and variance P_1 of the initial state vector S_1 are assumed to be known (conditioned upon) and the error terms V_t and E_t are assumed to be serially uncorrelated and independent of each other at all points in time. Note that E_t is an $n^{ss} \times 1$ matrix (where $n^{ss} \leq n^s$). As equations (D1)–(D3) constitute a linear Gaussian state-space model, the unknown state variables in S_t can be filtered using the standard Kalman filter. Sampling $S = [S_1, \dots, S_T]$ from its conditional distribution can then be done using the multimove Gibbs sampler of Carter and Kohn (1994).

D.1.2: Step 1. In step 1 of the Gibbs sampler, we sample the state z_t conditional on the data and parameters in the state-space system equations (10) and (11), namely σ_s^2, ρ_z , and σ_z^2 . We have $n^s = 1$ and $n^{ss} = 1$. The system matrices are given by $Y_t = \Delta s_{t+1} - \tilde{i}_t$, $Z_t = 1$, $S_t = z_t$, $V_t = \varepsilon_{t+1}^s$, $H_t = \sigma_s^2$, $Q_{t+1} = \sigma_z^2$, $T_t = \rho_z$, $K_{t+1} = 1$, $E_{t+1} = \varepsilon_{t+1}^z$, $s_1 = 0$, and $P_1 = \frac{\sigma_z^2}{1 - \rho_z^2}$.

D.1.3: Step 2. In step 2 of the Gibbs sampler, we sample the time-varying structural parameters β_t from the state-space model equations (12) and (13), that is, we sample the K states in β_t conditional on the data and parameters in the system, namely ρ_x, σ_x^2 , and $\sigma_{\beta_k}^2$ (with $k = 1, \dots, K$). We have $n^s = 2K$ and $n^{ss} = K$. The system matrices are given by $Y_t = \tilde{y}_t = (1 - \rho_x L)y_t$, $Z_t = [f_t - \rho_x f_{t-1}]$ (a $1 \times 2K$ matrix), $S_t = [\beta_t \ \beta_{t-1}]'$ (the $2K \times 1$ state vector), $V_t = \varepsilon_t^x$, $H_t = \sigma_x^2$, $T_t = \begin{bmatrix} I_K & 0_K \\ I_K & 0_K \end{bmatrix}$ (a $2K \times 2K$ matrix), $K_{t+1} = \begin{bmatrix} I_K \\ 0_K \end{bmatrix}$ (a $2K \times K$ matrix), $E_{t+1} = [\varepsilon_{1,t+1}^\beta \ \dots \ \varepsilon_{K,t+1}^\beta]'$ (a $K \times 1$ matrix), $Q_{t+1} = \begin{bmatrix} \sigma_{\beta_1}^2 & & & \\ & \mathbf{0} & & \\ & & \ddots & \\ & & & \mathbf{0} \\ \mathbf{0} & & & & \sigma_{\beta_K}^2 \end{bmatrix}$ (a $K \times K$ matrix), $s_1 = [0 \ \dots \ 0]'$ (a $2K \times 1$ vector), and $P_1 = 10^6 \times I_{2K}$ (a $2K \times 2K$ matrix).

D.2: OLS Regressions (Steps 1, 2, and 4)

D.2.1: General approach. Regression parameters (intercept, slope parameters, and error variance) can be sampled from a standard regression model,

$$y = w^r \gamma^r + \chi, \tag{D4}$$

where y is a $T \times 1$ vector containing T observations on the dependent variable, w^r is a $T \times M$ matrix containing T observations of M predictor variables, γ^r is the $M \times 1$ parameter vector, and χ is the $T \times 1$ vector of error terms for which $\chi \sim iid\mathcal{N}(0, \sigma_\chi^2 I_T)$. If there are no binary indicators ι in the regression or if all binary indicators in the regression ι are equal to 1, then $w^r = w$ and $\gamma^r = \gamma$ where w and γ are the unrestricted predictor matrix and the corresponding unrestricted coefficient vector, respectively. Otherwise, the restricted parameter vector γ^r and the corresponding restricted predictor matrix w^r contain only those elements of w and γ for which the corresponding binary indicators ι are equal to 1. The prior distribution of γ^r is given by $\gamma^r \sim \mathcal{N}(b_0^r, B_0^r \sigma_\chi^2)$ with b_0^r an $M \times 1$ vector and B_0^r an $M \times M$ matrix. The prior distribution of σ_χ^2 is given by $\sigma_\chi^2 \sim \mathcal{IG}(c_0, C_0)$ with scalars c_0 (shape) and C_0 (scale). The posterior distributions (conditional on y, w^r , and ι) of γ^r and σ_χ^2 are then given by $\gamma^r \sim \mathcal{N}(b^r, B^r \sigma_\chi^2)$ and $\sigma_\chi^2 \sim \mathcal{IG}(c, C^r)$ with,

$$B^r = [(w^r)'w^r + (B_0^r)^{-1}]^{-1}$$

$$b^r = B^r [(w^r)'y + (B_0^r)^{-1}b_0^r] \tag{D5}$$

$$c = c_0 + T/2$$

$$C^r = C_0 + \frac{1}{2} [y'y + (b_0^r)'(B_0^r)^{-1}b_0^r - (b^r)'(B^r)^{-1}b^r]$$

Following Frühwirth-Schnatter and Wagner (2010), we marginalize over the parameters γ when sampling ι and next draw γ^r conditional on ι . The posterior distribution of the binary indicators ι is obtained from Bayes' theorem as,

$$p(\iota|y, w, \sigma_\chi^2) \propto p(y|w, \sigma_\chi^2, \iota)p(\iota), \tag{D6}$$

where $p(\iota)$ is the prior distribution of ι and $p(y|w, \sigma_\chi^2, \iota)$ is the marginal likelihood of regression equation (D4) where the effect of the parameters γ has been integrated out. We refer to Frühwirth-Schnatter and Wagner (2010) [their equation (25)] for the closed-form expression of the marginal likelihood for the general regression model of equation (D4).

D.2.2: Step 1. The regressions estimated in step 1 are unrestricted so that in equation (D4) we have $w^r = w$ and $\gamma^r = \gamma$.

Sampling σ_s^2 conditional on the state z_t and the data is implemented by setting $y = \Delta s - \bar{i} - z, w^r = w = 0, \gamma^r = \gamma = 0, \sigma_\chi^2 = \sigma_s^2$ and $\chi = \varepsilon^s$ where $\Delta s, \bar{i}, z,$ and ε^s contain the stacked values of $\Delta s_{t+1}, \bar{i}_t, z_t,$ and ε_t^s over T . Sampling σ_s^2 is from the distribution $\sigma_s^2 \sim \mathcal{IG}(c, C)$ where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2} [y'y]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = \nu_0 T = 0.05T$ and $C_0 = c_0 \sigma_0^2 = 0.05T \times 0.5V(\Delta s - \bar{i})$.

Sampling ρ_z and σ_z^2 conditional on the state z_t is implemented by setting $y = z, w^r = w = z_{-1}, \gamma^r = \gamma = \rho_z, \sigma_\chi^2 = \sigma_z^2,$ and $\chi = \varepsilon^z$ where $z, z_{-1},$ and ε^z contain the stacked values of $z_t, z_{t-1},$ and ε_t^z over T . Sampling ρ_z is from the distribution $\mathcal{N}(b, B\sigma_z^2)$ with, from equation

(D5), $B = [w'w + (B_0)^{-1}]^{-1}$ and $b = B [w'y + (B_0)^{-1}b_0]$ where, as noted in Section 4.3.2 in the main text, $b_0 = 0$ and $V_0 = B_0\sigma_z^2 = 1$ so that $B_0 = \frac{1}{\sigma_z^2}$ where we use the prior belief σ_0^2 for σ_z^2 . Sampling σ_z^2 is from the distribution $\sigma_z^2 \sim \mathcal{IG}(c, C)$ where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2} [y'y + (b_0)'(B_0)^{-1}b_0 - b'(B)^{-1}b]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = \nu_0 T = 0.05T$ and $C_0 = c_0\sigma_0^2 = 0.05T \times 0.5V(\Delta s - \bar{i})$. Note that first σ_z^2 is sampled from $\mathcal{IG}(c, C)$ and then, given a draw for σ_z^2 , ρ_z is sampled from $\mathcal{N}(b, B\sigma_z^2)$.

D.2.3: Step 2. The regressions estimated in step 2 are unrestricted so that in equation (D4) we have $w^r = w$ and $\gamma^r = \gamma$.

Sampling $\sigma_{\beta_k}^2$ (for $k = 1, \dots, K$) conditional on the state β_{kt} is implemented by setting $y = \beta_k - \beta_{k-1}$, $w^r = w = 0$, $\gamma^r = \gamma = 0$, $\sigma_x^2 = \sigma_{\beta_k}^2$, and $\chi = \varepsilon_k^\beta$ where β_k , β_{k-1} , and ε_k^β contain the stacked values of β_{kt} , $\beta_{k,t-1}$, and ε_{kt}^β over T . Sampling $\sigma_{\beta_k}^2$ is from the distribution $\sigma_{\beta_k}^2 \sim \mathcal{IG}(c, C)$ where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2} [y'y]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = 0.05T$ and $C_0 = c_0\sigma_0^2 = 0.05T \times 0.01$.

Sampling ρ_x and σ_x^2 conditional on the calculated $x_t = y_t - f_t\beta_t$ is implemented by setting $y = x$, $w^r = w = x_{-1}$, $\gamma^r = \gamma = \rho_x$, $\sigma_x^2 = \sigma_x^2$, and $\chi = \varepsilon^x$ where x , x_{-1} , and ε^x contain the stacked values of x_t , x_{t-1} , and ε_t^x over T . Sampling ρ_x is from the distribution $\mathcal{N}(b, B\sigma_x^2)$ with, from equation (D5), $B = [w'w + (B_0)^{-1}]^{-1}$ and $b = B [w'y + (B_0)^{-1}b_0]$ where, as noted in Section 4.3.2 in the main text, $b_0 = 0$ and $V_0 = B_0\sigma_x^2 = 1$ so that $B_0 = \frac{1}{\sigma_x^2}$ where we use the prior belief σ_0^2 for σ_x^2 . Sampling σ_x^2 is from the distribution $\sigma_x^2 \sim \mathcal{IG}(c, C)$ where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2} [y'y + (b_0)'(B_0)^{-1}b_0 - b'(B)^{-1}b]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = 0.05T$ and $C_0 = c_0\sigma_0^2 = 0.05T \times 0.5V(y_t)$. Note that first σ_x^2 is sampled from $\mathcal{IG}(c, C)$ and then, given a draw for σ_x^2 , ρ_x is sampled from $\mathcal{N}(b, B\sigma_x^2)$.

D.2.4: Step 4 (parts b, c, and d). We first sample the binary indicators $\iota = \delta$ in equation (20). In particular, we follow George and McCulloch (1993) and Frühwirth-Schnatter and Wagner (2010) and use a single-move sampler in which the binary indicators δ_k are sampled one-by-one for $k = 1, \dots, K$. We calculate the marginal likelihoods $p(y|\delta_k = 1, \delta_{-k}, w, \sigma_\varepsilon^2)$ and $p(y|\delta_k = 0, \delta_{-k}, w, \sigma_\varepsilon^2)$ [see Frühwirth-Schnatter and Wagner (2010) for the correct expressions based on a regression of the type of equation (D4) with priors as discussed below]. Upon combining the marginal likelihoods with the Bernoulli prior distributions of the binary indicators $p(\delta_k = 1) = p_0$ and $p(\delta_k = 0) = 1 - p_0$, the posterior distributions $p(\delta_k = 1|y, \delta_{-k}, w, \sigma_\varepsilon^2)$ and $p(\delta = 0|y, \delta_{-k}, w, \sigma_\varepsilon^2)$ are obtained from which the probability $prob(\delta_k = 1|y, \delta_{-k}, w, \sigma_\varepsilon^2) = \frac{p(\delta_k=1|y, \delta_{-k}, w, \sigma_\varepsilon^2)}{p(\delta_k=1|y, \delta_{-k}, w, \sigma_\varepsilon^2) + p(\delta_k=0|y, \delta_{-k}, w, \sigma_\varepsilon^2)}$ is calculated which is used to sample δ_k , that is, draw a random number r from a uniform distribution with support between 0 and 1 and set $\delta_k = 1$ if $r < prob(\cdot)$ and $\delta_k = 0$ if $prob(\cdot) < r$.

We then sample the following parameters from equation (20): the intercept c , the residual variance σ_ε^2 , and the slope coefficients ϕ_k for which the corresponding binary indicators δ_k are equal to 1. The dependent variable y in equation (D4) contains the stacked values of $(1 - \rho_\varepsilon L)\tilde{s}_t$ over T . The restricted predictor matrix w^r in equation (D4) contains a $T \times 1$ vector of ones for the intercept and the stacked (over T) values of the regressors $(1 - \rho_\varepsilon L)\tau_{kt}\tilde{f}_{kt}$ for those k for which the binary indicators are equal to 1. The coefficient vector γ^r in equation (D4) contains c and the coefficients ϕ_k for those k for which the

binary indicators are equal to 1. The error term in equation (D4) is given by $\chi = \varepsilon$ where ε contains the stacked values of ε_t and $\sigma_\chi^2 = \sigma_\varepsilon^2$. Sampling γ^r is from the distribution $\mathcal{N}(b^r, B^r \sigma_\varepsilon^2)$ where B^r and b^r are defined by equation (D5) for which, as noted in Section 4.3.2 in the main text, we have $b_0 = 0$ (an $M \times 1$ vector of zeros) and $V_0 = B_0 \sigma_\varepsilon^2 = 10I_M$ so that $B_0 = \frac{10I_M}{\sigma_\varepsilon^2}$ where we use the prior belief σ_0^2 for σ_ε^2 . Sampling σ_ε^2 is from the distribution $\sigma_\varepsilon^2 \sim \mathcal{IG}(c, C^r)$ where c and C^r are given by equation (D5) with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = 0.05T$ and $C_0 = c_0 \sigma_0^2 = 0.05T \times 0.01$. Note that first σ_ε^2 is sampled from $\mathcal{IG}(c, C^r)$ and then, given a draw for σ_ε^2 , γ^r is sampled from $\mathcal{N}(b^r, B^r \sigma_\varepsilon^2)$.

Finally, we sample the parameter ρ_ε in equation (19) conditional on ε_t and on σ_ε^2 . Estimates for ε_t are obtained from $\varepsilon_t = \tilde{y}_t - c' - \sum_{k=1}^K \phi_k \tau_{kt} \tilde{J}_{kt}$. The regression is unrestricted so that in equation (D4) we have $w^r = w$ and $\gamma^r = \gamma$. We set $y = \varepsilon$, $w = \varepsilon_{-1}$, $\gamma = \rho_\varepsilon$, $\sigma_\chi^2 = \sigma_\varepsilon^2$, and $\chi = \varepsilon^*$ where ε , ε_{-1} , and ε^* contain the stacked values of ε_t , ε_{t-1} , and ε_t^* over T . Sampling ρ_ε is from the distribution $\mathcal{N}(b, B \sigma_\varepsilon^2)$ with, from equation (D5), $B = [w'w + (B_0)^{-1}]^{-1}$ and $b = B [w'y + (B_0)^{-1}b_0]$ where, as noted in Section 4.3.2 in the main text, $b_0 = 0$ and $V_0 = B_0 \sigma_\varepsilon^2 = 1$ so that $B_0 = \frac{1}{\sigma_\varepsilon^2}$ where we use the prior belief σ_0^2 for σ_ε^2 . When sampling ρ_ε from $\mathcal{N}(b, B \sigma_\varepsilon^2)$ we use the draw for σ_ε^2 obtained when estimating equation (20).

APPENDIX E: CONVERGENCE ANALYSIS OF GIBBS SAMPLER

We analyze the convergence of the MCMC sampler using the simulation inefficiency factors as proposed by Kim et al. (1998) and the convergence diagnostic of Geweke (1992) for equality of means across subsamples of draws from the Markov chain [see Groen et al. (2013) for a similar convergence analysis].

For each fixed parameter estimate and for every point-in-time estimate of the unobserved states, we calculate the inefficiency factor as $IF = 1 + 2 \sum_{l=1}^m \kappa(l, m) \hat{\theta}(l)$ where $\hat{\theta}(l)$ is the estimated l th-order autocorrelation of the chain of retained draws and $\kappa(l, m)$ is the kernel used to weigh the autocorrelations. We use a Bartlett kernel with bandwidth m , that is, $\kappa(l, m) = 1 - \frac{l}{m+1}$ where we set m equal to 4% of the retained sampler draws $D - B = 30,000$ (see Section 4.3.1). If we assume that n draws are sufficient to cover the posterior distribution in the ideal case where draws from the Markov chain are fully independent, then $n \times IF$ provides an indication of the minimum number of draws that are necessary to cover the posterior distribution when the draws are not independent. For example, if n is set to 100 then an inefficiency factor equal to 20 suggests that we need at least 2000 draws from the sampler for a reasonably accurate analysis of the parameter of interest. Additionally, we also compute the p values of the Geweke (1992) test which tests the null hypothesis of equality of the means of the first 20% and last 40% of the retained draws obtained from the sampler for each fixed parameter and for every point-in-time estimate of the unobserved component. The variances of the respective means are calculated using the Newey and West (1987) robust variance estimator using a Bartlett kernel with bandwidth equal to 4% of the respective sample sizes.

In Table E1, we present the convergence analysis of the results reported in Section 5 per country/currency and for the different steps of the estimation, that is, estimation of the risk

TABLE E1. Inefficiency factors and convergence diagnostics

Currency	Step	Para- meters/ States	Number	Inefficiency factors (Stats distribution)					Convergence (Rejection rates)	
				Median	Min	Max	5%	10%	5%	10%
AUD	z_t	ρ_z	1	3.36	3.36	3.36	—	—	0.00	0.00
		σ_z^2	1	6.31	6.31	6.31	—	—	0.00	0.00
		σ_s^2	1	6.39	6.39	6.39	—	—	0.00	0.00
		z	52	1.17	0.80	6.87	0.87	2.46	0.10	0.11
	β_t and x_t	ρ_x	1	0.73	0.73	0.73	—	—	0.00	0.00
		σ_x^2	1	1.45	1.45	1.45	—	—	0.00	0.00
		σ_β^2	4	11.93	10.02	12.47	—	—	0.00	0.00
		x	52	0.82	0.74	1.22	0.75	1.11	0.00	0.00
		β	208	0.98	0.78	1.24	0.78	1.23	0.00	0.00
	$E_t(\beta_t)$	δ	4	1.19	1.00	1.65	—	—	0.00	0.00
		$\{c, \phi\}$	5	1.19	0.96	1.45	—	—	0.20	0.40
		ρ_ε	1	2.26	2.26	2.26	—	—	0.00	0.00
		σ_ε^2	1	0.98	0.98	0.98	—	—	0.00	0.00
	CAD	z_t	ρ_z	1	2.27	2.27	2.27	—	—	0.00
σ_z^2			1	3.27	3.27	3.27	—	—	0.00	0.00
σ_s^2			1	3.53	3.53	3.53	—	—	0.00	0.00
z			52	1.13	0.84	3.33	0.91	1.76	0.04	0.11
β_t and x_t		ρ_x	1	0.62	0.62	0.62	—	—	0.00	0.00
		σ_x^2	1	1.56	1.56	1.56	—	—	0.00	0.00
		σ_β^2	4	12.62	10.69	16.15	—	—	0.00	0.00
		x	52	0.89	0.79	1.13	0.82	1.05	0.04	0.12
		β	208	1.01	0.82	1.07	0.82	1.07	0.00	0.00
$E_t(\beta_t)$		δ	4	1.06	0.88	1.13	—	—	0.00	0.25
		$\{c, \phi\}$	5	1.09	0.93	1.41	—	—	0.00	0.20
		ρ_ε	1	3.38	3.38	3.38	—	—	0.00	0.00
		σ_ε^2	1	0.98	0.98	0.98	—	—	0.00	0.00
EUR		z_t	ρ_z	1	5.77	5.77	5.77	—	—	0.00
	σ_z^2		1	10.41	10.41	10.41	—	—	0.00	0.00
	σ_s^2		1	10.25	10.25	10.25	—	—	0.00	0.00
	z		52	1.89	0.98	8.68	1.05	5.15	0.00	0.04
	β_t and x_t	ρ_x	1	0.65	0.65	0.65	—	—	0.00	0.00
		σ_x^2	1	1.61	1.61	1.61	—	—	0.00	0.00
		σ_β^2	4	16.10	13.85	21.35	—	—	0.00	0.00
		x	52	1.01	0.79	1.33	0.83	1.27	0.00	0.00
		β	208	1.04	0.84	1.30	0.84	1.30	0.00	0.00

TABLE E1. Continued

Currency	Step	Para- meters/ States	Number	Inefficiency factors (Stats distribution)					Convergence (Rejection rates)	
				Median	Min	Max	5%	10%	5%	10%
JPY	$E_t(\beta_t)$	δ	4	1.07	1.02	1.20	—	—	0.00	0.25
		$\{c, \phi\}$	5	0.98	0.88	1.19	—	—	0.00	0.00
		ρ_ε	1	2.47	2.47	2.47	—	—	0.00	0.00
		σ_ε^2	1	1.03	1.03	1.03	—	—	0.00	0.00
	z_t	ρ_z	1	2.64	2.64	2.64	—	—	0.00	0.00
		σ_z^2	1	6.04	6.04	6.04	—	—	0.00	0.00
		σ_s^2	1	5.65	5.65	5.65	—	—	0.00	0.00
		z	52	1.43	0.87	3.99	0.91	2.52	0.02	0.04
	β_t and x_t	ρ_x	1	0.72	0.72	0.72	—	—	0.00	0.00
		σ_x^2	1	0.99	0.99	0.99	—	—	0.00	0.00
		σ_β^2	4	12.11	8.30	18.14	—	—	0.00	0.00
		x	52	0.99	0.84	1.38	0.88	1.18	0.10	0.19
	β	208	1.12	0.85	1.42	0.85	1.42	0.00	0.25	
GBP	$E_t(\beta_t)$	δ	4	1.00	0.87	1.13	—	—	0.00	0.25
		$\{c, \phi\}$	5	1.10	0.99	1.23	—	—	0.00	0.00
		ρ_ε	1	1.86	1.86	1.86	—	—	0.00	0.00
		σ_ε^2	1	0.93	0.93	0.93	—	—	0.00	0.00
	z_t	ρ_z	1	2.44	2.44	2.44	—	—	0.00	0.00
		σ_z^2	1	3.95	3.95	3.95	—	—	0.00	0.00
		σ_s^2	1	5.48	5.48	5.48	—	—	0.00	0.00
		z	52	1.11	0.90	3.98	0.91	2.35	0.04	0.08
	β_t and x_t	ρ_x	1	0.68	0.68	0.68	—	—	0.00	0.00
		σ_x^2	1	1.78	1.78	1.78	—	—	0.00	0.00
		σ_β^2	4	15.05	10.86	20.09	—	—	0.00	0.25
		x	52	1.07	0.71	1.71	0.76	1.62	0.15	0.27
	β	208	0.96	0.91	1.19	0.91	1.19	0.00	0.23	
SGD	$E_t(\beta_t)$	δ	4	1.05	0.85	1.24	—	—	0.25	0.25
		$\{c, \phi\}$	5	1.02	0.83	2.03	—	—	0.00	0.20
		ρ_ε	1	2.22	2.22	2.22	—	—	0.00	0.00
		σ_ε^2	1	1.00	1.00	1.00	—	—	0.00	0.00
	z_t	ρ_z	1	3.92	3.92	3.92	—	—	0.00	0.00
		σ_z^2	1	5.44	5.44	5.44	—	—	0.00	0.00
		σ_s^2	1	6.62	6.62	6.62	—	—	0.00	0.00
		z	52	1.33	0.88	5.53	0.96	3.14	0.08	0.27

TABLE E1. Continued

Currency	Step	Para- meters/ States	Number	Inefficiency factors (Stats distribution)					Convergence (Rejection rates)	
				Median	Min	Max	5%	10%	5%	10%
KRW	β_t and x_t	ρ_x	1	0.77	0.77	0.77	—	—	0.00	0.00
		σ_x^2	1	1.41	1.41	1.41	—	—	0.00	0.00
		σ_β^2	4	13.64	7.40	17.60	—	—	0.25	0.50
		x	52	0.89	0.68	1.09	0.72	1.06	0.00	0.06
		β	208	0.95	0.74	1.18	0.74	1.18	0.00	0.00
	$E_t(\beta_t)$	δ	4	0.96	0.89	1.17	—	—	0.00	0.00
		$\{c, \phi\}$	5	1.08	0.85	2.06	—	—	0.00	0.40
		ρ_ε	1	2.35	2.35	2.35	—	—	0.00	0.00
		σ_ε^2	1	0.96	0.96	0.96	—	—	0.00	0.00
	z_t	ρ_z	1	2.79	2.79	2.79	—	—	0.00	0.00
		σ_z^2	1	5.96	5.96	5.96	—	—	0.00	0.00
		σ_s^2	1	6.57	6.57	6.57	—	—	0.00	0.00
		z	52	1.23	0.93	6.28	0.97	2.07	0.08	0.11
	β_t and x_t	ρ_x	1	0.70	0.70	0.70	—	—	0.00	0.00
		σ_x^2	1	1.38	1.38	1.38	—	—	0.00	0.00
		σ_β^2	4	12.07	8.39	12.86	—	—	0.00	0.00
		x	52	0.99	0.86	1.15	0.87	1.13	0.06	0.13
		β	208	0.97	0.93	1.00	0.93	0.99	0.25	0.25
	$E_t(\beta_t)$	δ	4	0.97	0.88	1.37	—	—	0.00	0.00
		$\{c, \phi\}$	5	1.07	0.96	1.77	—	—	0.00	0.00
ρ_ε		1	2.15	2.15	2.15	—	—	0.00	0.00	
σ_ε^2		1	0.95	0.95	0.95	—	—	0.00	0.00	
ZAR	z_t	ρ_z	1	2.31	2.31	2.31	—	—	0.00	0.00
		σ_z^2	1	3.85	3.85	3.85	—	—	0.00	0.00
		σ_s^2	1	2.80	2.80	2.80	—	—	0.00	0.00
		z	52	1.07	0.75	2.90	0.82	1.59	0.06	0.06
β_t and x_t	ρ_x	1	1.10	1.10	1.10	—	—	0.00	0.00	
	σ_x^2	1	1.66	1.66	1.66	—	—	0.00	0.00	
	σ_β^2	4	13.71	11.68	20.52	—	—	0.00	0.00	
	x	52	0.87	0.77	1.08	0.79	1.04	0.00	0.00	
	β	208	0.99	0.91	1.23	0.91	1.23	0.00	0.00	

TABLE E1. Continued

Currency	Step	Para- meters/ States	Number	Inefficiency factors (Stats distribution)					Convergence (Rejection rates)	
				Median	Min	Max	5%	10%	5%	10%
	$E_t(\beta_t)$	δ	4	1.15	1.02	1.58	—	—	0.00	0.00
$\{c, \phi\}$		5	0.98	0.94	1.06	—	—	0.00	0.20	
ρ_ε		1	3.40	3.40	3.40	—	—	0.00	0.00	
σ_ε^2		1	0.97	0.97	0.97	—	—	0.00	0.00	

Note: No statistics are reported for $E_t(\beta)$ as these series are entirely determined by the parameters ϕ for which convergence diagnostics are included in the table. The statistics of the distribution of the inefficiency factors are presented in columns 5–9 for every parameter or group of parameters. The inefficiency factors are calculated for every fixed parameter and for every point-in-time estimate of the unobserved states using a Bartlett kernel with bandwidth equal to 4% of the 30000 retained sampler draws. The reported distribution statistics are identical when parameters are considered individually as only one inefficiency factor is calculated in these cases. The rejection rates of the Geweke (1992) test conducted at the 5% and 10% levels of significance are reported in columns 10 and 11. These rates are equal to the number of rejections of the null hypothesis of the test per parameter group divided by the number of parameters in a parameter group. These rates are either 1 or 0 for parameters that are considered individually. They are based on the p-value of the Geweke test of the hypothesis of equal means across the first 20% and last 40% of the 30000 retained draws which is calculated for every fixed parameter and for every point-in-time estimate of the unobserved component. The variances of the respective means in the Geweke (1992) test are calculated with the Newey and West (1987) robust variance estimator using a Bartlett kernel with bandwidth equal to 4% of the respective sample sizes.

premium z_t , the estimation of the time-varying structural parameters β_t and the unobserved component x_t , and the estimation of the scapegoat effects $E_t(\beta_t)$. Convergence results are reported for individual parameters or for groups of parameters or states. The point-in-time estimates of a particular state (i.e. a time series) are grouped as well. For example, the states z_t and x_t are time series of length $T = 52$ so that the groups z and x in the table are of dimension 52 while the state β is a multidimensional state of dimension $K = 4$ so that the group β in the table is of dimension $T \times K = 208$. Since $K = 4$ the groups δ and $\{c, \phi\}$ are of dimension 4, respectively 5. We note that no statistics are reported for $E_t(\beta_t)$ as these series are entirely determined by the parameters ϕ for which convergence diagnostics are included in the table. In both tables, we report statistics of the distributions of the inefficiency factors for every individual parameter or group of parameters/states, that is, median, minimum, maximum, and—for the states z_t , x_t , and β_t —the 5% and 10% quantiles. Obviously, these statistics are all identical for the non-grouped parameters (For the group β , the 5% and 10% quantiles are often almost identical to minimum and maximum which stems from the fact that there is almost no time variation in the β 's—as reported in Figure 3—so that inefficiency factors are different for every k (per t) but very similar for every t (per k) which implicitly reduces the dimension of the group β from $T \times K = 52 \times 4$ to $K = 4$). The tables also report the rejection rates of the Geweke tests conducted both at the 5% and 10% levels of significance. These rates are equal to the number of rejections of the null hypothesis of the test per parameter group divided by the number of parameters in a parameter group. These rates can only be zero or one for individual (non-grouped) parameters but can lie between zero and one for the grouped parameters.

The calculated inefficiency factors for all currencies and steps of the estimation procedure suggest that the MCMC sampler performs well and that all parameters and states are well converged using our retained 30,000 draws. In fact, according to the inefficiency factors, an accurate analysis could have been conducted with less than 30,000 draws. We prefer an analysis based on 30,000 retained draws rather than, say, 10,000 retained draws because the Geweke (1992) test for equality of means across subsamples of the retained draws performs better with 30,000 draws than with 10,000 draws. The Geweke (1992) rejection rates reported in the table are, with few exceptions, equal or close to zero and therefore strongly suggest that the means of the first 20% and last 40% of the retained draws are equal. In a few instances, somewhat higher rejection rates are observed but, upon comparing different countries/currencies, there is no evidence that one parameter or state is systematically affected. In fact, most often the higher rejection rates are due to the particular sample of draws and are not indicative of non-convergence as these rejection rates are not withheld when we rerun the sampler using another seed. Hence, in general, we conclude that the convergence of the sampler for the retained number of draws is satisfactory.