

THE APPLICATION OF EXPECTED-UTILITY THEORY TO THE CHOICE OF INVESTMENT CHANNELS IN A DEFINED-CONTRIBUTION RETIREMENT FUND

BY

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ABSTRACT

This study examines the practical application of a system for the derivation of member utility functions for the purpose of recommending investment-channel choice to members of a defined-contribution retirement fund. The utility functions of post-retirement benefits from members of a defined-contribution fund are elicited. The risk aversion of each member is measured and the results are compared with a standard risk-tolerance assessment method.

KEYWORDS

Defined contribution, Investment-channel choice, Risk tolerance, Risk aversion, Utility function

1. INTRODUCTION

In South Africa, as in other countries, there has been a large movement from defined benefit (DB) schemes to defined contribution (DC) schemes since the 1980s (Andrew, 2004). Today the majority of employees in the South African private sector belong to DC schemes (Kamionsky & Bashe, 2001). This movement has resulted in a substantial transfer of risk from the employer (who traditionally sponsored a DB scheme) to the individual member of the DC scheme. These factors and the risks that they pose to DC scheme members are well documented e.g. Blake, Cairns & Dowd (2003) and Daykin (2002).

Arguably, trustees and members of a retirement fund have very little control over these factors, with the exception of the chosen investment strategy (normal retirement age and contribution rates are often determined by the rules of the fund). The investment strategy chosen for a fund member is thus critical in determining the ultimate benefit received, and hence the risk faced, by a member of a DC fund.

In South Africa most DC funds have one investment strategy in place for all members. There is, however, a move to offer member investment choice

(Kamionsky & Bashe, 2001). Typically this is offered by establishing a set of investment channels and allowing members to allocate various proportions of their accumulated retirement funds to these channels. In this instance, members are responsible for deciding their own investment strategy.

Ultimately, the existence of member investment choice should allow members to tailor an investment strategy to their unique objectives and shifting financial circumstances. It is an individual optimisation problem that the trustees of a retirement fund are unable to undertake. However, it is debatable whether individuals have the skills, knowledge or expertise to make such investment decisions (Bodie, 2004).

There is thus a need for an instrument that assists members to tailor an investment strategy to their objectives and financial circumstances so that they may derive the benefits of investment choice. Because the movement from DB to DC funds has resulted in devolution of financial risk from employers to employees, the onus on actuaries is increasingly to establish bases for the management of risks by members (Andrew, 1994). While this paper does not offer an actuarial model of those risks, it addresses issues relating to the implementation of systems that would facilitate the use of such models in the management of those risks.

The first aim of this study was to determine the feasibility of using Thomson's (2003b) system as a mechanism for eliciting the utility functions of individual members in a DC fund. While Thomson (2003b) succeeded in eliciting such functions for a different sample, no attempt was made in that study to systematise the process. In this study, which is reported more fully in Levitan (unpublished), the utility function of members was elicited through a computer system, using responses from a slightly modified version of Thomson's (2003b) questionnaire. The construction and interpolation techniques proposed by Thomson (2003b) were used.

As reported in Thomson (2003a), certain authors (e.g. Clarkson, 1990, 1996 and Ramsay, 1993) have rejected the use of expected-utility theory for actuarial applications. While Thomson (2003a) defends such use for the purposes envisaged in this study, that defence addresses its validity largely as normative theory. It may still be questioned whether, in practice, it adequately measures a subject's attitude to risk. Conversely, the risk-assessment systems typically used in the market are without theoretical foundation. Practitioners therefore have grounds for scepticism about both approaches.

The second aim of the study was to establish whether the levels of risk aversion implied by the utility functions elicited are consistent with the results of a risk-assessment system typical of those used in the market. The point of this aim is that, if in certain circumstances the two approaches generally produce consistent results, then, at least in those circumstances, not only is their validity enhanced, but there may also be scope for the complementary use of both approaches. This was recently explored by Faff et al (2008). They examine the consistency of financial risk-tolerance scores derived from survey data and a risk-aversion measure derived from lottery experiments.

Both aims were addressed in a limited way, both with regard to the sampling method and with regard to the research questions. There were two reasons for this. First, if for either of the aims the results were negative, then further research would arguably be unwarranted. Secondly, the design of research to answer research questions that might arise from positive results would have been much more demanding both on the subjects and on the financial resources available for the research. The purpose of the study was therefore largely to determine the need for further research.

2. RISK ASSESSMENT

2.1. Risk-assessment instruments

An investment strategy should be devised by considering the objectives of the investor and their attitudes towards risk (Lo & MacKinlay, 2001). There are a number of risk-assessment instruments that are used by investment consultancies and financial planners. However, they are often proposed without rationale or motivation (Roszowski et al, 1993).

Grable & Lytton (1999: 164) state that

“The need for a widely accepted and commonly used instrument is as great today as any time in the past. Without such an instrument financial service providers and researchers have been forced to use other assessment techniques that may not adequately measure the underlying construct of financial risk tolerance”.

Accordingly, they present an instrument that can be used to assess financial risk tolerance. It consists of thirteen multiple-choice questions. The questionnaire was designed to measure seven dimensions of risk, each question measuring one or more dimension.

Grable & Lytton (1999) developed their questionnaire by considering over 100 different risk-assessment items from various academic and trade publications. They developed the final thirteen-item instrument through a combination of pilot studies and statistical analyses.

The instrument presented meets the requisite criteria proposed by Macrimmon & Wehrung (1986), i.e. that all risk-assessment instruments should exhibit:

- relevance to respondents;
- ease of administration;
- adequate validity and reliability;
- the ability to derive a risk measure; and
- embodiment of a central concept of risk.

It may be noted that these criteria make no mention of consistency over time. While subjects' attitudes to risk may change over time, a measure of risk tolerance that exhibits relative stability over time, while nevertheless reflecting

differences between individuals, may be considered more reliable for the purposes envisaged in this study.

The response to each of the thirteen multiple-choice questions is given a weight (in the range of one to four) according to the 'riskiness' of the response. The higher the weight assigned, the more risky the choice. An index score is calculated by adding up the weights for each of the thirteen questions. A high score suggests a high risk tolerance whilst a low score suggests a lower risk tolerance.

Grable & Lytton (1999) conclude that individually, none of the questions alone was sufficient to accurately measure risk tolerance. However, when the questions are combined, the questionnaire provided a tool that could accurately measure an individual's financial risk tolerance.

Grable & Lytton (2003) presented further empirical evidence regarding the validity of their risk-assessment instrument. A sample of individuals who made their own investment decisions were asked to complete the questionnaire and indicate their portfolio of investment assets. The analysis indicated that the risk-assessment instrument scores were correlated with portfolio asset ownership as explained by modern portfolio theory. These findings offered additional support for the validity of the assessment instrument in measuring financial risk tolerance.

2.2. Expected-utility Theory

In Thomson (2003b) a system for deciding on an appropriate investment strategy for members of DC schemes that offer their members investment channel choice is proposed. This system is based on the 'equally likely certainty equivalent' method of Anderson, Dillon & Hardaker (1977). Whilst other papers have focused on the appropriate investment strategy for a fund as a whole using a mean-variance framework and downside risk measures (e.g. Arts & Vigna, 2003, Blake, Cairns & Dowd, 2001 and Kamionsky & Bashe, 2001), Thomson (op. cit.) considers individual preferences using expected-utility theory.

One characteristic that may be obtained from a member's utility function is the relative risk aversion (Pratt, 1964). This measures how the percentage of wealth invested in risky assets changes as the preferences of the member change. It refers to the change in the percentage investment in risky assets as the retirement benefit increases.

Relative risk aversion is measured using the function:

$$R(x) = -x \frac{u''(x)}{u'(x)}; \quad (1)$$

where $u(x)$ is the utility associated with a retirement benefit of x .

2.3. The Use of Expected-utility Theory in Setting an Investment Strategy

Hanna, Gutter, and Fan (2001) mention at least four methods of inferring investment risk tolerance. These include the observation of the actual investment

choice of the individual, the posing of hypothetical questions and the use of so-called attitudinal measures. They state that an individual's utility function is typically assumed to resemble one with constant relative risk aversion. Barsky et al. (1997) construct measures of the Arrow-Pratt concept of risk aversion using responses to hypothetical income questions. They concluded that households differ markedly in their willingness to assume risk and that risk aversion had considerable predictive power on the risky choices made by households.

The principle of expected-utility maximisation states that a rational investor should pursue that investment strategy which maximises their expected utility (Mitchell et al, 2001). For a given stochastic actuarial model of the returns on the investment channels available and a particular member's utility function, Thomson (2003b) shows how this principle can be applied to the recommendation of an optimal allocation between those investment channels.

Arguably, a fund with no investment choice or the default investment strategy of a fund with member investment choice should pursue that investment strategy which maximises the expected utility of as many members as possible. As members generally have different utility functions, it will generally be impossible to maximise the expected utility of more than one member.

Although theoretically utility functions can be used to determine an appropriate investment strategy for a member, it is not trivial to specify the utility function of a member. An incorrect specification of a member's utility function can result in the recommendation of an incorrect investment strategy.

For example, although lifestyle strategies have previously been evaluated using expected-utility theory, the choice of utility functions has differed. Booth & Ong (1994) assumed that members have an exponential utility function and Khorasane & Smith (1997) elicited utility functions that violate the axioms of expected utility theory and were restrictive, whilst more recently Vigna & Haberman (2001) made the assumption that members have quadratic utility functions. Accordingly, they have reached conflicting conclusions on the merits of a lifestyle investment strategy. The choices of utility functions have been unrealistic and thus no conclusion on a lifestyle investment strategy in the context of expected-utility theory can be inferred.

Thus, the ability to use expected-utility theory in determining an optimal investment strategy relies on the successful specification of a member's utility function.

2.4. The Elicitation of Utility Functions

Thomson (op. cit.) proposes an interactive system that can be used to elicit member-specific utility functions. The system is interactive, in that it requires a member's response to a series of questions in order to derive the member's utility function. The utility function obtained from the system may be applied to the post-retirement income or any other retirement benefit of members of retirement funds.

Thomson's (2003b) proposed framework thus allows for a direct comparison of various investment strategies. This is achieved by comparing the expected utility that members would derive from their benefit at retirement under competing investment strategies.

Whilst utility theory is appealing, the success in obtaining a member's utility function as well as the merits of applying utility theory have been mixed. For example, Elton & Gruber (1995: 214) state that "A number of brokerage firms and banks have developed programs to extract the utility functions of investors... These have not been particularly successful."

Thomson (2003a) defends the use of expected utility for the recommendation of an investment strategy for an individual member in a DC fund against arguments that have been levelled against it. However, he does raise some caveats regarding the use of expected-utility theory.

A theory of decision making under uncertainty may be justified as a 'descriptive', 'normative', or 'prescriptive' theory, evaluated respectively by its empirical validity, theoretical adequacy or pragmatic value (Bell, Raiffa & Tversky, 1999: 17). Thomson (2003b) justifies the use of expected-utility theory on normative grounds. He suggests that trained investment personnel should assist members with the implementation of the system and should explain the implications of the recommendations.

For the purposes of this research, it is assumed that expected-utility theory is an appropriate framework with which to compare competing investment strategies. It is also assumed that each member's preference for an outcome can be measured using an appropriate utility function.

Whilst theoretically appealing, it could be argued that such a system is not easy for members to understand and could potentially be misused by members, which would result in misstatement of their utility functions. As in the case of the risk-assessment instruments discussed in section 2.1, the issue of consistency over time was not discussed by Thomson (2003a). The ability to compare various investment strategies and derive an optimal, and reasonably stable, investment strategy using member-specific utility functions clearly relies on the successful elicitation of these functions and on the consistency of members' responses over time.

3. RESEARCH DESIGN

Thomson's (2003b) questionnaire was administered to members of a selected South African retirement fund. All members of the chosen retirement fund were approached for this study.

This is an improvement over the sampling method used in Thomson (2003b), which was based on responses acquired by students of the University of Witwatersrand. Each third-year actuarial science student was required to select a member of a retirement fund who was not yet on pension.

As at the date of the study, the fund chosen had the following characteristics:

- it was a DC fund with 2891 members;
- it was a provident fund; and
- the members were employees of a company offering employee-benefit services to retirement funds.

There is reason to believe that, on average, the members of the chosen fund have higher levels of investment knowledge than those of other funds and might therefore more easily complete the questionnaire. Also, if members with a basic level of investment knowledge were unable to use the system, it could imply that the system might be too complex in its current form. However, if members appeared to be comfortable with the system, it could suggest that where necessary investment training could be provided to members of other funds so that the system can be used.

The fund constituted a large group of members with varying demographic factors. However, the choice of retirement fund was not necessarily representative of retirement funds in South Africa.

3.1. Questionnaire Administration

An email was sent to each member of the fund informing them of the survey. The survey was then administered using the internet. After a respondent completed the questionnaires, a summary of the responses was displayed, emailed to the researcher and stored in an online database.

In total, 89 members responded to the study. This represented approximately 3.1% of the fund's membership. The response rate to the survey conducted by Grable & Lytton (2003) was also approximately 3%. Because of the method of administration, the small sample size may have resulted in a bias.

The majority (86.5%) of the participants were younger than 45. This shows no material bias; 83% of fund members were younger than 45. However, 63% of the respondents were younger than 35, compared with 28% of fund members. The average age of the sample was thus younger than that of the fund. One reason for this could be the method of survey distribution.

A significant number of respondents (87%) had some form of tertiary education, 62% having some form of university degree.

3.1.1. Questionnaires

Minor changes were made to the original questionnaires proposed by Thomson (2003b) and Grable & Lytton (1999). The section below details the changes and the rationale behind them.

This research used the approach proposed by Thomson (2003b) to elicit a member-specific utility function. Thomson's (2003b) questionnaire requires

members to choose between a series of risky prospects. It also refers to ‘benefits’ that a member may receive. The benefit chosen for the purpose of this study was the level of pension received in retirement, which was expressed in the form of a member’s net replacement ratio (NRR).

The NRR at retirement is that level of annual pension (after tax) that a member can afford using his/her accumulated retirement fund savings expressed as a proportion of the member’s annual income at retirement (after tax). The pension is calculated by dividing the fund credit at the date of retirement by an annuity factor at a real (before price inflation) rate of interest (Faculty & Institute of Actuaries, 2002). The annuity is a notionally purchased one as members may decide to purchase so-called ‘living annuities’, with-profit annuities, index-linked annuities or a variant of the traditional fixed-income annuities.

The NRR at the member’s retirement date is defined as

$$\frac{f/a_x}{s}$$

In this definition:

- f is the fund credit of the member at the retirement date;
- a_x is the average cost of an index-linked annuity of 1 per annum for a member retiring at age x ; and
- s is the member’s salary at the retirement date.

3.1.2. Changes made to Thomson’s questionnaire

The utility function derived and hence the questions used to elicit the member’s utility function must be phrased in terms of the benefit that the member may receive at retirement.

The retirement benefit may take a number of forms. For example, it could be:

- a lump sum in the form of the member’s accumulated fund credit at the date of retirement;
- the annual pension that can be purchased using the ultimate fund credit at retirement (this pension could be level, increase at a fixed rate or indexed to increase by some measure of inflation); and
- a combination of a lump sum and pension.

The full version of the questionnaire that was administered is provided in Appendix A.

3.1.3. Grable & Lytton’s instrument for financial risk tolerance

The questionnaire proposed by Grable & Lytton (1999) was amended for use in a South African context. The amended questionnaire is provided in Appendix B.

3.2. Survey website

A website was developed exclusively for the purpose of administering the questionnaires. Access to the questionnaire was kept as simple as possible.

All members of the retirement fund had access to computers, frequently used the internet and had at least a basic level of computer literacy. Therefore the usual problems associated with an internet-based survey did not apply.

The interactive nature of the questionnaire proposed by Thomson (2003b) makes a computer program the easiest way to administer the questionnaire. For example, questions in Thomson's (2003b) system are directly dependent on the answers to previous questions. An internet-based questionnaire permits the use of complex question-skipping logic and other features, which is not possible with paper-administered questionnaires.

Other advantages of using this method of questionnaire administration over other methods were that:

- the speed of the distribution of the survey was increased as all members of the fund were contacted simultaneously via email;
- it assured the member of confidentiality as their identity is not known to the researchers;
- it eliminated the response bias that can materialise if questionnaires are administered interactively as the internet asks each question in the same way;
- it enabled members to respond to the survey at their leisure;
- it electronically recorded results of questions and thereby mitigated the risk of incorrect data capturing;
- it could potentially result in more honest answers to sensitive topics such as salary, when giving their answers online instead of to an interviewer;
- it did not require a drop-off point as would be the case with paper-administered questionnaires; and
- it might have stimulated a higher response rate because of the novelty element of an internet survey.

There are some disadvantages to this choice of questionnaire administration method. These are mentioned below.

Thomson (2003b) proposes assistance for a member. Members who had problems completing the questionnaire did not have an interviewer with whom they could discuss their problems. However, the contact details of the researcher were provided in the email. These details could be accessed at all times during survey completion from the website. Three members did contact the researcher regarding problems they had in completing the survey.

There is a problem of self-selection using this method; the sample of respondents is likely to be biased using this method (Schonlau, 2002). This is further discussed in section 5.2.

Other possible disadvantages are that:

- members could quit or be disturbed in the middle of a questionnaire as the survey required between fifteen and twenty minutes to complete;

- the response rate was not guaranteed;
- the internet might be slow or unreliable;
- there was no way of ensuring that each member only completed the questionnaire once;
- members may have regarded the initial email as spam.

The website was designed to prevent non-responses or invalid responses to each of the four questionnaires. Examples of this included the requirement that each respondent answer all questions on a page before being permitted to proceed to the next page and asking respondents to reconsider their responses to the questionnaire if the answers provided were not within an appropriate range.

The website was also designed to avoid excessive information and explanation. If the system is to be used in practice, it may be advisable to make additional information available, without cluttering the web page, by means of a link to a help facility or to frequently asked questions.

3.3. Analysis of Respondents' Attitudes towards Risk

3.3.1. Analysis of risk aversion from Thomson's questionnaire

The questionnaire proposed by Thomson (2003b) provides a mechanism for obtaining five points on a member's utility curve. Two are arbitrary and specified in advance and the other three are specified by the member. The arbitrary points x_0 and x_4 were set at 40% and 90% as in question 1 of the questionnaire in Appendix A. The elicited points x_1 , x_2 and x_3 were set at the values of responses two, one and three respectively, as specified in that Appendix. These points are all that are required from the member to specify their utility function u , where x is a particular level of NRR and $u(x)$ is the member's utility associated with obtaining an NRR of x .

The utility curve for each member was derived using the process described in Thomson (2003b), as set out in Appendix C. The statistical package MATLAB was used to determine the parameters of each member's utility function.

Thomson's (2003b) questionnaire was used to derive member-specific utility functions. The aim of this study is to determine whether the utility function elicited accurately captures the member's attitude to financial risk. This differs from that of Faff et al (op. cit.) as they use real rather than hypothetical gambles to elicit the utility function. Also, the outcomes are one-off prizes as opposed to post-retirement income.

One method of doing this is to compare a measure of risk aversion derived from the elicited utility function with a measure derived from an alternative set of questions that uses simpler language. For this purpose, the questionnaire proposed by Grable & Lytton (1999) was used as it is widely used and has been academically motivated. A single measure of risk aversion or risk tolerance was therefore needed from each of the instruments.

A summary statistic of a member's risk aversion can be obtained from the member's utility function by using the relative risk aversion $R(x)$ as defined in equation (1). A measure of the overall risk aversion of the member may be evaluated by calculating an average of the function $R(x)$ over the range of retirement benefits considered. As explained in Thomson (2003b), this range comprises five values, denoted x_1 to x_5 .

The measure adopted, referred to below as the 'average risk aversion', is defined as \tilde{R} and is evaluated using the equation

$$\tilde{R} = \frac{1}{x_5 - x_1} \int_{x_1}^{x_5} R(x) dx. \quad (2)$$

The form chosen for the utility function in Thomson (2003b) means that the definite integral in this equation will not generally have a closed-form solution. The trapezoidal method (e.g. Hoffmann & Bradley, 1996: 487) was used to calculate \tilde{R} . The range of retirement benefits considered was divided into 100,000 sub intervals for the purposes of calculating the integral. In all but one case, this resulted in convergence of the integral to three decimal places. The case where convergence did not occur is discussed below.

3.3.2. Analysis of risk tolerance from Grable & Lytton's questionnaire

Grable & Lytton's (1999) instrument has been developed to calculate a single index score reflecting the member's risk tolerance. This is achieved by adding up the weights associated with the member's responses to the thirteen questions administered. The risk-tolerance scale proposed by Grable & Lytton (1999) is shown in Table 1.

TABLE 1.
RISK-TOLERANCE SCALE FOR GRABLE & LYTTON'S QUESTIONNAIRE

Question	Score			
	a	b	c	d
Question 1	a = 4	b = 3	c = 2	d = 1
Questions 2, 3, 6, 7, 8, 11, 13	a = 1	b = 2	c = 3	d = 4
Questions 4, 5, 12	a = 1	b = 2	c = 3	
Questions 9, 10	a = 1	b = 3		

3.3.3. Consistency of the measures

In this section, the method used for comparing the measures obtained from Thomson's (2003b) system and Grable & Lytton's (1999) questionnaire is described.

The utility function of each member was determined using the method specified above. \tilde{R} , as specified in equation (2), was calculated using the member-specific utility function. The risk-tolerance score from Grable & Lytton's (1999) questionnaire was determined for each member. The data points were inspected for possible outliers.

4. RESULTS

4.1. Member Responses

There were 89 recorded responses to the survey. These responses were received within two weeks of the initial email communication to members. The majority of the responses (67%) were received within the first three days of email distribution.

Although 89 responses were received, only 75 could be used for the elicitation of utility functions. This accounts for approximately 2.6% of the Fund's membership. Fourteen of the 89 responses could not be used as the members' responses contained one or more missing values for the questionnaire.

Three members of the fund contacted the researcher because of difficulties they experienced in responding to the survey. In all cases, the difficulty was with the specification of the certainty equivalent in Thomson's (2003b) questionnaire.

4.2. Results from Thomson's Questionnaire

The analysis reported in this section is on the responses received from the remaining 75 members. Figure 1 plots the utility curves of the 75 valid member responses over the NRR range of 40% to 90%. The figure clearly illustrates that there is variability in the utility functions elicited from members. It also illustrates the flexibility of the utility function form specified by Thomson (2003b) as the shapes of the utility functions can vary widely.

The reader's attention is drawn to two particular curves in Figure 2 that are notably different from the others: the curve that reaches 1 at an NRR of 50% and that which exhibits a downward slope between 50% and 70%. Further comment is made on these cases below.

The average risk aversion, \tilde{R} , was calculated for each of the valid member responses.

Figure 2 depicts the distribution of the average risk aversion, \tilde{R} , obtained for each valid member response in the sample.

As can be seen in Figure 2, there is a member with a risk-aversion of 29. This is possibly an outlier in the sample. This point is significantly higher than the second highest risk-tolerance score of 11.7, which could possibly also be an outlier compared with the remaining 73 members who have scores between -1.2 and 7.1. These two points affect the scale of the histogram shown in Figure 2.

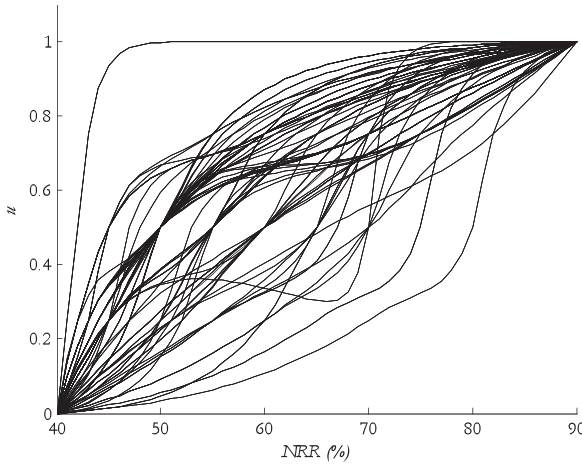


FIGURE 1: Utility functions derived from all valid member responses.

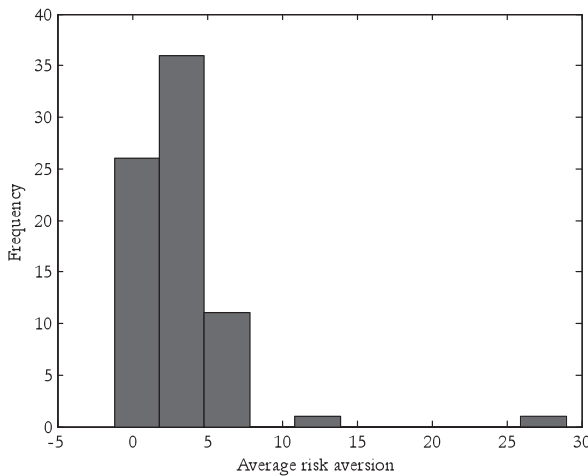


FIGURE 2: Histogram of average risk aversion \tilde{R} .

4.3. Results from Grable & Lytton’s Questionnaire

The risk-tolerance score for Grable & Lytton’s (1999) questionnaire is determined by adding up the weights associated with the member’s responses to the thirteen questions administered. The maximum possible score is 47 and the minimum possible score is 13. The higher the score, the more risk tolerant the member is assumed to be.

Figure 3 is a histogram of the Grable & Lytton risk-tolerance score obtained for the members in the sample.

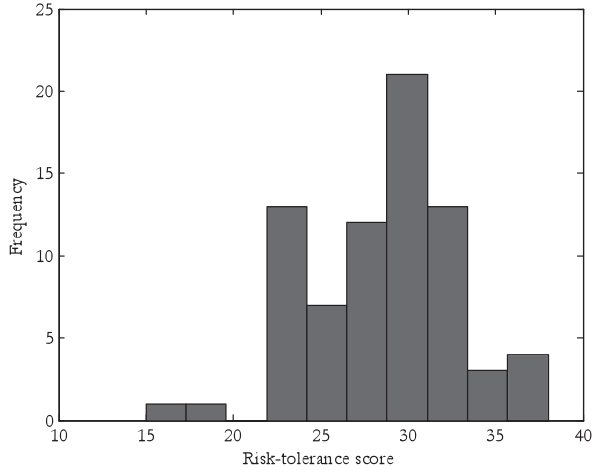


FIGURE 3: Histogram of Grable & Lytton's risk-tolerance score.

Most of the risk-tolerance scores observed are greater than 20. The histogram highlights the variability of risk-tolerance scores obtained within the sample.

4.4. Relationship between the Measures

In this section, the relationship between the risk-tolerance scores obtained from Grable & Lytton's (1999) and the average risk aversions obtained from Thomson's (2003b) questionnaire is investigated.

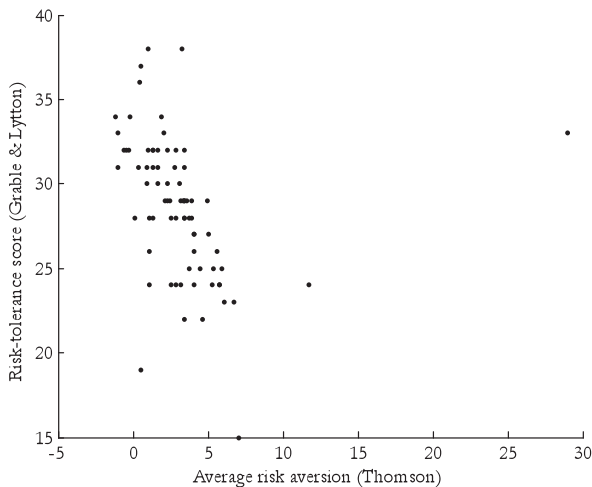


FIGURE 4: Scatter plot of average risk aversion and the risk-tolerance score.

A member with high risk tolerance should have a low average risk aversion but a high Grable & Lytton risk-tolerance score. Assuming both questionnaires accurately determine a member's financial risk tolerance, a negative relationship should exist between the two measures.

A graphical analysis is done first. Figure 4 is a scatter plot of the two measures. The average risk aversion \tilde{R} is plotted on the horizontal axis and Grable & Lytton's risk-tolerance score is plotted on the vertical axis. The scatter plot clearly illustrates a negative relationship between the scores obtained from the respective questionnaires.

The sample correlation coefficient of the risk-tolerance measures is -0.2498 . A one-tailed hypothesis test on the correlation coefficient was performed. The null hypothesis was that there is no correlation between the risk-tolerance scores. The alternative hypothesis was that there is a negative correlation between the risk-tolerance scores. The p -value calculated for this hypothesis test is 0.0153 . Hence, notwithstanding the small sample size, the null hypothesis of no correlation can be rejected at the 5% level of significance.

4.4.1. Investigation of possible outliers

Figure 4 indicates that two points should be investigated further to determine if they are possible outliers. These two points are indicated by rectangles in Figure 5. They correspond to respondents 5 and 42.

Although the responses of respondents 5 and 42 appear to be outliers, they cannot be removed from the data set without good reason. Furthermore, since the null hypothesis was rejected at the 5% level even with the apparent outliers, it is not necessary to remove them for the sake of the test of that hypothesis.

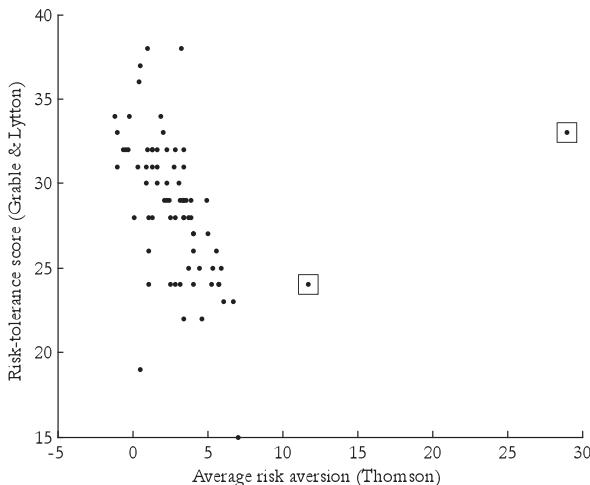


FIGURE 5: Scatter plot of average risk aversion and the risk-tolerance score indicating possible outliers.

Nevertheless, it is instructive to consider these responses as they illustrate problems encountered by respondents in their responses to the questionnaire. In this section, the responses of these members are investigated further to determine if they are outliers. A check on the remaining data points was performed to determine if there were any other outliers.

4.4.2. Investigation of member response 5

The Grable & Lytton risk-tolerance score of member response 5 was 33 compared with the median sample score of 29, whilst \tilde{R} was evaluated as 29.0 compared with a median value of 2.9.

The responses of member 5 to Thomson's (2003b) questionnaire are investigated.

Member 5 provided the following responses to Thomson's (2003b) questionnaire.

Respondent #5	NRR (%)
Response 1	41
Response 2	41
Response 3	42

This response was the only one where more than one of the responses of a member to Thomson's (2003b) questionnaire was equal. When deriving the utility function of the member, x_2 was set equal to 41%, x_3 was set equal to 42% and x_4 was set equal to 43%.

The utility function of member response 5 is plotted in Figure 6. It may be noted that this is one of the responses shown in Figures 1 and 2 to which attention is drawn in section 4.2.

The utility function illustrates that the member appears to have a strong aspiration level at an NRR of approximately 41%.

The member's first response to Thomson's (2003b) questionnaire was that she would choose a certain NRR of 60% over a 50–50 gamble of an NRR of 40% and 90%. The next question asked to what value the certain NRR had to be decreased in order to change her answer. Her answer was 41%.

The next question asked the member to choose between a certain NRR of 40% and a 50–50 gamble of an NRR of 40% and 41%. The member thus chose the gamble. The question that followed asked to what value the certain NRR of 40% had to be increased in order to change her answer. The only acceptable answer to this question would have been 41%. The member's answer to the first question thus forced the member to answer the second question in a particular way.

The low aspiration level results in a utility function tending very strongly to 1. The responses of the member indicate that she is very conservative, which

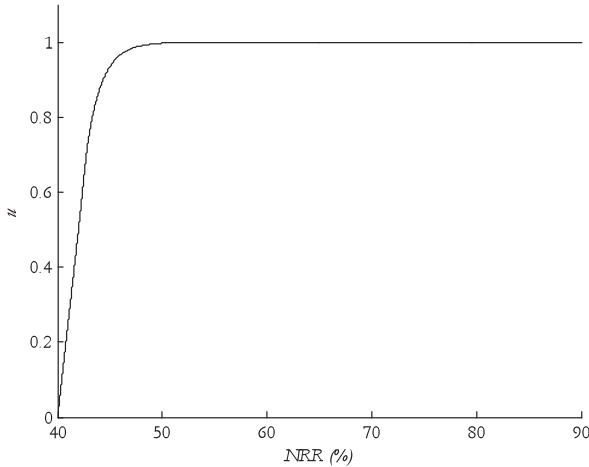


FIGURE 6: Utility function of member response 5.

results in a very large value for \tilde{R} . This level of extreme risk aversion is inconsistent with the member's Grable & Lytton risk-tolerance score of 33.

In the light of the above observations, the response is regarded as being an outlier in the sample.

4.4.3. Investigation of member response 42

The Grable & Lytton risk-tolerance score of member 42 was 24, whilst \tilde{R} was evaluated as 11.7. Both scores suggest that the member is less risk tolerant than other members in the sample. Member 42 provided the following responses to Thomson's (2003b) questionnaire.

Respondent #42	NRR (%)
Response 1	70
Response 2	45
Response 3	71

The utility function of member 42 is shown in Figure 7. It may be noted that this is the other response shown in Figures 1 and 2 to which attention is drawn in section 4.2.

The member's responses suggest an aspiration level at an NRR of 70%. Figure 7 indicates that the utility function is downward sloping for part of the retirement-benefit range considered. Thomson (2000) acknowledges that there are circumstances under which the derived utility function may not be monotonically increasing.

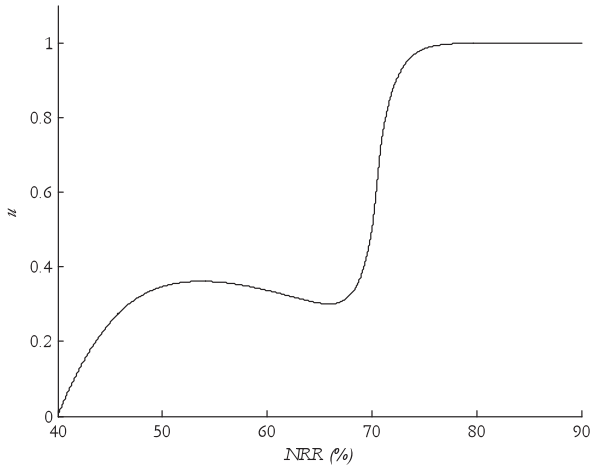


FIGURE 7: Utility function of member response 42.

As discussed in section 3.1, the risk-tolerance score \tilde{R} is calculated using the trapezoidal rule. The convergence obtained using different divisions of this interval is compared for member response 56 (analysed in section 3.2.1) and member response 42. The results are shown in Table 2.

TABLE 2.

EVALUATION OF \tilde{R} USING THE TRAPEZOIDAL RULE FOR DIFFERENT DIVISIONS OF THE NRR INTERVAL

Number of divisions	\tilde{R} for member response 42
1,000	10.115
2,000	18.966
3,000	9.957
5,000	3.093
7,500	22.424
100,000	11.173

Convergence does not occur for member 42. This is due to a discontinuity in the function $R(x)$. The value obtained for $\tilde{R}(x)$ is unreliable. For this reason, member response 42 is also regarded as an outlier. Where such difficulties occur, or where the elicited utility function is not monotonically increasing, the member should be given assistance in the completion of the Thomson (1993b) questionnaire.

4.4.4. Conclusions regarding outliers

Thomson (2000) rejected all respondents whose utility functions did not exhibiting non-satiation over the full range of benefits considered. The utility

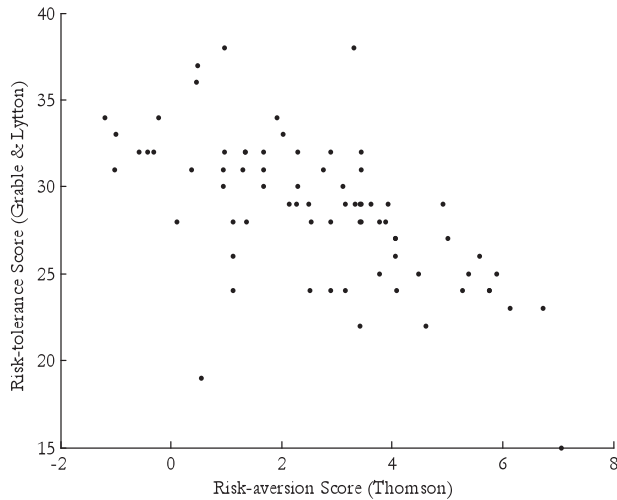


FIGURE 8: Revised scatter plot of average risk aversion and the risk-tolerance score.

functions of all the members were investigated for non-satiation over the range of retirement benefits considered.

Member response 5 is an example of a utility function tending strongly to one. Investigation of the other utility functions showed that this was the only such case.

The average risk aversion, \tilde{R} , for each member was tested for convergence to three decimal places. With the exception of member 42, convergence was achieved in all cases. An investigation of the data revealed that responses 5 and 42 are outliers in the sample. No other points in the sample are outliers when evaluated on the criteria presented in this section.

4.4.5. Comparison between measures (no outliers)

In this section, the outliers (respondents 5 and 42) are removed and the analysis performed in section 4.7 is repeated. A revised scatter plot of the scores is shown in Figure 8.

The sample correlation coefficient of the risk-tolerance measures after removing the two outliers is -0.6157 .

A one-tailed hypothesis test on the correlation coefficient was performed. The null hypothesis was that there is no correlation between the measures. The alternative hypothesis was that there is a negative correlation between them. The p -value calculated was $3.3655 \cdot 10^{-9}$. The null hypothesis of no correlation can thus be rejected at a 0.001% level of significance and it can be concluded that there is a highly significant negative relationship between the two measures.

5. DISCUSSION

This study was born out of Thomson's (2003b) proposed system. It is a study that aimed to determine if the questionnaire proposed by Thomson (2003b) could be used for the purpose of deriving the utility functions of individual members of a DC fund. It was thought that the results of the study might provide further insight into the reliability of Thomson's (2003) system as a mechanism for recommendations of investment channel choice.

This section provides a discussion of the results obtained. Limitations of the research and ideas for future research are also included.

5.1. Evaluation of Thomson's Questionnaire

This research investigated whether the questionnaire proposed by Thomson (2003b) allows for the correct specification of a member's utility function.

The relationship between the risk-tolerance score obtained from Grable & Lytton's (1999) questionnaire and the average risk aversion derived from the utility function elicited through Thomson's (2003) questionnaire is statistically significant. The correlation coefficient between the measures was shown to be strongly negative. This suggests that these risk-assessment instruments consistently assess the financial risk-tolerance of an individual member.

The Grable & Lytton (1999) questionnaire is a commonly used instrument for the assessment of financial risk tolerance. It is based on descriptive studies, whilst Thomson's (2003b) method used to elicit the utility function of a member is based on underlying theory. Assuming that the Grable & Lytton (1999) questionnaire reliably assesses the financial risk tolerance of a member, then this research provides support for the use of Thomson's (2003b) questionnaire as a basis for the derivation of a member's utility function.

It could be argued that if the financial risk-tolerance score of a member provides a reliable assessment of a member's risk tolerance, then there is no need for the relatively complicated questionnaire proposed by Thomson (2003b). This is because it should be possible to determine the shape of the member's utility function from Grable & Lytton's (1999) questionnaire, which is simpler to answer. However, this would ignore investment risk.

Actuaries define investment risk as the risk of not meeting liabilities (Booth et al, 1998). Members of a DC fund will have expectations of their ultimate retirement benefit. Thomson's (2003b) questionnaire provides meaningful information on the aspiration level of the member concerned. Grable & Lytton's (1999) questionnaire does not provide an indication of the level of retirement benefit required by the member. It is thus possible for two members to obtain the same risk-tolerance score on Grable & Lytton's (1999) questionnaire but have different retirement income requirements.

In addition, the financial circumstances of the members may be very different. Thomson's (2003b) questionnaire derives the member's utility function

by requiring the member to specify the certainty equivalent for a risky prospect. This provides valuable information on the income requirements of the member.

As explained in section 2.3, Thomson's questionnaire allows the optimal investment strategy to be determined using expected-utility theory. The utility function is mathematically tractable and provides for the optimisation of a member's investment strategy. The risk-tolerance score of Grable & Lytton (1999) provides no basis for the identification of an optimal choice of investment channels.

5.2. Potential for Bias in the Sample

The sample size used in the analysis was too small, and not sufficiently representative of retirement funds in general, to obtain statistically significant results regarding the entire population of retirement-fund members.

In the first place, as mentioned in section 3, there is reason to believe that, on average, the members of the chosen fund have higher levels of investment knowledge than those of other funds and might therefore more easily complete the questionnaires. They might also have different attitudes to risk, or different types of risky behaviour, than those of employees in other companies within the same industry, or in other industries. It might be possible to extend the study to other employers in South Africa so as to obtain a representative sample of employees. Besides the problems of access to employees and expense that would arise, however, it is questionable whether that would be more informative, unless the outcomes were to be analysed by industry. Such an analysis would have turned the study into a more substantial exercise than could be conveniently reported in one article; furthermore, it would beg the question whether similar relationships would apply in other countries. In the absence of such an analysis, significant relationships that might be found in some industries might be masked by aggregation with others.

Secondly, as mentioned in section 3.2, there is a problem of self-selection using this method; the sample of respondents is likely to be biased (Schonlau, *op. cit.*). For example, younger members may be considered more likely to respond to internet-based surveys than older members as they are more computer literate. The self-selection bias means that statistical methods such as inference are not appropriate. As mentioned in section 3.1, the response rate was only 3.1%. Because of the method of administration, the small sample size may have resulted in a bias. In particular, the average age of the sample was younger than that of the fund.

In certain respects, the bias in the sampling process might correspond to the bias in usage that would occur if the system were to be implemented. For example, just as younger members might have been more willing to participate in the study, so the same members might be more willing to use the proposed system. To that extent, the sample may be representative of the sub-population

that is relevant to the use of the system. There may, on the other hand, be other age-related effects. For example, older employees may hold more senior positions, and, in comparison with younger members, may be less able to justify the time taken to participate in the study than the time that would be taken to use the proposed system for the purposes of actual decision-making.

It would be possible to improve the sampling method, but that would involve greater commitment by the employer and the employees. It would still raise the questions whether the sample selected would be representative of the sub-population that would use the proposed system if it were implemented and whether the answers given to the study would correspond to the answers that would be given if the employees were actually going to use the results as a basis for decision-making.

The best way of solving this problem would be to implement the system and gather data of members' answers to the questions, the resulting recommendations of optimal allocations and the decisions actually made. An unbiased sample could thus be obtained and careless or frivolous use of the system could be eliminated from the analysis of that sample by ignoring any use of the system that did not result in consistent decision-making. The purpose of this study was to establish whether such an exercise might be worth while.

Arguably the best place in which to implement the system in the first instance is in the offices of a provider of financial services. Such an employer would have a greater interest in the use of the system than those in other industries since, if it is successful, it might be profitable to develop it into a product for sale to clients. The overhead costs of implementation could therefore be more easily justified, and benefits could be reaped from the experience of staff in using the system. Apart from ease of access by the authors, the selection of such an employer for this study serves to contextualise the research to the initial implementation of the proposed system amongst the employees of such an employer.

5.3. Other Limitations of the Study

A limitation of the study was that the same range of NRRs was specified in the first question of Thomson's (2003b) questionnaire for each member. In this case, all members were required to choose between a certain NRR of 60% and a 50–50 chance of an NRR of 40% or 90%. However, every member of a fund will have a different set of financial circumstances, level of accumulated retirement savings, current salary and scale of salary progression. Although two members may each have a benefit requirement of a 70% NRR, they may have very different likelihoods of achieving that target.

An improvement of the study would have been to incorporate the financial circumstances of the member by using their salary and accumulated fund savings. The NRRs shown in the first question of Thomson's (2003b) questionnaire could then have been tailored to the member's circumstances. The

member would thus be rejecting or accepting a hypothetical prospect based on a realistic scenario of what is obtainable. This was not possible to incorporate into the study as the chair of the trustees of the fund in the study did not want confidential information to be obtained from fund members. In practical applications, however, the use of this information might improve the elicitation of the utility function.

It would also have been of interest to revisit the study after some time both to test the stability of the measures and to eliminate possibly careless or frivolous responses. Because of the constraints required for the sake of confidentiality, it was not possible to do this.

5.4. Suggestions for Further Research

Further research could consider using both questionnaires to make investment-channel recommendations. The answers to Grable & Lytton's (1999) questionnaire could provide a broad indication of the shape of the member's utility function whilst parts of Thomson's (2003b) questionnaire could provide both insight into the level of income required by the member in retirement and more detail of the shape of the member's utility function.

The research would investigate whether it is possible to ask only the first of Thomson's (2003) three questions. Arguably, the complexity of Thomson's (2003b) questionnaire is the repeated requirement that the member specify their certainty equivalents over various ranges of retirement benefit.

It would be of interest to investigate how the utility function derived for a member changes as the additional information from subsequent questions in Thomson's (2003b) questionnaire is incorporated into the utility function. The shape and characteristics of a utility function derived from say one question and three questions could be compared. Of more importance would be how the investment strategy recommendation changes as additional information is provided.

Research could also be done on whether it is possible for the aspiration levels or NRR requirements of the member to be elicited from Grable & Lytton's (1999) questionnaire. This could address the actuarial risk referred to above.

Further research on Grable & Lytton's (1999) questionnaire could include tests to establish whether all of the 13 questions are required or whether there is a sub-set of questions that determines a member's financial risk tolerance with sufficient accuracy. A principal-components analysis or cluster analysis on the responses to these questions could provide useful insight into the individual questions. This could include the demographic characteristics of the member.

Further research on the relationship between Grable & Lytton's (1999) questionnaire and Thomson's questionnaire could include the establishment of circumstances in which the Grable & Lytton (1999) questionnaire gives a reliable estimate of the utility function, so that the utility function need be elicited only in other circumstances.

6. CONCLUSIONS

This study aimed to determine if the system proposed by Thomson (2003b) could be used for the purpose of deriving the utility functions of individual members of a DC fund, and thereby an actuarial basis for the management of financial risk by the members of such funds. The utility functions of members in a South African DC fund were elicited. While not all subjects who started answering the questionnaire were able to complete it, a substantial majority were able to do so. The inability of the remainder to complete it might not in all cases have been due to problems relating to the questionnaires themselves. Of those who did complete the questionnaires, there were two who exhibited unreliable results. In practice it would be necessary to afford such members the assistance of trained investment personnel.

The average risk aversion obtained from the utility functions elicited was found to be consistent with a risk-tolerance score obtained by means of an alternative instrument that is simpler to understand. For a sub-population corresponding to the sample measured, this relationship was statistically significant. This indicates that, apart from the manifestly unreliable type of results referred to in the preceding paragraph, Thomson's (2003b) questionnaire could be considered for the purpose of eliciting utility functions.

The strong relationship between the results of Grable & Lytton's (1999) questionnaire and those of Thomson's (2003b) method vindicates both of these approaches to the measurement of risk for the purposes of investment channel choice, at least for a sub-population of the type that was sampled in study. It should be recognised, though, that the sample tested cannot be regarded as representative of retirement funds in general. On the other hand, the sub-population effectively tested may correspond more closely to the sub-population that would use the system if it were implemented. The importance of the results of this study lies in the establishment of the existence of a sub-population of a particular type of fund for which the relationship between the risk measures is significant. In this paper the focus is on the use of expected-utility theory for normative purposes in the context of decision-making about post-retirement income. Since its submission, Faff et al (*op. cit.*), have addressed the consistency of expected-utility theory as a descriptive theory. Nevertheless, their results strengthen the findings of this paper.

Resulting from this finding, it is evident that both approaches may have a place in the process of advising retirement-fund members on the choices available, and a number of questions arise for further research. In the mean time, the methods used in this study may be replicated by consultants who wish to test whether, in the funds that they advise, the proposed system could be implemented, together with risk-assessment systems typically used in the market, to enhance the choice of investment channels in defined-contribution retirement funds. In the first instance the system could be implemented by providers of financial services amongst their own employees.

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APPENDIX

A. THOMSON'S QUESTIONNAIRE (AMENDED)

Question 1.

If you could choose your investment strategy

(1) So that there is a 50–50 chance that your annual income after retirement (after tax) will be:

(a) **40%**; or

(b) **90%**;

of your annual salary at retirement (after tax); or

(2) so that your income after retirement (after tax) will be equal to **60%** of your salary at retirement (after tax) with certainty;

Which of (1) and (2) would you choose?

Please select (1) or (2), or if you are indifferent, select option (3) below.

(3) I am indifferent between choices (1) and (2).

⇒ If the member's answer to Question 1 is (1) then Question 2 follows, and thereafter Question 4.

⇒ If the member's answer to Question 1 is (2) then Question 3 follows, and thereafter Question 4.

⇒ If the member's answer to Question 1 is (3) then Question 4 follows.

Question 2.

To what value would the percentage in choice (2) have to be **increased** in order to change your answer? _____

The member will be asked to reconsider their answer if the response does not exceed 60 or is not smaller than 90.

Question 3.

To what value would the amount in (2) have to be **decreased** in order to change your answer? _____

The member will be asked to reconsider their answer if the response is not smaller than **60** and greater than **40**.

Response one to the questionnaire is the answer given by the member to Question 2 or Question 3 or **60** in the case when (3) in Question 1 is chosen.

Question 4.

If you could choose your investment strategy

- (1) So that there is a 50–50 chance that your annual income after retirement (after tax) will be:
 (a) **40%**; or
 (b) **X%**;
 of your annual salary at retirement (after tax); or

- (2) so that your income after retirement (after tax) will be equal to **a%** of your salary at retirement (after tax) with certainty;

Which of (1) and (2) would you choose?

Please select (1) or (2), or if you are indifferent, select option (3) below.

- (3) I am indifferent between choices (1) and (2).

- o The value for **X** in Question 4 is the answer to Question 2 or Question 3, or **60** in the case where the member chose (3) in Question 1
- o The value for **a** is the geometric mean of **40%** and **X%**
- ⇒ If the member's answer to Question 4 is (1) then Question 5 follows, and thereafter Question 7.
- ⇒ If the member's answer to Question 4 is (2) then Question 6 follows, and thereafter Question 7.
- ⇒ If the member's answer to Question 4 is (3) then Question 7 follows.

Question 5.

To what value would the percentage in choice (2) have to be **increased** in order to change your answer? _____

The member will be asked to reconsider their answer if the response does not exceed **a** or is not smaller than **X**.

Question 6.

To what value would the amount in (2) have to be **decreased** in order to change your answer? _____

The member will be asked to reconsider their answer if the response is not smaller than **a** and greater than **40**.

Response two to the questionnaire is the answer given by the member to Question 5 or Question 6 or **a** in the case when (3) in Question 4 is chosen.

Question 7.

If you could choose your investment strategy

(1) So that there is a 50–50 chance that your annual income after retirement (after tax) will be:
 (a) **X%**; or
 (b) **90%**;
 of your annual salary at retirement (after tax); or

(2) so that your income after retirement (after tax) will be equal to **c%** of your salary at retirement (after tax) with certainty;

Which of (1) and (2) would you choose?

Please select (1) or (2), or if you are indifferent, select option (3) below.

(3) I am indifferent between choices (1) and (2).

o The value for **c** is the geometric mean of **X%** and **90%**.

⇒ If the member's answer to Question 7 is (1) then Question 8 follows.

⇒ If the member's answer to Question 7 is (2) then Question 9 follows.

⇒ If the member's answer to Question 7 is (3) then no further questions follow.

Question 8.

To what value would the percentage in choice (2) have to be **increased** in order to change your answer? _____

The member will be asked to reconsider their answer if the response does not exceed **c** or is not smaller than **Y**.

Question 9.

To what value would the amount in (2) have to be **decreased** in order to change your answer? _____

The member will be asked to reconsider their answer if the response is not smaller than c and greater than X .

Response three to the questionnaire is the answer given by the member to Question 8 or Question 9 or c in the case when (3) in Question 7 is chosen.

**B. GRABLE & LYTTON'S INSTRUMENT FOR THE ASSESSMENT OF
FINANCIAL RISK TOLERANCE**

- 1. In general, how would your best friend describe you as a risk taker?**
 - a) A real gambler
 - b) Willing to take risks after completing adequate research
 - c) Cautious
 - d) A real risk avoider

- 2. You are on a TV game show and can choose one of the following. Which would you take?**
 - a) R 5,000 in cash
 - b) A 50% chance at winning R 50,000
 - c) A 5% chance at winning R 500,000

- 3. You have just finished saving for a once-in-a-lifetime vacation. Three weeks before you plan to leave, you lose your job. You would:**
 - a) Cancel the vacation
 - b) Take a much more modest vacation
 - c) Go as scheduled, reasoning that you need the time to prepare for a job search
 - d) Extend your vacation, because this might be your last chance to go first class

- 4. If you unexpectedly received 6 month's salary to invest, what would you do?**
 - a) Deposit it in a bank account or money market account
 - b) Invest it in safe, high-quality bonds or bond unit trusts
 - c) Invest it in shares or share unit trusts

- 5. In terms of experience, how comfortable are you investing in shares or share unit trusts?**
 - a) Not at all comfortable

- b) Somewhat comfortable
 - c) Very comfortable
6. When you think of the word “risk”, which of the following words comes to mind first?
- a) Loss
 - b) Uncertainty
 - c) Opportunity
 - d) Thrill
7. Some experts are predicting prices of assets such as gold, collectibles, and property (tangible assets) to increase in value; bond prices may fall, however, experts tend to agree that government bonds are relatively safe. Most of your investment assets are now in government bonds. What would you do?
- a) Hold all the bonds
 - b) Sell the bonds, put half the proceeds into cash, and the other half into tangible assets
 - c) Sell the bonds and put the total proceeds into tangible assets, and borrow additional money to buy more
8. Given the best-and worst-case returns of the four investment choices below, which would you prefer?
- a) R 1,000 gain best case; R 0 worst case
 - b) R 4,000 gain best case; R 1,000 loss worst case
 - c) R 13,000 gain best case; R 4,000 loss worst case
 - d) R 24,000 gain best case; R 12,000 loss worst case
9. In addition to whatever you own, you have been given R50 000. You are now asked to choose between:
- a) A sure gain of R25,000
 - b) A 50% chance to gain R50,000 and a 50% chance to gain nothing
10. In addition to whatever you own, you have been given R100 000. You are now asked to choose between:
- a) A sure loss of R25,000
 - b) A 50% chance to lose R50,000 and a 50% chance to lose nothing
11. Suppose a relative left you an inheritance of R500,000, stipulating in the will that you invest *ALL* the money in *ONE* of the following choices. Which one would you select?
- a) A savings account
 - b) A unit trust that owns shares and bonds
 - c) A portfolio of 15 listed shares
 - d) Commodities like gold, silver, oil and cattle

12. If you had one year of annual salary to invest, which of the following investment choices would you find most appealing?

- a) 60% in low-risk, 30% in medium-risk and 10% in high-risk investments
- b) 30% in low-risk, 40% in medium-risk and 30% in high-risk investments
- c) 10% in low-risk, 40% in medium-risk and 50% in high-risk investments

13. Your trusted friend and neighbour, an experienced geologist, is putting together a group of investors to fund an exploratory gold-mining venture. The venture could pay back 50 to 100 times the investment if successful. If the mine does terribly, the entire investment is worthless. Your friend estimates the chance of success is only 20%. If you had the money, how much would you invest?

- a) Nothing
- b) One month's salary
- c) Three month's salary
- d) Six month's salary

C. THE INTERPOLATION PROCESS

The following formulation is taken from Thomson (2003b). We denote the five values of the NRR, determined as described in section 3.3.1, as x_0, \dots, x_4 in increasing order of magnitude and the corresponding observed utility values as:

$$u(x_i) = \frac{i}{4}.$$

The utility function is then:

$$u(x) = v_i(x) \text{ for } x \in S_i, \quad i = 1, \dots, 4;$$

where:

$$S_i = (0, x_1] \text{ and } v_i(x) = u_1(x) \text{ for } i = 1;$$

$$S_i = [x_3, \infty) \text{ and } v_i(x) = u_3(x) \text{ for } i = 4;$$

$$S_i = [x_{i-1}, x_i) \text{ and } v_i(x) = \frac{(x_i - x)u_{i-1}(x) + (x - x_{i-1})u_i(x)}{x_i - x_{i-1}} \text{ otherwise;}$$

$$u_i(x) = a_i \ln x + b_i \text{ if } n_i \approx 0;$$

$$a_i x^{n_i} + b_i \text{ otherwise;}$$

and n_i, a_i and b_i are determined so that, for $i = 1, 2, 3$:

$$u_i(x_{i-1}) = u(x_{i-1})$$

$$u_i(x_i) = u(x_i); \text{ and}$$

$$u_i(x_{i+1}) = u(x_{i+1}).$$

Except where $n_i = 0$, the value of n_i may be determined from the formula:

$$w_i^{n_i} - 2 + w_{i-1}^{-n_i} = 0; \quad (3)$$

where:

$$w_i = \frac{x_{i+1}}{x_i}.$$

Equation (3) can be solved for n_i by a Newton-Raphson method. The values of a_i and b_i may then be determined by means of the equations:

$$a_1 = \frac{1}{4\Delta x_0^{n_1}};$$

$$a_2 = \frac{1}{4\Delta x_1^{n_2}}, \text{ or } \frac{1}{4\Delta x_2^{n_2}};$$

$$a_3 = \frac{1}{4\Delta x_3^{n_3}}.$$

$$b_1 = u(x_1) - a_1 x_1^{n_1};$$

$$b_2 = u(x_2) - a_2 x_2^{n_2}; \text{ and}$$

$$b_3 = u(x_3) - a_3 x_3^{n_3}.$$

If $n_i \approx 0$, then:

$$u_i(x) = a_i \ln x + b_i.$$

The values of a_i and b_i are then:

$$a_1 = \frac{1}{4(\ln x_1 - \ln x_0)};$$

$$a_2 = \frac{1}{4(\ln x_2 - \ln x_1)};$$

$$a_3 = \frac{1}{4(\ln x_4 - \ln x_3)};$$

$$b_1 = u(x_1) - a_1 \ln x_1;$$

$$b_2 = u(x_2) - a_2 \ln x_2; \text{ and}$$

$$b_3 = u(x_3) - a_3 \ln x_3.$$

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