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Influence of ion and variable dust charge on electron-dust bremsstrahlung emission spectrum in complex plasmas

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The bremsstrahlung process is studied in complex plasmas including the influence of ions and variable dust charge. The electron-dust particle bremsstrahlung radiation cross-section (e-D-BRCS) is obtained with the analytic expression for the effective dust charge in terms of the Debye length and the temperature ratio. The e-D-BRCS is found to be reduced with either the decrease of ion temperature or increase of electron temperature. The ion density effect is found to be more important in the small electron temperature domain. Interestingly, the influence of ion temperature and density is found to be independent of the bremsstrahlung emission energy. The effective dust charge is also found to decrease with an increase of the ratio of the electron temperature to the ion temperature. In addition, it is found that the effective dust charge increases with an increase of the ratio of the electron density to the ion density. Moreover, the e-D-BRCS is found to be increased with the decrease of ion density.

Key words: complex plasmas, election-dust particle bremsstrahlung

1. Introduction

In the atomic and plasma communities, the bremsstrahlung spectrum (Totsuji 1985; Hakopian 1991; Jung & Jeong 1996; Jackson 1999; Riffert, Klingler, & Ruder 1999; Fujimoto 2004; Gould 2006; Jung & Murakami 2009; Embréus, Stahl, & Fülöp 2016) has received significant attention since the bremsstrahlung process is known as the one of the most basic atomic processes in plasmas. In dusty plasmas, the radiation processes have been extensively investigated to explore various plasma parameters since the collective interactions in complex plasmas are known to be ubiquitous in a variety of astrophysical and laboratory plasmas (Mendis & Rosenberg 1994; Bouchoule 1999; Shukla & Mamum 2002; Ramazanov *et al.* 2008, 2010; Shukla & Eliasson 2009). In most conventional complex plasmas, the plasma consists of a three-component complex plasma: thermal electrons, ions and negatively charged dust particles. In a previous work (Jung & Murakami 2009), the influence of electron temperature and density was investigated on

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the ion-dust grain bremsstrahlung process in dusty plasmas. However, the more relevant bremsstrahlung process, known as the electron-dust particle bremsstrahlung process, in terms of the effect of variable dust charge under the influence of ion temperature and density variations has not been investigated. Hence, in this work, we investigated the influence of variable dust change including ion temperature and density effects on the kinetic radiation process due to the interaction of electrons with dust particles in a three-component complex plasma.

In § 2, we derive the analytic expression for the bremsstrahlung radiation cross-section (BRCS). In addition, we discuss the effective interaction potential in complex plasmas. In § 3, we obtain the BRCS in complex plasmas. In § 4, we obtain the effective charge of dust particles in a complex plasma. Moreover, the influence of ion temperature and density on the electron-dust particle bremsstrahlung process is investigated. Finally, § 5 provides the conclusion.

2. Bremsstrahlung cross-section in complex plasmas

In the quantum Born analysis, the differential electron-dust particle bremsstrahlung cross-section (e-D-BCS) $\mathrm{d}^2\sigma_b$ can be derived from the non-relativistic perturbation analysis (Gould 2006)

$$\mathrm{d}^2\sigma_b = \mathrm{d}\sigma_C\,\mathrm{d}W_\omega, \tag{2.1}$$

where $\mathrm{d}\sigma_C/\mathrm{d}q$ is the differential elastic scattering cross-section per momentum transfer $\mathrm{d}q$

$$\frac{\mathrm{d}\sigma_C}{\mathrm{d}q} = \frac{1}{2\pi\hbar v_0^2}q\left|\int \mathrm{d}^3\boldsymbol{r}\,\mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(\boldsymbol{r})\right|^2, \tag{2.2}$$

$\boldsymbol{q}(=\boldsymbol{k}_0-\boldsymbol{k}_f)$ is a vector for the momentum transfer with $\boldsymbol{k}_0$ and $\boldsymbol{k}_f$ being the initial and final state wave vectors of the projectile electron, respectively, $\hbar$ is the Planck constant, v_0 is the initial electron velocity and $V(\boldsymbol{r})$ is the interaction potential between electrons and negatively charged dust particles. The integral $\int \mathrm{d}^3\boldsymbol{r}\,\mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(\boldsymbol{r})[=\tilde{V}(\boldsymbol{q})]$ is the Fourier transform of $V(\boldsymbol{r})$. Here, $\mathrm{d}W_\omega/\mathrm{d}\Omega$ stands for the differential photon emission probability per unit differential sold angle $\mathrm{d}\Omega$ for the direction of the radiation photon within the frequency interval $\mathrm{d}\omega$

$$\frac{\mathrm{d}W_\omega}{\mathrm{d}\Omega} = \frac{\alpha\lambda_C^2}{4\pi^2}\sum_{\hat{\boldsymbol{e}}}|\hat{\boldsymbol{e}}\cdot\boldsymbol{q}|^2\frac{\mathrm{d}\omega}{\omega}, \tag{2.3}$$

where $\alpha(=e^2/\hbar c\cong 1/137)$ is the fine structure constant with e and c being the elementary charge and the speed of light, respectively, $\hat{\boldsymbol{e}}$ is the photon polarization unit vector, $\lambda_C(=\hbar/m_ec)$ is the Compton wavelength and m_e is the mass of the electron. By integrating over all directions of the bremsstrahlung photons and by summing over the photon polarization directions, the electron-dust particle bremsstrahlung radiation cross-section (e-D-BRCS) per frequency interval $\mathrm{d}\omega$ is then obtained by

$$\frac{\mathrm{d}^2\sigma_b}{\mathrm{d}\omega} = \frac{1}{3\pi^2}\frac{\alpha}{(m_ec^2)^2\beta_0^2}\left|\int \mathrm{d}^3\boldsymbol{r}\,\mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(\boldsymbol{r})\right|^2\frac{q^3\mathrm{d}q}{\omega}, \tag{2.4}$$

where $\beta_0 = v_0/c$. In a complex plasma composed of singly charged ions, electrons and negatively charged dust particles, the equilibrium quasi-neutral condition (Vidhya Lakshmi, Bharuthram, & Shukla 1993) is determined by $n_{e0}+Zn_{d0}=n_{i0}=n_0$, where n_{j0} is the equilibrium density of species j ($=e$, i, d for electrons, ions and dust particles, respectively), Z is the charge number of the dust particles and n_0 is the plasma total density. The effective Debye length (Vidhya Lakshmi *et al.* 1993) λ_{eff} in a three-component complex plasma is then found to be

$$\lambda_{\text{eff}} = \lambda_{De}\left(\frac{T_e}{T_i} + \frac{n_{e0}}{n_0}\right)^{-1/2}, \tag{2.5}$$

where $\lambda_{Dj}(= k_B T_j/4\pi n_{j0} q_j^2)^{1/2}$ is the Debye length of a particle j. The quantity $(T_e/T_i + n_{e0}/n_0)^{-1/2}$ indicates a correction factor that reflects the influences of the temperature and density of electrons and ions on λ_{De}. The effective interaction potential $V_{\text{eff}}(r)$ with charge $-Ze$ in a three-component complex plasma is expressed in the Yukawa potential form with an effective Debye length λ_{eff} such as $V_{\text{eff}}(r) = (Ze^2/r)exp(-r/\lambda_{\text{eff}})$. The Fourier transform $\tilde{V}_{\text{eff}}(q)$ of the effective electron-dust particle interaction potential is determined by the lower cutoff r_L of the integration. Typically, the lower cutoff is given as $r_L = \max\{a, r_c\}$ where a is the spherical dust particle radius, $r_c(= Ze^2/E_0)$ is the distance of closest approach for the electron-dust collision and E_0 is the kinetic energy of the electrons that are flowing past the dust particle. In the Born limit, we have $a > r_c$ since $E_0 > Ze^2/a$. Therefore, the lower cutoff of the integral for high-energy electron-dust particle encounters is given by the dust particle size, $r_L = a$. After some mathematical manipulation, the Fourier transformation $\tilde{V}_{\text{eff}}(q)$ of the ion-dust particle interaction potential is then obtained by

$$\begin{aligned}\tilde{V}_{\text{eff}}(q) &= Ze^2 \int_{r \geq r_L = a} \mathrm{d}^3\boldsymbol{r}\, \exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r} - r/\lambda_{\text{eff}})\frac{1}{r} \\ &= \frac{4\pi Ze^2 a}{q[(a/\lambda_{\text{eff}})^2 + (qa)^2]}\left[\frac{a}{\lambda_{\text{eff}}}\sin(qa) + qa\cos(qa)\right]\exp(-a/\lambda_{\text{eff}}).\end{aligned} \tag{2.6}$$

Taking the limit as $a \to 0$, the result is equivalent to the standard Fourier transform of the Yukawa potential for a screened point charge, i.e. the same as the conventional electron-ion bremsstrahlung case.

3. Bremsstrahlung radiation cross-section in complex plasmas

The e-D-BCS in a complex plasma is represented in the form

$$\frac{\mathrm{d}^2\sigma_b}{\mathrm{d}\omega} = \frac{16}{3}\frac{Z^2\alpha^3 a_0^2}{\bar{E}_0}\left\{\frac{\exp(-1/\bar{\lambda}_{\text{eff}})}{1/\bar{\lambda}_{\text{eff}}^2 + \bar{q}^2}\left[\bar{q}\cos\bar{q} + \left(\frac{1}{\bar{\lambda}_{\text{eff}}}\right)\sin\bar{q}\right]\right\}^2 \frac{\bar{q}\,\mathrm{d}\bar{q}}{\omega}, \tag{3.1}$$

where $\bar{E}_0(= E_0/Ry = m_e v_0^2/2Ry)$ is the initial kinetic energy of the flowing electron scaled by the Rydberg constant $Ry(= m_e e^4/2\hbar^2 \approx 13.6\ \text{eV})$, $a_0(= \hbar^2/m_e e^2)$ is the Bohr radius, $\bar{\lambda}_{\text{eff}}(= \lambda_{\text{eff}}/a)$ is the effective Debye length scaled by the dust particle radius and $\bar{q}(= qa)$ is the scaled momentum transfer. The validity of the Born approximation can be secured using the Massey parameter (Gould 2006), $\eta_M = |V|/E_0$, where $|V|$ is the magnitude of the interaction potential. For typical laboratory complex plasmas, the numerical parameters are known to be $Z \approx 100 \sim 1000$, $a \approx 0.01 \sim 1\ \mu\text{m}$ and $\lambda_D/a \approx 5 \sim 100$ (Bliokh, Sinitsin, & Yaroshenko 1995). The Born analysis (Weinberg 2015) is therefore quite reliable for exploring the electron-dust bremsstrahlung process in a complex plasma because the Massey parameter η_M for the electron-dust interaction is usually less than 1. Additionally, the correction obtained by the Elwert–Sommerfeld factor (Gould 2006) as the ratio of the absolute square of the final and the initial Coulomb s-wave functions at the surface of the dust particle becomes unity since the Coulomb focusing near $r \geq a(\gg a_0)$ is negligible. Therefore, (3.1) is fairly reliable if the kinetic energy of the flowing electron (E_0) is greater than the interaction energy of the electron and dust particle at the surface of the dust particle (Ze^2/a). We also show that the physical properties of the bremsstrahlung emission spectrum can be found from BRCS (Weinberg 2015) expressed as $\mathrm{d}^2\chi_b/\mathrm{d}\bar{\varepsilon}\,\mathrm{d}\bar{q} \equiv \hbar\omega(\mathrm{d}\sigma_b/\hbar\,\mathrm{d}\omega\,\mathrm{d}\bar{q})$, where $\bar{\varepsilon}(=\varepsilon/Ry)$ is the scaled photon energy and $\varepsilon(=\hbar\omega)$ is the photon energy. Then, after some mathematical manipulations, the scaled e-D-BRCS $\bar{\chi}_{Z,\bar{\varepsilon}}[=(\mathrm{d}^2\chi_b/\mathrm{d}\bar{\varepsilon})/\pi a_0^2]$ for a fixed dust charge Z in three-component complex plasmas is found to be

$$\bar{\chi}_{Z,\bar{\varepsilon}} = \frac{16}{3\pi}\frac{Z^2\alpha^3}{\bar{E}_0}\int_{\bar{q}_{\min}}^{\bar{q}_{\max}} \mathrm{d}\bar{q}\left\{\frac{\bar{q}\exp(-2/\bar{\lambda}_{\mathrm{eff}})}{(1/\bar{\lambda}_{\mathrm{eff}}^2+\bar{q}^2)^2}\left[\left(\frac{1}{\bar{\lambda}_{eff}^2}\right)\sin^2(\bar{q}\bar{\lambda}_{\mathrm{eff}}) + \left(\frac{2\bar{q}}{\bar{\lambda}_{\mathrm{eff}}}\right)\sin(\bar{q}\bar{\lambda}_{\mathrm{eff}})\cos(\bar{q}\bar{\lambda}_{\mathrm{eff}}) + \bar{q}^2\cos^2(\bar{q}\bar{\lambda}_{\mathrm{eff}})\right]\right\}, \tag{3.2}$$

where the quantities $\bar{q}_{\min}\{(=q_{\min}a) = (a/a_0)[\bar{E}_0^{1/2} - (\bar{E}_0-\bar{\varepsilon})^{1/2}]\}$ and $\bar{q}_{\max}\{(\equiv q_{\max}a) = (a/a_0)[\bar{E}_0^{1/2} + (\bar{E}_0-\bar{\varepsilon})^{1/2}]\}$ are the minimum and the maximum momentum transfer scaled by the dust radius, respectively. If the lower bound of the integration in (3.2) is used as λ_{eff} instead of the radius of the dust particle a in order to avoid the capture of the flowing electron by the target dust particle, the curly bracket in (3.2) is then replaced by $\{\bar{q}e^{-2}(1/\bar{\lambda}_{\mathrm{eff}}^2+\bar{q}^2)^{-2}[\bar{\lambda}_{\mathrm{eff}}^{-2}\sin^2(\bar{q}\bar{\lambda}_{\mathrm{eff}}) + 2\bar{q}\bar{\lambda}_{\mathrm{eff}}^{-1}\sin(\bar{q}\bar{\lambda}_{\mathrm{eff}})\cos(\bar{q}\bar{\lambda}_{\mathrm{eff}}) + \bar{q}^2\cos^2(\bar{q}\bar{\lambda}_{\mathrm{eff}})]\}$. Since the time scale of the bremsstrahlung process ($\sim 2\lambda_{\mathrm{eff}}/v_0$) is generally shorter than that of the dust charging process ($\sim 10^{-3}$ s), the dust charge is regarded as a constant charge in the electron-dust particle bremsstrahlung process. The e-D-BRCS with the lower bound 'λ_{eff}' will decrease to approximately 20 % of the e-D-BRCS with the lower bound 'a' because the change of the lower bound in the potential Fourier transform reduces the interaction range of the bremsstrahlung process. An excellent discussion (Khrapak, Klumov, & Morfill 2008) shows the additional part of the electrostatic potential that includes the effects of plasma absorption and ion-neutral collisions. It was also found that, in the absence of ion flux on the surface of the dust particle, i.e. non-absorbing dust particles, the electrostatic potential only appears in the standard Debye–Hückel form. In this work, we only retain the Debye–Hückel interaction potential because we adopt the bremsstrahlung emission process owing to the scattering of electrons by non-absorbing dust particles. Investigation of the bremsstrahlung process by electrons and absorbing dust particles will be addressed elsewhere.

4. Effective charge of dust particles and ion temperature effect

The dust particle charge is known to be associated with the potential difference φ_d between the particle potential φ_g and the plasma potential φ_p (Shukla & Mamum 2002). It is also found that the orbital-motion-limited charging current I_j of species j is $I_j = 4\pi a^2 n_i q_j (k_B T_j/2\pi m_j)^{1/2}(1 - q_j\varphi_d/k_B T_j)$ for $q_j\varphi_d < 0$ and $I_j = 4\pi a^2(n_j - Zn_d)q_j(k_B T_j/2\pi m_j)^{1/2} exp(-q_j\varphi_d/k_B T_j)$ for $q_j\varphi_d > 0$ and $Zn_d \ll n_i$ (Shukla & Mamum 2002). From the condition $I_e + I_i = 0$ and the quasi-neutral condition, the dust charge ($-Ze$) in the charging processes can be obtained with variables of the electron (T_e) and ion (T_i) temperatures by the relation $(T_i m_e/T_e m_i)^{1/2}(1 + Ze^2/ak_B T_i) exp(Ze^2/ak_B T_e) - n_{e0}/n_{i0} = 0$ since $\varphi_d = -Ze/a$. The analytic expression of the effective dust charge $Z_{\mathrm{eff}}[= Z_{\mathrm{eff}}(\lambda_{De}, \lambda_{Di}, T_i, T_e)]$ is then obtained in terms of the Lambert W-function (Corless *et al.* 1996) with variations of the electron Debye length λ_{De}, ion Debye length λ_{Di} and

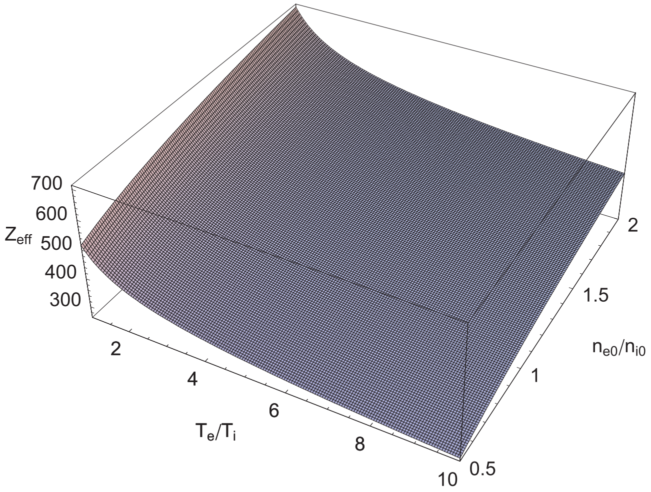

FIGURE 1. The surface plot of the effective dust charge Z_{eff} with variables the temperature ratio T_e/T_i, and the density ratio n_{e0}/n_{i0} for $k_BT_e = 2.5$ eV.

temperature ratio T_i/T_e

$$Z_{\mathrm{eff}}(\lambda_{De}, \lambda_{Di}, T_i, T_e) = \frac{ak_BT_e}{e^2}\left\{W\left[\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{\lambda_{Di}}{\lambda_{De}}\right)e^{T_i/T_e}\right] - \frac{T_i}{T_e}\right\}, \tag{4.1}$$

where the special function $W(x)$ stands for the Lambert W-function. For strongly coupled complex plasmas, the effective dust charge Z'_{eff} is then found to be $Z'_{\mathrm{eff}} = (ak_BT_e/e^2)W[(m_i/m_e)^{1/2}\lambda_{Di}/\lambda_{De}]$. In order to determine the charge of dust particles in a complex plasma with a radiation field, the interaction between radiation and dust particles must be considered. This is because the work function is known to be approximately 4 to 6 eV for the dust materials (Tielens 2005). However, the contribution of photoelectric ejection has been neglected because we assume that the bremsstrahlung photon density in the above effective dust charge Z_{eff} is small compared with the density of the electrons in a complex plasma. Moreover, the consequence of ion temperature on the e-D-BRCS process has not yet been investigated. If we include the influence of dust charge and ion temperature on the bremsstrahlung emission spectrum, the scaled e-D-BRCS $\bar{\chi}_{Z_{\mathrm{eff}},\bar{\varepsilon}}$ is given by

$$\begin{aligned}\bar{\chi}_{Z_{\mathrm{eff}},\bar{\varepsilon}} = {}& \frac{16}{3\pi}\frac{\alpha^3}{\bar{E}_0}\left(\frac{ak_BT_e}{e^2}\right)^2\{W[(m_i/m_e)^{1/2}(\lambda_{Di}/\lambda_{De})e^{T_i/T_e}] - T_i/T_e\}^2 \\ & \times \int_{\bar{q}_{\mathrm{min}}}^{\bar{q}_{\mathrm{max}}} d\bar{q}\left\{\frac{\bar{q}\exp(-2/\bar{\lambda}_{\mathrm{eff}})}{(1/\bar{\lambda}_{\mathrm{eff}}^2 + \bar{q}^2)^2}\left[\left(\frac{1}{\bar{\lambda}_{\mathrm{eff}}^2}\right)\sin^2(\bar{q}\bar{\lambda}_{\mathrm{eff}})\right.\right. \\ & \left.\left. + \left(\frac{2\bar{q}}{\bar{\lambda}_{\mathrm{eff}}}\right)\sin(\bar{q}\bar{\lambda}_{\mathrm{eff}})\cos(\bar{q}\bar{\lambda}_{\mathrm{eff}}) + \bar{q}^2\cos^2(\bar{q}\bar{\lambda}_{\mathrm{eff}})\right]\right\}.\end{aligned} \tag{4.2}$$

Hence, this equation is very useful for exploring the dust charging and ion temperature effects on the bremsstrahlung spectra in complex plasmas. Changes in the bremsstrahlung spectra including the ion-wake field (Kompaneets, Morfill, & Ivlev 2009) will be discussed elsewhere.

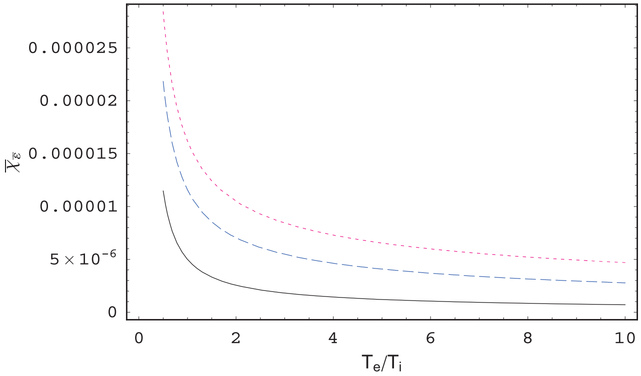

FIGURE 2. The scaled e-D-BRCS $\bar{\chi}_{\bar{\varepsilon}}$ with variables of the temperature ratio T_e/T_i for $\bar{E}_0 = 10$ and $\bar{\varepsilon} = 2$. The solid, dashed and dotted curves show the cases of $n_{e0}/n_0 = 0.1$, $n_{e0}/n_0 = 0.4$ and $n_{e0}/n_0 = 0.8$, respectively.

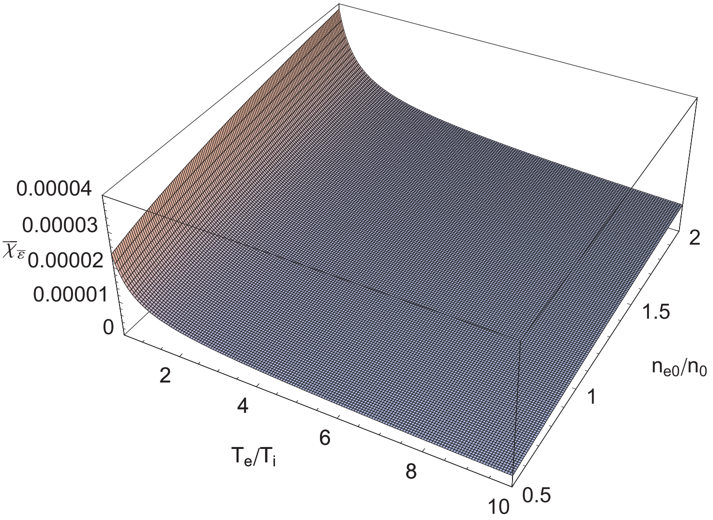

FIGURE 3. The surface scheme of the scaled e-D-BRCS $\bar{\chi}_{\bar{\varepsilon}}$ with variables of the temperature ratio T_e/T_i and the density ratio n_{e0}/n_0 for $\bar{E}_0 = 10$ and $\bar{\varepsilon} = 2$.

We set $a = 0.1$ µm and $\lambda_D/a = 50$ to explore specifically the effects of ion temperature and density on the bremsstrahlung emission processes in complex plasmas. Figure 1 presents the surface plot of effective dust charge Z_{eff} with variations of T_e/T_i and n_{e0}/n_{i0}. It is shown that the effective dust charge Z_{eff} decreases as T_e/T_i increases, but increases as n_{e0}/n_{i0} increases. Therefore, the variable effective charge Z_{eff} (4.1) is very useful for investigating the appearances of the charge variation as well as the electron-dust bremsstrahlung process in complex plasmas, since the BRCS for the dust charged case is given as $\bar{\chi}_{Z_{\text{eff}},\bar{\varepsilon}} \propto Z_{\text{eff}}^2$. Figure 2 shows the change of e-D-BRCS $\bar{\chi}_{\bar{\varepsilon}}$ with T_e/T_i for various values of n_{e0}/n_0. As shown, it is found that the cross-sectional area increases as the density ratio increases. Thus, we expect the bremsstrahlung emission power to be enhanced by the increase of electron density but suppressed by the increase of dust density since the charge neutrality condition is given by $Zn_{d0}/n_0 = 1 - n_{e0}/n_0$. Figure 3 also shows a surface plot of the scaled e-D-BRCS $\bar{\chi}_{\bar{\varepsilon}}$ with variations of T_e/T_i and n_{e0}/n_0. It can be

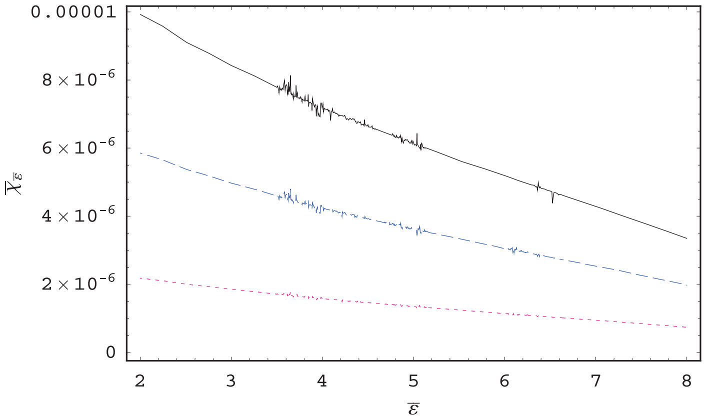

FIGURE 4. The scaled e-D-BRCS $\bar{\chi}_{\bar{\varepsilon}}$ with variables of the scaled bremsstrahlung emission energy $\bar{\varepsilon}$ for $\bar{E}_0 = 10$ and $n_{e0}/n_0 = 0.3$. The solid, dashed and dotted curves show the cases of $T_e/T_i = 1$, $T_e/T_i = 2$ and $T_e/T_i = 10$, respectively.

seen from these figures that the density effect on the e-D-BRCS reduces as the temperature ratio increases. We therefore find that the density effect is more substantial in the lower electron temperature or higher ion temperature domains. Therefore, it can be expected that the bremsstrahlung emission power in one-temperature plasmas ($T_e/T_i \approx 1$), such as dust burning processes, is much stronger than in conventional complex plasmas ($T_e/T_i \gg 1$) due to the effect of ion temperature. Figure 4 depicts the electron-dust BRCS $\bar{\chi}_{\bar{\varepsilon}}$, scaled by πa_0^2, with variation of the scaled bremsstrahlung emission energy $\bar{\varepsilon}$ and the temperature ratio T_e/T_i. The effect of ion temperature on the e-D-BRCS $\bar{\chi}_{\bar{\varepsilon}}$ is found to decrease with increasing bremsstrahlung emission energy. Also, the e-D-BRCS is significantly suppressed as the emission energy increases.

5. Conclusions

We studied the temperature effect on the spectrum of electron-dust particle bremsstrahlung radiation processes including variable dust charge in complex plasmas. We used the Born analysis to obtain e-D-BRCS in terms of bremsstrahlung radiation energy, collision energy, Debye length, dust charge, dust radius, ion temperature and plasma density. In this work, we found that the e-D-BRCS reduces with decreasing ion temperature or increasing electron temperature. We also found that the ion temperature influence is independent of the bremsstrahlung emission energy. Since the size of dust particle is much larger than the wavelength of photons of the UV and soft X-ray bremsstrahlung radiation, where the electron temperature is in the range of a few eVs in complex plasma, the bremsstrahlung radiation photons will be significantly absorbed. On the other hand, if the particle size is smaller than the wavelength of the bremsstrahlung emission, photon scattering will follow the Rayleigh scattering law. This absorption and scattering of the bremsstrahlung radiation then reduces the radiation intensity along the line of sight, causing dust particle extinction (Padmanabhan 2001). We obtain the analytic expression of the variable dust charge $Z_{\text{eff}}\{= (ak_BT_e/e^2)[W[(m_i/m_e)^{1/2}(\lambda_{Di}/\lambda_{De})e^{T_i/T_e}] - T_i/T_e]\}$ in terms of the Lambert W-function (Younsi & Tribeche 2008) in this work. Hence, we obtain the e-D-BRCS $\bar{\chi}_{Z_{\text{eff}},\bar{\varepsilon}}$ with the analytic expression for the effective dust charge Z_{eff} in terms of the Debye length and the temperature ratio. We also found that the effective dust charge decreases with an increase of the temperature ratio T_e/T_i. Moreover, we have found that that the effective dust charge increases with an increase of the density ratio n_{e0}/n_{i0}. Additionally, we found that the e-D-BRCS increases as the electron-ion density ratio increases. We therefore found that both the temperature and the density of ions play an essential role in the electron-dust bremsstrahlung emission process. It is also interesting that the electron-dust particle bremsstrahlung emission process could be a candidate mechanism for terahertz radiation since they can produce coherent bremsstrahlung emission spectrum in the terahertz range when the flowing electrons encounter uniformly distributed charged dust particles. A detailed exploration of terahertz radiation from electron-dust particle bremsstrahlung radiation can be studied in future work. The results in this paper provide useful knowledge of the bremsstrahlung emission spectra of complex plasmas.

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Declaration of interest statement

Competing interests: The authors declare none.

Data availability

The data that support the findings of this study are available upon reasonable request from the authors.

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