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ROBUST POLICIES IN A STICKY INFORMATION ECONOMY

FRANCESCO GIULI
Sapienza University of Rome

This paper analyzes the behavior of a central bank under strong ("Knightian") uncertainty when the short-run trade-off between output and inflation is represented by the sticky information Phillips curve proposed by Mankiw and Reis [*Quarterly Journal of Economics* 117(4), 1295–1328 (2002)]. By solving the robust control problem analytically, we show why model uncertainty does not affect the optimal monetary policy response to demand and productivity shocks, whereas it causes a stronger reaction of the monetary policy instrument to a cost-push (i.e., markup) shock. Differently from what occurs in sticky price models, the antiattenuation effect can result in a degree of price level stabilization that is greater or less than that experienced in the rational expectation model, depending on the central bank's degree of conservatism. These results dramatically affect the rationale for delegating monetary policy to a central banker more conservative than the society.

Keywords: Robust Control, Sticky Information, Minmax Policies, Delegation

1. INTRODUCTION

Increasing research activity has recently focused on the effects of uncertainty in the conduct of monetary policy. Whereas the traditional literature dealt with uncertainty by adding exogenous disturbances to a linear–quadratic economic framework, which guarantees that the certainty equivalence holds true, a more interesting kind of uncertainty faced by central bankers, known as model uncertainty, is progressively attracting the interest of scholars.

In Brainard's (1967) analysis, model uncertainty, understood in the sense that the marginal effect of a policy instrument on a macroeconomic outcome is described by a parameter distribution, leads to cautious policy. This result has recently been challenged by a body of research investigating how monetary policy should be conducted when the central bank knows the structural relations of the economy (the reference model) but faces uncertainty about the parameter values or the stochastic structure of the model. This problem has been tackled using robust control techniques: a minmaximizing central bank, aiming to avoid poor performances associated with "unfortunate" parameter configurations, derives robust

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monetary policy rules under the worst possible specification (the worst-case model or scenario). In dynamic stochastic general equilibrium models with monopolistic competitive producers experiencing frictions in price setting that cause monetary non-neutrality (i.e., the New Keynesian approach), this behavior leads, in general, to an antiattenuation result (i.e., the policy maker reacts more vigorously to shocks), but this conclusion does not always apply.

The antiattenuation result typically emerges in closed economies where uncertainty is modeled through a set of additive perturbations controlled by a fictitious "evil agent" who tries to maximize the central bank's loss by setting a specification error of bounded size [i.e., the *unstructured model uncertainty* proposed by Hansen and Sargent, HS (2004)], whereas the opposite may occur in open economies [Giordani and Soderlind (2004); Leitemo and Söderström (2008a, 2008b)]. In any case, the optimal trade-off between targets is not affected by the preference for robustness [i.e., Walsh's (2004) equivalence result]. Yet, even if the desired trade-off is the same, target variables turn out to be more volatile. The same nonunivocal conclusions are reached in forward-looking New Keynesian models under *parameter uncertainty* where, differently from HS, the evil agent controls one or more coefficients of the structural equations [Giannoni (2002)]. In such a context, the robust policy may need to react more or less strongly than in the certainty-equivalence environment to fluctuations in inflation and in the output gap, according to the assumptions that are made on the central bank's objective function or on the parameter that is subject to uncertainty [Kara (2002)]. Min-max policy prescriptions are hence fragile with respect to changes in the description of the economy, starting from the very characterization of uncertainty.

The economic framework used by most of the modern monetary policy literature, and hence by most of the robust control literature, is based on the New Keynesian sticky price Phillips curve (SPPC), that is, a relation between inflation and marginal cost derived from Calvo's (1983) hypothesis of a time-contingent price-adjustment rule. This approach faces, however, several difficulties in explaining some stylized facts of inflation dynamics: the need to wait for several periods before obtaining the maximum impact of policies (the dynamic response of output and inflation is hump-shaped), as emphasized by Mankiw (2001); the fact that disinflation is always contractionary [Ball (1994)]; the fact that inflation is highly serially correlated [Fuhrer and Moore (1995)]. In the face of these difficulties, Mankiw and Reis (2002) proposed to replace the traditional stickiness of Calvo's price model with a friction in the dissemination and gathering of new information. Their starting point is that information dissemination is slow, so that firms cannot update it every period. Prices are flexible, but information is sticky. By introducing this hypothesis into a monopolistically competitive framework à la Blanchard and Kiyotaky (1987), Mankiw and Reis (2002) derived a Phillips curve able to overcome the major shortcomings of the sticky price models.

The relevance of uncertainty to monetary policy setting and the ongoing debate[1] on the opportunity to replace the traditional SPPC with the sticky information

Phillips curve (SIPC) motivate our analysis of a central bank under strong uncertainty facing a short-run trade-off between output and inflation given by the SIPC. In the model with VAR representation and rational expectations formed in different periods in the past, to be presented below, the robust control problem can be solved analytically, allowing us to show how and why the robust monetary policy in this environment differs from the optimal one identified by Ball, Mankiw, and Reis [BMR] (2005) when different kinds of shocks hit the economy. We rate this as an important contribution of the paper, because SPPC models can be analytically solved only when shocks are i.i.d., which exacerbates the lack of persistence of variables.

The interest for this analysis is also motivated by the different targeting regimes that are found to be optimal in the two (sticky price and sticky information) contexts. The second-order approximation of households' utility is a function of the variability of output around its flexible price level and of the cross-sectional variability of output across different firms, which, in turn, depends on the cross-sectional variability of prices [Woodford (2003)]. Under sticky prices, the latter variability is determined by current and lagged values of squared inflation, which leads to the optimality of some inflation targeting, whereas under sticky information it is linked to the current and lagged values of squared price-level variations, which leads to the optimality of price-level targeting. Our analysis hence makes it possible to understand the effect of strong uncertainty when a price-level targeting regime is optimally adopted. Whereas under inflation targeting the optimal policy after an inflationary shock never requires a decrease in the price level, and thus in the nominal policy instrument (i.e., the money supply in our context), we shall show below that under price-level targeting the optimal policy can require either a decrease or an increase in the nominal policy instrument, depending on the central bank's degree of conservatism. For given central bank preferences, the antiattenuation result in the use of the policy instrument produced by model uncertainty can hence bring about radically different economic performances in the two monetary policy regimes.

Finally, our analysis contributes to the debate on monetary policy delegation. Model uncertainty justifies the appointment of a central banker with a loss function different from the social one even in the absence of inflation or stabilization biases. Our results will show that under sticky information it is optimal to appoint an independent central banker who is more or less conservative than society according to society's preferences.

The paper is organized as follows. In Section 2 we summarize BMR's (2005) model, highlighting the optimality of a price targeting rule and discussing the shocks affecting the economy. In Section 3 we compute the robust policy under the unstructured approach. In line with most of the literature, if the central bank and the evil agent act simultaneously, in the worst-case model target variability increases with the preference for robustness [see Walsh (2004); Leitemo and Söderström (2008b)]. When the policy maker employs this robust instrument rule but the actual misspecification is zero (the *approximated model* solution), an antiattenuation result emerges: the policy maker varies her instrument more vigorously than she would in the certainty equivalent case. This overreaction produces a greater or smaller stabilization of the price level depending on the value of the desired trade-off in the rational expectation model. When in this model the policy maker reacts to the cost shock by injecting (mopping up) money, in the approximated model she reacts by injecting (mopping up) even more money. Thus, whereas in sticky price economies where it is optimal to target inflation the central bank never reacts to a cost shock by producing a price deflation, this may well happen in a sticky information economy under a price-targeting regime. If the central bank acts as a Stackelberg leader with respect to the evil agent, Walsh's (2004) equivalence result is no longer valid: the policy maker is less aggressive in the stabilization of the price level. The macroeconomic consequences of this behavior in the approximating model solutions are driven by the interaction of two forces that may operate in opposite directions: the first is the antiattenuation effect in the use of the nominal policy instrument (the same at play under the Nash timing); the second is due to the new desired trade-off, which favors output stabilization. In Section 4 we tackle the issue of the optimal design of institutions, that is, the degree of central bank conservatism that maximizes social welfare. Section 5 concludes.

2. THE BALL–MANKIW–REIS MODEL

Mankiw and Reis (2002) propose a relation between output and inflation based on the idea that symmetric monopolistically competitive firms choose their optimal prices in each period, but the information set they use when solving the profit maximization problem may not be the current one. Information is sticky in the sense that firms update it sporadically. The process that guides information gathering is similar to that formulated by Calvo (1983) for price adjustment: in each period only a fraction $(1-\omega)$ of firms can adjust their prices. In Mankiw and Reis (2002), in each period only a fraction $(1-\omega)$ of firm obtain a new information set, whereas the remaining ω continue to fix prices on the basis of the old one. The log-linearization of the first-order conditions of consumers' and producers' optimization problems yield the SIPC and the price-level equations:[2]

$$\pi_t = \frac{(1-\omega)}{\omega}\left[a x_t + u_t\right] + (1-\omega)\sum_{k=0}^{\infty}\omega^k E_{t-1-k}\left(\pi_t + a\Delta x_t + \Delta u_t\right), \qquad \textbf{(1)}$$

$$p_t = (1-\omega)\sum_{k=0}^{\infty}\omega^k E_{t-k}\left(p_t + a x_t + u_t\right). \qquad \textbf{(2)}$$

Inflation (π_t) depends on the current output gap ($x_t = y_t - y_t^n$), on the cost-push shock (u_t),[3] and on the past expectations of current inflation, of the growth of the output gap, and of the shock. a is a combination of fundamental parameters that represents the sensitivity of the firm's optimal price to the expected deviation of output gap. The price level depends on current and past expectations on the output gap and cost push shock.

As Mankiw and Reis (2002) and BMR (2005) assume, the demand side of the economy is represented by a quantity theory equation linking the real output to the real money balance,

$$y_t = m_t - p_t, \tag{3}$$

where m_t is the log of the money supply.[4] The policy instrument for period t is chosen at time $t-1$; policy decisions hence affect the economy with a lag (the central bank cannot respond contemporaneously to the shock).

As for the welfare function, BMR (2005) follow Woodford's (2003) methodology and derive the welfare objective function from the average level of utility across all households. A second-order approximation of households' loss is found to be a function of the variability of output around its flexible price level [Var(x_t)] and of the cross-sectional variability of output across different firms [Var$_j(x_{jt})$]:[5]

$$w_t = w[\text{Var}(x_t), \text{Var}_j(x_{jt})].$$

As the natural output differs from the efficient output only by a constant independent of policy, the variability of output around the natural level also measures the variability around the efficient level. The cross-sectional variability of output across different firms enters the loss function because variability at the firm level is inefficient because it creates variability in the labor supply around the efficient level. The period loss can be formally written as

$$w_t = \text{Var}\,(x_t) + bE[\text{Var}_j(p_{jt} - p_t)] + K, \tag{4}$$

where b is a combination of fundamental parameters and K collects the terms independent of policy (for the sake of simplicity, we set $K = 0$). In equation (4), we have used

$$\text{Var}_j(x_{jt}) = \zeta^2 \text{Var}_j(p_{jt} - p_t)$$

to replace the cross-sectional variability of output across different firms with the price variability (ζ is the elasticity of demand).

Because it can be shown that $\text{Var}_j(p_{jt} - p_t) = \sum_{j=1}^{\infty} f(\omega, j)(p_{t+k} - E_{t+k-j}\,p_{t+k})^2$, the central bank attempts to minimize

$$W_t = \sum_{k=0}^{\infty} \beta^k w_{t+k} = \sum_{k=0}^{\infty} \beta^k x_{t+k}^2 + b \sum_{j=1}^{\infty} f(\omega, j)\,(p_{t+k} - E_{t+k-j}\,p_{t+k})^2, \tag{5}$$

where $f(\omega, j) = \frac{(1-\omega)\omega^j}{(1-\omega^j)(1-\omega^{j+1})}$.

Equation (5) shows that the terms in the loss function depend on aggregate variables. Under sticky information, social welfare departs from the common microfounded representation derived under sticky prices, where the cross-sectional variability of output is determined by current and lagged values of squared inflation.

The optimal monetary policy can be found by minimizing the loss function (5) with respect to the policy instrument subject to the demand side and the supply side of the model. BMR show that by combining equations (2) and (3) it is possible to determine a linear relation between p_t and the money supply target m_t. Because m_t is determined one period in advance, there exists a linear relation between the money supply and the expectation formed at time $t-1$ (when the policy instrument is set) of the price level at time t ($E_{t-1}p_t$), which can hence be interpreted as the policy instrument.

The general moving average, MA(∞), representation of the price level and the output equilibrium paths are, respectively, $p_t = \sum_{j=0}^{\infty} \phi_j \epsilon_{t-j}$ and $y_t = \sum_{j=0}^{\infty} \varphi_j \epsilon_{t-j}$, where ϵ_t are the innovations of the model and ϕ_j and φ_j are unknown coefficients. Because of the lag in the policy transmission mechanism, the robust policy process in MA(∞) form is given by $E_{t-1}p_t = \sum_{j=1}^{\infty} \phi_j \epsilon_{t-j}$ and $E_{t-1}y_t = \sum_{j=1}^{\infty} \varphi_j \epsilon_{t-j}$.

By substituting the MA(∞) representations of the relevant variables, the price level equation (2) can be written as

$$\sum_{j=1}^{\infty} \phi_j \epsilon_{t-j} = \sum_{j=1}^{\infty} \Lambda^j (\phi_j \epsilon_{t-j} + a\varphi_j \epsilon_{t-j} + \rho_j \epsilon_{t-j}),$$

where $\Lambda^j = (1-\omega) \sum_{k=0}^{j} \omega^k$ and $u_t = \sum_{j=0}^{\infty} \rho_j \epsilon_{t-j}$ is a cost-push shock that follows an arbitrary stationary process.

Because this expression must hold for all possible realizations of ϵ_{t-j}, it follows that

$$\varphi_j = \frac{1}{a}\left(\frac{1-\Lambda^j}{\Lambda^j}\phi_j - \rho_j\right). \tag{6}$$

Because $p_t - E_{t-i}p_t = \sum_{j=0}^{i-1} \phi_j \epsilon_{t-j}$, we can write the objective function as

$$L = \left[\sum_{j=1}^{\infty} \varphi_j^2 + b\sum_{i=1}^{\infty}\left(\frac{1}{\Lambda^{i-1}} - \frac{1}{\Lambda^i}\right)\sum_{j=1}^{i-1} \phi_j^2\right]\sigma_\epsilon^2. \tag{7}$$

By minimizing (7) with respect to ϕ_j subject to (6), we obtain the optimal coefficients

$$\phi_j^* = \frac{1}{a^2 b + \frac{1-\Lambda^j}{\Lambda^j}}\rho_j \quad \text{for } j > 0 \tag{8}$$

and

$$\varphi_j^* = -\frac{ab}{a^2 b + \frac{1-\Lambda^j}{\Lambda^j}}\rho_j \quad \text{for } j > 0. \tag{9}$$

Due to the lag in the policy transmission mechanism, the central bank cannot reply contemporaneously to the shock; hence in period $t = 0$, price and output

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coefficients are given by

$$\phi_0 = \frac{1-\omega}{1-(1-\omega)(1-a)} \quad \text{and} \quad \varphi_0 = -\phi_0. \tag{10}$$

The relation between the optimal coefficients is

$$\varphi_j^* = -ab\phi_j^*, \tag{11}$$

which implies the targeting rule

$$E_{t-1}p_t = k_t - \frac{1}{ab}E_{t-1}x_t, \tag{12}$$

where k_t can be zero or any deterministic path. As stressed by Hall (1984) and BMR (2005), optimal policy can be described as an *elastic price standard*; that is, the central bank allows the price level to deviate from its target when output deviates from its natural rate. To understand this result, consider a cost-push shock at time $t = 0$. The informed firms immediately raise their prices and the aggregate price level increases. If the shock is autocorrelated, in the following period the uninformed firms continue to set prices at zero, but for the informed firms policy now produces its effect and induces them to set prices according to the optimal trade-off (12). As long as the shock persists, the central bank allows output and price level to deviate from their targets. This implies that under price level targeting, when the shock vanishes, all firms set the same zero price, whereas, under inflation targeting, in order to avoid disinflation, the central bank must induce the informed firms to set positive prices. This would cause output variability, because informed and uninformed firms are still setting different prices.

The desired targets' trade-off is implemented through the following instrument rule in MA(∞) representation:

$$m_t^* = \sum_{j=1}^{\infty} \eta_j^* \epsilon_{t-j},$$

where the coefficients η_j^* are given by

$$\eta_j^* = \phi_j^* + \varphi_j^* = \frac{1-ab}{a^2b + \frac{1-\Lambda^j}{\Lambda^j}}\rho_j. \tag{13}$$

For the purpose of our analysis it is important to highlight that the optimal policy predicts a trade-off between target variables that can be implemented though either an increase or a reduction in the nominal policy instrument, depending on the value of ab [see equation (13)]. If $ab < 1$, the policy maker reacts to an inflationary shock by increasing the nominal money supply, because the desired trade-off implies that an increase in the price level requires a less than proportional decrease in the output gap. By contrast, if $ab > 1$, the optimal trade-off predicts a more than proportional decrease in the output gap, which, in its turn, implies a negative response of the nominal money supply. Under an inflation targeting regime such policy behavior is suboptimal, as inflation stabilization never requires a decrease in the price level.

In this model, cost-push shocks pose a nontrivial policy stabilization problem, whereas other kinds of shocks, such as productivity or demand shocks, result in complete price level and output gap stabilization in the first period after the shock.[6] This is due to a well-known characteristic of the model; that is, stabilizing the price level (around zero or around any deterministic path) is equivalent to stabilizing the welfare-relevant output gap.[7] As stressed by BMR, the non-neutrality of monetary policy stems from innovations not immediately observed by all price setters. Due to the presence of a lag in the transmission mechanism of monetary policy, in the first period after a demand shock both output and prices increase, but in the following period strict price targeting makes all the effects disappear completely, because informed and uninformed firms set the same price. A productivity shock does not affect the gap between the natural level of output and its first-best level, so that by stabilizing the price level the policy maker also stabilizes both the natural and the welfare-relevant output gap. The same result occurs in the case of a demand shock, because the policymaker is able to counteract any shift in the aggregate demand equation by an appropriate adjustment of the policy instrument. In contrast, in the face of a cost-push shock, a complete price-level stabilization does not instead guarantee output stabilization, as implied by equation (2). Hence equation (12) is also the optimal monetary policy in an economy that may experience demand, technology, and cost-push (i.e., markup) shocks. In the first two cases, the policy maker successfully reaches $E_{t-1}x_t = 0$ by committing to a zero or predetermined price-level path; in the third case, the gap between the natural level and the first-best level of output is not constant, and policy makers must optimally trade off the stabilization of the different objective variables.

3. THE *UNSTRUCTURED* MODEL UNCERTAINTY APPROACH

In this section we apply HS's (2004) robust control techniques to the New Keynesian model with sticky information summarized in the previous section. The *unstructured* model uncertainty approach conceives the specification errors as a serially correlated shock process stemming from omitted variables in the structural model of the economy. These shocks are assumed to be of bounded size, which can be viewed as a measure of the central bank's preference for robustness. The lack of a prior distribution for the shock process and the preference for robustness induce the policy maker to adopt a minmax strategy that can be represented through a "mind" game played by the central bank and a fictitious "evil agent," who represents the policy maker's fear concerning specification errors. The "evil agent" chooses the amount of misspecification to maximize the central bank loss. In the following section we assume that the two players act simultaneously, whereas in Section 3.2 we analyze the Stackelberg equilibrium.

In this model, just as in BMR's, productivity or demand shocks do not affect the gap between the natural level of output and its first-best level, so that the central bank does not have to exploit the SIPC trade-off; hence it does not worry about misspecification errors.[8] For this reason, we continue to focus only on cost-push shocks.

3.1. The Nash Timing

To model parameter uncertainty, we introduce in the price equation (2) a second source of disturbance denoted by z_t, which is added to the central bank's reference model. The timing of the model is the same as in BMR: the policy instrument for period t is chosen at time $t-1$ (the policy maker continues to affect the economy only in the first period after the shock); in each period the policy maker observes the realized shock, forms her expectation of the next period price, which depends on her expectation of the next period shock (formed according to BMR's assumption of a known stochastic structure), and sets the money supply accordingly; the private agents also observe the shock and choose their reaction. The introduction of the evil agent requires specifying the timing of its actions as well, but, being a mind projection of the central bank's fear, it must share the central bank's information set and act according to the same timing, so that the specification error also affects the economy with a one-period lag, that is, contemporaneously to the policy instrument.

The misspecified price equation can be written as follows:

$$p_t = (1-\omega)\sum_{k=0}^{\infty}\omega^k E_{t-k}\left(p_t + \alpha x_t + u_t + z_t\right). \tag{14}$$

Similarly to the other variables, the general MA(∞) representation for the "optimal misspecification" chosen by the evil agent can be written as $z_t = \sum_{j=1}^{\infty}\gamma_j\epsilon_{t-j}$, with γ_j unknown coefficients. By substituting it into the price level equation, we get

$$\sum_{j=1}^{\infty}\phi_j\epsilon_{t-j} = \sum_{j=1}^{\infty}\Lambda^j[\phi_j\epsilon_{t-j} + a\varphi_j\epsilon_{t-j} + (\rho_j\epsilon_{t-j} + \gamma_j\epsilon_{t-j})]. \tag{15}$$

Because this expression must hold for all possible realizations of ϵ_{t-j}, it follows that

$$\varphi_j = \frac{1}{a}\left(\frac{1-\Lambda^j}{\Lambda^j}\phi_j - \rho_j - \gamma_j\right). \tag{16}$$

Because $p_t - E_{t-i}p_t$ is still given by $\sum_{j=0}^{i-1}\phi_j\epsilon_{t-j}$, we can rewrite the loss function as

$$L_s = \left[\sum_{j=1}^{\infty}\varphi_j^2 + b\sum_{i=1}^{\infty}\left(\frac{1}{\Lambda^{i-1}} - \frac{1}{\Lambda^i}\right)\sum_{j=0}^{i-1}\phi_j^2\right]\sigma_\epsilon^2 - \sum_{j=1}^{\infty}\theta\gamma_j^2\sigma_\epsilon^2,$$

where θ represents the preference for robustness, or equivalently the set of models available to the evil agent against which the policy maker wants to be robust. When

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the degree of misspecification goes to zero, θ goes to infinity and we turn to the standard rational expectations model. An increase in the preference for robustness means a decrease in θ.[9]

By minimizing the new objective function, subject to equation (16), we obtain

$$\phi_j = \frac{1}{a^2 b + \frac{1-\Lambda^j}{\Lambda^j}}(\rho_j + \gamma_j) \tag{17}$$

and thus

$$\varphi_j = -\frac{ab}{a^2 b + \frac{1-\Lambda^j}{\Lambda^j}}(\rho_j + \gamma_j). \tag{18}$$

Even if γ_j is still undetermined, in this stage we can note that the relation between the output gap and the price level coefficients is given by

$$\varphi_j = -ab\phi_j. \tag{19}$$

Hence, the optimal price–output trade-off is not affected by the preference for robustness; that is, the robust optimal targeting rule is the same as the optimal rule without fear of specification errors [see equation (12)]. This is the equivalence result derived by Walsh (2004) in sticky price New Keynesian models.

The optimal amount of misspecification is obtained by maximizing the loss function with respect to γ_j, subject to equation (16):

$$\gamma_j = -\frac{\frac{1-\Lambda^j}{\Lambda^j}}{a^2\theta - 1}\phi_j + \frac{1}{a^2\theta - 1}\rho_j. \tag{20}$$

By substituting equation (17) into (20), we get

$$\gamma_j^{rn} = \frac{b}{\theta\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j}\right) - b}\rho_j, \tag{21}$$

where the superscript rn denotes the robust Nash equilibrium values in the worst-case model. The amount of misspecification is increasing in the central bank's preference for robustness (θ), and it is negatively related to a, which captures the sensitivity of firms' optimal price to the expected deviation of output (when a decreases, larger movements in output are needed in order to affect inflation). As for parameter b (the weight associated with price variability in the loss function), we have $\partial\gamma_j^{rn}/\partial b > 0$; this means that when offsetting relative price variability becomes more costly, the amount of misspecification increases.

In Appendix B, we show that in the case of a productivity shock, by committing to a zero or predetermined price-level path, the central bank completely stabilizes the output gap. Hence it has no reason to exploit the trade-off offered by the SIPC, it does not fear specification errors (because they are optimally set equal to zero by the evil agent), and the robust policy coincides with the optimal one described by equation (12). In the light of what we clarified at the end of the previous section, the same occurs in the case of demand shocks.

By substituting equation (21) into equations (17) and (18), we obtain the robust Nash equilibrium price and output coefficients in the worst-case scenario:[10]

$$\phi_j^{rn} = \frac{1}{a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}} \rho_j, \tag{22}$$

$$\varphi_j^{rn} = -\frac{ab}{a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}} \rho_j. \tag{23}$$

When $\theta \to \infty$, ϕ_j^{rn} and φ_j^{rn} collapse to the optimal coefficients in the absence of model misspecification [see equations (8) and (9)]; when the fear of misspecification is "low" (θ is "high"), it is $\phi_j^{rn} > 0$ and $\varphi_j^{rn} < 0$, and a cost-push shock increases inflation and decreases output. It might seem that if θ were so small that ϕ_j^{rn} and φ_j^{rn} would be negative and positive, respectively, the policy maker would face a quite unrealistic situation, as a cost-push shock would cause deflationary pressure. The second-order conditions ensure, however, that whereas the central bank always minimizes the loss function, the evil agent maximizes rather than minimizes the loss function only for $\theta > a^{-2}$ (see Appendix A).[11] Hence, when the min–max Nash equilibrium exists, the denominators of (22) and of (23) are always positive, so that ϕ_j^{rn} is never negative and φ_j^{rn} is never positive.

The relations between the misspecification and the output and price coefficients are, respectively,

$$\frac{\gamma_j^{rn}}{\varphi_j^{rn}} = -\frac{1}{a\theta}, \tag{24}$$

$$\frac{\gamma_j^{rn}}{\phi_j^{rn}} = \frac{b}{\theta}. \tag{25}$$

Moreover, we can note that $\partial|\varphi_j^{rn}|/\partial\theta < 0$ and $\partial\phi_j^{rn}/\partial\theta < 0$. Thus, in line with standard results, both the price level and the output gap are more volatile in the worst-case scenario, when the preference for robustness increases.

Remember that the analytical solution in the worst-case model is derived by assuming the central bank's worst fears about parameter configuration results to be justified ex post. This implies that we cannot say whether the greater volatility of macroeconomic variables (in comparison with the rational expectations solution) represents a welfare cost due to the central bank's fear about misspecification or due to the effective realization of specification errors. In other words, in the worst-case model, we cannot isolate the effect of the pure fear for misspecification on the policy behavior from the effect of the actual realization of a misspecified Phillips curve.[12] Hence, any comparison between the rational expectations and the worst-case model must be made carefully, because they are slightly different models. This is why, in order to isolate the consequences for the economy of the fear of misspecification, we need to focus on the *approximating model solution.* This is found by assuming that the central banker sets her instrument with the aim of its being robust against model misspecification, but that the actual misspecification is zero.

Given the simple demand side of the model, we can immediately find the MA(∞) representation for the instrument rule under the worst-case scenario,

$$m_t^{rn} = \sum_{j=1}^{\infty} \eta_j^{rn} \epsilon_{t-j},$$

where the coefficients η_j^{rn} are given by

$$\eta_j^{rn} = \phi_j^{rn} + \varphi_j^{rn} = \frac{1-ab}{a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}} \rho_j. \tag{26}$$

The approximating model solution under the Nash timing (denoted by the superscript "*an*") is then obtained by substituting equation (26) into the price-level equation (15), setting $\gamma_j = 0$:

$$\phi_j^{an} = \frac{a + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)} \rho_j \tag{27}$$

and

$$\varphi_j^{an} = -\frac{a^2 b + ab\frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)} \rho_j. \tag{28}$$

By comparing the coefficients of the rational expectations and the approximating model, we get

$$\phi_j^{an} - \phi_j^{*} = \frac{\frac{b}{\theta}(1-ab)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(\frac{1}{a}\frac{1-\Lambda^j}{\Lambda^j} + 1\right)\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j}\right)} \rho_j \gtrless 0 \quad \text{if } b \lessgtr a^{-1}, \tag{29}$$

$$\begin{aligned} \left|\varphi_j^{an}\right| - \left|\varphi_j^{*}\right| &= -\frac{\frac{1-\Lambda^j}{\Lambda^j}\frac{b}{\theta}(1-ab)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j}\right)} \rho_j \\ &\lessgtr 0 \quad \text{if } b \lessgtr a^{-1}, \end{aligned} \tag{30}$$

and

$$\eta^{rn} - \eta^{*} = \frac{\frac{b}{\theta}(1-ab)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j}\right)} \rho_j \gtrless 0 \quad \text{if } b \lessgtr a^{-1}. \tag{31}$$

Thus, the consequence for the economy of an instrument rule set with the aim of being robust against model misspecification when the actual misspecification is zero depends upon the central bank's degree of conservatism. When $ab < 1$, the central bank in the original rational expectations model reacts to a cost-push shock by expanding the nominal money supply [see equation (13)]. In fact, the desired trade-off [see equation (11)] implies that an increase in the price level is counterbalanced by a less then proportional contraction in output, that is, in the real money supply. In contrast, when $ab > 1$, the central bank will mop up money in order to induce a more than proportional output contraction. In the approximating model, the central bank tries to implement the optimal targets trade-off [see equation (19)] but, due to the concerns for model misspecification, the implied instrument rule is not designed for the approximating model but for the worst-case one. This fear of misspecification produces an antiattenuation result in the use of the policy instrument: when in the original model the policy maker reacts to a cost shock by injecting money, in the approximating model she overreacts by injecting even more money. If the optimal policy is to tighten the nominal policy instrument, in the approximating model the nominal money reduction will be even greater [see equation (31)].

The consequences of this behavior for the economy are to allow the price level (output gap) to deviate from the steady state more (less) than under the rational expectation model when $b < a^{-1}$; the opposite is true when $b > a^{-1}$ [see equations (29) and (30)]. When $ab = 1$, both models produce the same results, as the antiattenuation principle cannot emerge when there is no policy reaction in the original model, that is, $\eta^* = 0$. This means that for this particular parameter configuration, the endogenous-model trade-off between the price level and the output gap exactly reflects the central bank's desired trade-off.

To summarize, the effect of the fear of misspecification on the target variables is not univocal. Due to the price targeting regime, the antiattenuation principle in the use of the policy instrument can result in a price-level stabilization that can be less than, greater than, or the same as that obtained in the rational expectations model, depending on the target trade-off in the central bank objective function. This, in turn, implies that an increase in the preference for robustness causes the output gap and the reaction of the policy instrument (price level) to be less (more) volatile when $b < a^{-1}$, and vice versa when $b > a^{-1}$, as shown by the signs of the following derivatives:

$$\frac{\partial \phi_j^{an}}{\partial \theta} = -\frac{\frac{b}{\theta^2}\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right) a\,(1-ab)}{\left[\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)\right]^2}\rho_j \lesseqgtr 0 \quad \text{if } b \lesseqgtr a^{-1},$$

$$\frac{\partial \left|\varphi_j^{an}\right|}{\partial \theta} = \frac{\frac{b}{\theta^2}\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)(1-ab)\,\frac{1-\Lambda^j}{\Lambda^j}}{\left[\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)\right]^2}\rho_j \gtreqless 0 \quad \text{if } b \lesseqgtr a^{-1},$$

$$\frac{\partial \eta^{rn}}{\partial \theta} = \frac{-\frac{b}{\theta^2}\,(1-ab)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)^2} \lesseqgtr 0 \quad \text{if } b \lesseqgtr a^{-1}.$$

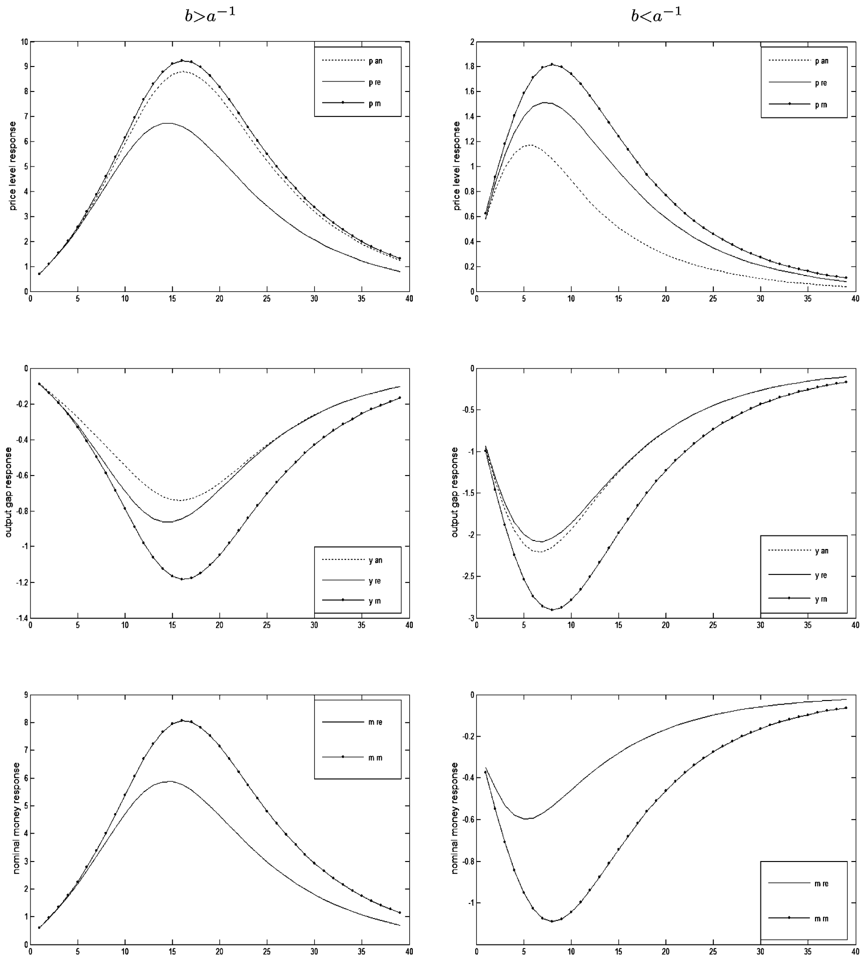

FIGURE 1. Antiattenuation result under the Nash timing.

The above results are illustrated in Figure 1, which shows the impulse responses of the price level, the output gap, and the money supply under the approximating, the rational expectations, and the worst-case model. The graphs in the left column are drawn under our benchmark parameterization,[13] and thus $b < a^{-1}$; the right-hand column considers the same parameterization but with $b = 10 > a^{-1}$.

3.2. The Stackelberg Solution

In this section, we assume that the central bank acts as a Stackelberg leader, and so designs its policies taking into account the evil agent's optimal decision for misspecification.[14]

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From the preceding section we know that the evil agent's reaction function is

$$\gamma_j = -\frac{\frac{1-\Lambda^j}{\Lambda^j}}{a^2\theta - 1}\phi_j + \frac{1}{a^2\theta - 1}\rho_j.$$

By substituting it into the price level equation constraint (16) and rearranging, we get

$$\varphi_j = \frac{1}{a}\frac{a^2\theta}{1 - a^2\theta}\left(-\frac{1-\Lambda^j}{\Lambda^j}\phi_j + \rho_j\right). \tag{32}$$

By minimizing the central bank loss function (7) with respect to ϕ_j subject to equation (32), we obtain the robust price coefficients under the Stackelberg timing in the worst-case model, denoted by the superscript "rs":

$$\phi_j^{rs} = \frac{1}{\left(\frac{a^2\theta - 1}{a^2\theta}\right)^2 a^2 b + \frac{1-\Lambda^j}{\Lambda^j}}\rho_j. \tag{33}$$

Using the above equation in the constraint (32) we obtain the robust output coefficients:

$$\varphi_j^{rs} = -\frac{\left(\frac{a^2\theta - 1}{a^2\theta}\right) ab}{\left(\frac{a^2\theta - 1}{a^2\theta}\right)^2 a^2 b + \frac{1-\Lambda^j}{\Lambda^j}}\rho_j. \tag{34}$$

Note first that when $\theta \to \infty$, both ϕ_j^{rs} and φ_j^{rs} collapse to the optimal ones [see equations (8), (9)]; second, as $(\frac{a^2\theta - 1}{a^2\theta}) \leq 1$, the robust price coefficients are always greater than the optimal ones, that is, $|\phi_j^{rs}| > |\phi_j^*|$, but the difference between the robust and the optimal output coefficients is nonlinear:

$$\left|\varphi_j^{rs}\right| \lesseqgtr |\varphi_j^*| \quad \text{if } \frac{1-\Lambda^j}{\Lambda^j} \gtrless b\frac{(a^2\theta - 1)}{\theta}.$$

Because $\frac{1-\Lambda^j}{\Lambda^j}$ is a combination of probabilities that decreases over time, the robust output coefficients can be initially smaller than the optimal ones (as it happens with our benchmark parameterization), and the opposite is true after some periods. By comparing the Stackelberg robust coefficients with the ones obtained under the Nash timing [equations (22) and (23)], we obtain that $|\phi_j^{rs}| > |\phi_j^{rn}|$ and $|\varphi_j^{rs}| < |\varphi_j^{rn}|$.

Thus, irrespective of the central bank's degree of conservatism, output gap stabilization under the Stackelberg timing is greater than under the Nash timing, and the opposite is true for price-level deviations. We can clarify the economic mechanism that leads to this unusual result after showing the specification error and the robust targeting rule. Using the robust price coefficient in the evil agent's

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reaction function, the specification error under the Stackelberg timing is

$$\gamma_j^{rs} = \frac{\left(\frac{a^2\theta - 1}{a^2\theta}\right)^2 a^2 b}{(a^2\theta - 1)\left[\left(\frac{a^2\theta - 1}{a^2\theta}\right)^2 a^2 b + \frac{1-\Lambda^j}{\Lambda^j}\right]}\rho_j.$$

As it happens under the Nash timing, γ_j^{rs} is positively related to the preference for robustness, that is, $\partial\gamma_j^{rs}/\partial\theta < 0$, but after some manipulations it is easy to verify that the first-mover advantage of the central bank leads to a lower specification error, that is, $|\gamma_j^{rs}| < |\gamma_j^{rn}|$.

This result is obtained by employing the following targeting rule, that is, the ratio between equations (34) and (33):

$$\varphi_j^{rs} = -\left(\frac{a^2\theta - 1}{a^2\theta}\right) ab\phi_j^{rs}. \tag{35}$$

The relations between the misspecification and the output and price coefficients are

$$\frac{\gamma_j^{rs}}{\varphi_j^{rs}} = -\frac{1}{a\theta}, \tag{36}$$

$$\frac{\gamma_j^{rs}}{\phi_j^{rs}} = \frac{a^2\theta - 1}{a^2\theta}\frac{b}{\theta}. \tag{37}$$

In terms of the endogenous variables, the robust policy rule can be written as

$$E_{t-1}p_t = k_t - \frac{1}{ab}\left(\frac{a^2\theta}{a^2\theta - 1}\right)E_{t-1}x_t. \tag{38}$$

Again, in the face of demand or productivity shocks, strict price-level targeting implies that $E_{t-1}x_t = 0$, whereas in the case of cost-push shocks the robust trade-off between the target variables (35) is affected by the preference for robustness: contrary to what occurs under the Nash timing, the equivalence result does not apply because the central bank anticipates the evil agent's behavior.

The relation between the specification error and the output coefficient is the same as under the Nash timing [see equations (36) and (24)], whereas the ratio between the specification error and the price coefficient is now lower [see equations (37) and (25)]. It follows that in reaction to a cost-push shock the central bank allows the price level to deviate from the target more than it does under the Nash timing [see equations (19) and (35)]. This less aggressive attitude in stabilizing the price level stems from the central bank's awareness that a more stable output gap dampens the persistence of the inflationary process and of the shock error. This can be achieved only by allowing the price level to absorb a higher part of the cost-push shock. The central bank foresees that the specification error is not randomly chosen but is "optimally" designed by the evil agent, whose objective

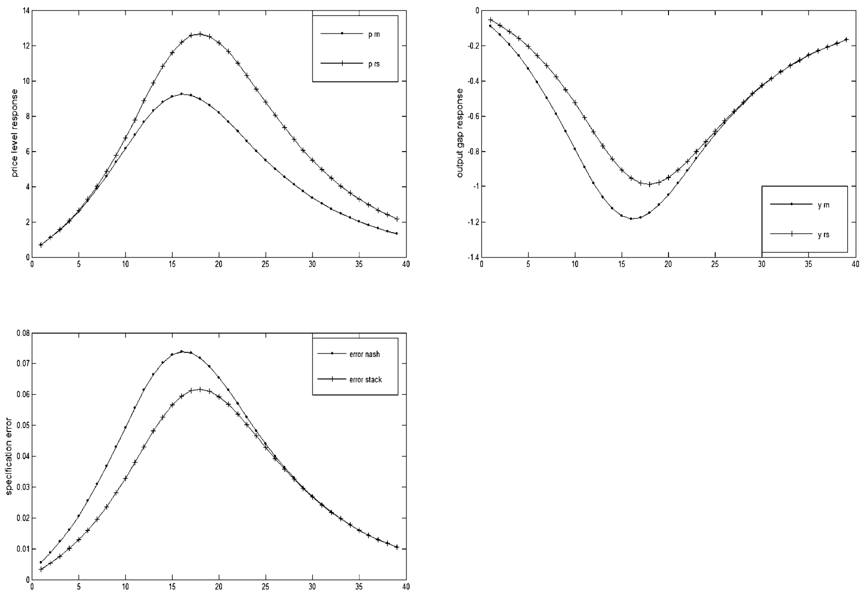

FIGURE 2. Nash vs. Stackelberg in the worst-case model.

is to maximize the target's variability. By internalizing the evil agent's reaction function, the policy maker optimally trades off the marginal benefit, in terms of a lower specification error due to output gap stabilization, and the marginal cost of a higher price variability.

These results are illustrated in Figure 2, which shows the impulse responses of the price level, the output gap, and the specification error under both the Nash and the Stackelberg timing for a given preference for robustness ($\theta = 100$). In the first quarters after the shock, when deviations from targets count for more in the loss function, under the Stackelberg timing there is a consistent gain in terms of output gap stabilization. This, in its turn, causes a lower specification error, whereas the price-level pattern is quite similar to the price-level response under the Nash timing. After some periods, the price level exhibits higher variability, but output gap deviations are still smaller than under the Nash timing. Hence, the gain in terms of target stabilization in the first periods after the shock more than offsets the higher price variability in the following periods.

How can the central bank reach this result in the first periods after the shock? The reason lies in the dynamic properties of the SIPC. Coibion (2006) has emphasized two features that play a key role in determining the inertial behavior of the inflation process in the sticky information model, the frequency of information updating and the degree of real rigidities. Both features produce small price adjustments in the first periods after the shock because few firms know the shock and their optimal price is largely unaffected. A high degree of real rigidity means in fact a low elasticity of the firm's optimal price to expected deviation in the output gap or, in other words, "firms care relatively more about the overall price level . . . than about . . . the aggregate demand" [see Coibion (2006), p. 4]. In such a situation, a central bank that foresees the evil agent's behavior will put more weight on output gap stabilization in order to immediately dampen the specification error without provoking a sudden increase in price-level variability.[15]

In order to focus on the consequences for the economy of a central bank policy action designed to be robust against model misspecification when the actual misspecification is zero, we now solve the approximating model under the Stackelberg timing.

The robust instrument rule in MA(∞) representation is

$$m_t^{rs} = \sum_{j=1}^{\infty} \eta_j^{rs} \epsilon_{t-j}$$

with

$$\eta_j^{rs} = \phi_j^{rs} + \varphi_j^{rs} = \frac{1 - \left(\frac{a^2\theta - 1}{a^2\theta}\right) ab}{\left(\frac{a^2\theta - 1}{a^2\theta}\right)^2 a^2 b + \frac{1-\Lambda^j}{\Lambda^j}} \rho_j \tag{39}$$

and thus $\eta_j^{rs} \gtrless 0$ if $b \lessgtr \frac{1}{a}(\frac{a^2\theta-1}{a^2\theta})^{-1}$. By substituting the coefficients (33), (34), and (39) into the price-level equation without specification errors, we obtain

$$\phi_j^{as} = \frac{a + \frac{1-\Lambda^j}{\Lambda^j} - \left(\frac{a^2\theta-1}{a^2\theta}\right) a^2 b \left(1 - \frac{a^2\theta-1}{a^2\theta}\right)}{\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)\left[\left(\frac{a^2\theta-1}{a^2\theta}\right)^2 a^2 b + \frac{1-\Lambda^j}{\Lambda^j}\right]} \rho_j$$

and

$$\varphi_j^{as} = -\frac{\left(\frac{a^2\theta-1}{a^2\theta}\right) ab \left[\frac{1-\Lambda^j}{\Lambda^j} + \left(\frac{a^2\theta-1}{a^2\theta}\right) a\right]}{\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)\left[\left(\frac{a^2\theta-1}{a^2\theta}\right)^2 a^2 b + \frac{1-\Lambda^j}{\Lambda^j}\right]},$$

where the superscript "*as*" denotes the approximating model solution under the Stackelberg timing. By comparing the coefficients of the rational expectations model and of the approximating model, after some algebra, we obtain the following conditions:

$$\phi_j^{as} - \phi_j^* > 0 \quad \text{if } 0 < b < b_j^{as},$$

$$\phi_j^{as} - \phi_j^* < 0 \quad \text{if } b > b_j^{as},$$

and

$$\left|\varphi_j^{as}\right| - |\varphi_j^*| > 0 \quad \text{if } b > b_j^{as},$$

$$\left|\varphi_j^{as}\right| - |\varphi_j^*| < 0 \quad \text{if } 0 < b < b_j^{as},$$

where

$$b_j^{as} = \frac{1}{a}\left[1 + \left(\frac{a^2\theta - 1}{a^2\theta}\right)^{-1}\left(1 + \frac{1}{a}\frac{1-\Lambda^j}{\Lambda^j}\right)\right].$$

Because $1 + (\frac{a^2\theta-1}{a^2\theta})^{-1}(1 + \frac{1}{a}\frac{1-\Lambda^j}{\Lambda^j}) > 1$, when the central bank acts as a Stackelberg leader, the range of values of the degree of conservatism that allow the emergence of a higher (lower) price level (output gap) variability is greater than that under the Nash timing (i.e., $b < a^{-1}$). The reason is that under the Stackelberg timing the policy maker implements a trade-off between the targets that stabilizes the output gap more than under the Nash timing, because this policy behavior minimizes the specification error. Hence, the consequences for the economy are given by the interaction of two forces: On one hand, the antiattenuation result in the use of the nominal policy instrument induces the central bank to react to the cost shocks in the same direction as in the rational expectations model but with greater intensity (see the previous section); on the other hand, and differently from the Nash timing, the central bank now seeks to implement a target trade-off that depends on its preference for robustness and it always stabilizes the output gap more than under the Nash timing. We name this result the *trade-off effect*. To show the consequences of this interaction, we can divide the central bank's degree of conservatism into four regions:

1. $0 < b \leqslant 1/a$. When $b < 1/a$ we know that in the rational expectations model the central bank reacts to a cost-push shock by increasing the nominal money supply [equation (13)]. Hence, under the Stackelberg timing in the approximating model, both the antiattenuation result and the desired targets trade-off act in the direction of increasing the policy instrument response, making it possible to reach a greater stabilization of the output gap; that is, $\phi_j^{as} > \phi_j^*$, $|\varphi_j^{as}| < |\varphi_j^*|$, and $\eta_j^{rs} > \eta_j^* > 0$ (Figure 3a). When $b = 1/a$, the antiattenuation result does not operate, because in the rational expectations model the central bank does not react to the cost shock, but the desired trade-off induces the central bank to react to the cost shock by increasing the nominal money supply and thus we obtain again $\phi_j^{as} > \phi_j^*$, $|\varphi_j^{as}| < |\varphi_j^*|$, and $\eta_j^{rs} > \eta_j^*$.
2. $\frac{1}{a} < b \leqslant \frac{1}{a}(\frac{a^2\theta-1}{a^2\theta})^{-1}$. In this region the antiattenuation result and the trade-off effect act in opposite directions. The first effect decreases the nominal policy instrument (as in the rational expectation model, we have $\eta_j^* < 0$), whereas the trade-off effect leads to the opposite reaction [equation (35)]. The latter effect dominates over the former, leading to higher output stabilization, i.e., $\phi_j^{as} > \phi_j^*$, $|\varphi_j^{as}| < |\varphi_j^*|$, and $\eta_j^{rs} \geqslant 0 > \eta_j^*$. When $b = \frac{1}{a}(\frac{a^2\theta-1}{a^2\theta})^{-1}$ we get $\eta_j^{rs} = 0$, meaning that the central bank does not react to the cost shock because the two forces (for what concerns the instrument reaction) are in exact balance. Anyway, given that $\eta_j^* < 0$, we still have $0 = \eta_j^{rs} > \eta_j^*$, and thus $\phi_j^{as} > \phi_j^*$ and $|\varphi_j^{as}| < |\varphi_j^*|$ (Figure 3a).
3. $\frac{1}{a}(\frac{a^2\theta-1}{a^2\theta})^{-1} < b < b_j^{as}$. The antiattenuation result and the trade-off effect act in opposite directions. The latter continues to dominate over the former, that is, $\phi_j^{as} > \phi_j^*$ and $|\varphi_j^{as}| < |\varphi_j^*|$, but in contrast to the previous case, the antiattenuation result is great enough to induce a contraction in the policy instrument, i.e., $\eta_j^{rs} < 0$. In any

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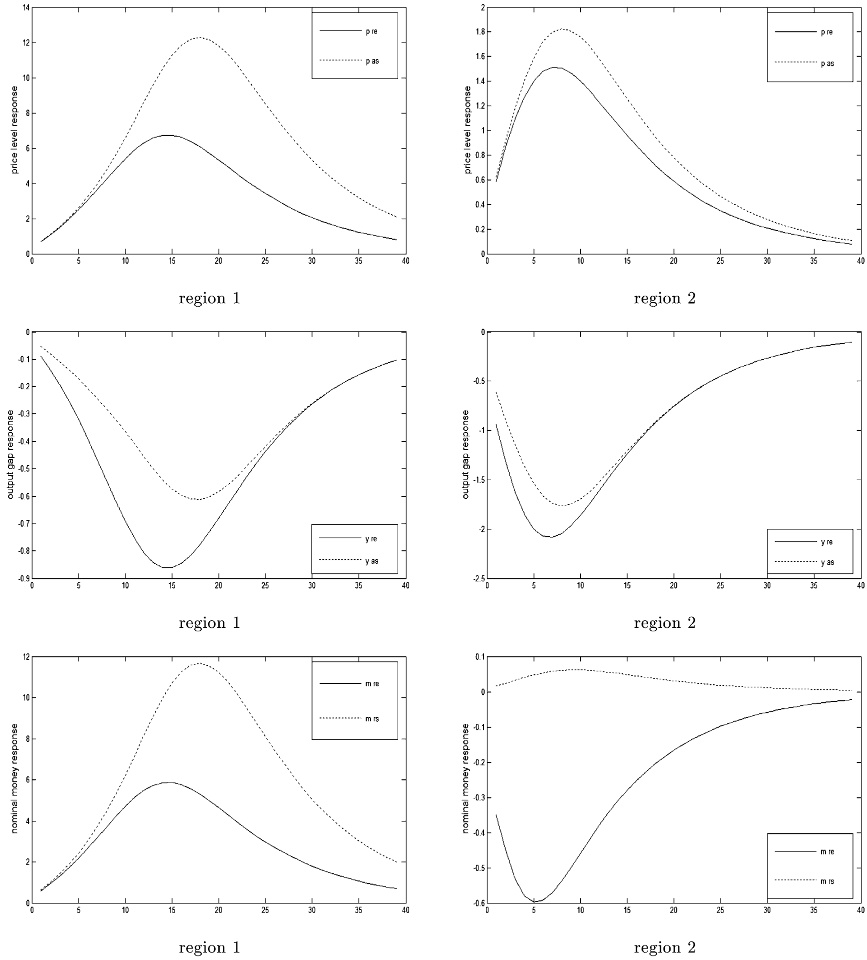

(a)

FIGURE 3. Approximating model under Stackelberg timing.

case, the nominal money contraction is smaller than under the rational expectation model; that is, $|\eta_j^{rs}| < |\eta_j^*|$ (Figure 3b).

4. $b > b_j^{as}$. Opposite to regions 2 and 3, now the antiattenuation result dominates over the trade-off effect. Hence the policy instrument reaction ($\eta_j^{rs} < 0$) is greater than that under the rational expectations model, leading to a higher price-level stabilization; that is, $\phi_j^{as} < \phi_j^*$, $|\varphi_j^{as}| > |\varphi_j^*|$, and $|\eta_j^{rs}| > |\eta_j^*|$. When $b = b_j^{as}$, the two models produce the same results, i.e., $\phi_j^{as} = \phi_j^*$ and $|\varphi_j^{as}| = |\varphi_j^*|$, meaning that the consequences for the economy (and not for the policy instrument, as happens when $b = \frac{1}{a}(\frac{a^2\theta - 1}{a^2\theta})^{-1}$) of the interaction of the two forces are exactly countervailed; that is, $\eta_j^{rs} = \eta_j^* < 0$. It is worth noticing that b_j^{as} is not fixed once and for all, but it depends on $\frac{1-\Lambda^j}{\Lambda^j}$, which is a combination of probabilities that decreases over time. It is hence possible that

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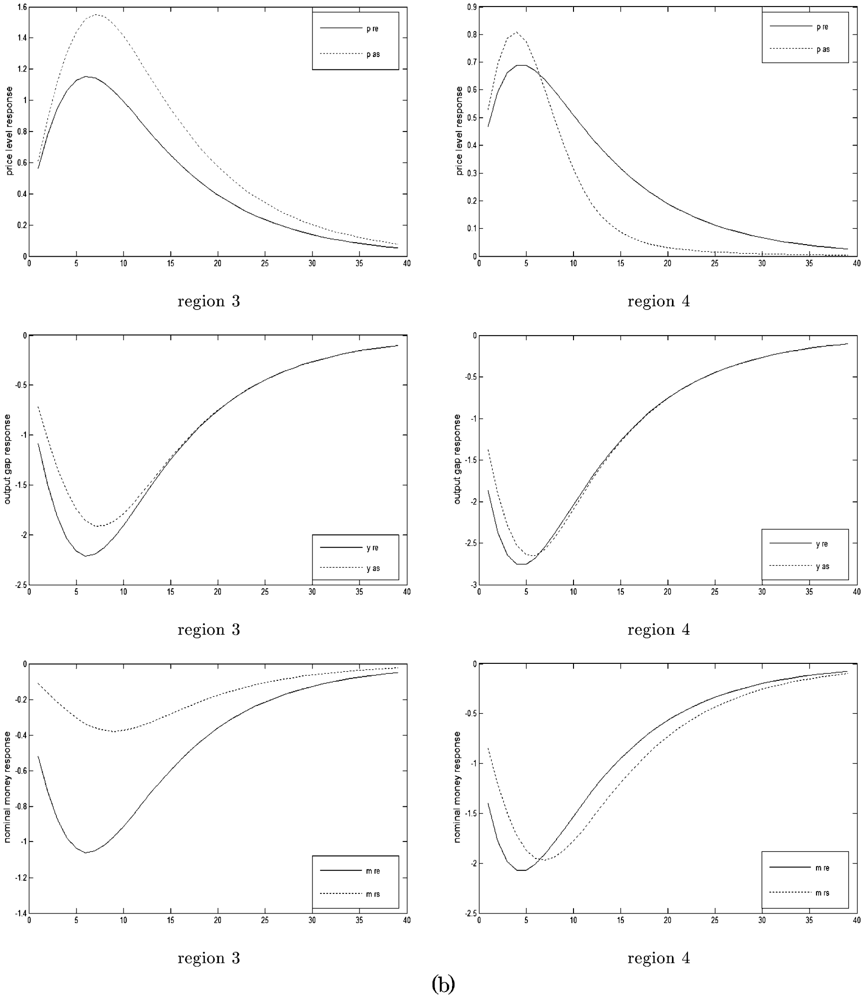

FIGURE 3. Continued.

a given degree of conservatism falls in the third region in the first quarters after the shock and in the fourth later on. In such a situation, the trade-off effect dominates over the antiattenuation result as long as $b < b_j^{as}$ and the opposite is true when $b > b_j^{as}$.[16] This is the case shown in Figure 3b.

By comparing the Stackelberg robust coefficients under the approximating model with the ones obtained under the Nash timing, it is easy to check that $\phi_j^{as} > \phi_j^{an}$ and $|\varphi_j^{as}| < |\varphi_j^{an}|$. In both schemes we have the same antiattenuation result. Its sign depends on the desired trade-off in the rational expectations model, that is, $ab \gtrless 1$. When the central bank has the first mover advantage, it stabilizes

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the output gap more than it does under the Nash timing and this force acts always in the same direction, leading to $\eta_j^{rs} > \eta_j^{rn}$.

4. WELFARE ANALYSIS

This section analyzes the rationale for delegating monetary policy to an independent central banker. Since Rogoff's (1985) contribution, it has been widely acknowledged that in the presence of an inflation bias (produced by the policy maker's desire to push output above its natural level), the monetary authorities should be more conservative than society in order to maximize a microfounded social welfare function. The sticky price New Keynesian models reach the same conclusion, but the rationale is different. Even though in this kind of model (usually) there is no inflation bias, the forward-looking nature of inflation and the presence of a time inconsistency problem induce a "stabilization bias" that justifies the appointment of a conservative central banker [see, among others, Clarida et al. (1999)]. In BMR, there is neither an inflation bias, because the central bank does not have an overly ambitious output target, nor a stabilization bias, because the lag in the policy transmission mechanism and the backward-looking nature of the SIPC assure that the discretionary and commitment solutions coincide. In this context there is no scope for appointing a Rogoff-conservative central banker: the social preferences have to be translated into the objective function of the central banker. We now wish to determine whether and how the model misspecification affects this result or, to put it differently, whether model uncertainty justifies conservatism. Clearly, if this were the case, the reason for appointing a central banker with different preferences could not rely upon the traditional reasons, but upon the strategic interaction between the central bank and the evil agent.

Welfare analysis in the robust control literature is in its infancy. The existing contributions on this topic[17] consider either a social planner with the same preference for robustness as the policy maker, in which case the social planner chooses the central bank's degree of conservatism taking into account the worst-case model solution, or a social planner with no concern for robustness. In the latter case, the social planner chooses a degree of central bank conservatism different from the social one in order to offset the distortions introduced into the economy by the robust behavior of both the private agents and the central bank.[18] We shall consider here both possibilities.

A microfounded loss function can be written in the form

$$L_s = \left[\sum_{j=1}^{\infty} \varphi_j^2 + b^s \sum_{i=1}^{\infty} \left(\frac{1}{\Lambda^{i-1}} - \frac{1}{\Lambda^i} \right) \sum_{j=1}^{i-1} \phi_j^2 \right] \sigma_\epsilon^2, \tag{40}$$

where b^s is the weight society assigns to relative price variability.

A social planner concerned with robustness minimizes equation (40) with respect to b, subject to the worst case model solution. Under the Nash timing, we substitute equations (22) and (23) into (40) and, after some algebra, obtain the

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following relation linking the socially desired trade-off (b^S) to the optimal one (b^w) (see Appendix C):

$$b^w = \left(1 - \frac{1}{a^2\theta}\right) b^S. \quad (\mathbf{41})$$

According to equation (41), the minimization of the social loss makes it necessary to appoint a central banker who is more populist than society; the optimal degree of conservatism decreases when the preference for robustness increases. Of course, under the rational expectations model ($\theta \to \infty$), we have $b^w = b^S$.

How can we interpret this unconventional result? We know that (i) the specification error is negatively related to the output gap; and (ii) when the central bank can exploit the first mover advantage (Stackelberg timing), it chooses to stabilize the output gap more than it would under the Nash timing. This result is due to the central bank's desire to trade off more inflation variability with smaller output gap deviations, thus damping the specification error. The same rationale is at the basis of our delegation result. By substituting b^w into the robust policy rule under the Nash timing (12), we obtain the robust policy rule under the Stackelberg timing (38). This shows that, in order to minimize the specification error, the social planner concerned with robustness does not try to offset the robust behavior of the central banker, but it assigns her that relative weight b^w (greater than that of society) which guarantees that she acts as if she was the Stackelberg leader vis-à-vis the evil agent. In contrast, under the Stackelberg timing there are no information advantages that the social planner can exploit and that are not considered by the central bank. The preference of the central bank must hence be the same as that of society, as we can check by substituting the coefficients (33) and (34) into the social loss function and minimizing with respect to b.

If the social planner is not a robust decision maker, it will minimize equation (40) subject to the approximating model solution, with the aim of undoing the distortions introduced into the economy by the robust behavior of both the private agents and the central bank. Unfortunately, a closed form solution to this problem does not exist, but we can provide a clear intuition of the central planner's behavior in the Nash timing case (confirmed by the FOC of this problem, derived in Appendix D): a central banker who is more conservative than society is appointed when $b^s < a^{-1}$. In this case it is optimal from a social point of view (the rational expectations model solution) to react to an inflationary shock by injecting money. Due to the antiattenuation principle in the use of the policy instrument, an uncertainty-averse central banker with the same degree of conservatism as society would, however, overreact by injecting more money, hence producing an output gap stabilization greater than that under the rational expectation model [see equations (29), (30), and (31) and the related discussion]. The nonrobust social planner, who dislikes this result, offsets the antiattenuation principle by appointing a more conservative central banker. Similar reasoning can be followed to explain why, if $b^s > a^{-1}$, the social planner appoints a central banker who is less conservative than society.[19]

5. CONCLUSIONS

This paper has confirmed most of the results obtained in sticky price models, including the fragility of the robust policy prescription: as in all the other existing models, the policies we have obtained are robust in the face of a *given* model misspecification, but need not be robust if this is differently specified. However, our analysis also elucidates the economic mechanisms at play in a sticky information economy under strong uncertainty and provides some new results, which can be summarized as follows.

1. If the central bank and the evil agent act simultaneously, in the worst-case model, target variability increases with the preference for robustness, whereas the desired trade-off between targets is the same as in the rational expectations model: Walsh's equivalence result holds. In line with the standard interpretation, this result stems from the central bank's overestimation of the inflationary consequences of cost shocks due to its fear of misspecification. This, in turn, causes a greater output contraction. When the central bank wants to be robust against specification errors but the actual misspecification is zero (the approximating model), the robust instrument rule over-reacts to the cost shocks in the following way. When in the rational expectations model the policy maker responds to the cost shock by injecting money (the desired trade-off implies that an increase in the price level is counterbalanced by a less-then-proportional contraction in output), in the approximating model she overreacts by injecting more money, thus leading to greater output stabilization. By contrast, when in the rational expectations model the price targeting regime requires to respond to the cost shock by decreasing money (the desired trade-off implies that an increase in the price level is counterbalanced by a more-then-proportional contraction in output), in the approximating model the robust instrument rule induces the central bank to overreact and to decrease the money supply even more, thus producing a greater price-level stabilization.
2. If the central bank has the first-mover advantage (Stackelberg timing), the optimal trade-off between targets is affected by the preference for robustness because the central bank internalizes the evil agent's behavior. To minimize the specification error, the central banker, who can now exploit the trade-offs between the specification error and the target variables, seeks to stabilize the output gap at the cost of higher price variability (the trade-off effect). The consequences for the economy in the approximating model solution depend on the interaction between the trade-off effect, which always acts in the same direction, and the antiattenuation effect, which acts as in the Nash timing. Given this interaction, there exists a range of degrees of conservatism for which price level stabilization is initially lower with respect to the rational expectations model, whereas the opposite is true after some periods. The explanation of the trade-off effect relies on the dynamic properties of the sticky information model, whereas the possibility for the anti-attenuation effect to produce either tighter or looser policies is due to the price targeting regime adopted by the central bank.
3. If the social planner shares the same concern for robustness as the central banker and the Nash timing is employed, a central banker who is less conservative than society should be appointed. The optimal degree of conservatism decreases when the preference for robustness increases. This findings rely neither on the "inflation

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bias" nor on the "stabilization bias" motivation. Knowing the relation between the output gap and the specification error, a robust welfare-maximizer social planner assigns to the central banker a weight on price-level stabilization that is lower than that of society in order to induce her to act as if she was a Stackelberg leader. When there are no information advantages the social planner can exploit (Stackelberg timing), the policy maker's preference must instead be the same as that of society.

4. If the social planner is not a robust decision maker, it tries to offset the distortions introduced by the robust behavior of both the private agents and the central banker. Hence, under the Nash timing, when the overreaction of the robust instrument rule leads to a greater output (price) stabilization than in the RE model, the planner should appoint a central banker more (less) conservative than society. In the former (latter) case the optimal degree of conservatism increases (decreases) with the central bank preference for robustness.

NOTES

1. See, e.g., Keen (2007), Kiley (2007), Klenow and Willis (2007), and Korenok (2008).

2. For a complete derivation of the SIPC see, among others, Trabandt (2007) and Khan and Zhu (2006).

3. The reason that, contrary to BMR's original contribution, we here focus only on cost-push shocks and disregard demand or productivity shocks is discussed at the end of the next section.

4. BMR (2005) also consider a control error shock, but we omit it for simplicity.

5. See BMR (2005, Appendix) for the analytical derivation of the welfare function.

6. BMR consider a cost-push shock stemming from random variation in taxes that causes variations in farmers' markups and we can interpret the shock u_t in the same way. For a discussion of this kind of shocks see, e.g., Clarida et al. (2002) and Woodford (2003).

7. In the context of sticky price models, Blanchard and Galì (2007) defined this characteristic as the "divine coincidence."

8. In Appendix B we show that the robust policy under productivity shocks coincides with the optimal one.

9. See Giordani and Soderlind (2004) and Hansen and Sargent (2004).

10. Due to the lag in the transmission mechanisms, the time 0 impact of a unit shock is independent of policy and it is still given by equation (10).

11. See, e.g., Giordani and Soderlind (2004) and Leitemo and Söderström (2008b) for further discussions.

12. Equivalently, we cannot say whether the implementation of the optimal trade-off (which is not affected by the preference for robustness) is indeed reached through a more or less aggressive response to shocks.

13. As a baseline for the whole paper, we assume a calibration that is commonly used in the sticky information literature: $a = 0.16$ for the sensitivity of price to expected output gap; $(1 - \omega) = 0.25$ for the sticky information parameter; $b = 0.8$ for the weight assigned to the relative price variability in the central bank's objective function; $\rho = 0.9$ is the coefficient of the AR(1) process for the cost-push shock; $\theta = 100$ is the preference for robustness.

14. See Hansen and Sargent (2003).

15. In the working paper version of this contribution [Giuli (2006)] we show that the same conclusion is reached by employing the parametric approach proposed by Giannoni (2002). In this case, the central bank has multiple priors about the distribution of the parameter a, which represents the sensitivity of the firm's optimal price to the expected deviation of output. Once again, the policy maker adopts a min–max strategy that can be represented by a "mind game" played by the central

bank and the fictitious "evil agent," where the latter now controls the numerical values of a over a given domain. The evil agent's best response is obtained by studying the sign of the first derivative of the equilibrium value of the central-bank objective function with respect to a. In such a situation the central banker is more cautious in stabilizing the price level because the gain that is obtained in terms of price stabilization of a unit output loss is lower than in the rational expectations model, as the evil agent's best response is to set the lower bound of a. The robust trade-off is given by $\varphi_j^r = -a_{\text{low}} b \phi_j^r$, which is similar to equation (35). A simple intuition for this attenuation result is the following: a positive cost push shock leads the central bank to offset it through an output contraction. When robustness concerns on the slope of the Phillips curve are introduced, the central bank is aware that part of its effort will be frustrated by the evil agent's best response (i.e., $a = a_{\text{low}}$). This reduction in the effectiveness of the policy action induces the policy maker to place more weight on the target variable for which the evil agent's action is less harmful, that is, output stabilization.

16. Due to the stickiness in the diffusion of information, the peak of the policy instrument response occurs after some periods. Hence, in the first quarters after the shock the small (absolute) value of the policy instrument in the rational expectation model generates a mild overreaction (region 3), which increases over time (region 4).

17. See Kilponen (2003), Gaspar et al. (2005), and Tillmann (2009).

18. Both cases rely upon the hypothesis that the policy maker and the private sector share the same reference model and the same degree of preference for robustness. Otherwise there would be a discrenpancy between the policy maker's and the private sector's expectations about future inflation and output. As stressed by Walsh, "An interesting area for future research would be to allow private agents and the policy maker to have different worst-case models" [see Walsh (2004)]. Even though this would enrich the analysis of the optimal institutional design in the presence of strong uncertainty, in this preliminary welfare analysis in a sticky information model, we follow the existing literature on this topic.

19. An intuition of the social planner's behavior under the Stackelberg timing is prevented by the need to take into account the distorsions produced in this case by both the antiattenuation principle and the trade-off effect, which may act in opposite directions (see the discussion at the end of the previous section). The space available suggests disgregarding the numerical simulations that must be performed to overcome this difficulty.

REFERENCES

Ball, Laurence (1994) Credible disinflation with staggered price setting. *American Economic Review* 84, 282–289.

Ball, Laurence, N. Gregory Mankiw, and Ricardo Reis (2005) Monetary policy for inattentive economies. *Journal of Monetary Economics* 52(4), 703–725.

Blanchard, Olivier and Jordi Galì (2007) Real wage rigidities and the New Keynesian model. *Journal of Money, Credit, and Banking* 39(1), 35–65.

Blanchard, Olivier and Nobuhiro Kiyotaki (1987) Monopolistic competition and the effects of aggregate demand. *American Econnomic Review* 77(4), 647–666.

Brainard, William (1967) Uncertainty and the effectiveness of policy. *American Economic Review* 57, 411–425.

Calvo, Guillermo A. (1983) Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12, 383–398.

Clarida, Richard, Jordi Gali, and Mark Gertler (1999) The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature* 37(4), 1661–1707.

Clarida, Richard, Jordi Gali, and Mark Gertler (2002) A simple framework for international monetary policy analysis. *Journal of Monetary Economics* 49(5), 879–904.

Coibion, Olivier (2006) Inflation inertia in sticky information models. *Contributions to Macroeconomics* 6(1), 1–29.

Fuhrer, Jeffrey and George Moore (1995) Inflation persistence. *Quarterly Journal of Economics* 110(1), 127–160.

Gaspar, Vitor, Otmar Issing, Oreste Tristani, and David Vestin (2005) *Imperfect Knowledge and Monetary Policy*. Cambridge, UK: Cambridge University Press.

Giannoni, Marc P. (2002) Does model uncertainty justify caution? Robust optimal monetary policy in a forward looking model. *Macroeconomic Dynamics* 6(1), 111–114.

Giordani, Paolo and Paul Soderlind (2004) Solution of macromodels with Hansen–Sargent robust policies: Some extensions. *Journal of Economic Dynamics and Control* 28(12), 2367–2397.

Giuli, Francesco (2006) Robust Control in a Sticky Information Economy. Working paper 98, Sapienza University of Rome, Department of Public Economics.

Hall, Robert E. (1984) Monetary Strategy with an Elastic Price Standard. In Price Stability and Public Policy: A Symposium Sponsored by the Federal Reserve Bank of Kansas City.

Hansen, Lars P. and Thomas J. Sargent (2002) Robust Control and Model Uncertainty in Macroeconomics. Mimeo, Stanford University.

Hansen, Lars P. and Thomas J. Sargent (2003) Robust control of forward looking models. *Journal of Monetary Economics* 50(3), 581–604.

Hansen, Lars P. and Thomas J. Sargent (2004) *Misspecification in Recursive Macroeconomic Theory*. Chicago: University of Chicago.

Kara, Hakan (2002) Robust Targeting Rules for Monetary Policy. Working paper 60, Central Bank of Turkey.

Keen, Benjamin D. (2007) Sticky price and sticky information price setting models: What is the difference? *Economic Inquiry* 45(4), 770–786.

Khan, Hashmat and Zhenhua Zhu (2006) Estimates of the sticky-information Phillips curve for the United States. *Journal of Money, Credit, and Banking* 38(1), 195–208.

Kiley, Michael T. (2007) A quantitative comparison of sticky-price and sticky-information models of price setting. *Journal of Money, Credit, and Banking* 39(1), 101–125.

Kilponen, Juha (2003) A Positive Theory of Robust Monetary Policy and Robust Control. Discussion paper 18, Bank of Finland.

Klenow, Peter J. and Jonathan L. Willis (2007) Sticky information and sticky prices. *Journal of Monetary Economics* 54(1), 79–99.

Korenok, Oleg (2008) Empirical comparison of sticky price and sticky information models. *Journal of Macroeconomics* 30(3), 906–927.

Leitemo, Kai and Ulf Söderström (2008a) Robust monetary policy in a small open economy. *Journal of Economic Dynamics and Control* 32(10), 3218–3252.

Leitemo, Kai and Ulf Söderström (2008b) Robust monetary policy in the New-Keynesian framework. *Macroeconomic Dynamics* 12(1), 126–135.

Mankiw, N. Gregory (2001) The inexorable and mysterious tradeoff between inflation and unemployment. *Economic Journal* 111(471), 45–61.

Mankiw, N. Gregory and Ricardo Reis (2002) Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve. *Quarterly Journal of Economics* 117(4), 1295–1328.

Rogoff, Kenneth S. (1985) The optimal degree of commitment to an intermediate monetary target. *Quarterly Journal of Economics* 100(4), 1169–1189.

Tillmann, Peter (2009) Does model uncertainty justify conservatism? Robustness and the delegation of monetary policy. *The B.E. Journal of Macroeconomics* 9(1), article 26.

Trabandt, Mathias (2007). Sticky Information vs. Sticky Prices: A Horse Race in a DSGE Framework. Kiel Working Papers 1369, Kiel Institute for the World Economy.

Walsh, Carl E. (2004) Robust optimal interest rule and robust control: An equivalence result. *Journal of Money, Credit, and Banking* 36, 1105–1114.

Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

APPENDIX A

A.1. THE WORST-CASE MODEL SOLUTION UNDER THE NASH TIMING

The control problem of the central bank is as follows:

$$\min_{\phi_j} \max_{\gamma_j} L_r = \left[\sum_{j=1}^{\infty}\varphi_j^2 + b\sum_{i=1}^{\infty}\left(\frac{1}{\Lambda^{i-1}} - \frac{1}{\Lambda^i}\right)\sum_{j=0}^{i-1}\phi_j^2\right]\sigma_\epsilon^2 - \sum_{j=1}^{\infty}\theta\gamma_j^2\sigma_\epsilon^2,$$

$$\text{s.t. } \varphi_j = \frac{1}{a}\left(\frac{1-\Lambda^j}{\Lambda^j}\phi_j - \rho_j - \gamma_j\right).$$

The FOC with respect to ϕ_j is

$$\frac{\partial L_r}{\partial \phi_j} = \varphi_1\frac{\partial\varphi_1}{\partial\phi_1}\sigma_\epsilon^2 + \varphi_2\frac{\partial\varphi_2}{\phi_2}\sigma_\epsilon^2 + \cdots + \left(\frac{1}{\Lambda^0} - \frac{1}{\Lambda^1}\right)\phi_0 + b\sigma_\epsilon^2\left(\frac{1}{\Lambda^1} - \frac{1}{\Lambda^2}\right)(\phi_0+\phi_1) + \cdots$$
$$+ b\sigma_\epsilon^2\left(\frac{1}{\Lambda^2} - \frac{1}{\Lambda^3}\right)(\phi_0+\phi_1+\phi_2) + b\sigma_\epsilon^2\left(\frac{1}{\Lambda^3} - \frac{1}{\Lambda^4}\right)(\phi_0+\phi_1+\phi_2+\phi_3) + \cdots = 0.$$

Because $1/\Lambda^\infty = 1$, after some manipulations we get

$$\varphi_1\frac{\partial\varphi_1}{\partial\phi_1}\sigma_\epsilon^2 + \varphi_2\frac{\partial\varphi_2}{\partial\phi_2}\sigma_\epsilon^2 + \cdots + b\sigma_\epsilon^2\frac{1-\Lambda^0}{\Lambda^0}\phi_0 + b^s\sigma_\epsilon^2\frac{1-\Lambda}{\Lambda}\phi_1 + b^s\sigma_\epsilon^2\frac{1-\Lambda^2}{\Lambda^2}\phi_2 + \cdots = 0.$$

By using $\varphi_j = \frac{1}{a}(\frac{1-\Lambda^j}{\Lambda^j}\phi_j - \rho_j - \gamma_j)$ and $\frac{\partial\varphi_j}{\partial\phi_j} = \frac{1}{a}\frac{1-\Lambda^j}{\Lambda^j}$, after some manipulations we obtain the jth coefficients (17) and (18) in the main text.

The second-order condition is always positive:

$$\frac{\partial^2 L_r}{\partial\partial\phi_j} = \left(\frac{1}{a}\frac{1-\Lambda^1}{\Lambda^1}\right)^2\sigma_\epsilon^2 + \left(\frac{1}{a}\frac{1-\Lambda^2}{\Lambda^2}\right)^2\sigma_\epsilon^2 + \cdots + b\sigma_\epsilon^2\frac{1-\Lambda^0}{\Lambda^0}$$
$$+ b\sigma_\epsilon^2\frac{1-\Lambda^1}{\Lambda^1} + b\sigma_\epsilon^2\frac{1-\Lambda^2}{\Lambda^2} + \cdots.$$

This ensures that the central bank is minimizing the loss function.

The FOC with respect to γ_j is

$$\varphi_1\frac{\partial\varphi_1}{\partial\gamma_1}\sigma_\epsilon^2 + \varphi_2\frac{\partial\varphi_2}{\gamma_2}\sigma_\epsilon^2 + \cdots - \theta\gamma_1\sigma_\epsilon^2 - \theta\gamma_2\sigma_\epsilon^2 - \cdots = 0.$$

By substituting the constraints and partial derivatives ($\partial\varphi_j/\partial\gamma_j = -1/a$) into the FOC and rearranging, we get equation (20) in the main text.

The second-order condition ($\frac{\partial^2 L_r}{\partial\partial\gamma_j} = \frac{1}{a^2}\sigma_\epsilon^2 + \frac{1}{a^2}\sigma_\epsilon^2 + \cdots - \theta\sigma_\epsilon^2 - \theta\sigma_\epsilon^2 - \cdots$) is negative provided that $\theta > a^{-2}$. This ensures that the evil agent is maximizing the loss function. This is in line with Hansen and Sargent's (2002) proof stating the existence of a cut-off value for θ above which the expected value of the loss function is finite and the second-order conditions are satisfied.

Combining the FOCs, we get equations (22) and (23) in the main text.

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APPENDIX B

B.1. THE CASE OF PRODUCTIVITY AND DEMAND SHOCK

Productivity shocks affect the natural level of output (y_t^n) that can be expressed in a moving average representation as $y_t^n = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$, where ψ_j are unknown coefficients representing the effect of the technology shock on the natural level of output. Hence the output gap ($x_t = y_t - y_t^n$) equilibrium path is now given by $\hat{x}_t = \sum_{j=0}^{\infty} (\varphi_j - \psi_j)\epsilon_{t-j}$. Because of the lag in the policy transmission mechanism, the policy process is given by $E_{t-1} p_t = \sum_{j=1}^{\infty} \phi_j \epsilon_{t-j}$ and $E_{t-1} x_t = \sum_{j=1}^{\infty} (\varphi_j - \psi_j)\epsilon_{t-j}$.

Given the equation for the price level $[p_t = (1-\omega) \sum_{k=0}^{\infty} \omega^k E_{t-k}(p_t + a x_t)]$, by substituting the MA(∞) representations we obtain

$$\sum_{j=1}^{\infty} \phi_j \epsilon_{t-j} = \sum_{j=1}^{\infty} \Lambda^j [\phi_j + a(\varphi_j - \psi_j)]\epsilon_{t-j},$$

where $\Lambda^j = (1-\omega) \sum_{k=0}^{j} \omega^k$.

To deal with the fear of misspecification, the specification error is disguised by the technology shock; otherwise it would be detected immediately. Hence the evil agent now controls $z_t = \sum_{j=1}^{\infty} \gamma_j \epsilon_{t-j}$, which is added to the technology shock in the central bank's reference model:

$$\sum_{j=1}^{\infty} \phi_j \epsilon_{t-j} = \sum_{j=1}^{\infty} \Lambda^j \{\phi_j + a[\varphi_j - (\psi_j + \gamma_j)]\}\epsilon_{t-j}.$$

Because this expression must hold for all possible realizations of ϵ_{t-j}, it follows that

$$\varphi_j - \psi_j = \frac{1}{a} \frac{1-\Lambda^j}{\Lambda^j} \phi_j + \gamma_j. \tag{A.1}$$

The control problem of the central bank is now given by

$$\min_{\phi_j} \max_{\gamma_j} L_r = \left[\sum_{j=1}^{\infty} (\varphi_j - \psi_j)^2 + b \sum_{i=1}^{\infty} \left(\frac{1}{\Lambda^{i-1}} - \frac{1}{\Lambda^i} \right) \sum_{j=0}^{i-1} \phi_j^2 \right] \sigma_\epsilon^2 - \sum_{j=1}^{\infty} \theta \gamma_j^2 \sigma_\epsilon^2$$
$$\text{s.t. } (A.1).$$

The FOC with respect to ϕ_j is

$$\frac{\partial L_r}{\partial \phi_j} = (\varphi_1 - \psi_1) \frac{\partial (\varphi_1 - \psi_1)}{\partial \phi_1} \sigma_\epsilon^2 + (\varphi_2 - \psi_2) \frac{\partial (\varphi_2 - \psi_2)}{\phi_2} \sigma_\epsilon^2$$
$$+ \cdots + b \left(\frac{1}{\Lambda^0} - \frac{1}{\Lambda^1} \right) \phi_0 + b\sigma_\epsilon^2 \left(\frac{1}{\Lambda^1} - \frac{1}{\Lambda^2} \right) [\phi_0 + \phi_1] + \cdots.$$

Because $1/\Lambda^{\infty} = 1$, by substituting $\varphi_j - \psi_j = \frac{1}{a}\frac{1-\Lambda^j}{\Lambda^j}\phi_j + \gamma_j$ and $\frac{\partial(\varphi_j - \psi_j)}{\partial \phi_j} = \frac{1}{a}\frac{1-\Lambda^j}{\Lambda^j}$, after some manipulations we get

$$\frac{\partial L_r}{\partial \phi_j} = \left(\frac{1}{a}\frac{1-\Lambda^1}{\Lambda^1}\phi_1 + \gamma_1\right)\frac{1}{a}\frac{1-\Lambda^1}{\Lambda^1}\sigma_\epsilon^2 + \left(\frac{1}{a}\frac{1-\Lambda^2}{\Lambda^2}\phi_2 + \gamma_2\right)\frac{1}{a}\frac{1-\Lambda^2}{\Lambda^2}\sigma_\epsilon^2$$
$$+ \cdots + b\sigma_\epsilon^2\frac{1-\Lambda^0}{\Lambda^0}\phi_0 + b^s\sigma_\epsilon^2\frac{1-\Lambda}{\Lambda}\phi_1 + b^s\sigma_\epsilon^2\frac{1-\Lambda^2}{\Lambda^2}\phi_2 + \cdots = 0.$$

After some algebra we obtain the jth coefficients:

$$\phi_j = -\frac{a}{\frac{1-\Lambda^j}{\Lambda^j} + a^2 b}\gamma_j,$$

$$\varphi_j = \psi_j + \left(\frac{\frac{1-\Lambda^j}{\Lambda^j}(1-a) + a^2 b}{\frac{1-\Lambda^j}{\Lambda^j} + a^2 b}\right)\gamma_j.$$

The FOC with respect to γ_j is

$$\frac{\partial L_r}{\partial \gamma_j} = (\varphi_1 - \psi_1)\frac{\partial(\varphi_1 - \psi_1)}{\partial \gamma_1}\sigma_\epsilon^2 + (\varphi_2 - \psi_2)\frac{\partial(\varphi_2 - \psi_2)}{\partial \gamma_2}\sigma_\epsilon^2$$
$$+ \cdots - \theta\gamma_1\sigma_\epsilon^2 - \theta\gamma_2\sigma_\epsilon^2 - \cdots = 0.$$

By substituting the constraints and partial derivatives $\frac{\partial \varphi_1}{\partial \gamma_1} = 1$ into the FOC and rearranging, we get $\gamma_j = \frac{1}{a(\theta-1)}\frac{1-\Lambda^j}{\Lambda^j}\phi_j$. Combining the FOCs yields

$$\left[1 + \frac{a}{\frac{1-\Lambda^j}{\Lambda^j} + a^2 b}\frac{1}{a(\theta-1)}\frac{1-\Lambda^j}{\Lambda^j}\right]\phi_j = 0 \Rightarrow \phi_j = 0,$$

and thus $\varphi_j - \psi_j = 0$ and $\gamma_j = 0$.

Because the technology shock does not pose a relevant policy trade-off, the central bank does not fear this kind of shock (i.e., $\gamma_j = 0$). Both the optimal [see BMR (2005)] and the robust policy result in a complete stabilization of the objective variables the first period after the shock. In period $t = 0$ the price and the output coefficients are still given by equation (10). As usual, the same result occurs in the case of a demand shock, because the policymaker is able to counteract any specification error in the aggregate demand equation by an appropriate adjustment of the policy instrument. Therefore, in the absence of cost channels in the monetary policy transmission mechanism, the central bank does not fear this kind of errors.

APPENDIX C

In this Appendix we demonstrate the delegation result under the Nash timing when the social planner is concerned with robustness (Section 4).

By substituting the robust coefficients [(20) and (21)] into the society's loss function, we obtain

$$L_s^{rn} = \sum_{j=1}^{\infty}\left(\frac{ab}{a^2b+\frac{1-\Lambda^j}{\Lambda^j}-\frac{b}{\theta}}\rho_j\right)^2\sigma_\epsilon^2 + b^s\sum_{i=1}^{\infty}\left(\frac{1}{\Lambda^{i-1}}-\frac{1}{\Lambda^i}\right)$$
$$\times\sum_{j=0}^{i-1}\left(\frac{1}{a^2b+\frac{1-\Lambda^j}{\Lambda^j}-\frac{b}{\theta}}\rho_j\right)^2\sigma_\epsilon^2.$$

The optimal degree of central bank conservatism is obtained by minimizing the above expression with respect to parameter b:

$$\frac{\partial L_s^{rn}}{\partial b} = \frac{\partial\left(\varphi_1^{rn}\right)^2}{\partial b}\sigma_\epsilon^2 + \frac{\partial\left(\varphi_2^{rn}\right)^2}{\partial b}\sigma_\epsilon^2 + \cdots + b^s\left(\frac{1}{\Lambda^0}-\frac{1}{\Lambda^1}\right)\frac{\partial\left(\phi_0^{rn}\right)^2}{\partial b}$$
$$+ b^s\sigma_\epsilon^2\left(\frac{1}{\Lambda^1}-\frac{1}{\Lambda^2}\right)\left[\frac{\partial\left(\phi_0^{rn}\right)^2}{\partial b}+\frac{\partial\left(\phi_1^{rn}\right)^2}{\partial b}\right] + b^s\sigma_\epsilon^2\left(\frac{1}{\Lambda^2}-\frac{1}{\Lambda^3}\right)$$
$$\times\left[\frac{\partial\left(\phi_0^{rn}\right)^2}{\partial b}+\frac{\partial\left(\phi_1^{rn}\right)^2}{\partial b}+\frac{\partial\left(\phi_2^{rn}\right)^2}{\partial b}\right]+\cdots = 0.$$

After some manipulations, we get

$$\frac{\partial\left(\varphi_1^{rn}\right)^2}{\partial b}\sigma_\epsilon^2 + \frac{\partial\left(\varphi_2^{rn}\right)^2}{\partial b}\sigma_\epsilon^2 + \cdots + b^s\sigma_\epsilon^2\left(\frac{1}{\Lambda^0}-\frac{1}{\Lambda^1}+\frac{1}{\Lambda^1}-\frac{1}{\Lambda^2}+\frac{1}{\Lambda^2}-\frac{1}{\Lambda^3}\right.$$
$$\left.+\cdots+\frac{1}{\Lambda^\infty}\right) + b^s\sigma_\epsilon^2\left(\frac{1}{\Lambda^1}-\frac{1}{\Lambda^2}+\frac{1}{\Lambda^2}-\frac{1}{\Lambda^3}+\cdots+\frac{1}{\Lambda^\infty}\right)\frac{\partial\left(\phi_1^{rn}\right)^2}{\partial b}+\cdots = 0.$$

Because $1/\Lambda^\infty = 1$, in general notation we can write

$$\sum_{j=1}^{\infty}\frac{\partial\left(\varphi_j^{rn}\right)^2}{\partial b}\sigma_\epsilon^2 + \sigma_\epsilon^2 b^s\sum_{j=0}^{\infty}\frac{1-\Lambda^j}{\Lambda^j}\frac{\partial\left(\phi_j^{rn}\right)^2}{\partial b} = 0.$$

Substituting the partial derivatives $\frac{\partial(\varphi_j^{rn})^2}{\partial b}$ and $\frac{\partial(\phi_j^{rn})^2}{\partial b}$ and rearranging, we obtain

$$\sigma_\epsilon^2\sum_{j=1}^{\infty}\left[\frac{a^2b\left(a^2b+\frac{1-\Lambda^j}{\Lambda^j}-\frac{b}{\theta}\right)-\left(a^2-\frac{1}{\theta}\right)a^2b^2}{\left(a^2b+\frac{1-\Lambda^j}{\Lambda^j}-\frac{b}{\theta}\right)^3}\right]\rho_j^2$$
$$-\sigma_\epsilon^2 b^s\sum_{j=1}^{\infty}\left(\frac{1}{\Lambda^j}-1\right)\left[\frac{a^2-\frac{1}{\theta}}{\left(a^2b+\frac{1-\Lambda^j}{\Lambda^j}-\frac{b}{\theta}\right)^3}\right]\rho_j^2 = 0.$$

Hence, $b^s/b = a^2/(a^2-\frac{1}{\theta})$; that is, we obtain the delegation result in the main test.

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APPENDIX D

In this Appendix we demonstrate the delegation result under the Nash timing when the social planner is not concerned with robustness (Section 4):

$$L_s^{an} = \frac{1}{2}\left[\sum_{j=1}^{\infty}\left(\varphi_j^{an}\right)^2 + b^s \sum_{i=1}^{\infty}\left(\frac{1}{\Lambda^{i-1}} - \frac{1}{\Lambda^i}\right)\sum_{j=0}^{i-1}\left(\phi_j^{an}\right)^2\right]\sigma_\epsilon^2.$$

By differentiating with respect to b and manipulating as in Appendix B, we get

$$\sum_{j=1}^{\infty}\frac{\partial\left(\varphi_j^{an}\right)^2}{\partial b}\sigma_\epsilon^2 + \sigma_\epsilon^2 b^s \sum_{j=0}^{\infty}\frac{1-\Lambda^j}{\Lambda^j}\frac{\partial\left(\phi_j^{an}\right)^2}{\partial b} = 0.$$

Substituting the partial derivatives and manipulating, we get

$$\sum_{j=1}^{\infty}\left[\left(\frac{\left(a^2 b + ab\frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)\left(a^2 + a\frac{1-\Lambda^j}{\Lambda^j} - \frac{1}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j}\right)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)^3\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)^2}\rho_j^2\right)\right]\sigma_\epsilon^2$$

$$+ b^s \sum_{j=1}^{\infty}\left[\frac{-\left(a + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)a\left(a^2 + a\frac{1-\Lambda^j}{\Lambda^j} - \frac{1}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j}\right)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)^3\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)^2}\rho_j^2\right]\sigma_\epsilon^2 = 0.$$

Thus

$$\frac{b}{b^s} = \frac{\sum_{j=1}^{\infty}\left[\frac{\left(a^2 + a\frac{1-\Lambda^j}{\Lambda^j} - \frac{1}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j}\right)\left(a^2 + a\frac{1-\Lambda^j}{\Lambda^j} - \frac{ab}{\theta}\right)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)^3\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)^2}\rho_j^2\right]}{\sum_{j=1}^{\infty}\left[\left(\frac{\left(a^2 + a\frac{1-\Lambda^j}{\Lambda^j} - \frac{1}{\theta}\right)\left(\frac{1-\Lambda^j}{\Lambda^j}\right)\left(a^2 + a\frac{1-\Lambda^j}{\Lambda^j} - \frac{1}{\theta}\right)}{\left(a^2 b + \frac{1-\Lambda^j}{\Lambda^j} - \frac{b}{\theta}\right)^3\left(\frac{1-\Lambda^j}{\Lambda^j} + a\right)^2}\rho_j^2\right)\right]};$$

the numerator and the denominator differ only for the last terms in brackets, that is, ab/θ and $1/\theta$, and hence $b^s = b$ only if $b = a^{-1}$, $b < b^s$ only if $b > a^{-1}$, and $b > b^s$ only if $b < a^{-1}$.