SOURCE-NOISE IN RADIO SYNTHESIS IMAGES OF POLARISED SOURCES

A.A. DESHPANDE
Australia Telescope National Facility,
P.O. Box 76, Epping, NSW 2121, Australia
&
Raman Research Institute
Bangalore 560 080, India.

Abstract. We discuss the distribution of Source- and receiver-noise in radio synthesis images of polarised sources. Analytical expressions are presented for the rms noise at any location in the polarisation images. We compare these results with those for the images of unpolarised sources and discuss the usefulness of deconvolution of snap-shot images in reducing the source-noise from the off-source regions in the images.

1. Introduction

The noise at the output of a radio telescope contains, apart from the contribution from the receiver-noise, the noise contributed by the source itself (the so called 'source-noise'). While the receiver-noise from different antennas of a synthesis array are mutually independent (uncorrelated), the fluctuations due to the source are partially/fully correlated between different antennas depending on the structure of the source and the baselines. Hence, the receiver-noise (on its own) contributes uniformly to the fluctuations across a synthesis image, where as the source-noise across the image has a high degree of correlation with the synthesis image itself.

A number of theoretical papers have discussed the aspect of source-noise in detail (Anantharamaiah *et al.* 1989, Kulkarni 1989, Crane & Napier 1989, Vivekanand & Kulkarni (1991), and Anantharamaiah *et al.* (1991)). The last two papers also present expressions, derived using independent approaches, for distributions of the noise in synthesis images. However, the discussion so far is applicable either to total-intensity images of unpolarised sources or images made using only single-polarisation feeds. In this paper, we will attempt to extend this discussion to describe the noise distribution in a general case of synthesis mapping (complete in all polarisation parameters) of a partially polarised source.

2. Synthesis Images and polarisation parameters

We consider a synthesis array with N antennas each of which is equipped with orthogonal, linear feeds (X & Y). For simplicity, we consider snap-shot images S_I, S_Q, S_U and S_V corresponding to the Stokes parameters in a 'phased-array' mode (i.e. the all the zero-spatial-frequency measurements are also used). The complex output voltages corresponding to the two polarisation channels (X,Y) of a phased-array phased in a direction (l_o,m_o) can be written as

$$\Psi^X(l_o,m_o) = \frac{1}{N}\sum_{i=1}^{N}(\Psi_i^X + \eta_i^X) \tag{1a}$$

J.G. Robertson and W.J. Tango (eds.), Very High Angular Resolution Imaging, 95–99.

$$\Psi^Y(l_o, m_o) = \frac{1}{N}\sum_{i=1}^{N}(\Psi_i^Y + \eta_i^Y) \tag{1b}$$

where η_i^X & η_i^Y are the complex voltages due to the receiver-noise (from the X & Y channels respectively) of i^{th} antenna and Ψ_i^X & Ψ_i^Y are the corresponding complex voltages due to the source.

In general, for a partially polarised source, we can write

$$\Psi_i^X = \sum_{l,m}[\alpha\Psi_{op}(l,m) + \Psi_{ou}^X(l,m)]e^{-j\phi} \tag{2a}$$

$$\Psi_i^Y = \sum_{l,m}[\sqrt{(1-\alpha^2)}\Psi_{op}(l,m) + \Psi_{ou}^Y(l,m)]e^{-j\phi} \tag{2b}$$

where $\phi = [x_i(l - l_o) + y_i(m - m_o)]$ for a location (x_i, y_i) of the i^{th} antenna , $\alpha\Psi_{op}(l,m)$ & $\sqrt{(1-\alpha^2)}\Psi_{op}(l,m)e^{j\delta}$ are the complex voltages received at (x,y) = (0,0) from the polarised component, and $\Psi_{ou}^X(l,m)$ & $\Psi_{ou}^Y(l,m)$ are the corresponding volatges due to the unpolarised component.

The images resulting from the correlations of $\Psi^X(l_o, m_o)$ & $\Psi^Y(l_o, m_o)$ are related to the Stokes parameter images (S_I, S_Q, S_U & S_V) as

$$S_{XX} =<\Psi^X\Psi^{X*}>= S_I + S_Q cos2\chi + S_U sin2\chi \tag{3a}$$

$$S_{YY} =<\Psi^Y\Psi^{Y*}>= S_I - S_Q cos2\chi - S_U sin2\chi \tag{3b}$$

$$S_{XY} =<\Psi^X\Psi^{Y*}>= -S_Q sin2\chi - S_U cos2\chi + jS_V \tag{3c}$$

$$S_{YX} =<\Psi^{X*}\Psi^Y>= -S_Q sin2\chi - S_U cos2\chi - jS_V \tag{3d}$$

χ is the angle beween the X feed and the meridian to the North celestial pole and is measured in the direction celestial North towards East.

The image S_I in 'phased-array' mode is then

$$S_I(l_o, m_o) = S_p(l_o, m_o) + S_u(l_o, m_o) + S_r/N \tag{4}$$

where S_r is the equivalent unpolarised flux/beam from the receiver-noise from a single antenna. $S_p(l_o, m_o)$ and $S_u(l_o, m_o)$ are the contributions in the dirty image S_I from the polarised and the unpolarised source distributions respectively.

3. NOISE DISTRIBUTION IN SYNTHESIS IMAGES

3.1. Case 1: Completely unpolarised source

We will first recall the expression for the distribution of the noise in the synthesis image (this, unless mentioned otherwise, will be assumed to include also the total-power outputs from each of the N antennas and that it is a snap-shot image) of an unpolarised source (Anantharamaiah *et al.* (1991), and Vivekanand & Kulkarni (1991)). The rms noise (σ) in the at any location (l_o, m_o) is given by

$$\sigma(l_o, m_o) = \frac{1}{\sqrt{B\tau}}(S_u(l_o, m_o) + S_r/N) \tag{5}$$

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where B is the pre-detection bandwidth in Hz and τ is the post-detection integration time in seconds.

Note that the magnitude of the source-noise follows the structure in the image, while the receiver-noise contributes uniformly across the image. (Although it is possible to treat the receiver-noise as an additional uniform-background flux as far such noise analysis is concerned, we will treat it separate from the source distribution for clarity.)

Anantharamaiah *et at.* (1991) make an important point that with the inclusion of total-power outputs from each of the antennas in the image, the noise in the image is a simple sum of the noise from each of the sources of noise and hence deconvolution of the snap-shot "total-power" image can completely remove the source-noise from the off-source region.

The image S_I (which is an average of the images S_{XX} and S_{YY}) will have the noise fluctuations that are $\sqrt{2}$ times smaller than those in the images averaged as the signals in the X channels are not correlated with those in the Y channels. The other images (Q,U,V) althought will have no non-zero contributions on the average, the distributiion of the noise in them is expacted to be same as that in S_I. The actual noise, however, will be uncorrelated between any two these 4 images. It important to note that among the considered images of a completely unpolarised source, the images of only S_I (and S_{XX} & S_{YY}) have the possibility of completely removing the source-noise from the off-source regions if snap-shot images are deconvolved.

The above arguments about the polarisation images are unaltered when measurements with dual-circular feeds are considered.

3.2. Case 2: A fully linearly polarised point source

Now let us consider a field containing only a fully linearly polarised point source which is observed using pairs of orthogonal linear-polarisation feeds (X,Y) in a synthesis array. For simplicity, let us assume that the reciever-noise (S_r) is absent, and that the position angle of source polarisation matches with the angle of the X (dipole) feed. The total-power image (S_{XX}) of the X channel will have fluctuations whose rms value at the source position is $S/\sqrt{B\tau}$ where S is the source intensity.

Absence of any signal in the Y channels will result in the outputs of the other measured images, namely S_{YY}, S_{XY} and S_{YX} being identically zero.

If the feed pair is rotated by an angle ϕ so that the signals in the X & Y channels are non-zero, then the fluctuations in S_{XY} and S_{YX} at the position of the source are given by $(S/2).Sin(2.\phi)/\sqrt{B\tau}$ while those in S_{XX} and S_{YY} will be $S.Cos^2(\phi)/\sqrt{B\tau}$ and $S.Sin^2(\phi)/\sqrt{B\tau}$ respectively. It is important to note that these fluctuations in the above products will be fully correlated and therefore the image in Stokes parameter V will be zero at all times.

Thus, in the case of polarisation measurements, we need to note this correlation in fluctuations resulting from the polarisation characteristics of the source explicitly in addition to the correlations due to the source structure.

3.3. A GENERAL CASE OF A PARTIALLY POLARISED SOURCE

Using the approach similar to that described in Anantharamaiah *et al.* (1991), expressions are derived to describe the noise distribution in the polarisation images of a partially polarised source. The involved algebra is lengthy but simple. The results for the different images have very similar forms and hence can be described as

$$\sigma^2_{S_I}(l_o, m_o) = \frac{1}{2B_T}.[S_p^2(l_o, m_o) + (S_p(l_o, m_o) + S_u(l_o, m_o) + S_r/N)^2] \quad (6)$$

and

$$\sigma^2_{S_k}(l_o, m_o) = \frac{1}{2B_T}.[2.S_k^2(l_o, m_o) - S_p^2(l_o, m_o) + (S_p(l_o, m_o) + S_u(l_o, m_o) + S_r/N)^2] \quad (7)$$

where k can be Q, U or V.

It is worth noting that the variance of the noise in these polarisation images does not depend on the angle χ. However, it should be remembered that the vector (Q,U) has already been corrected (de-rotated) for the feed orientations.

The similarity in the form of the above expressions for the Q,U and V images is not surprising considering the fact that they are interchangable with the rotation of the complex-coordinate system. Simple arguments show that these results should be valid even when circular feeds are considered.

4. Discussion and Conclusions

From the eq.s (6,7) it is clear that, in general, the rms noise fluctations are not directly proportional to the average image in all the four cases. Let us consider the eq. (6) for variance of the noise in the image S_I in particular. The two images S_{XX} and S_{YY} that are combined to obtain the image S_I have the noise contribution from the polarised component which is completely correlated between the two images, hence making the contribution from that term higher by a factor of 2. Alternatively, it can be shown that the effective number of independent measurements available for the estimation of the polarised component are a factor of two less than those for other components. In the case of the unpolarised components, two orthogonal polarisation channels provide twice the number of independent estimates than any one polarisation channel. For a given arbitrary elliptical polarisation of the polarised part, it is in principle possible to think of two orthogonal polarisation states, one of which matches with the polarisation characteristic of the polarised component. In such a case, the non-matching (orthogonal) channel fails to provide an estimate of the polarised component.

As the rms noise in these images does not vary linearly with the average values of the components, deconvolution of the snap-shot images will not be able to completely remove the source-noise from the off-source regions. However, as we already noted above, deconvolution of the snap-shot images of S_{XX} and S_{YY} can remove the source-noise from the off-source regions. This suggests that the source-noise in off-source regions of the images S_I & S_Q can be removed if the snap-shot images

S_{XX} & S_{YY} are deconvolved before they are combined. It is not immediatly obvious whether equivalent argument would be true in the case of other images. But similar argument in the case of two circular feeds can be applied for the deconvolvability of the source-noise in images S_I & S_V. By considering the two linear feeds rotated by $\pi/4$, similar is possible for the image S_U also. Thus by identifying the appropriate component images and deconvolving them before combining to produce the images in Stokes parameters is should be possible to remove the source-noise from the off-source regions in the derived images.

These conclusions will not apply strictly if only the correlations between different antennas are used to produce the image. In the case of unpolarised sources, as N becomes large we expect the difference between noise in the images made using the 'phased-array' mode and correlation array to reduce (Anantharamaiah *et al.* (1991). Although similar behaviour may be expected for partially polarised sources, further investigation of this case for correlation arrays would be still useful.

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Discussion:

Perley:

For this effect to be important, one must have a strongly polarized source whose flux dominates the system temperature. Are there such sources, and will you be able to measure this effect?

Deshpande:

Yes. Fortunately, there are such sources at metre and longer wavelengths. To see the effect I talked about, the source structure need not be very compact. Basically, what we want is the uncertainty in estimation of the extra noise due to the polarised component to be a few times this extra noise itself. If we observe with a synthesis array (N elements) and in spectral mode (M spectral channels to enable more reliable estimation of the rms noise in the images) then a polarised flux per synthesised beam of a few times $S_D/\sqrt{M}$ may be sufficient, where S_D the mean contribution in the dirty image ('phased-array' mode) at that location. Observations of the Vela pulsar in gated mode would be a very interesting case for seeing this effect.