

# What's wrong with Tooley's argument from evil?

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**Abstract:** Michael Tooley has recently advanced a novel formulation of the problem of evil. The argument primarily intends to address sceptical theist responses to the problem of evil by giving a theoretical argument that prima facie evils are probably ultima facie evils. He thus argues from a single prima facie evil to the conclusion that God probably does not exist, before extending his argument to encompass many prima facie evils using Carnapian inductive logic. In this article I respond to Tooley's two arguments. I improve the first by clearing up some ambiguities, before noting a variety of problems. In particular, the fatal problem is that, contra Tooley, the occurrence of some event is in fact evidence that such an event is not in fact impermissible for God to allow. I then challenge Tooley's extension using Carnapian induction, offering a parody which suggests that Carnapian induction of this variety leads to manifestly absurd results.

## Introduction

Formulating the argument against God's existence from the evil present in the world has historically been a controversial and difficult task. Philosophers have, in the past, held that there is a logical inconsistency between the propositions that God exists and that evil exists. More recently, they have tried to formulate the argument evidentially, utilizing various principles of inductive logic to argue that the particular evils observed in the world constitute significant, perhaps overwhelming, evidence against theism. Perhaps the most sophisticated example to date is Tooley's (Plantinga & Tooley (2008)) use of Carnapian inductive logic to argue, first, that the probability of theism is below 0.5, and second, that the probability of theism is at most  $1/(n + 1)$ , where  $n$  is the number of states of affairs in the actual world each of which, judged by known rightmaking and wrongmaking properties (hereafter, RMPs and WMPs), it would be morally wrong to allow.<sup>1</sup>

Here, I offer a detailed critique of Tooley's argument. I offer some improvements to the argument, as well as some reasons to think that the argument is more

fundamentally mistaken. In particular, Tooley's first argument that the probability of any given prima facie evil being ultima facie evil is greater than 0.5 fails to consider all the relevant evidence. I will argue that the fact that an event has happened can, at least in principle, be good evidence that the permission of that event by God is morally permissible. More importantly, the extent to which the occurrence of a prima facie evil state of affairs confirms or disconfirms the thesis that the permission of that state of affairs would be morally permissible by God depends crucially on the probability of theism given all the other evidence we have. Unless Tooley has some other argument to demonstrate that theism is sufficiently improbable given the rest of our evidence, then his argument does not succeed in establishing the overall improbability of theism.

### **The structure of the argument**

#### *The argument*

Tooley's argument can be formulated as follows:<sup>2</sup>

Let 'There are no RMPs that are known to be counterbalancing' stand for the more complex phrase, 'There are no RMPs that we know of such that we are justified in believing both that the action in question has those RMPs, and that those properties are sufficiently serious to counterbalance any relevant, known, WMP (or WMPs)'.<sup>3</sup>

- (1) Any action<sup>4</sup> of choosing not to prevent the Lisbon earthquake has a known WMP such that there are no RMPs that are known to be counterbalancing.<sup>5</sup>

Note that this premise does *not* say that the Lisbon earthquake has a known WMP such that there are no RMPs whatsoever which counterbalance it. That would simply be to state the conclusion, and would be unacceptable to any theist. Rather, he is saying that we do not *know* of any RMPs such that the Lisbon earthquake has those RMPs and such that we know those RMPs outweigh the WMP. It allows for the possibility that there is an *unknown* RMP which counterbalances the WMP. But his next premise argues that for an RMP of any given weight, it is just as a priori likely that the action also has a further WMP of equal weight. It follows that it is more likely than not that a prima facie wrong action is also ultima facie wrong.

This might be best seen algebraically. Suppose a value of 0 represents moral indifference. The known WMP gives the action a value of, say,  $-5$  units. The unknown RMPs and WMPs also have an aggregative value, says Tooley. Call it  $k$ . Then the ultima facie value of the action is  $k-5$ . But, he says, it is just as likely that  $k$  is a given positive value as its negative equivalent. For example, it is just as likely that  $k = 10$  as that  $k = -10$ . If  $|k| = 5$  or larger, it is equally likely that the overall value of the action is positive as that it is negative. Good news for the theist. But if  $|k|$  is less than 5, the overall value of the action will be negative

regardless of whether  $k$  is positive or negative. And so it trivially follows that it is more probable that the overall value of the action is negative.

Hence Tooley's next premise:

- (2) For any action whatever, the logical probability that that action is morally wrong, all things considered, given that the action has a WMP that we know of, and that there are no RMPs that are known to be counterbalancing, is greater than one half.<sup>6</sup>

Since, according to (1), an action of permitting the Lisbon earthquake meets the criteria presented in (2), the probability of permitting the Lisbon earthquake being morally wrong, given (1), is greater than one half. Thus, in Tooley's words:

- (3) The logical probability that an action of choosing not to prevent the Lisbon earthquake is morally wrong, all things considered, given that choosing not to prevent the Lisbon earthquake has a WMP that we know of, and that there are no RMPs that are known to be counterbalancing, is greater than one half.

Since the moral wrongness of any action of permitting the Lisbon earthquake and the occurrence of the Lisbon earthquake jointly entail atheism, it follows from (3) that:

- (4) The logical probability that God does not exist, given that choosing not to prevent the Lisbon earthquake has a WMP that we know of, and that there are no RMPs that are known to be counterbalancing, is greater than one half.

I will respond first by offering some preliminary thoughts challenging premise (1) or, at least, challenging the certainty of it. I will then highlight some ambiguities in Tooley's formulation, doing my best to clarify and improve the argument. I will then discuss at length the most fatal problem with the argument, discussing a crucial ambiguity in premise (2). Finally, I will examine and refute Tooley's extension of the argument from a single case of evil to many cases.

### *Objections in the literature*

It will be helpful to give a brief survey of other responses first. The primary responses to Tooley in the literature are due to Plantinga (in Plantinga & Tooley (2008)) and Otte (2013).

Both authors place considerable weight on what we might call 'probabilistic scepticism': the idea that we could reasonably avoid Tooley's argument by withholding judgement about certain probabilistic claims. Hence, Plantinga, an externalist, offers a number of concerns about the concept of logical probability and its relationship to epistemic justification. Both offer scepticism about Tooley's symmetry principle underlying (2) (essentially, that unknown RMPs and WMPs are

of equal weight and distribution), saying that it is reasonable simply to withhold judgement with respect to it. Otte goes further and suggests that the privation theory of evil is a positive reason to reject the symmetry principle.

There is a problem for optimists about probabilistic or Bayesian epistemology. Many theists (myself included) think that we are committed to at least rough probabilistic judgements, especially judgements of equiprobability where different possibilities seem roughly symmetrical, as in this case (that is, there seems no more reason to think that unknown RMPs are more abundant or weighty than unknown WMPs, and so we should think that their distribution is equiprobable). Even for those who aren't as committed to probabilism, there are problems with these responses: for example, while it might be that the privation theory of evil gives reason to think that the distribution of unknown RMPs and WMPs is unequal, if it offers no more reason to think that it is unknown RMPs, specifically, which are more abundant or weighty, it seems just as likely that it is unknown WMPs which are more abundant or weighty. And then Tooley's argument is unaffected. We do not need to know with much certainty that unknown RMPs and WMPs are equally weighty for Tooley's argument to work. All we need is no more reason to think that unknown RMPs are more abundant or weighty than unknown WMPs. And it seems clear that, aside from independent evidence or justification for God's existence, we have no *obvious* such reason. Indeed, it is hard to see how withholding judgement in the way Plantinga and Otte suggest would not make ordinary moral judgements impossible. For if we cannot even commit ourselves to my suggested symmetry thesis concerning unknown RMPs and WMPs, on what basis can we say that *any* prima facie wrong actions by anyone at all are ultima facie wrong? For if we are reasonable in withholding all judgement about the likely distribution of unknown RMPs and WMPs, it is difficult to see how we could have any confidence that a prima facie wrong action committed by an ordinary human being does not have an unknown RMP vindicating it. The most Plantinga and Otte could say is that although we would be *justified* in withholding judgement, we might also be justified in making such a judgement in this case. But surely we are not only justified in thinking that Hitler's actions were morally wrong; we are obliged to think so. And there is an apparent consistency problem in not applying the same reasoning to God's actions.

In any case, there will be some theists who, depending on its interpretation (which this article will clarify), find such a symmetry principle entirely convincing, as well as being swayed by probabilistic reasoning in general. Tooley's argument may therefore have considerable force for optimistic Bayesian theists. The benefit of this article is, therefore, that it grants considerable common ground to Tooley: it grants the framework of logical probability and the relevant symmetry considerations. But I show that there are still fatal technical flaws in Tooley's argument.

There are other responses Plantinga and Otte offer. Plantinga is sceptical of (1) since theists who find theodicies convincing or have alternative justification for

theism could reasonably reject it. I am inclined to agree, but this article shows exactly how a theist could reasonably reject it within an internalist framework – even those theists who don't have a compelling theodicy and who think that some states of affairs in the world are such that, had they not occurred, they would think it impermissible for God to permit them. Hence, my argument concedes considerable ground to Tooley and shows that his argument still fails.

### **Preliminary problems**

#### *The potential success of theodicies*

An important preliminary consideration in interpreting Tooley's argument charitably is that it is primarily an argument against sceptical theists who concede that theodicies are generally unsuccessful. In particular, there are sceptical theists who think that there is at least one state of affairs in the world such that, *had it not occurred*, we would expect God not to permit it (on the basis of its WMPs). But, say these sceptical theists, our cognitive and epistemic limitations imply that it is unreasonable to conclude from our knowledge of these states of affairs that the permission of these states of affairs is overall evil, and so that God does not exist. Tooley's argument is an attempt to show that this inference is indeed reasonable – that our knowledge regarding certain prima facie evil states of affairs is good reason to think that, probably, the permission of these states of affairs is overall evil. Since this conclusion entails atheism, it follows from the probability calculus that, probably, God does not exist.

Note, therefore, that Tooley does not give a detailed defence of premise (1) – conceded by this variety of sceptical theist – that any action of choosing not to prevent the Lisbon earthquake has a known WMP such that there are no RMPs known to be counterbalancing.<sup>7</sup> Of this premise, Tooley writes:

[This statement] makes a claim that would be challenged by philosophers who respond to the evidential argument from evil by offering a theodicy. Nevertheless, the claim seems very reasonable, given the relevant facts about the world, together with the moral knowledge that we possess. For what rightmaking properties can one point to that one has good reason to believe would be present in the case of an action of allowing the Lisbon earthquake, and that would be sufficiently serious to counterbalance the wrongmaking property of allowing more than 50,000 ordinary people to be killed? (Plantinga & Tooley (2008), 122)

This relatively cursory dismissal of theodicies will be seen as deeply unsatisfactory by many philosophers, and is a relatively conspicuous shortcoming of Tooley's argument. There are many intelligent, educated people who are sincerely persuaded that they are aware of some reason which justifies the suffering in the world. One might think that the goods of free will, forgiveness, or redemption are all such important goods that, were they to be instantiated, would outweigh the wrongmaking properties of the action of allowing the negative corollaries of such goods to obtain. It is not obvious that these goods are justifying, but Tooley has not demonstrated that they are not. Importantly, Tooley needs this

premise to be certain for his argument to work. For Tooley's intermediate conclusion is of the form  $P(X|Y) > 0.5$ , where  $X$  =<sub>df</sub> an action of choosing not to prevent the Lisbon earthquake is morally wrong, all things considered, and  $Y$  =<sub>df</sub> choosing not to prevent the Lisbon earthquake has a WMP that we know of, and that there are no RMPs that are known to be counterbalancing. But we do not know  $Y$  for certain, so we have to factor in our uncertainty into our final evaluation of  $P(X)$ . This is done by weighting  $P(X)$  by  $P(Y)$ :

$$P(X) = P(X|Y) \times P(Y) + P(X|\neg Y) \times P(\neg Y)$$

If  $P(Y) = 1$ , then  $P(X) = P(X|Y)$ , and so  $P(X) > 0.5$  (assuming the argument is otherwise correct). But suppose  $P(Y)$  is only 0.6. Then  $P(X) = P(X|Y) \times 0.6 + P(X|\neg Y) \times 0.4$ . But then knowing that  $P(X|Y) > 0.5$  does not show us that  $P(X) > 0.5$ . It shows us only that  $P(X) > 0.3$ . And this conclusion does not help Tooley much. So even if  $Y$  is more probable than not, it may not suffice for the argument in the presence of significant uncertainty about  $Y$ . So the various kinds of uncertainty about premise 1 – uncertainty about whether God's allowing the Lisbon earthquake has a WMP, or uncertainty about whether a known RMP is true of the Lisbon earthquake or whether a known RMP is justifying – can be appealed to here.

#### *Improving the premise*

There are other problems with this premise as it stands. One is that Tooley's understanding of deontological properties can easily be doubted, since it assumes that the moral value of certain properties is insensitive to context. For example, the property of involving forcibly taking someone's money is sometimes a WMP, but could be an RMP in the case of taxes or justly imposed fines.

Tooley defines RMPs and WMPs as properties such that if an action had a WMP and no RMPs, it would be wrong, and vice versa. Of course, some actions (and probably every action of relevance to the problem of evil) have a more complex mixture of properties. So, Tooley suggests, RMPs and WMPs can be weighted so that an action is wrong if the WMPs outweigh the RMPs, and vice versa. This is clear enough.

But one might easily doubt this logic of value more generally. Perhaps the value of a state of affairs is not an additive function of the values of its components. Perhaps the values of its components are determined by the overall context. Chisholm (1990) points to certain cases – pleasure at another's displeasure, displeasure at one's previous misdeed, an ugly (when considered in isolation) part of a beautiful painting – where the good component is, in the context of the whole, bad (and vice versa). Such evil wider states of affairs *defeat* rather than merely *counterbalance* the component's goodness.<sup>8</sup> We might adopt on this basis a *contextualist* view of evil, or of WMPs. It may be that whether a feature of an action is an RMP or WMP (or neither) depends entirely on the whole

picture. A theist – whether sceptical or not – might thereby deny that we know premise 1 with any certainty at all – it might be that the unknown properties of the action do not just *outweigh* the WMP but actually negate the wrongmakingness of the WMP altogether, by integrating it into a complex whole in which it no longer has weight as a WMP. A non-sceptical theist can say this is possible, perhaps even likely. A sceptical theist can say that, given our cognitive limitations, we have no way of judging whether the WMP in the Lisbon earthquake really is a WMP at all.

One way Tooley might redeem his argument here is by adopting an epistemic conception of RMPs: an RMP is just a property which increases a reasonable subject's confidence that the action was a good one. This might seem more promising. Again, however, whether a property performs that function is also context-relative, in the same way that evidence in general is relative to background knowledge. Unlike the ontic conception of rightmaking, however, the epistemic conception at least has a clear apparatus to account for changes in background knowledge. If Tooley can show that relative to the specific background knowledge we have, the property of involving the killing of 60,000 ordinary people ought to lower our credence in the proposition that allowing the Lisbon earthquake is permissible, then his argument may have some force. The difficulty here is that making the premise so sensitive to background knowledge and context would require so much argumentative work that the considerations which he thinks are doing the real work in the argument (*viz.* the distribution of unknown morally relevant properties) could potentially be overshadowed.

Perhaps this apparatus is, in fact, unnecessary. For what Tooley is fundamentally appealing to is quite clear: it is the sense that many sceptical theists would grant that, in the absence of any knowledge about the world, our theoretical notion of love would lead us to expect that an all-loving being would permit the Lisbon earthquake, on the basis that it involved the death of 60,000 people. It might be that unknown features of God's permitting the Lisbon earthquake offer a counterbalancing RMP to a genuine WMP. Or it might be that unknown features of the action convert the WMP into an RMP or into a neutral feature. It might be that unknown features of the action, if known, would prevent the ostensible WMP from increasing a reasonable subject's confidence that the action was a wrong one. But one thing that can be agreed on by some theists is that, if we did not know whether the action had occurred, and all we knew about the action was that it involved the death of 60,000 people, we would probably judge the action wrong. The probability that God would perform such an action is therefore less than 0.5. And Tooley wants to say that it is just as likely that the unknown features of such an action render the action as a whole more permissible or less permissible. It seems to me that this argument can be just as reasonably made without the controversial conception of RMPs and WMPs which he seems to endorse.<sup>9</sup> So far so good.

One final difficulty with premise (1), which makes the premise more plausible but which is obscured later in the argument – is the ambiguity in 'known to be

counterbalancing'. Tooley interprets this as the claim that there are no RMPs that we know of such that we are justified in believing both that the action in question has those RMPs, and that those properties are sufficiently serious to counterbalance any relevant, known, WMPs. As such, this premise is relatively (though not entirely) weak and easy to accept. But again, Tooley's failure to account for uncertainty makes his argument weaker than it need be. Take a disjunction of weighty RMPs. For each individual RMP, one might not be confident that it holds of the Lisbon earthquake, or one might not be confident that it is weightier than the WMP. But one might be virtually certain that the *disjunction* of RMPs is both true of the Lisbon earthquake and sufficiently weighty. In that case, premise (1) would be true, but it is hard to imagine that a compelling argument from evil could be mounted on the basis of such an example. Nevertheless, I suspect that the argument could be modified to avoid this sort of case. Tooley just needs the premise I gave above: that theoretically, we would not expect an all-loving God to permit the Lisbon earthquake, in the absence of other information.

Some of these problems derive merely from ambiguity in Tooley's argument. Others are more organic. Nevertheless, there is a reasonable way to preserve the gist of the argument while alleviating most of these problems. That said, highlighting these ambiguities may limit the audience of Tooley's argument. For, given the considerations outlined here, there might well be plenty of theists who simply wouldn't accept the more robust premise I have suggested. For those theists, Tooley may have to concede that he has failed to demonstrate the key premise, or he may offer further support for it. But there are, I suspect, plenty of theists who would accept it. Tooley should therefore restrict his argument to those theists who accept that there are some states of affairs in the world which, had they not occurred, they would think were probably impermissible by God. The view is that, considering only the known features of the action of God's permitting the Lisbon earthquake, with no knowledge of whether the earthquake occurred, we would judge the action to be probably impermissible by God.<sup>10</sup> This is the gist of Tooley's premise, and is acceptable to a number of theists. In any case, I will now argue that even given this premise, the argument still does not do the work that Tooley intends.

### **A more serious problem**

This leads us to the most fatal problem of Tooley's argument from single cases. The issue is not, in fact, with his argument from premise (2). That is fine as far as it goes. Conceive of an action that it seems it would be wrong for God to permit – perhaps, permitting every created human to suffer eternal conscious torment. Most of us would confidently agree that it is likely that this is impermissible. And if someone objected that there might be unknown RMPs outweighing the *prima facie* wrongness of this action, we would respond that there might well be, but the *likelihood* is still that the action is impermissible. So to that



extent Tooley's suggestions pertaining to the equiprobability of similarly weighted unknown RMPs and WMPs are not particularly objectionable.

The problem, however, is that Tooley does not just say that, in the absence of such an event occurring, we should judge that *prima facie* evils are *ultima facie* evils. His argument seems to require that we *must* say the same once such an event has actually occurred. And although this might seem innocuous initially, there is a compelling case that the occurrence of such an event does, in fact, change the probability that that event is permissible by God. Let me explain.

Armed with the premise that the probability that the action of God's permitting a Lisbon earthquake would probably be morally wrong *given only* the known RMPs and WMPs of permitting the Lisbon earthquake, with no knowledge of whether the earthquake occurred, is greater than 0.5, we can give a clearer formalization of the argument to highlight the more troublesome mistake. This premise can be formalized:

$$P(W|WMP) > 0.5$$

$W =_{df}$  It would be wrong for God to permit a Lisbon earthquake

$WMP =_{df}$  Our knowledge regarding the RMPs and WMPs of God's permitting a Lisbon earthquake (most notably, that it has an epistemically weighty WMP)

My contention is that even this probabilistic claim does not do the required work. To see why, we can formalize the rest of the argument.

WMP is taken as data. We know that  $P(W|WMP) > 0.5$ . And since  $W$  and  $L$  jointly entail  $\sim T$  (atheism), it can be shown from the Kolmogorov axioms that  $P(\sim T|WMP\&L) > P(W|WMP\&L)$ . So as long as  $P(W|WMP\&L)$  is also greater than 0.5 – that is, as long as knowing that the Lisbon earthquake occurred makes no difference to the probability that it would be morally permissible for God to allow it – it follows that  $P(\sim T|WMP\&L)$  is also greater than 0.5. This is just to say that the probability of atheism given WMP and  $L$  is greater than 0.5. And WMP and  $L$  appear not to be in doubt. The argument succeeds. But it makes the assumption that  $L$  does not change the probability of  $W$ . Tooley assumes, that is, that  $P(W|WMP) = P(W|WMP\&L)$ . This implicit assumption is crucial to the argument, but is in fact probably false.

While Tooley briefly acknowledges that there might be 'countervailing positive evidence in support of the existence of God', his probabilistic treatment neglects the impact that the prior probability of theism and evidence for theism can, in principle, have on  $P(W)$ . Interestingly, he also neglects the impact that the occurrence of the Lisbon earthquake can have on  $P(W)$ . Indeed, I shall show that his formalization of the argument actually seems to preclude these considerations, and so is in tension with his claim that there might be countervailing positive evidence in support of theism.

This, it turns out, is a critical flaw in Tooley's argument. To see this, consider the following analogy: suppose you are hiding in a trench in a battle. To exit the trench and run out into the open seems, on the basis of your knowledge of military tactics, fatal, and unlikely to help your side at all. It seems reasonable to conclude on this basis that, probably, your good and loyal military commander would not command you to do such a thing, since the command would have a serious WMP with no known counterbalancing RMPs. Suppose you are then commanded to exit the trench and run out into the open. According to the known RMPs and WMPs properties of this command, the command is probably an evil one. Since a good and loyal military commander could not command something evil (provided we define 'good' in this context with sufficient care), this entails that the probability that your military commander is good is less than 0.5. But this clearly ignores any positive evidence you might have for thinking that your military commander is good, and the prior probability that your military commander is good. It is perfectly conceivable that *the occurrence of the command itself* ends up modifying the probability that such a command is morally permissible. But if Tooley's argument works, it is difficult to see how any evidence for the goodness of your commander could outweigh that, in theory, the command seemed wrong to you. Even if you had amazing evidence for your commander's goodness, the fact that, on balance, this command seemed wrong to you before it was given shows that your commander simply isn't good. This is an absurd result. So Tooley's approach seems to contradict the seeming importance of positive evidence or prior probability at this juncture.<sup>11</sup>

Another counter-intuitive consequence of Tooley's method is the following: suppose, for whatever reason, that we consider theism to have an overwhelming prior probability (whether because of a high intrinsic probability or because of strong evidence), but that we are aware of just one prima facie evil state of affairs. This might be a completely trivial evil: I drop an ice cream, and I am unaware of a counterbalancing good that comes from this event. This, according to Tooley, has so much weight that, regardless of any considerations of prior probability, the probability of theism drops to below 0.5. Again, this seems implausible. Not only that, but it is in tension with his claim that countervailing positive evidence might raise the probability of theism above 0.5. On Tooley's schema, it is not clear how this is possible.

What is going on here? The issue is that Tooley is not really looking for  $P(\sim T|WMP)$ , as his prose suggests, but rather for  $P(\sim T|WMP\&L)$ . That is, he wants to know the probability of atheism given WMP *and* that the Lisbon earthquake actually occurred. Starting with  $P(W|WMP) > 0.5$ , Tooley seems to think that  $P(W|WMP) = P(W|WMP\&L)$ , and so  $P(W|WMP\&L)$  is likewise greater than 0.5. That is, he assumes that L makes no difference to the probability of W. Then, since  $P(W|WMP\&L) = P(W\&L|WMP\&L)$ , it follows that  $P(W\&L|WMP\&L) > 0.5$ . And because W&L entails  $\sim T$ , it follows that  $P(\sim T|WMP\&L) > 0.5$ . This is just a formalization of Tooley's argument as described in section 2.1.

We noted earlier that it is not clear how evidence for God even *could* be accommodated in Tooley's framework: after all, in Tooley's framework, whether W is probably true surely just depends on the known WMP and the symmetry principle. And before we knew that the Lisbon earthquake occurred, we might well have agreed. Even with evidence of God's existence – perhaps very strong evidence – we might still judge that God would not have allowed the Lisbon earthquake, and so be surprised when the Lisbon earthquake occurs. That is the basic premise Tooley is appealing to. But then how does evidence for God relate to Tooley's argument? The answer is that the prior probability of theism (which will depend on the intrinsic probability of theism and evidence for or against theism) is channelled through the probability of the Lisbon earthquake occurring. This is to say that although a theist could reasonably agree that evidence for God makes no difference to W in itself (that is, unless one knows that the earthquake has occurred, a theist could think that W is probably true, and that the evidence for God makes no difference to this), the evidence for God *does* make a difference to the probability of W when one finds out that the earthquake has actually occurred – just as one can consider the military commander's command wrong in itself (and the evidence for the goodness of the commander is irrelevant until the command is given), but change one's opinion when one finds out that the command has actually been given. A consequence of this is that the occurrence of the Lisbon earthquake affects the probability of the permission of the Lisbon earthquake being morally permissible by God. Thus, it is false that  $P(W|WMP) = P(W|WMP\&L)$ . There is in fact a relatively simple mathematical proof available of this:

$$P(W|WMP\&L) = P(W|WMP\&L\&T) \times P(T|WMP\&L) + P(W|WMP\&L\&\sim T) \times P(\sim T|WMP\&L)$$

$P(W|WMP\&L\&T)$  is 0, since theism and the Lisbon earthquake's occurring entail that God's permitting the Lisbon earthquake is permissible (so W is false). Thus:

$$P(W|WMP\&L) = P(W|WMP\&L\&\sim T) \times P(\sim T|WMP\&L)$$

$P(W|WMP\&L\&\sim T)$  is the probability that an action of God's permitting the Lisbon earthquake would be wrong, given its uncounterbalanced WMP, given that it has occurred, and given that God does not exist. But Tooley seems to think that L makes no difference to the probability of W. And it is not clear that learning  $\sim T$  should increase one's confidence in W, at least on Tooley's account, since for Tooley, the probability of W is judged only by WMP. So, for Tooley,  $P(W|WMP) = P(W|WMP\&L) = P(W|WMP\&L\&\sim T)$ . But if so, then:

$$P(W|WMP\&L) = P(W|WMP\&L) \times P(\sim T|WMP\&L)$$

This is consistent only if  $P(\sim T|WMP\&L) = 1$ . But since  $P(\sim T|WMP\&L)$  is not 1 (because the Lisbon earthquake's having an uncounterbalanced WMP and its occurrence does not *entail* atheism), we have a contradiction. Thus, it cannot be the case that L makes no difference to the probability of W.

On reflection, this is entirely intuitive. For in the military case, it is intuitively clear that learning that a captain has given a command does, in fact, affect the probability that that command is good (specifically, if we have good evidence that the captain is good, it increases it).

We can get more of a grip on how L might affect the probability of W by the odds form of Bayes' Theorem. At this point, it will be helpful for ease of reading to exclude WMP from the conditional. We will take it as a given that WMP is part of the background knowledge.

$$\frac{P(W|L)}{P(\sim W|L)} = \frac{P(L|W)}{P(L|\sim W)} \times \frac{P(W)}{P(\sim W)}$$

According to this, adding L into our conditional lowers the probability of W just if the first multiplicand is less than 1. So it is worth comparing  $P(L|W)$  and  $P(L|\sim W)$ . These are the probabilities that the Lisbon earthquake would occur if it would be wrong for God to permit it and if it would not be wrong for God to permit it. Intuitively, these should be different: if it would be wrong, then there are fewer ways the earthquake could occur – it could not occur in any world where God exists, for example. But if it is not wrong, there are more ways the earthquake could occur: in a variety of worlds where God exists. This is borne out by looking more closely at the probabilities in question:

$$P(L|W) = P(L|W\&T) \times P(T|W) + P(L|W\&\sim T) \times P(\sim T|W)$$

$$P(L|\sim W) = P(L|\sim W\&T) \times P(T|\sim W) + P(L|\sim W\&\sim T) \times P(\sim T|\sim W)$$

Turning this into intuitive judgements is not easy. But it can be seen how  $P(L|\sim W)$  could exceed  $P(L|W)$ : for the first conjunct in the first line is 0, whereas the corresponding first conjunct in the second line is more than 0. How much more than 0? That depends on the values of its constituents. But one of these constituents is  $P(T|\sim W)$ , which is just the prior probability of theism given that permitting the Lisbon earthquake is morally acceptable. This could well be high if there is good evidence for theism. So  $P(L|\sim W)$  may be considerably higher than  $P(L|W)$  if theism is independently probable. The only reason  $P(L|\sim W)$  would not exceed  $P(L|W)$  is if the second conjunct in the first line exceeds the second conjunct in the second line by the same magnitude. I leave this consideration aside for reasons of space, only to note that I have thought carefully about whether it does so and can see no reason to think it should. And in any case, that is Tooley's burden. So, quite clearly,  $P(L|\sim W)$  and  $P(L|W)$  could be discrepant, and

hence Tooley has to factor L in to his judgement on W. He has not done so. And if he did do so, he would have to provide a reason to think that  $P(T|\sim W)$  is not high.<sup>12</sup>

Part of this complexity arises because Tooley has not framed his argument in a standard Bayesian format. So we can briefly consider whether there is a strictly Bayesian (i.e. in the sense of using Bayes' Theorem) argument to be made from his premises. Tooley's argument is essentially intended to vindicate our intuitive judgement that we would not expect God to perform a certain action from a sceptical theist rebuttal. Even if theism is true, we wouldn't expect this of God. So his premise can be understood as claiming that  $P(W|T)$  (with WMP implicitly understood in the background knowledge)  $> 0.5$ .

- (1)  $P(W|T) > 0.5$  (premise)
- (2)  $P(\sim W|T) < 0.5$  (from 1)
- (3)  $P(L|T) = P(L|T \& W) \times P(W|T) + P(L|T \& \sim W) \times P(\sim W|T)$  (theorem)
- (4)  $P(L|T) = P(L|T \& \sim W) \times P(\sim W|T)$  (from 3, since  $P(L|T \& W) = 0$ )
- (5)  $P(L|T) < P(L|T \& \sim W) \times 0.5$  (from 2 and 4)

What this generates is an argument that  $P(L|T)$  is less than 0.5 – perhaps considerably less than 0.5, depending on the value of  $P(L|T \& \sim W)$ . That is, on theism, we would not expect the Lisbon earthquake. This could form the basis of a simple Bayesian argument:

$$\frac{P(T|L)}{P(\sim T|L)} = \frac{P(L|T)}{P(L|\sim T)} \times \frac{P(T)}{P(\sim T)}$$

Tooley could therefore argue that since  $P(L|T) < 0.5$ , and  $P(L|\sim T)$  is plausibly higher,<sup>13</sup> the final ratio exceeds the first ratio, and so L increases the probability of atheism. If Tooley has an argument that the final ratio is 1 or lower, then the final result will be that atheism is probable. But this is a weak result. For it depends crucially on the prior probabilities of T and  $\sim T$ , which might be modified by other evidence. And even if Tooley can establish this prior probability, he still needs to show that  $P(L|\sim T)$  exceeds  $P(L|T)$ . He has shown *at most* that  $P(L|T)$  is less than 0.5. But this shows nothing on its own. So the result is a relatively uninteresting one in the absence of these other premises. The best case scenario, in the absence of a discussion of prior probability and independent evidence, is that L supports atheism to some extent. But many theists already grant this premise, and it is not typically thought to deal theism a mortal wound. So a simple Bayesian reconstruction of Tooley's argument suggests that it is not powerful.

### **Structure-descriptions and the generalization to many prima facie evils**

Tooley does offer an extension of the argument to encompass the number of events in the world which seem, given their RMPs and WMPs, impermissible by

God. The aim is to make God's existence very improbable indeed, not just less probable than not. This argument suffers from all the same problems as the previous argument, but I will also show that it is vulnerable to at least two further criticisms, one of which is redeemable and the other of which is fatal. The argument is as follows:<sup>14</sup>

$n$  = number of prima facie evil events in the world  
 $k$  = number of unknown WMPs and RMPs

Define a Q-predicate as a predicate which is maximal with respect to U, the set of unknown WMPs and RMPs. A predicate is maximal with respect to U if, when applied to an individual, it indicates, for every property in U, whether or not that individual has that property or not. So if there are three basic properties P, Q, and R in U, the following predicates would be maximal with respect to U:

$(P \ \& \ Q \ \& \ R)$	$(P \ \& \ Q \ \& \ \sim R)$
$(P \ \& \ \sim Q \ \& \ R)$	$(P \ \& \ \sim Q \ \& \ \sim R)$
$(\sim P \ \& \ Q \ \& \ R)$	$(\sim P \ \& \ Q \ \& \ \sim R)$
$(\sim P \ \& \ \sim Q \ \& \ R)$	$(\sim P \ \& \ \sim Q \ \& \ \sim R)$

As can be seen, maximal predicates offer a helpful way of enumerating all the possible combinations of predicates or properties applying to any given object.

In this case, there are  $k$  members of U.<sup>15</sup> U is the set of unknown morally relevant properties. Since  $y$  basic properties generate  $2^y$  maximal predicates, it follows that there are  $2^k$  Q-predicates.<sup>16</sup> And each of these Q-predicates will have its own moral weight. If the RMPs in a given Q-predicate outweigh the WMPs, then the Q-predicate is 'positive'. But, as far as we know, there is no reason to think that there are more unknown RMPs than unknown WMPs or vice versa. And we have no reason to think that unknown RMPs are weightier on average than unknown WMPs. And so these symmetry considerations<sup>17</sup> suggest that *no more than half* of all Q-predicates are positive (in fact, less than half will be positive, since some will be neutral). Assuming, charitably, that half of all Q-predicates are positive, the number of positive Q-predicates is  $2^{k-1}$ .

Predicates can be applied to objects. And there is a simple formula governing how many possible combinations there are, for any given number of basic properties and any number of objects. So, if we have one basic property P and three objects a, b, and c, we have the following possibilities:

$Pa \ \& \ Pb \ \& \ Pc$	$Pa \ \& \ Pb \ \& \ \sim Pc$
$Pa \ \& \ \sim Pb \ \& \ Pc$	$Pa \ \& \ \sim Pb \ \& \ \sim Pc$
$\sim Pa \ \& \ Pb \ \& \ Pc$	$\sim Pa \ \& \ Pb \ \& \ \sim Pc$
$\sim Pa \ \& \ \sim Pb \ \& \ Pc$	$\sim Pa \ \& \ \sim Pb \ \& \ \sim Pc$

These are called state-descriptions, and the general rule is that the number of state descriptions is equal to  $m^n$ , where  $m$  is the number of maximal predicates and  $n$  is the number of objects (which in our case, are states of affairs). So in this case, where one basic property generates two maximal predicates,  $m = 2$ . And since  $n = 3$ , we have  $2^3 = 8$  state-descriptions.

These state-descriptions can be organized into structure-descriptions, where state-descriptions within a structure-description differ only in the permutations of objects therein. For example:

Structure-description 1: Pa & Pb & Pc

Structure-description 2: Pa & Pb & ~Pc

Pa & ~Pb & Pc

~Pa & Pb & Pc

Structure-description 3: Pa & ~Pb & ~Pc

~Pa & Pb & ~Pc

~Pa & ~Pb & Pc

Structure-description 4: ~Pa & ~Pb & ~Pc

Tooley, following Carnap, proposes that the correct prior probability distribution applies a principle of indifference not across state-descriptions but across structure-descriptions, so that the prior probability of Pa & Pb & Pc is not 1/8 but 1/4.

There is a helpful formula given by Carnap which calculates the number of structure-descriptions given by  $m$  maximal predicates and  $n$  objects. That formula is:

$$\frac{(n + m - 1)!}{n!(m - 1)!}$$

Again, since in this case  $m = 2$  and  $n = 3$ , the total amounts to 4, which fits with our enumeration of structure-descriptions. The formula generalizes for any integral number of maximal predicates and any integral number of objects.

We have said that there are  $2^k$  maximal predicates, where  $k$  is the number of unknown, basic RMPs and WMPs. So the total number of structure-descriptions is:

$$\frac{(n + 2^k - 1)!}{n!(2^k - 1)!}$$

Now, in order for the theist to say that no prima facie evil event in the world is ultima facie evil (by which I mean that it would be impermissible for God to permit the occurrence of that event), they will have to hold that for every such event, the unknown RMPs and WMPs outweigh the known WMPs of the event. And that can only be the case if a positive Q-predicate is predicated of every one of those events. So we will be looking for structure-descriptions which involve only positive Q-predicates. Since there are  $2^{k-1}$  positive Q-predicates, the number

of structure-descriptions which attribute positive Q-predicates to every prima facie evil event is:

$$\frac{(n + 2^{k-1} - 1)!}{n!(2^{k-1} - 1)!}$$

Now the crucial step. We know that there are a certain number of structure descriptions for all prima facie evils and a given number of unknown moral properties. And we know that there is a certain number of structure descriptions on which all these prima facie evils come out as justifiable. The probability that they do all come out as justifiable – call this  $P(k,n)$  – is, according to Tooley, simply the ratio of these:

$$\frac{(n + 2^{k-1} - 1)!/n!(2^{k-1} - 1)!}{(n + 2^k - 1)!/n!(2^k - 1)!}$$

Tooley shows that this is a very small number as follows. It can then be seen by comparing  $P(k,n)$ ,  $P(k,n + 1)$  and  $P(k + 1,n)$  that  $P(k,n)$  is a monotonically decreasing function of  $k$  and  $n$ , except where  $n = 1$ . That is, as  $k$  or  $n$  increases,  $P(k,n)$  decreases. This is because the ratios  $P(k,n)/P(k,n + 1)$  and  $P(k,n)/P(k + 1,n)$  are both greater than 1 (except where  $n = 1$ ). Since  $P(k,n)$  is therefore at a maximum when  $k = 1$ , we can set an upper bound for  $P(k,n)$  as  $P(1,n)$ :

$$\begin{aligned} & \frac{(n + 2^{1-1} - 1)!/n!(2^{1-1} - 1)!}{(n + 2^1 - 1)!/n!(2^1 - 1)!} \\ &= \frac{n!/n!}{(n + 1)!/n!} \\ &= \frac{1}{n + 1} \end{aligned}$$

Since this is a maximum value for  $P(k,n)$ , we can say that  $P(k,n)$  is at most  $1/(n + 1)$ , where  $n$  is the number of prima facie evil events in the world. Since there are many such events,  $P(k,n)$  is very low. Conversely, the probability that there is at least one ultima facie evil event in the world is very high. And since the existence of at least one ultima facie evil event in the world entails God's non-existence, the probability that God does not exist is even higher. Given that this argument involves some assumptions charitable to the theist, Tooley says,  $1/(n + 1)$  is a significant *overestimate* of the probability that all prima facie evils are justifiable, and so an even more significant overestimate of the probability of theism.

As I previously mentioned, this argument succumbs to all my criticisms of Tooley's first argument. For example, Tooley again neglects the fact that whether or not an event has occurred may drastically change the probability that that event is permissible by God. But even setting aside those difficulties,



the Carnapian method is demonstrably unreasonable. One problem is that it fails to adequately represent the extreme conditional dependence of the evilness of various  $n_i$ . That is, finding out that there are some prima facie evils should make it very likely that there are other prima facie evils of similar kinds. But Tooley's argument seems insensitive to this fact – a fact which severely limits the force of the argument from evil against theism.<sup>18</sup>

The clearest problem, however, is that the improbability here is not generated by any enormous tension between theism and suffering in the world, but simply by the magnitude of  $n$  and  $k$ . And there are parodies which can show exactly why this is so problematic. I will give just one. Consider the following scenario:

Humans exist in a world populated only with prima facie good states of affairs. They manage to individuate 1,000 independent such states of affairs. Then  $n = 1,000$ . There are just 6 unknown morally relevant properties, and so there are 64 Q-predicates. Thus, the total number of structure descriptions is:

$$\frac{(1,000 + 64 - 1)!}{1,000!(64 - 1)!} = \frac{(1,063) \dots (1,001)}{(63) \dots (1)}$$

Suppose humans are in the possession of the following knowledge: the unknown RMPs and WMPs are heavily balanced in favour of RMPs, and RMPs are both weightier and more common than WMPs. RMPs and WMPs are balanced such that only *one* of the 64 Q-predicates is sufficient to render these prima facie good states of affairs evil. Call the 63 Q-predicates preserving goodness preserver Q-predicates, and call the other predicate the evil Q-predicate. The details of these RMPs and WMPs remain unknown, of course.

This case is *obviously* favourable to theism. The world contains nothing but prima facie good states of affairs, and humans even have the knowledge that the unknown RMPs and WMPs of their world are balanced in favour of good. Few things in philosophy are as clear as that people in this world should not be able to generate an inductive argument from evil against the existence of God.

If Tooley's approach is correct, however, these people can do exactly that. For, as Tooley does, they note that a single instantiation of the evil Q-predicate is sufficient for God's non-existence. And they point out to the theists of their world that God exists only if the correct structure-description of the world contains only preserver Q-predicates. But, they say, there are  $(1,063) \dots (1,001)/(63) \dots (1)$  structure-descriptions. And only a small proportion of them contain only preserver Q-predicates. To be precise, the number of structure-descriptions containing only preserver Q-predicates is:

$$\frac{(1,000 + 63 - 1)!}{1,000!(63 - 1)!} = \frac{(1,062) \dots (1,001)}{(62) \dots (1)}$$

The proportion of total structure-descriptions which only contain preserver Q-predicates is therefore equal to:

$$\frac{(1, 062) \dots (1, 001) \times (63) \dots (1)}{(62) \dots (1) \times (1, 063) \dots (1, 001)} = \frac{63}{1, 063}$$

So it is very likely that at least one of these 1,000 prima facie good states of affairs actually exemplifies the evil Q-predicate. And so it is very likely that God does not exist.

According to this inductive method, the probability of all of these seemingly permissible states of affairs being permissible is *still* incredibly low even when there is no prima facie evil in the world, and even when almost all Q-predicates render these states of affairs permissible. This improbability is generated simply by the fact that there are a large number of states of affairs, and by the fact that there are a modest number of unknown RMPs and WMPs. But it is painstakingly obvious that there simply being a large number of states of affairs should not count as such compelling and apparently indefeasible evidence against theism. This parody therefore shows Tooley's Carnapian inductive method to be unreasonable.

### Conclusion

The basic point of Tooley's argument is the symmetry principle, which suggests that we have no good reason to suppose that unknown RMPs are more abundant or weighty than unknown WMPs. This seems to me correct, and mandates the equiprobability assumption he endorses. Short of the evidence of the event occurring, and without a theodicy which would lead us to expect these sorts of evils, Tooley's symmetry principle is certainly reasonable and warrants the intermediate conclusion  $P(W|WMP) > 0.5$ . Insofar as Tooley's argument is intended only to rebut certain sceptical theist responses to the problem of evil, therefore, his argument seems reasonable, despite the negative conclusion of this article.

But in going any further, Tooley's argument fails for multiple reasons. His treatment of RMPs and WMPs and the structure of morality is controversial, at least. More problematically, Tooley neglects the importance of uncertainty regarding considerations such as the existence of God, independent evidence for God, the existence of certain RMPs, and the value of certain RMPs. He neglects the potential success of theodicies and fails suitably to limit his argument to those who already accept one of the key premises. Fatally, he neglects the fact that the occurrence of a prima facie evil may affect the probability that that event is permissible by God.

When generalizing his argument to multiple prima facie evils, Tooley makes one redeemable technical error. However, his inductive framework has at least one decisive paradoxical example against it, such that it is difficult to see how the

argument can be redeemed. Whatever weight the problem of evil has against theism, it does not appear that Tooley's formalization goes any significant way towards establishing this.<sup>19</sup>

## References

- CHISHOLM, RODERICK M. (1990) 'The defeat of good and evil', in M. M. Adams & R. M. Adams (eds) *The Problem of Evil* (Oxford: Oxford University Press), 53–68.
- OTTE, RICHARD (2013) 'A Carnapian argument from evil', in D. Howard-Snyder & J. P. McBrayer (eds) *The Blackwell Companion to the Problem of Evil* (Chichester: John Wiley & Sons), 83–97.
- PLANTINGA, ALVIN & TOOLEY, MICHAEL (2008) *Knowledge of God* (Oxford: Blackwell).

## Notes

1. We can describe such states of affairs as 'prima facie wrongs'. Tooley has advanced his argument elsewhere, but Plantinga and Tooley (2008) is the most elaborate account of the argument.
2. Otte (2013) offers a helpful simplification of the argument.
3. Tooley goes into considerably more detail defining rightmaking and wrongmaking properties and their relation to the overall wrongness of an action. Some of this may be objected to – and I note such objections shortly – but since it is immaterial to my primary doubt, we may – at least in this article – accept it for the sake of argument.
4. It is not clear what the scope of Tooley's 'Any' is – whether it includes possible actions or only actual actions. This makes a significant difference. As I shall subsequently demonstrate, knowing whether or not such an action has actually happened may affect the probability that it was wrong to permit, and so the plausibility of this premise depends on the scope of 'Any' here.
5. This corresponds to Tooley's premise (15).
6. This corresponds to Tooley's premise (19).
7. This last clause is to be interpreted, as explained above, as the claim that there are no RMPs that we know of such that we are justified in believing both that the action in question has those RMPs, and that those properties are sufficiently serious to counterbalance any relevant, known, WMPs.
8. This might be especially important in developing a non-consequentialist theodicy. Consequentialist gods are often subject to criticism for using their creatures as instruments *in order to achieve* greater goods – often involving intense suffering. But a theodicy based on defeat of evils rather than counterbalancing of evils can entertain the possibility of those evils being *converted* to goods rather than just outweighed by later (or previous) goods.
9. It is possible that this is what Tooley meant all along, as his conception of RMPs and WMPs is fairly loosely characterized. If so, this discussion can be interpreted as a neutral clarification rather than an objection.
10. This resolves the problem of uncertainty regarding the premise, since the premise just *is* a probability distribution, rather than a proposition.
11. It is open to Tooley to argue that the prior probability of theism is no more than 0.5, and that this is simply before considering the other evidence. Once other evidence is considered, the probability of theism might increase above 0.5. But this would, in the first place, require an argument for such a prior probability, and hardly shows that atheism is true. For many theists will already concede that evil constitutes *some* evidence against theism. But, they suppose, the positive evidence for theism significantly outweighs such negative evidence. And in any case, it is far from clear that Tooley's argument even shows this much.
12. Tooley could object that his argument only aims to show that the probability of theism is less than 0.5 given only considerations of intrinsic probability and the problem of evil, and that the theist is therefore not entitled to a high value for  $P(T|\sim W)$ . But this is not a very substantial conclusion: I suspect that almost all theists would say that the intrinsic (epistemic) probability of theism is less than 0.5, and that evil probably does not push it above 0.5. What Tooley presumably wants to show is that theism is *strong* evidence against theism. But for that he needs his second argument, to which we will respond shortly.
13. This is by no means clear, or argued for, and is itself open to doubt.

14. Again, Otte (2013) offers a very helpful simplified version of the argument.
15. The negations of these properties are not counted among  $k$ .
16. This is because adding one basic property doubles the number of maximal predicates. It does not merely increase the number of maximal predicates by 2. So, in our example, with 3 basic properties (P, Q, and R), we can see that there are 8 ( $= 2^3$ ) maximal predicates. Tooley makes his first mistake here, saying that there are  $2k$  maximal predicates. He relies on  $2k$  for some of the key proofs of his argument, and so his proofs are mistaken. By good fortune, however, the conclusions he needs from those proofs (viz. that  $P(1,n) = 1/(n + 1)$  and that  $P(k,n)$  is a monotonically decreasing function of  $k$ ) still happen to be correct, though the requisite proofs are more complicated than the ones Tooley provides. I therefore relegate these to the appendix. I will also amend Tooley's argument in the main text accordingly.
17. This is the respect in which Tooley's two arguments form the same basic argument: both use a symmetry consideration concerning unknown RMPs and WMPs, the force of which is basically to show that sceptical theism makes no difference to our judgements concerning the probable wrongness of actions, as long as we apply a sort of principle of indifference across unknown moral properties. In this central insight, I think Tooley is actually correct. He is just mistaken to suppose that this clinches the deal for the problem of evil.
18. I thank an anonymous reviewer for pointing me to Otte's (2013) work, which develops this objection more clearly and forcefully. But I am not sure that Otte's suggestion is wholly correct when he says it is equiprobable, on this account, that God has reason to permit any given number of events. Maybe Carnap's logic works out that way, but I cannot see it.
19. My thanks to John DePoe, Liz Jackson, and the Tyndale Fellowship Philosophy of Religion Study Group for their helpful comments on this article.

**Appendix: Proofs that  $P(k,n)$  decreases monotonically with  $k$  and that  $P(1,n) = 1/n + 1$**

*$P(k,n)$  decreases monotonically with  $k$*

$$P(k, n) = \frac{(n + 2^{k-1} - 1)!}{n!(2^{k-1} - 1)!} \div \frac{(n + 2^k - 1)!}{n!(2^k - 1)!} = \frac{(n + 2^{k-1} - 1)!n!(2^k - 1)!}{n!(2^{k-1} - 1)!(n + 2^k - 1)!}$$

$$= \frac{(n + 2^{k-1} - 1)!(2^k - 1)!}{(2^{k-1} - 1)!(n + 2^k - 1)!}$$

$$P(k + 1, n) = \frac{(n + 2^k - 1)!}{n!(2^k - 1)!} \div \frac{(n + 2^{k+1} - 1)!}{n!(2^{k+1} - 1)!} = \frac{(n + 2^k - 1)!n!(2^{k+1} - 1)!}{n!(2^k - 1)!(n + 2^{k+1} - 1)!}$$

$$= \frac{(n + 2^k - 1)!(2^{k+1} - 1)!}{(2^k - 1)!(n + 2^{k+1} - 1)!}$$

$$\frac{P(k + 1, n)}{P(k, n)} = \frac{(n + 2^k - 1)!(2^{k+1} - 1)!}{(2^k - 1)!(n + 2^{k+1} - 1)!} \div \frac{(n + 2^{k-1} - 1)!(2^k - 1)!}{(2^{k-1} - 1)!(n + 2^k - 1)!}$$

$$= \frac{(n + 2^k - 1)!(2^{k+1} - 1)!(2^{k-1} - 1)!(n + 2^k - 1)!}{(2^k - 1)!(n + 2^{k+1} - 1)!(n + 2^{k-1} - 1)!(2^k - 1)!}$$

$$= \frac{(n + 2^k - 1)!}{(2^k - 1)!} \times \frac{(2^{k+1} - 1)!}{(n + 2^{k+1} - 1)!} \times \frac{(2^{k-1} - 1)!}{(n + 2^{k-1} - 1)!} \times \frac{(n + 2^k - 1)!}{(2^k - 1)!}$$

$$= \frac{(n + 2^k - 1) \dots (2^k)}{1} \times \frac{1}{(n + 2^{k+1} - 1) \dots (2^{k+1})}$$

$$\times \frac{1}{(n + 2^{k-1} - 1) \dots (2^{k-1})} \times \frac{(n + 2^k - 1) \dots (2^k)}{1}$$

$$= \frac{(n + 2^k - 1) \dots (2^k)}{(n + 2^{k+1} - 1) \dots (2^{k+1})} \times \frac{(n + 2^k - 1) \dots (2^k)}{(n + 2^{k-1} - 1) \dots (2^{k-1})}$$

$$= \frac{(n + 2^k - 1)(n + 2^k - 1)}{(n + 2^{k+1} - 1)(n + 2^{k-1} - 1)} \times \frac{(n + 2^k - 2)(n + 2^k - 2) \dots (2^k)(2^k)}{(n + 2^{k+1} - 2)(n + 2^{k-1} - 2) \dots (2^{k+1})(2^{k-1})}$$

In the case of  $n = 1$ , then this resolves to 1. But it can be shown that when  $n > 1$ ,  $P(k + 1, n) < 1$ .

Each multiplicand in this equation exhibits a more general pattern. Where  $b$  is some non-negative integer (in the first multiplicand, for example,  $b = n - 1$ ):

$$\frac{(2^k + b)(2^k + b)}{(2^{k+1} + b)(2^{k-1} + b)}$$

Now, let  $a = 2^{k-1}$ . Since  $2^k = 2(2^{k-1})$  and  $2^{k+1} = 4(2^{k-1})$ , it follows that  $2^k = 2a$  and  $2^{k+1} = 4a$ . Therefore:

$$\frac{(2^k + b)(2^k + b)}{(2^{k+1} + b)(2^{k-1} + b)} = \frac{(2a + b)(2a + b)}{(4a + b)(a + b)} = \frac{4a^2 + 4ab + b^2}{4a^2 + 5ab + b^2}$$

The difference between the numerator and denominator is  $ab = b2^{k-1}$ . Since  $b$  is a positive integer except where it is 0 (it cannot go below  $(n-n)$ ), and since  $2^{k-1}$  is positive,  $ab$  must be positive. So it follows that the denominator is greater than the numerator for each term in the equation (except the last term). It follows, therefore, that the total denominator is greater than the total numerator, and so that  $P(k,n) > P(k,n + 1)$  for all  $n > 1$ . This establishes that  $P(k,n)$  is a monotonically decreasing function of  $k$ , as long as  $n > 1$ .

$$P(1, n) = 1/n + 1$$

The proof here is somewhat easier.

$$\begin{aligned} P(1, n) &= \frac{(n + 2^{1-1} - 1)!}{n!(2^{1-1} - 1)!} \div \frac{(n + 2^1 - 1)!}{n!(2^1 - 1)!} = \frac{(n + 2^{1-1} - 1)!n!(2^1 - 1)!}{n!(2^{1-1} - 1)!(n + 2^1 - 1)!} \\ &= \frac{(n + 2^{1-1} - 1)!(2^1 - 1)!}{(2^{1-1} - 1)!(n + 2^1 - 1)!} \\ &= \frac{n!1!}{0!(n + 1)!} = \frac{n!}{(n + 1)!} = \frac{(n)(n - 1) \dots (1)}{(n + 1)(n)(n - 1) \dots (1)} = \frac{1}{n + 1} \end{aligned}$$