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On the multifractality of plasma turbulence in the solar wind

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Abstract. In this work we have analyzed turbulent plasma in the kinetic scale by the characterization of magnetic fluctuations time series. Considering numerical Particle-In-Cell (PIC) simulations we apply a method known as MultiFractal Detrended Fluctuation Analysis (MFDFA) to study the fluctuations of solar-wind-like plasmas in thermodynamic equilibrium (represented by Maxwellian velocity distribution functions), and out of equilibrium plasma represented by Tsallis velocity distribution functions, characterized by the *kappa* (κ) parameter, to stablish relations between the fractality of magnetic fluctuation and the kappa parameter.

Keywords. Plasma, turbulence, multifractality

1. Introduction

The upper atmosphere of the Sun is continuously releasing a stream of charged particles which constitutes the solar wind. This ejected plasma gives an extent of interesting phenomena in plasma physics. One of the fundamental problems in this area is the understanding of the relaxation process in a collisionless plasma and the resultant state of the electromagnetic turbulence, in particular, at kinetic scales.

In this work we are applying a method known as *MultiFractal Detrended Fluctuation Analysis* (MFDFA) (Kantelhardt *et al.* 2002) to study the magnetic fluctuations of solar-wind-like plasmas in thermodynamic equilibrium (represented by Maxwell velocity distribution functions), and out of equilibrium plasma represented by Tsallis distribution functions, characterized by a parameter κ. In first place we studied magnetic fluctuations through *Particle in Cell* (PIC) (Viñas *et al.* 2014) simulations of a magnetized plasma compound by ions and electrons, where we calculated the multifractality in time series, extracted in the simulations, to establish relations between the multifractality of the system and the kappa parameter.

The MFDFA method allows us to study and extract valuable information of any time series, that is the reason it has been applied in different research areas, for example, there are studies in the biology field associated to DNA sequences (Peng *et al.* 1994) and also heartbeat time series (Peng *et al.* 1995); furthermore, there are studies in seismic complex networks (Pastén *et al.* 2018) and different applications in economics and finance (Grech 2016). Those applications and results show that this method is an interesting tool to explore in different areas of science.

This article organizes as follows: section 2 describes the computer simulation where the magnetic fluctuation data were obtained, associated to each velocity distribution functions; section 3 describes the MFDFA method and the steps we have to carry out; in section 4 we show the preliminary results when we applied the MFDFA method to the magnetic fluctuation data. Finally, in section 5 we summarize our main conclusions.

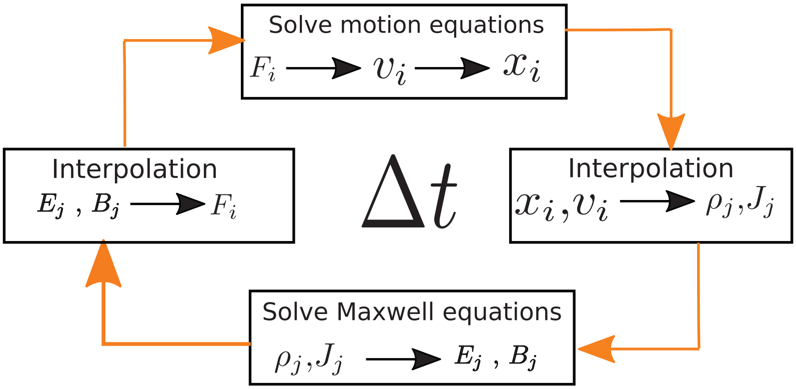

Figure 1. The figure shows how the electromagnetic fields are calculated in each cell. Starting with the position and the velocity of the particles it is possible to determine the density and the current generated in each cell. Then, Maxwell equations are solved to determine the fields in each cell.

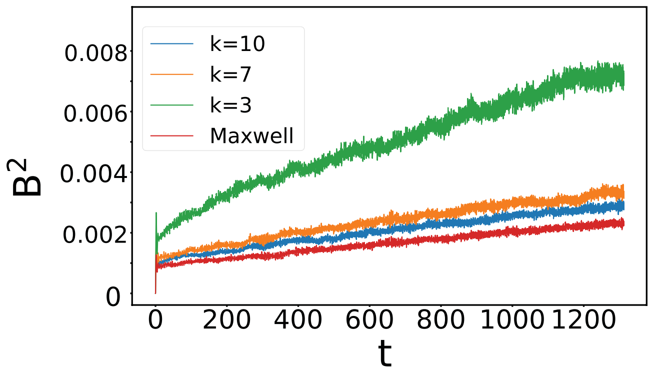

Figure 2. Magnetic fluctuations for different velocity distribution functions. We can observe the results associate with each value of κ. Fields and time are dimensionless. Time is normalized to the electron gyrofrequency.

2. Particle In Cell Simulation (PIC)

We have analyzed magnetic fluctuations obtained in a Particle In Cell (PIC) simulation (Viñas *et al.* 2014). We studied a turbulent plasma compound by ions and electron that are treated kinetically and periodic boundary conditions are imposed. Time is normalized in units of the electron cyclotron frequency and particle positions are normalized to the electron inertial lengths. Fig. 1 shows the general idea of the method.

Then, the resultants magnetic fluctuation associated to each velocity distribution functions: Maxwellian and Tsallis (for $\kappa = 3, 7, 10$) are shown in Fig. 2. We can observe that while the κ parameter increases, magnetic fluctuations converge to the Maxwellian equilibrium as is expected.

3. MultiFractal Detrended Fluctuation Analysis (MFDFA)

In order to determinate the fractality of times series, we have to carry out a series of steps (Kantelhardt *et al.* 2002). For this, let us consider that B_k is a time series of length N; first, we have to determine the profile:

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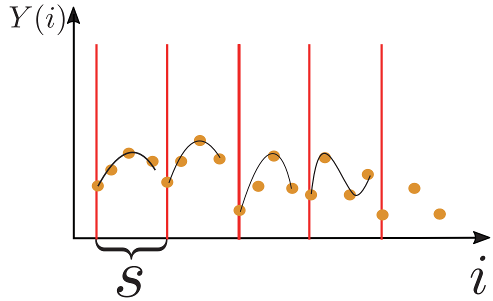

Figure 3. When the profile is determinated, polynomial adjust are calculated for each window. Note that we have $2N_s$ segments because of the data that may remain in the profile.

$$Y(i) = \sum_{k}^{i} B_k - < B >,$$

where $i = 1, ..., N$. Then, divide the profile into $N_s = \text{int}\ (N/s)$ nonoverlapping segments of equal length s. Since the length N is not often a multiple of the length s a short part of the profile may remain. To consider this part, the same analysis will be carry out starting from the end of the series. The next step is to determine the local trend for each segments using a polynomial adjust:

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} \left(Y\left[(v-1)\, s + i \right] - y_v\,(i) \right)^2 \ ,$$

where y_v is the polynomial adjust in each window. Finally, overage over all segments to obtain the qth order fluctuation function:

$$F_q(s) = \left(\frac{1}{2N_s} \sum_{v=1}^{2N_s} \left(F^2(v, s) \right)^{q/2} \right)^{1/q} , \tag{3.1}$$

where q and $F^2(v, s)$ are the generalize dimension index and the variance in each window, respectively.

The general idea of the method is shown in Fig. 3.

The objective is to determine how Eq. (3.1) is related with s for different values of q. With this function we can determine the generalized Hurst exponent $h(q)$, further more, it increases for large values of s as a power-law:

$$F_q(s) \sim s^{h(q)} .$$

4. Results

We applied the MFDFA method to magnetic fluctuations given by Fig. 2. Results are shown in Fig. 4. Here we compute Eq. (3.1) for different values of q (q = ±20, ±15, ±10, ±5, 0), then, in log-log plots we obtained values for the generalized Hurst exponent associated to each q.

5. Conclusions

In this article we studied the magnetic fluctuation of a collisionless plasma with different velocity distribution functions using the MFDFA method. First, we explained the simulation where the data were obtained; then, we describe the general ideas and steps of the MFDFA method to apply them in the simulation data. Finally, we computed the relations between the κ parameter and the generalize Hurst exponent.

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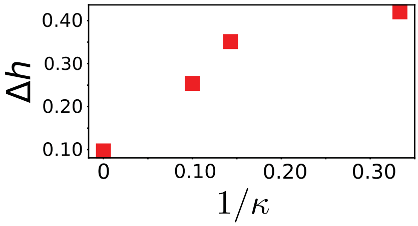

Figure 4. Relations between the generalize Hurst exponent of the time series and the velocity distribution functions. Here we took $\Delta h_\kappa = h_\kappa(q=10) - h_\kappa(q=-10)$ The limit $1/\kappa = 0$ represents the Maxwellian distribution.

The results obtained in Fig. 4 shows the multifractal spectrum of the time series analyzed. These results suggest a mono fractal behavior for the four time series, this is due that for each distribution Δh is lower than 0.5. It is interesting to notice that, while the κ parameter increases, the monofractality in the time series decreases. This result also suggests that, for lower values of κ there is no long-range correlations or, this correlations tend to zero quickly in these time series. Nevertheless, the results are still preliminary and there are different ways to corroborate and analyze them. In a future work we pretend to build the singularity spectra of fractal dimensions and analyze more time series to characterize this complex behavior.

The motivation of this research is to characterize turbulent plasma through time series analysis using the MFDFA method, with the objective of describe the dependencies associated to the different velocity distribution functions. We expect our analysis to be useful for the characterization of the electromagnetic turbulence in a collisionless space plasma, such as the solar wind. Furthermore, this tool may be useful to extract valuable information about the plasma when high resolution particle detectors are not available.

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