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Indirect simultaneous positioning of deformable objects with multi-pinching fingers based on an uncertain model

S. Hirai and T. Wada

Department of Robotics, Ritsumeikan University, Kusatsu, Shiga, 525–8577 (Japan)
E-mail: hirai@se.ritsumei.ac.jp

SUMMARY

A new approach to the control of indirect simultaneous positioning of deformable objects is presented. Many manufacturing processes that deal with deformable objects such as clothes and rubber sheets involve a positioning of multiple points on a deformable object. The multiple points should be guided simultaneously to the desired locations. Moreover, these positioned points cannot be manipulated directly. This operation is referred to as indirect simultaneous positioning. In this article, we will propose a new control law for indirect simultaneous positioning of a deformable object based on its uncertain model and will show the robustness of the proposed control law. First, a simplified physical model of a deformable object is developed for its positioning operation. Second, indirect simultaneous positioning of an extensible object is formulated. Based on a linearized model of an extensible object, we will propose a novel control law for indirect simultaneous positioning. Next, we will prove the robustness of the proposed control law theoretically. Finally, experimental results will show the robustness of our proposed control law against the discrepancy between a real fabric and its uncertain model.

KEYWORDS: Deformable objects, Positioning.

1. INTRODUCTION

There exist many manipulative operations that deal with deformable objects such as fabrics, wires, rubber sheets, and dough in various manufacturing including garment industry, electronic industry, and food industry. Due to deformability of manipulated objects, most these operations are performed by humans though automatic operations are strongly required in various industries. Many manipulative operations that deal with deformable objects result in a positioning of multiple points on a deformable object. In this positioning, multiple points on a deformable object should be guided to the desired locations simultaneously. Moreover, it is often impossible to manipulate the positioned points directly. For example, one operation called linking is involved in the manufacturing of seamless socks. In linking of fabrics, loops at the end of a fabric must be matched to loops of another fabric so that the two fabrics can be sewed seamlessly. These points cannot be manipulated so that a sewing needle be guided along the matched loops. Mating of a flexible part in electric industry also results in the positioning of mated points on the object. These points cannot be manipulated directly since the points in a mating part contact with a mated part. Consequently, we find that a positioning of multiple points on a deformable object is one of fundamental operations in the manipulation of deformable objects. Since the positioned points cannot be manipulated directly, the guidance of positioned points must be performed by controlling some points except the positioned points. This operation is referred to as *indirect simultaneous positioning*. In this paper, we will investigate the control law for indirect simultaneous positioning of deformable objects.

In indirect simultaneous positioning of a deformable object, multiple points should be guided simultaneously by controlling the motion of manipulated points. Motion of individual positioned points and that of manipulated points are interfered one another. This implies that the indirect simultaneous positioning is a multi-input and multi-output operation with coupling between inputs and outputs. Thus, a model of the deformable object is indispensable to determine the motion of manipulated points. On the other hand, it is difficult to build an exact model of a deformable object since its deformation may be nonlinear and often shows hysteresis. The goal of this research is not to build an exact model of a deformable object but to establish a control law for indirect simultaneous positioning of a deformable object. Thus, we will build a simple model of a deformable object and will establish a model-based control law for indirect simultaneous positioning, which is robust enough to deal with the discrepancy between a deformable object and its uncertain model.

Modeling of deformable objects has been studied in computer graphics[1-3] and virtual reality.[4] These researches focus on the modeling of object deformation, and manipulative operations of deformable objects are out of consideration. Manipulative operations of deformable objects have been recently studied in robotics literature. Automatic handling of deformable parts in garment industry and shoe industry has been experimentally investigated.[5] Zheng and Chen have proposed a strategy to insert a deformable beam into a hole.[6] Ono et al. have derived a strategy for unfolding a fabric using a touch sensor and a vision sensor.[7] These researches focus on handling of deformable objects, while positioning operations of deformable objects are out of focus. Positioning of a deformable object using two manipulators has been studied.[8-9] A law to control the position and the orientation of a

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deformable object has been proposed there. The control law can be applied to a positioning of one object but multiple points on a deformable object cannot be controlled through their approach.

In this paper, we will propose a new control method for indirect simultaneous positioning of a deformable object. First, a simplified physical model of an extensible object is developed for its positioning operation. Second, indirect simultaneous positioning of a deformable object is formulated. Based on a linearized model of an extensible object, we will propose a novel control law for its indirect simultaneous positioning. Next, we will discuss the robustness of our control law. Finally, experimental results will show the robustness of our proposed method when dealing with a discrepancy between a real fabric and its model.

2. INDIRECT SIMULTANEOUS POSITIONING

In this section, we will formulate indirect simultaneous positioning of a deformable object. In this operation, multiple points on a deformable object should be guided to their desired locations, as illustrated in Figure 1. Since it is impossible to manipulate the positioned points directly, a set of mechanical fingers pinch the object except the positioned points. The guidance is then performed by controlling the manipulated points, which are driven by machanical pinching fingers, as shown in the Figure 1.

A control system for indirect simultaneous positioning of a deformable object is illustrated in Figure 2. Recall that it is impossible to build an exact model of a deformable object since its deformation may be nonlinear and may have hysteresis. This implies that a feedforward control of the location of positioned points based on an object model cannot be realized. Detection of the location of positioned points is indispensable to perform indirect simultaneous positioning of a deformable object. Thus, real-time machine vision is introduced to measure the position of positioned points. The detected location is sent to a controller. The locations of manipulated points are controlled by mechanical pinching fingers. Recall that multiple positioned points should be controlled simultaneously through the motion of manipulated points in indirect simultaneous positioning. Since the location of positioned points and that of manipulated points interfere one with another, the indirect simultaneous positioning is a multi-input and multi-output operation with coupling between inputs and outputs. Thus, a model of the deformable object is indispensable to determine the motion of manipulated points and is built into the controller in advance. Consequently, the controller determines the motion of machanical fingers from the measured location of positioned points based on an object model, which may include discrepancy with an actual deformable object.

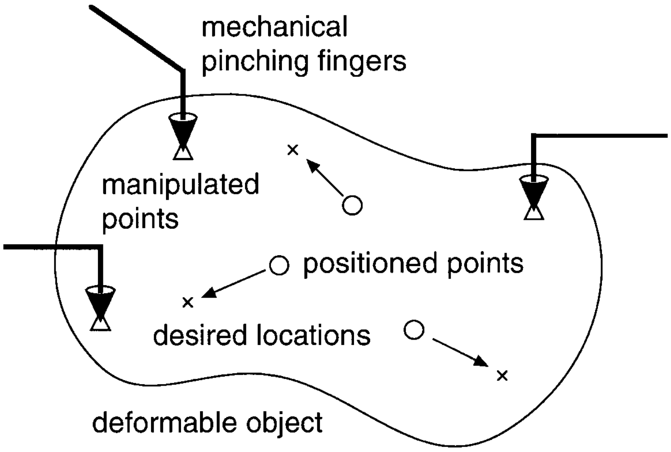

Fig. 1. Indirect positioning of a deformable object
Multiple points on a deformable object should be guided to their desired locations. These positioned points cannot be manipulated directly. The guidance must be performed by controlling manipulated points, which do not coincide with the positioned points.

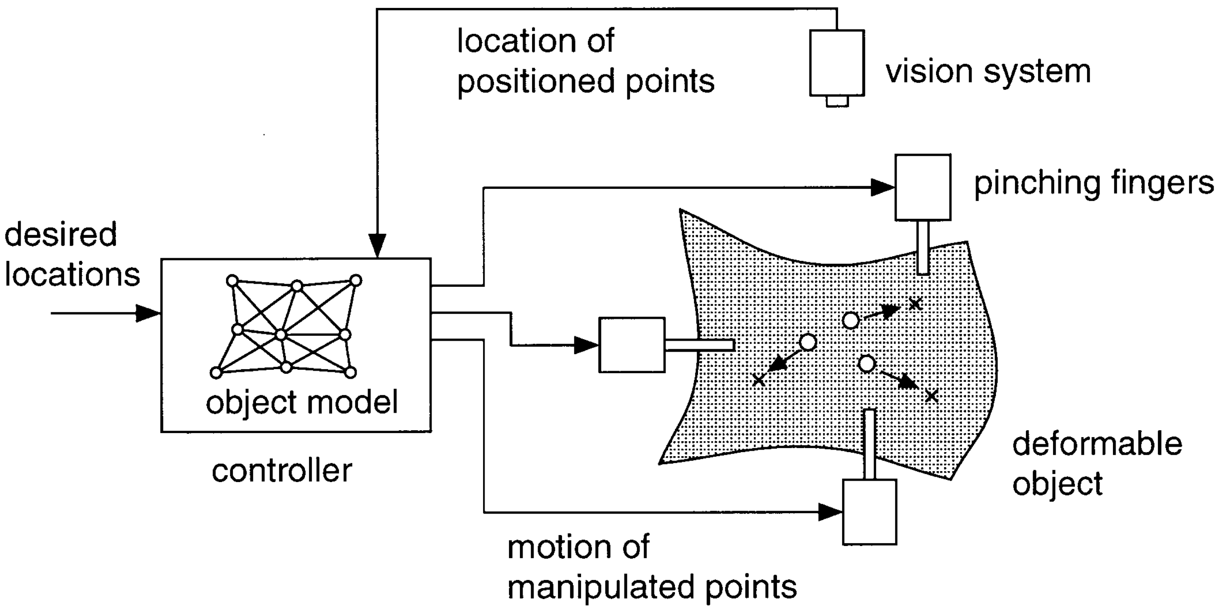

Fig. 2. Vision-based positioning system
Location of positioned points is measured by a vision system. The motion of manipulated points is determined from measurements based on an uncertain model of a deformable object.

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3. FORMULATION OF INDIRECT SIMULTANEOUS POSITIONING BASED ON OBJECT MODEL.

In this section, we will formulate indirect simultaneous positioning of an extensible object. First, we will develop a static model of an extensible object. Indirect simultaneous positioning of an extensible object is then formulated based on the object model. Equilibrium equations on the object are also formulated to derive a control law for indirect simultaneous positioning.

3.1 Static modeling of extensible objects

In this section, we will develop a static model of an extensible object such as fabrics and rubber sheets. The relationship between force and displacement in deformation of a deformable object is nonlinear and often shows hysteresis. It is difficult to model the nonlinearity and the hysteresis in the object deformation. Note that the purpose of this article is not to develop an *exact* model of the object deformation but to establish a control law for indirect simultaneous positioning of deformable objects. The object model can be simple and may involve uncertainty if a control law is robust enough to deal with the discrepancy between an actual object and an object model. Thus, we will build a simple object model and will construct a robust control law for indirect simultaneous positioning based on the simple model.

Let us formulate static behavior of an extensible object. For the simplicity of the modeling, we will apply a lattice modeling technique to an object model. Assume that an extensible object deforms in two-dimensional plane. Let us model the object by a set of lattice points and springs connecting the lattice points, as illustrated in Figure 3-(a). Assume that the object model consists of $(M+1)$ $(N+1)$ lattice points as shown the figure. Let $O-xy$ be a coordinate system on the two-dimensional plane. Let $\boldsymbol{p}_{i,j}=[x_{i,j}, y_{i,j}]^T$ be position vector of the (i,j)-th lattice point with respect to the coordinate system. The shape of an extensible object can be described by a set of position vectors $\boldsymbol{p}_{0,0}$ through $\boldsymbol{p}_{M,N}$.

Springs are located between neighboring lattice points on an object model, as illustrated in Figure 3-(b). Let us introduce a unit vector to describe the direction of each spring. Let $\mathbf{e}_{i,j}^{\alpha,b}$ be a unit vector between the (i, j)-th lattice point and the $(i+\alpha, j+\beta)$-th lattice point. Namely,

$$\boldsymbol{e}_{i,j}^{\alpha,\beta}=\frac{\boldsymbol{p}_{i+\alpha,j+\beta}-\boldsymbol{p}_{i,j}}{|\boldsymbol{p}_{i+\alpha,j+\beta}-\boldsymbol{p}_{i,j}|}.$$

Assume that springs involved in an object model linear. Let $k_{i,j}^{\alpha,\beta}$ be the spring constant of a spring between the (i, j)-th lattice point and the $(i+\alpha, j+\beta)$-th lattice point. Let $l_{i,j}^{\alpha,\beta}$ be the natural length of the spring. The extension of the spring is then described as follows:

$$d_{i,j}^{\alpha,\beta}=|\boldsymbol{p}_{i+\alpha,j+\beta}-\boldsymbol{p}_{i,j}|-l_{i,j}^{\alpha,\beta}$$

Now we can formulate forces acting on individual lattice points. Let $\boldsymbol{F}_{i,j}^{\alpha,\beta}$ be a force acting on lattice point $\boldsymbol{p}_{i,j}$ by a spring connecting two points $\boldsymbol{p}_{i,j}$ and $\boldsymbol{p}_{i+\alpha,j+\beta}$. Force $\boldsymbol{F}_{i,j}^{\alpha,b}$ is then described by

$$\boldsymbol{F}_{i,j}^{\alpha,\beta}=k_{i,j}^{\alpha,\beta}\,d_{i,j}^{\alpha,\beta}\,\boldsymbol{e}_{i,j}^{\alpha,\beta}$$

The result force acting on point $\boldsymbol{p}_{i,j}$ can be derived by summing all forces caused by springs connected to the point. Thus, the resultant force $\boldsymbol{F}_{i,j}$ is described by

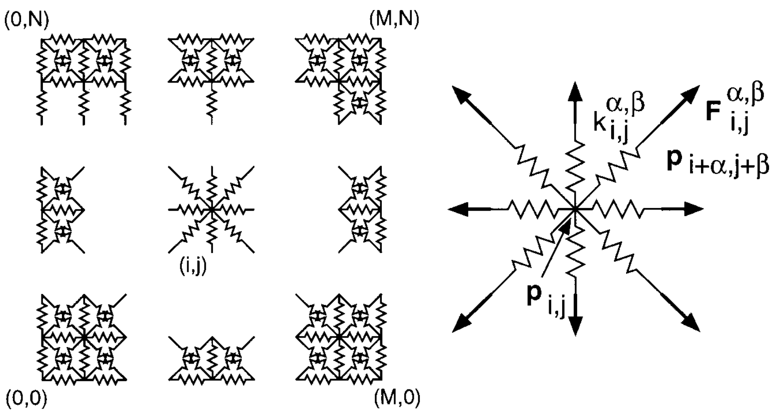
$$\boldsymbol{F}_{i,j}=\sum_{(\alpha,\beta)\in D}\boldsymbol{F}_{i,j}^{\alpha,\beta} \qquad (1)$$

where

$$D=\{(\alpha,\beta)\,|\,\alpha,\beta\in\{-1,0,1\},(\alpha,\beta)\neq(0,0)\}.$$

Set D defines the arrangement of springs and characterizes the static property of an extensible object.

In this paper, we will neglect hysteresis of an extensible object and its dynamic behavior to build a simple object model for indirect simultaneous positioning. Thus, we will build a static model of an extensible object. Let us formulate the potential energy of an extensible object to build its static model. Let U be the potential energy of the object. The potential energy is given by the sum of elastic energy of all springs composing of the object model. The potential energy U is thus given by



(a) Lattice structure (b) Neighboring points

Fig. 3. Spring model of an extensible object
Neighboring lattice points are connected by linear springs. The shape of the object can be described by coodinates of all lattice points. The potential energy at a deformed shape can be computed once all coordinates are given.

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$$U=\frac{1}{2}\sum_{i=0}^{M}\sum_{j=0}^{N}\sum_{(\alpha,\beta)\in D}\frac{1}{2}\,k_{i,j}^{\alpha,\beta}\{d_{i,j}^{\alpha,\beta}\}^{2}. \tag{2}$$

The summation in the above equation counts elastic energy of each spring twice. The summation should be halved to derive the potential energy. Note that force $\boldsymbol{F}_{i,j}$ is equal to the partial derivative of U with respect to position vector $\boldsymbol{p}_{i,j}$. That is,

$$\boldsymbol{F}_{i,j}=-\frac{\partial U}{\partial \boldsymbol{p}_{i,j}}.$$

The potential energy U is a function of a set of position vectors $\boldsymbol{p}_{0,0}$ through $\boldsymbol{p}_{\mathrm{M,N}}$. The potential energy U reaches its minimum at a static stable shape. Thus, we can compute the deformed shape of a fabric by minimizing the potential energy U with respect to position vectors $\boldsymbol{p}_{0,0}$ through $\boldsymbol{p}_{\mathrm{M,N}}$ under geometric constraints imposed on the object.

3.2. *Description of indirect simultaneous positioning*

In this section, we will formulate indirect simultaneous positioning of an extensible object based on the object model. Let us construct a lattice structure of an object model so that the positioned points and the manipulated points are located on lattice points on the model. Note that positioned points do not coincide with manipulated points.

The shape of a fabric is determined by coordinates $x_{i,j}$ and $y_{i,j}$, where $i\in[0, M]$ and $j\in[0, N]$. In the indirect simultaneous positioning of an extensible object, some coodinates should be guided to their desired values. These coordinates are referred to as *positioned coordinates*. This guidance should be performed by controlling some coordinates except positioned coordinates. These coordinates are referred to as *manipulated coordinates*. Coordinates except positioned coordinates or manipulated coordinates are referred to as *non-target coordinates*. As a result, we can classify a set of coordinates into three subsets; (1) manipulated coordinates, (2) positioned coordinates, and (3) non-target coordinates. For example, consider a positioning illustrated in Figure 4-(a). In this example, three points marked as circles should be guided to their desired locations marked as crosses. This guidance is performed by controlling three points marks as triangles. In this example, a set of positioned coordinates is given by $x_{1,1}$, $y_{1,1}$, $x_{1,2}$, $y_{1,2}$, $x_{2,2}$, and $y_{2,2}$ while a set of manipulated coordinates is given by $x_{0,3}$, $y_{0,3}$, $x_{1,2}$, $y_{1,2}$, $x_{3,3}$, and $y_{3,3}$. The desired values of positioned coordinates coincide to desired coordinates of positioned points. Consider a positioning illustrated in Figure 4-(b). In this example, three points marked as circles should be aligned on a target line perpendicular to the x-axis. Note that we must guide the x-coordinates of the three points to the x-intercept of the line, while we do not have to control the y-coordinates of the three points. Thus, a set of positioned coordinates in this example is given by $x_{1,1}$, $x_{1,2}$, and $x_{2,2}$. Coordinates $y_{1,1}$, $y_{1,2}$, and $y_{2,2}$, are involved in non-target coordinates. The desired values of positioned coordinates coincide to the x-intercept of the target line.

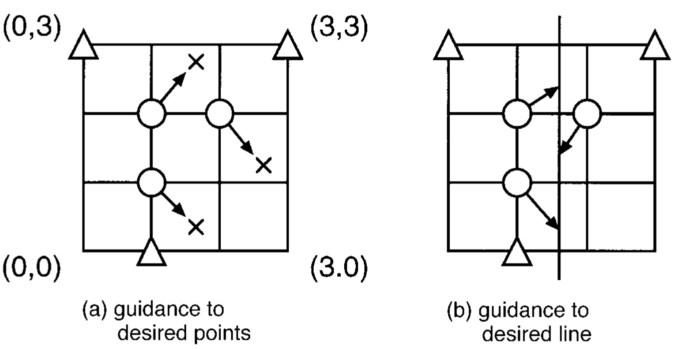

Fig. 4. Classification of coordinates
Coordinates can be classified into manipulated coordinates, positioned coordinates, and non-target coordinates. The classification depends not only on the configuration of positioned points and manipulated points but also on the desired region of positioned points.

Let $\boldsymbol{r}_m$ be a vector consisting of manipulated coordinates, $\boldsymbol{r}_p$ be a vector composed of positioned coordinates, and $\boldsymbol{r}_n$ be a vector consisting of non-target coordinates. Vectors $\boldsymbol{r}_m$, $\boldsymbol{r}_p$, and $\boldsymbol{r}_n$ are referred to as *manipulated coordinates vector*, *positioned coordinates vector*, and *non-target coordinates vector*, respectively. Let m, p, and n be dimension of vector $\boldsymbol{r}_m$, that of vector $\boldsymbol{r}_p$, and that of vector $\boldsymbol{r}_n$, respectively. For example, in a positioning shown in Figure 4-(a), we have

$$\begin{aligned}\boldsymbol{r}_m &= [x_{1,1}, x_{1,2}, y_{1,2}, x_{2,2}, y_{2,2}]^T,\\ \boldsymbol{r}_p &= [x_{0,3}, y_{0,3}, x_{1,2}, y_{1,2}, x_{3,3}, y_{3,3}]^T,\\ \boldsymbol{r}_n &= [x_{0,0}, y_{0,0}, x_{0,1}, y_{0,1}, \ldots, x_{3,2}, y_{3,2}]^T.\end{aligned}$$

Dimensions are given by $m=6$, $p=6$, and $n=24$. In a positioning shown in Figure 4-(b), we have

$$\begin{aligned}\boldsymbol{r}_m &= [x_{1,1}, x_{1,2}, x_{2,2}]^T,\\ \boldsymbol{r}_p &= [x_{0,3}, y_{0,3}, x_{1,2}, y_{1,2}, x_{3,3}, y_{3,3}]^T,\\ \boldsymbol{r}_n &= [y_{1,1}, y_{1,2}, y_{2,2}, x_{0,0}, y_{0,0}, \ldots, x_{3,2}, y_{3-2}]^T.\end{aligned}$$

Dimensions are given by $m=3$, $p=6$, and $n=27$.

Note that individual positioned coordinates should be guided to their desired values. This implies that all elements composing vector $\boldsymbol{r}_p$ have there desired values.

Let $\boldsymbol{r}_p^*$ be a vector consisting of the desired values of the positioned coordinates. Then, the goal of indirect simultaneous positioning is given by an equation; $\boldsymbol{r}_p=\boldsymbol{r}_p^*$. This goal must be achieved by controlling manipulated coordinates, $\mathbf{r}_m$.

3.3. *Force equilibrium during indirect simultaneous positioning*

In this section, we will derive equilibrium equations on an extensible object. During indirect simultaneous positioning of an extensible object, internal forces caused by elasticity of the object and external forces acting on it at manipulated points by mechanical pinching fingers are balanced. The location of positioned points is characterized by equilibrium equations on the object. Equilibrium equations are thus required to derive a control law for indirect simultaneous positioning.

Note that the potential energy of an extensible object can be computed from coordinates of lattice points on an object

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model. Moreover, individual coordinates are involved in $\boldsymbol{r}_m$, $\boldsymbol{r}_p$, or $\boldsymbol{r}_n$. This implies that the potential energy is a function of vectors $\boldsymbol{r}_m$, $\boldsymbol{r}_p$, and $\boldsymbol{r}_n$. Thus, let us describe the potential energy of an extensible object as $U(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n)$. Note that external forces are exerted on an object at manipulated points by machanical pinching fingers. Let $\boldsymbol{\lambda}_{i,j} = [\lambda^x_{i,j}, \lambda^y_{i,j}]^T$ be an external force at a manipulated point $\boldsymbol{p}_{i,j}$. Let $\boldsymbol{\lambda}$ be a vector consisting of all external forces at manipulated points. Vector $\boldsymbol{\lambda}$ is m-dimensional. Equations of equlibrium at manipulated points are then described collectively as follows:

$$\frac{\partial U}{\partial \boldsymbol{r}_m} - \boldsymbol{\lambda} = \mathbf{o}_m \tag{3}$$

No external forces are exerted on positioned points and non-controlled points. Equations of equilibrium at positioned points and non-controlled points are written as follows, respectively:

$$\frac{\partial U}{\partial \boldsymbol{r}_p} = \mathbf{o}_p \tag{4}$$

$$\frac{\partial U}{\partial \boldsymbol{r}_n} = \mathbf{o}_n \tag{5}$$

The above equations are satisfied at a stable state of an extensible object. Assuming that an object model is completely exact, we can compute the location of manipulated points that achieve the given simultaneous positioning by solving the above equations. However, because of the discrepancy between an actual object and its model, a model-inversion approach is impractical. Thus, it is necessary to develop a control law, which is robust to the discrepancy.

4. LINEARIZED ROBUST CONTROL LAW FOR INDIRECT SIMULTANEOUS POSITIONING

In this section, we will propose a novel control law for indirect simultaneous positioning of deformable objects. Applying the proposed control law, the guidance of positioned coordinates to their desired values can be achieved by controlling manipulated coordinates. First, we will derive a linearized model of an extensible object. Second, a iterative control law for indirect simultaneous positioning will be derived based on the linearized model of an extensible object. The robustness of the proposed control law is then investigated theoretically.

4.1. *Linearized model of extensible objects*

Let us derive a linearized model of extensible object in order to construct a linear control law for indirect simultaneous positioning. Let $\boldsymbol{r}_m$, $\boldsymbol{r}_p$, and $\boldsymbol{r}_n$ be manipulated coordinates vector, positioned coordinates vector, and non-target coordinates vector at an equilibrium, respectively. Consider the behavior of an extensible object in the neighborhood around the equilibrium. Let $\delta\boldsymbol{r}_m$, $\delta\boldsymbol{r}_p$, and $\delta\boldsymbol{r}_n$ be deviations from the equilibrium. Linearizing eqn. (4) around the equilibrium yields

$$\frac{\partial^2 U}{\partial \boldsymbol{r}_m \partial \boldsymbol{r}_p} \delta\boldsymbol{r}_m + \frac{\partial^2 U}{\partial \boldsymbol{r}_n \partial \boldsymbol{r}_p} \delta\boldsymbol{r}_n + \frac{\partial^2 U}{\partial \boldsymbol{r}_p \partial \boldsymbol{r}_p} \delta\boldsymbol{r}_p = \boldsymbol{o}_p.$$

The partial derivatives in this equation are evaluated at the equilibrium. Vector $\boldsymbol{o}_P$ is a p-dimensional zero vector. Linearizing eqn. (5) around the equilibrium, we have

$$\frac{\partial^2 U}{\partial \boldsymbol{r}_m \partial \boldsymbol{r}_n} \delta\boldsymbol{r}_m + \frac{\partial^2 U}{\partial \boldsymbol{r}_n \partial \boldsymbol{r}_n} \delta\boldsymbol{r}_n + \frac{\partial^2 U}{\partial \boldsymbol{r}_p \partial \boldsymbol{r}_n} \delta\boldsymbol{r}_p = \boldsymbol{o}_n.$$

Vector $\boldsymbol{o}_n$ is n-dimensional zero vector. Combining the above equations yields

$$A\delta\boldsymbol{r}_m + B\delta\boldsymbol{r}_n + C\delta\boldsymbol{r}_p = \boldsymbol{o}_{p+n} \tag{6}$$

where

$$A = \begin{bmatrix} \dfrac{\partial^2 U}{\partial \boldsymbol{r}_m \partial \boldsymbol{r}_p}(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n) \\ \dfrac{\partial^2 U}{\partial \boldsymbol{r}_m \partial \boldsymbol{r}_n}(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n) \end{bmatrix},$$

$$B = \begin{bmatrix} \dfrac{\partial^2 U}{\partial \boldsymbol{r}_n \partial \boldsymbol{r}_p}(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n) \\ \dfrac{\partial^2 U}{\partial \boldsymbol{r}_n \partial \boldsymbol{r}_n}(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n) \end{bmatrix},$$

$$C = \begin{bmatrix} \dfrac{\partial^2 U}{\partial \boldsymbol{r}_p \partial \boldsymbol{r}_p}(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n) \\ \dfrac{\partial^2 U}{\partial \boldsymbol{r}_p \partial \boldsymbol{r}_n}(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n) \end{bmatrix}.$$

Matrix A is a $(p+n)\times m$ matrix, B is a $(p+n)\times n$ matrix, C is a $(p+n)\times p$ matrix. Equation (6) is a collection of equilibrium equations corresponding to positioned coordinates and non-target coordinates, where no external forces are applied. Matrices A, B, and C are stiffness matrices, which depend on coordinates vectors $\boldsymbol{r}_m$, $\boldsymbol{r}_p$, and $\boldsymbol{r}_n$.

Let us derive the relationship between $\delta\boldsymbol{r}_m$ and $\delta\boldsymbol{r}_p$ as well as the relationship between $\delta\boldsymbol{r}_n$ and $\delta\boldsymbol{r}_p$ in order to derive a control law for positioning. Equation (6) can be rewritten as follows:

$$F\begin{bmatrix} \delta\boldsymbol{r}_m \\ \delta\boldsymbol{r}_n \end{bmatrix} = -C\delta\boldsymbol{r}_p$$

where $F = [A\ B]$. Matrix F is a $(p+n)\times(m+n)$ matrix, which depend on coordinates vectors $\boldsymbol{r}_m$, $\boldsymbol{r}_p$, and $\boldsymbol{r}_n$. Note that matrix F is a square matrix when dimension p is equal to m. This implies that matrix F is square when the number of positioned coordinates is equal to the number of manipulated coordinates. In addition, if the matrix F is invertible, we have the following equation:

$$\begin{bmatrix} \delta\boldsymbol{r}_m \\ \delta\boldsymbol{r}_n \end{bmatrix} = -F^{-1}C\delta\boldsymbol{r}_p.$$

This equation can be divided into the following two equations:

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$$\delta \boldsymbol{r}_m = -S_U F^{-1} C \delta \boldsymbol{r}_p, \tag{7}$$

$$\delta \boldsymbol{r}_n = -S_D F^{-1} C \delta \boldsymbol{r}_p, \tag{8}$$

where

$$S_U = [I_m \; O_{m\times n}],$$
$$S_D = [O_{n\times m} \; I_n].$$

Matrix S_U is a $m\times(m+n)$ matrix and S_D is a $n\times(n+m)$ matrix. From the above equations, we find that displacement $\delta \boldsymbol{r}_m$ can be determined uniquely for an arbitrary infinitesimal error of positioning coordinates, $\delta \boldsymbol{r}_p$, if matrix F is invertible.

4.2. *Iterative control law for indirect simultaneous positioning*

In this section, we will propose an iterative control law to achieve a given indirect simultaneous positioning of an extensible object. The control law will be derived based on the linearized model of an extensible object, that is, eqns. (7) and (8).

Note that a vision system is introduced to measure the current values of positioned coordinates. This implies that the current value of positioned coordinates vector $\boldsymbol{r}_m$ can be measured through a vision system. Moreover, recall that mechanical fingers pinch an extensible object and no slip between the fingers and the object occurs. Namely, the current value of manipulated coordinates vector $\boldsymbol{r}_m$ can be computed from the motion of mechanical pinching fingers. On the hand, the current values of non-target coordinates cannot be measured. Let $\boldsymbol{r}_m^k$ be the value of manipulated coordinates vector at the k-th iteration and $\boldsymbol{r}_n^k$ be the value of non-target coordinates vector at the k-th iteration. Let $\boldsymbol{r}_p^k$ be the value of positioned coordinates vector at the k-th iteration, which can be measured through a vision system. Let F_k and C_k be stiffness matrices at the k-th iteration, say, $F_k = F(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n)$ and $C_k = C(\boldsymbol{r}_m, \boldsymbol{r}_p, \boldsymbol{r}_n)$.

Let us derive a recursive law to update the manipulated coordinates vector based on eqn. (7). Assume that matrix F is invertible. Substituting deviation $\delta \boldsymbol{r}_m$ by difference $\boldsymbol{r}_m^k - \boldsymbol{r}_m^{k-1}$ and deviation $\delta \boldsymbol{r}_p$ by error $\boldsymbol{r}_p^* - \boldsymbol{r}_{k-1}$, we find the following equation:

$$\boldsymbol{r}_m^k = \boldsymbol{r}_m^{k-1} - g_p S_U F_{k-1}^{-1} C_{k-1} (\boldsymbol{r}_p^d - \boldsymbol{r}_p^{k-1}), \tag{9}$$

where scalar g_p is a gain. The right side of this equation can be evaluated at the $(k-1)$-th iteration. Thus, the value of manipulated coordinates vector at the $(k-1)$-th iteration can be updated into the value at the k-th iteration by this equation. Note that matrix C^{-1} depends not only on $\boldsymbol{r}_p$ and $\boldsymbol{r}_m$ but on $\boldsymbol{r}_n$. Thus, it is necessary to update the value of non-target coordinates vector $\boldsymbol{r}_n$. Let us derive a recursive law to update non-target coordinates vector based on eqn. (8). Let us substitute deviation $\delta \boldsymbol{r}_n$ by difference $\boldsymbol{r}_n^k - \boldsymbol{r}_n^{k-1}$. Note that we have to substitute deviation $\delta \boldsymbol{r}_p$ by a quantity which can be evaluated at the $(k-1)$-th iteration. Thus, we substitute deviation $\delta \boldsymbol{r}_p$ by difference $\boldsymbol{r}_p^{k-1} - \boldsymbol{r}_p^{k-2}$, which can be evaluated at the $(k-1)$-th iteration. These substitutions yield the following equation:

$$\boldsymbol{r}_n^k = \boldsymbol{r}_n^{k-1} - g_p S_D F_{k-1}^{-1} C_{k-1} (\boldsymbol{r}_p^{k-1} - \boldsymbol{r}_p^{k-2}). \tag{10}$$

Table I. Iterative control law for indirect simultaneous positioning.

Step 1	Measure r_p^k by vision system.
Step 2	Evaluate error $e(k) = \|r_p^k - r_p^*\|$. Stop if the error is smaller than a predefined value. Increase k otherwise.
Step 3	Compute $F_{k-1} = F(r_m^{k-1}, r_p^{k-1}, r_n^{k-1})$ and $C_{k-1} = C(r_m^{k-1}, r_p^{k-1}, r_n^{k-1})$.
Step 4	Update r_m^k and r_n^k.
Step 5	Command r_m^k to mechanical fingers.
Step 6	Goto Step 1.

The right side of this equation can be evaluated at the $(k-1)$-th iteration. Thus, the value of non-target coordinates vector at the k-th iteration can be updated into the value of the k-th iteration by this equation.

As a result, control law for indirect simultaneous positioning is summarized as shown in Table I. First, the current value of positioned coordinates vector is measured by a vision system. Second, stiffness matrices F_{k-1} and C_{k-1} are computed. Next, manipulated coordinates vector and non-target coordinates vector are updated using eqns. (9) and (10), respectively. Then, the value of manipulated coordinates vector $\boldsymbol{r}_m^k$ is commanded to a set of pinching fingers so that an extensible object can be deformed. This procedure is iterated until an error of positioning coordinates vector is close to zero.

4.3. *Robustness of linearized control law*

In this section, we will investigate the robustness of the proposed control law for indirect simultaneous positioning. Assume that the static behavior of an actual extensible object is given in eqns. (7) and (8). Supposing that differences $\boldsymbol{r}_m^k - \boldsymbol{r}_m^{k-1}$ and $\boldsymbol{r}_p^k - \boldsymbol{r}_p^{k-1}$ are sufficiently small, we find that the following equation is satisfied:

$$\boldsymbol{r}_m^k - \boldsymbol{r}_m^{k-1} = -S_U F_{k-1}^{-1} C_{k-1} (\boldsymbol{r}_p^k - \boldsymbol{r}_p^{k-1}).$$

The above equation can be described simply as follows:

$$\boldsymbol{r}_m^k = \boldsymbol{r}_m^{k-1} - Q_{k-1} (\boldsymbol{r}_p^k - \boldsymbol{r}_p^{k-1}), \tag{11}$$

$$Q_{k-1} = S_U F_{k-1}^{-1} C_{k-1}. \tag{12}$$

Recall that matrix F and C involve errors due to the discrepancy between an actual extensible object and its model. Assume that this discrepancy results in the identification errors of spring constants in the object model. Let us denote matrix F with errors by $\tilde{F}$ and matrix C with errors $\tilde{C}$. Then, an iterative control law given in eqn. (9) can be rewritten as follows:

$$\boldsymbol{r}_m^k = \boldsymbol{r}_m^{k-1} = -g_p S_U F_{\tilde{k-1}}^{-1} C_{\tilde{k-1}} (\boldsymbol{r}_p^* - \boldsymbol{r}_p^{k-1}).$$

The above equation can be described simply as follows:

$$\boldsymbol{r}_m^k = \boldsymbol{r}_m^{k-1} - \tilde{Q}_{k-1} (\boldsymbol{r}_p^* - \boldsymbol{r}_p^{k-1}) \tag{13}$$

$$\tilde{Q}_{k-1} = g_p S_U F_{\tilde{k-1}}^{-1} C_{\tilde{k-1}} \tag{14}$$

Subtracting eqn. (13) from eqn. (11) yields

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$$Q_{k-1}(\boldsymbol{r}_p^k - \boldsymbol{r}_p^{k-1}) = \tilde{Q}_{k-1}(\boldsymbol{r}_p^* - \boldsymbol{r}_p^{k-1}).$$

Since F is a regular matrix, matrix Q_{k-1} is invertible. Thus, we have

$$\boldsymbol{r}_p^k = \boldsymbol{r}_p^{k-1} + Q_{k-1}^{-1}\tilde{Q}_{k-1}(\boldsymbol{r}_p^* - \boldsymbol{r}_p^{k-1}).$$

From the above equation, we have the following equation:

$$\boldsymbol{r}_p^* - \boldsymbol{r}_p^k = (I_p - Q_{k-1}^{-1}\tilde{Q}_{k-1})(\boldsymbol{r}_p^* - \boldsymbol{r}_p^{k-1}). \tag{15}$$

Finally, we can obtain

$$\|\boldsymbol{r}_p^* - \boldsymbol{r}_p^k\| \le \|I_p - Q_{k-1}^{-1}\tilde{Q}_{k-1}\|\,\|\boldsymbol{r}_p^* - \boldsymbol{r}_p^{k-1}\|. \tag{16}$$

Therefore, we find that

$$\boldsymbol{r}_p^k \rightarrow \boldsymbol{r}_p^* \quad \text{as} \quad k \rightarrow \infty \tag{17}$$

if matrix $\tilde{Q}_k$ satifies

$$\|I_p - Q_k^{-1}\tilde{Q}_k\| < 1. \tag{18}$$

Thus, we find that condition for the convergence of the positioned coordinates vector is given by eqn. (18).

Individual elements of matrix $\tilde{Q}_k$ decrease when gain g_p decreases. This implies that the convergence of the proposed control law is satisfied by decreasing the gain g_p despite errors involved in matrix $\tilde{Q}_k$. The speed of convergence is, however, reduced when the gain g_p decreases.

5. EXPERIMENTAL VERIFICATION

In this section, we will experimentally verify the robustness of the proposed control law for indirect simultaneous positioning and will investigate the effect of model errors to the positioning.

The experimental setup is shown in Figure 5. Successive images of an extensible object are captured by a CCD camera, Toshiba IK-M43H. The images are sent to a vision board, Fujitsu Tracking Vision, which is installed on a PC. This vision board has a capacity of finding some patterns in an image captured by a CCD camera. Patterns are memorized in the board in advance. Using this capacity, we can measure the coordinates of positioned points. Three

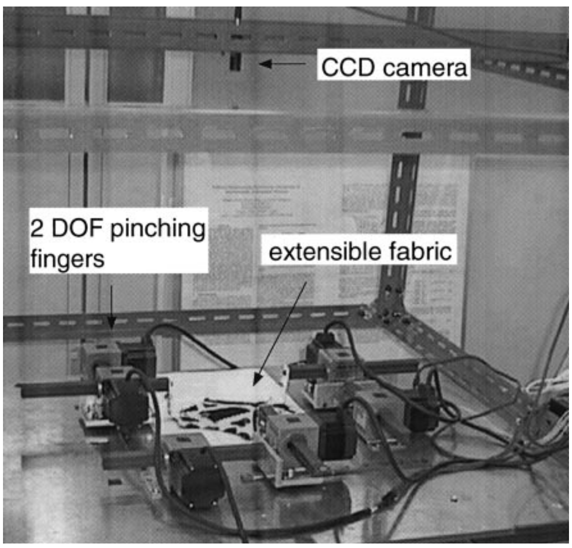

Fig. 5. Experimental setup
A CCD camera measures the location of positioned points. Three 2-DOF pinching fingers deform an extensible fabric.

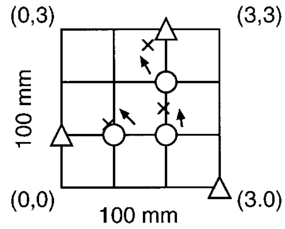

Fig. 6. Configuration of positioned points and manipulated points in experiments
Three positioned points, marked as circles, should be guided to their desired locations by controlling three manipulated points, marked as triangles.

2-DOF fingers are located around the extensible object. Each finger is driven by two stepping motors: Oriental motor LMS2B250PK-1, which is controlled by the PC through a motor control board, Adtek system science aISA-M59.

A knitted fabric of acrylic 85[%] and wool 15[%] with an area of 100[mm]$\times$100[mm] is used in the experiments. A model of the fabric is composed of 4×4 lattice points. Locations of positioned points and those of manipulated points are illustrated in Figure 6. As shown in the figure, we have three positioned points, marked as circles, and have three manipulated points, marked as triangles. Positioned coordinates vector $\boldsymbol{r}_p$ and manipulated coordinates vector $\boldsymbol{r}_m$ are thus given as follows:

$$\boldsymbol{r}_p = [x_{1,1}, y_{1,1}, x_{1,2}, y_{1,2}, x_{2,2}, y_{2,2}]^T,$$
$$\boldsymbol{r}_m = [x_{0,3}, y_{0,3}, x_{1,0}, y_{1,0}, x_{3,2}, y_{3,2}]^T.$$

The desired value of positioned coordinates vectors is assumed to be

$$\boldsymbol{r}_p^* = [30, 40, 65, 50, 54, 90]^T.$$

The desired locations of positioned points are marked as crosses in the figure.

Assume that all spring constants of horizontal springs coincide one with another. Also, we assume that all spring constants of vertical springs are equal to one another and that all spring constants of diagonal springs coincide one with another. Let k_c, k_w, and k_s be the spring constant of horizontal springs, that of vertical springs, and that of diagonal springs, respectively. From a tensile test of fabric, we have identified spring constants as $k_c = 4.17$[gf/mm], $k_w = 13.2$[gf/mm], and $k_s = 3.32$[gf/mm]. Note that the ratio of the three spring constants is critical in our control law. Thus, let us define $\alpha = k_c/k_s$ and $\beta = k_w/k_s$. From identified spring constants, we have $\alpha = 1.256$ and $\beta = 3.976$. In experiments, various values of α and β are used in the control law so that the robustness of proposed control law against model identification errors can be investigated. Moreover, values 0.1 and 0.5 of gain g_p are used in eqns. (9) and (10) to examine the effect of the gain.

In the following experiments, we will evaluate how positioned coordinates converge to their desired values. Let $e(k)$ be the square root of positioned coordinates errors at the k-th iteration. That is,

$$e(k) = \|\boldsymbol{r}_p^k - \boldsymbol{r}_p^*\|.$$

Let us plot the value of $e(k)$ with respect to the number of iterations. Experimental results when the gain g_p is equal to

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0.1 and the ratio α takes its original value are plotted in Figure 7-(a). In this experiment, the ratio β takes various values; the original value, ten times the value, one hundred times the value, one-tenth of the value, one-hundred of the value, and one-thousand of the value. As described in the figure, positioned coordinates converge to their desired values when ratio β is greater than or equal to one-hundred of its original value and is less than or equal to ten times of its original value. Positioning has failed when ratio β takes hundred times of its original value or one-thousand of its original value. Graphs corresponding to these values are not plotted in the figure. Experimental results when the gain g_p is equal to 0.1 and the ratio β takes its original value are plotted in Figure 7-(b). The ratio α takes various values; the original value, one-tenth of the value, one-hundred of the value, one-thousand of the value, ten times of the value, and one hundred times of the value. As described in the figure, positioning has succeeded except when the ratio α takes one hundred times of its original value or one-thousand of its original value.

Experimental results when the gain g_p is equal to 0.5 and the ratio α takes its original value are plotted in Figure 8-(a). It is found that the error norm diverges and that positioning fails when the ration β is equal to ten times of its original value or when it is equal to one-hundred of its original value. Graphs corresponding to these values are not plotted in the figure. Comparing with Figure 7-(a), we find that the admissable range of ratio β is narrow. Experimental results when the gain g_p is equal to 0.5 and the ratio β takes its original value are plotted in Figure 8-(b). It is found that the error norm diverges and that positioning fails when the ratio β is equal to hundred times of its original value or when is equal to one-tenth of its original value. Graphs corresponding to these values are not plotted in the figure. Comparing with Figure 7-(b), we find that the admissable range of ratios α is narrow.

As described in the figures, positioned coordinates converge to their desired values as long as ratios α and β take values closer of their indentified values. Positioned coordinates are oscillatory, or diverge if either α or β is far from the identified values. Figures 7(a) and 7-(b) show that positioned coordinates converge to their desired values when ratios α and β take 10 times of their original values or 0.01 times of their original values. From these results, we conclude that our proposed control law is robust against model errors. Comparing Figures 7 and 8, we find that the admissable range of ratios when the gain g_p is equal to 0.1 is wider than that when the gain g_p is equal to 0.5. The speed

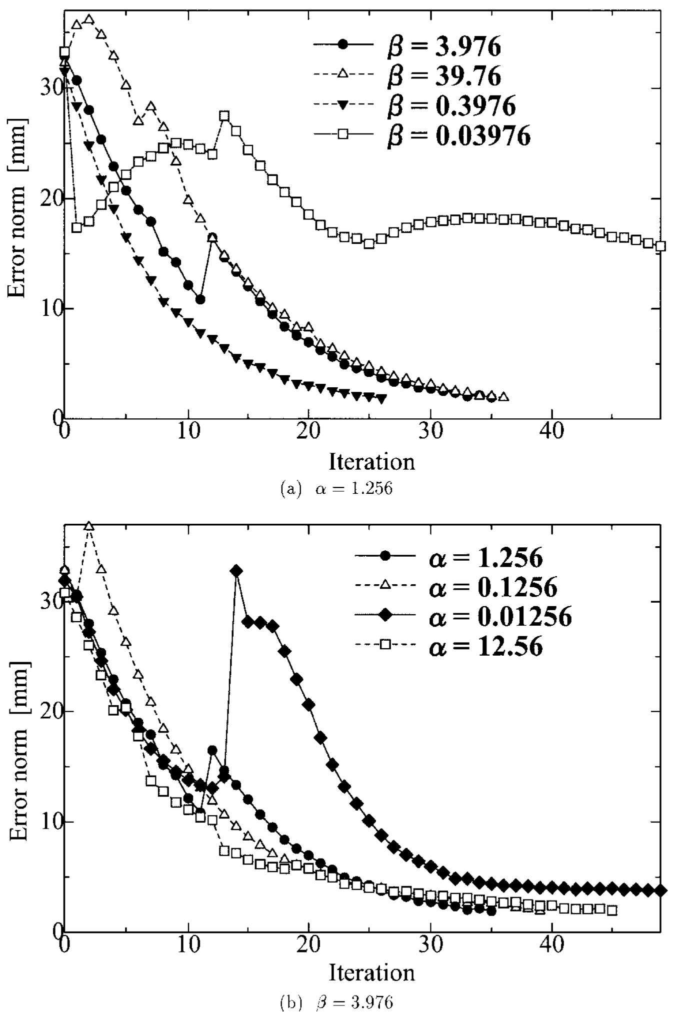

Fig. 7. Robustness of the control law against model errors for a small gain
Error $e(k)$ when g_p=0.1 is plotted for various values of α and β.

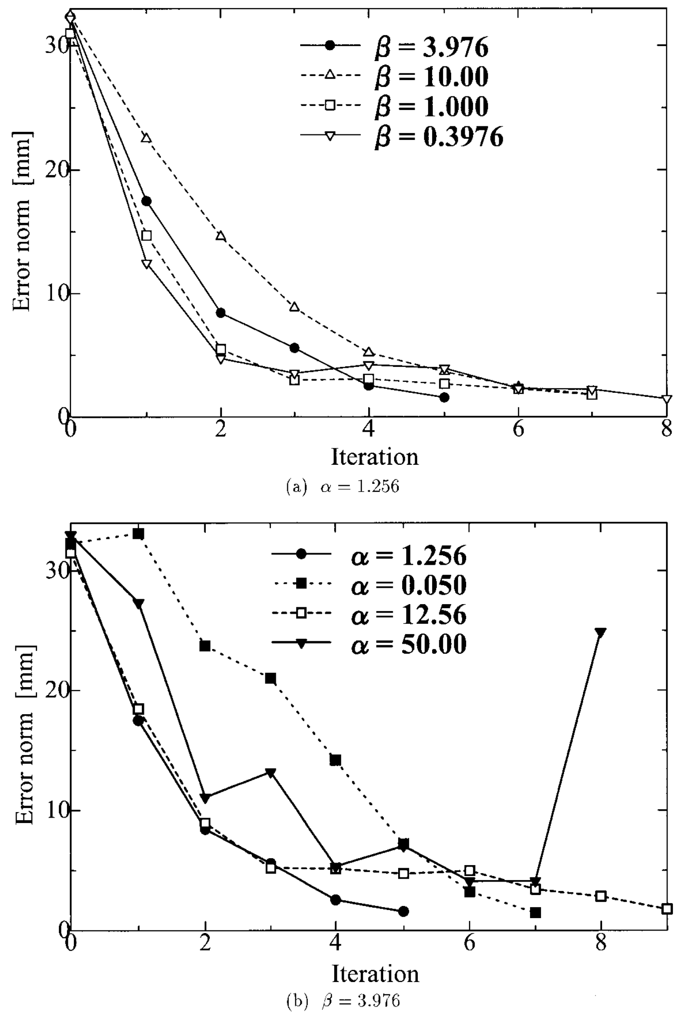

Fig. 8. Robustness of the control law against model errors for a large gain
Error $e(k)$ when g_p=0.5 is plotted for various values of α and β.

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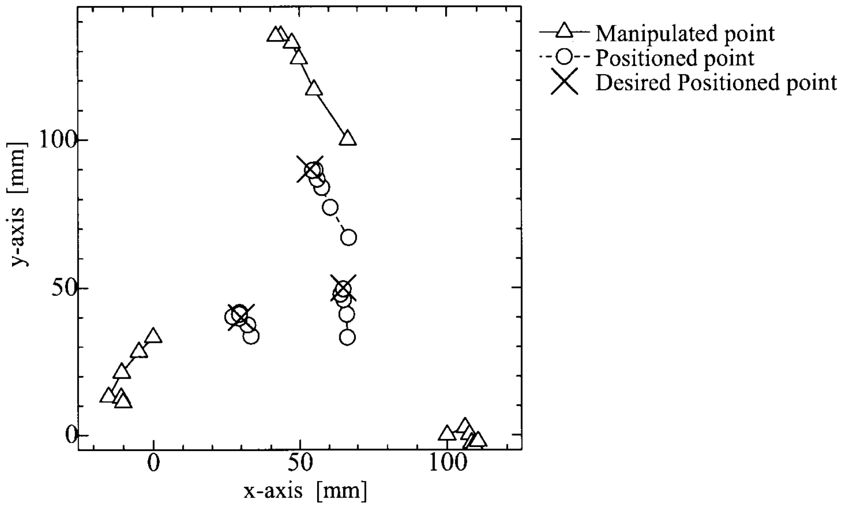

Fig. 9. Behavior of positioned points and manipulated points
Locations of positioned points and manipulated points when $g_p = 0.5$, $\alpha = 1.256$, and $\beta = 3.976$ are plotted. Three positioned points, marked as circles, are guided to their desired locations, marked as crosses, by moving three manipulated points, marked as triangles.

of convergence is, however, larger when the gain g_p is equal to 0.5 than when the gain g_p is equal to 0.1. Namely, the robustness is greater and the convergence is slower for a small gain, while the robustness is smaller and the convergence is faster for a large gain. Thus, we have to choose a small gain when model errors are supposed to be large while we can take a large gain when model errors are supposed to be small.

Figure 9 shows the behavior of positioned coordinates and manipulated coordinates when $g_p = 0.5$, $\alpha = 1.256$, and $\beta = 3.976$. The accuracy of the convergence to the desired values has reached the resolution of the vision sensor, which is almost 1[mm]. According to the above experimental results, we conclude that uncertain identified model parameters can be applied to our proposed control law in a practical indirect simultaneous positioning of extensible fabric.

6. CONCLUDING REMARKS

In this paper we have proposed a novel control law for indirect simultaneous positioning of deformable objects, and the robustness of the proposed control law against model uncertainty is shown theoretically and experimentally. First, we have proposed a mathematical model of an extensible object. Second, indirect simultaneous positioning of an extensible object has been formulated based on an object model. Next, we have proposed a novel iterative control law for indirect simultaneous positioning based on a linearized object model. Then, the robustness of the proposed control law has been analytically proved. Finally, we have investigated the robustness of the control law against model errors experimentally. We have found the proposed control law is robust against model errors and that roughly identified object model can be applied to the control of indirect simultaneous positioning.

The contribution of this paper is summarized as follows: (1) We have proposed a simple control law for indirect simultaneous positioning of deformable objects, and (2) We have shown that an uncertain object model is acceptable in the proposed control law for indirect simultaneous positioning. Our approach is to build a robust control system based on an uncertain model. Note that model uncertainty is inevitable in actual handling of deformable objects. Consequently, our approach can be applied to various handling including grasping, assembly, and positioning of deformable objects, as well as handling of objects by flexible fingers.

References

1. D. Terzopoulous and K. Fleisher, "Modeling inelastic deformation: Viscoelasticity, Plasticity, Fracture", *Computer Graphics* **22** No.4, 269–278, (May 1988).
2. H. Wakamatsu, S. Hirai and K. Iwata, "Modeling of linear objects considering Bend, Twist, and Extensional Deformations", *Proc. IEEE Int. Conf. on Robotics and Automation*, Nagoya (May 1995) **vol 1**, pp. 433–438.
3. A. Joukhader, A. Deguet and C. Laugie, "A collision model for rigid and deformable bodies", *Proc. IEEE Int. Conf. on Robotics and Automation*, Alberquerque, (May 1998) pp. 982–988.
4. Y. Chai and G.R. Luecke, "Virtual clay modeling using the ISU Exoskeleton", *Proc. IEEE Virtual Reality Annual Int. Symp*, (1998) pp. 76–80.
5. P.M. Taylor et al., *Sensory Robotics for the Handling of Limp Materials* (Springer-Verlag, Berlin, 1990).
6. Y.F. Zheng and C. Chen, "Strategies for automatic assembly of defromable objects", *Proc. IEEE Int. Conf. on Robotics and Automation*, (1991) pp. 2598–2603.
7. E. Ono et al., "Strategy for unfolding a fabric piece by cooperative sensing of touch and vision", *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, (1995) pp. 441–445.
8. K. Kosuge, H. Yoshida, T. Fukuda, M. Sakai and K. Kaitani, "Manipulation of flexible objects by dual manipulators", *Proc. IEEE Int. Conf. on Robotics and Automation*, (1995) pp. 318–323.
9. D. Sun, Y. Liu and J.K. Mills, "Cooperative control of a two-manipulator system handling a general flexible object", *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, (1997) pp. 5–10.

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