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OPTION BASED PORTFOLIO INSURANCE REVISITED

By R. Bouchaib

ABSTRACT

In recent years, Constant Proportion Portfolio Insurance (CPPI) has been the most widely recognised form of portfolio insurance among market practitioners, despite a lack of theoretical framework to support it. This paper presents a revised formulation of Option Based Portfolio Insurance (OBPI) and shows, through a case study, how it can be used as a structured product and applied in practice as a dynamic investment strategy for insurance and pensions funds such as with-profits funds. CPPI and the Revised Option Based Portfolio Insurance (ROBPI) technique adopted in this paper are similar in the sense that they rely on dynamic allocation between risky and risk-free assets to provide downside protection. Comparison between the two methods shows that ROPBI is more efficient and forward looking, giving more information about downside risk and producing less volatile asset allocation, which reduces transaction costs and any market impact.

KEYWORDS

Option Based Portfolio Insurance; Constant Proportion Portfolio Insurance; Option replicating portfolio; Structured products, Call option; Call-spread option; Dynamic Asset Allocation; With-profits funds; Risky-asset.

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1. Introduction

The formulation of option based portfolio insurance (OBPI) was first described by Leland & Rubinstein (1976). It is an investment strategy which combines conventional assets and vanilla options to achieve a diversified risky portfolio exposure and to protect the initial investment from downside risk. The following two investment strategies define OBPI, as outlined in portfolio management literature:
— hold a diversified risky portfolio and buy a put option; and
— hold a risk-free asset and buy a call option.

Based on a put approach, OBPI is expressed as follows:

$$V = S + \text{Put}$$

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where:
— V is the value of an investment portfolio;
— S is the diversified risky asset;
— Put is a put option on S with maturity T and strike K;
— K is the floor representing the level of protection for V at maturity; and
— T is the investment period.

The payout of this strategy at the end of the investment period is $V_T = S_T + (K - S_T)^+$. This is equal to:

$$\begin{aligned} V_T &= S_T \qquad \text{if } K \leq S_T \\ V_T &= K \qquad \text{if } K \geq S_T. \end{aligned}$$

The equivalent strategy based on a call approach can be expressed as follows:

$$V = ZC + \text{Call}$$

where:
— ZC is a risk-free zero coupon bond such as $ZC = K \cdot e^{-r \cdot T}$ and r is the risk-free rate; and
— Call is a call option on S with maturity T and strike K.

The payout of this strategy at the end of the investment period is:

$$V_T = K + (S_T - K)^+.$$

This is equal to:

$$\begin{aligned} V_T &= K \qquad \text{if } K \geq S_T \\ V_T &= S_T \qquad \text{if } K \leq S_T. \end{aligned}$$

Throughout this paper, it is assumed that:
— any dividends from the risky asset are not paid out, instead they are reinvested within the portfolio; and
— all options are European (however, the results of this paper are also valid for American style floors. The price of an American call option is equal to the European one when there are no dividend payouts).

To date, OBPI, as formulated in the literature, has not been widely used by financial institutions, despite its derivation from option pricing theory. This formulation of OBPI cannot be directly used in practice, since the value of the portfolio V is known in advance and ZC and the call option are priced by the market. The method proposed in this paper, ROBPI, is a self-funded formulation of OBPI, which renders it directly applicable to any investment portfolio.

It will be shown that the simplest form of ROBPI implies that the risky asset should be increased when it performs well and reduced when it underperforms. This is also the main dynamic process behind constant proportion portfolio insurance (CPPI). CPPI has been the most popular portfolio insurance technique among market practitioners to date, since there have been no better alternatives available in the literature. A comparison between ROBPI and CPPI in Section 2.7 will show that the former is more flexible and gives a less volatile asset allocation, which reduces transaction costs and adverse market impact.

2. Revised Option-Based Portfolio Insurance

This section introduces a revised formulation of OBPI which can be applied in practice as portfolio insurance. This formulation is derived from a pricing technique of structured products called guaranteed equity bonds (GEB) in the United Kingdom.

2.1 *ROBPI using Traded Options*

To ensure that the option based portfolio insurance formula is verified, it requires the introduction of an extra parameter which makes the cost of the portfolio insurance equal to the value of the investment portfolio. Since ZC provides the required floor level, the quantity of call options needs to be adjusted instead of being set arbitrarily equal to one unit. Therefore, after investing in the ZC to provide the floor, the remaining assets should be invested to buy a number of call options to provide participation in risky-asset growth. The number of call options depends on the value of the investment portfolio, the ZC and the call option price. Since the risky asset is non-dividend paying stock, this implies that the number of call options is always between zero and one. This could be further verified in the Black & Scholes' (B&S) call option formula (e.g. Hull, 2000). ROBPI is now defined as follows:

$$V = ZC + \lambda \cdot \text{Call}$$

where:

— λ is a call proportion, such as:

$$\left(\frac{V - ZC}{\text{Call}}\right)^{+} = \left(\frac{V - ZC}{V \cdot N(d_1) - N(d_2) \cdot ZC}\right)^{+};$$

— λ represents the rate of participation in the risky-asset growth;

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— Call is the Black & Scholes (1973) call option price;

— $d_1 = \dfrac{\ln\left(\dfrac{V}{ZC}\right) + \left(\dfrac{\sigma^2}{2}\right) \cdot T}{\sigma \cdot \sqrt{T}}$;

— $d_2 = d_1 - \sigma \cdot \sqrt{T}$;

— σ is the implied volatility of the risky asset; and

— $N(.)$ is the normal distribution.

The participation in risky-asset growth increases with the value of the portfolio V, but decreases with the ZC and the call option price. The formula above is used to price GEBs based on equity indices, which exclude divided payouts. This gives a higher participation in the risky-asset growth, which could be higher than 100%, depending on the level of the risk-free interest rate.

The payout of this strategy at the end of the investment period is:

$$V_T = K + \lambda \cdot (S_T - K)^+.$$

This is equal to:

$$\begin{aligned} V_T &= K && \text{if } K \geq S_T \\ V_T &= K + \lambda \cdot (S_T - K) && \text{if } K \leq S_T. \end{aligned}$$

2.2 *ROPBI using a Put Option*

ROBPI using a put option can be derived from a call formulation by using call-put parity. Replacing the call option by its equivalent put option structure gives the following portfolio:

$$V = \lambda \cdot (S + \text{Put}) + (1 - \lambda) \cdot ZC.$$

This portfolio indicates that an equity portfolio aiming to achieve self-funded portfolio insurance by purchasing a put option should also invest in the risk-free asset. The literature regarding this portfolio insurance implies that liquidating part of a risky-asset portfolio to finance the purchase of a put option is sufficient to provide the required level of protection K. In contrast, the ROBPI formula above shows that the protection will be incomplete without investing in the risk-free asset.

The payout of this strategy at the end of the investment period is:

$$V_T = (1 - \lambda) \cdot K + \lambda \cdot S_T + \lambda \cdot (K - S_T)^+.$$

This is equal to:

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$$V_T = K + \lambda \cdot (S_T - K) \qquad \text{if } K \leq S_T$$

$$V_T = K \qquad \text{if } K \geq S_T.$$

While the call and put option strategies are equivalent, the focus of this paper is the call option strategy, which provides a simpler and more flexible formulation using a call-spread strategy.

2.3 *ROBPI using Replicated Options*

The aim of this section is to convert the formula used in pricing GEBs into dynamic portfolio insurance, targeting the same payout profile as traded options. Under B&S assumptions, this can be achieved by using option replicating portfolios. Using the option replication formula in Baxter & Rennie (2000), portfolio insurance can be expressed as follows:

$$V_t = ZC \cdot B_t + \lambda \cdot [V_t \cdot N(d_1) \cdot S_t - N(d_2) \cdot ZC \cdot B_t]$$

where:

— $(V_t \cdot N(d_1) \cdot S_t - N(d_2) \cdot ZC \cdot B_t)$ is the B&S replicating portfolio of a call option;

— $B_t = e^{r \cdot t}$ is a deterministic cash bond price; and

— $d_1 = \dfrac{\ln\left(\dfrac{V_t}{ZC \cdot B_t}\right) + \left(\dfrac{\sigma^2}{2}\right) \cdot (T - t)}{\sigma \cdot \sqrt{(T - t)}}$.

This replicating portfolio contains three components:

— $\lambda \cdot V_t \cdot N(d_1)$ is an amount invested in the risky asset;

— ZC is an amount invested in the risk-free zero-coupon bond with maturity T; and

— $-\lambda \cdot N(d_2) \cdot ZC$ is a short position in the risk-free zero-coupon bond with maturity T.

Offsetting long and short positions in the risk-free asset leads to the following consolidated portfolio:

$$V_t = [1 - \lambda \cdot N(d_2)] \cdot ZC \cdot B_t + V_t \cdot [\lambda \cdot N(d_1)] \cdot S_t.$$

This portfolio is made up only of two components, and implies that the proportion invested in the risky asset should be increased when the risky-asset price increases and vice versa. This is consistent with the assumption that the risky-asset future price is independent from historical prices.

The proportion invested in the risky asset is:

$$w_M = \lambda_t \cdot N(d_1) = \frac{(V_t \cdot N(d_1) - ZC \cdot B_t \cdot N(d_1))^+}{V_t \cdot N(d_1) - ZC \cdot B_t \cdot N(d_2)}.$$

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The proportion invested in the risk-free asset is:

$$w_f = (1 - \lambda_t \cdot N(d_2)) \cdot \frac{ZC \cdot B_t}{V_t} = \left(\frac{(ZC \cdot B_t \cdot N(d_1) - ZC \cdot B_t \cdot N(d_2))}{V_t \cdot N(d_1) - ZC \cdot B_t \cdot N(d_2)} \right)$$

w_M and w_f proportions represent a complete portfolio as:

$$w_M + w_f = \frac{(V_t \cdot N(d_1) - ZC \cdot B_t \cdot N(d_1))}{V_t \cdot N(d_1) - ZC \cdot B_t \cdot N(d_2)} + \frac{(ZC \cdot B_t \cdot N(d_1) - ZC \cdot B_t \cdot N(d_2))}{V \cdot N(d_1) - ZC \cdot B_t \cdot N(d_2)} = 1.$$

The asset allocation proportions implied by ROBPI are:

— $w_f = \left(\dfrac{(N(d_1) - N(d_2))}{\dfrac{V}{ZC \cdot B_t} \cdot N(d_1) - N(d_2)} \right)$ invested in the risk-free asset; and

— $w_M = 1 - w_f$ invested in the risky asset.

The proportions invested in risky and risk-free assets are between zero and one, implying that no borrowing is required. In cases where investing the entire portfolio in the risk-free asset is not sufficient to provide the floor (when ZC is higher than V) external funding would be required to support any risky-asset investment. The ROBPI formula provides information about the likelihood of an investment portfolio being worth more than a certain minimum value over a certain period of time. As in an option pricing formula, ROBPI formulation is independent of risk preferences and is a forward looking strategy.

2.4 *Numerical Illustrations*

Table 1 shows some numerical examples of risky-asset exposure using ROBPI formula with different floors and investment periods. These examples are for illustration purposes and are based on 20% risky-asset volatility and a risk-free rate of 4%.

Table 1. Risky-asset exposure based on a call option strategy

	Time horizon							
Floor as % of V	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year
90%	64%	65%	67%	69%	70%	72%	74%	75%
95%	53%	56%	59%	62%	64%	67%	69%	71%
100%	40%	46%	50%	54%	57%	61%	63%	66%
105%	25%	34%	40%	46%	50%	54%	57%	61%
110%	9%	21%	29%	36%	42%	47%	51%	55%

Risky-asset exposure increases with the investment period and decreases with the level of downside risk protection. Using the normal shape of volatility term-structure of an equity index makes risky-asset exposure less sensitive to the floor level, especially for long-term investment periods. A typical term-structure volatility of an equity index has been used in the case study in Section 4.

2.5 *ROBPI using Call-Spread Strategy*

The call-spread could be treated as a general form of a vanilla call option strategy. The following formula represents the risk-free asset exposure using the call-spread strategy for $t = 0$:

$$w_f = \left(\frac{(N(d_1) - N(d_2)) - \alpha \cdot (N(d_1^\beta) - \beta \cdot N(d_2^\beta))}{\frac{V}{ZC \cdot B_1} \cdot N(d_1) - N(d_2) - \alpha \cdot \left(\frac{V}{ZC \cdot B_t} \cdot N(d_1^\beta) - \beta \cdot N(d_2^\beta) \right)} \right)$$

where:

— α is a positive call-spread proportion with a value between 0 and 1;
— K_1 and K_2 are the strikes of the call spread;
— β is a positive parameter such as $K_2 = K_1 \cdot \beta$ and $\beta \geq 1$;
— d_1 and d_2 are based on strike K_1; and
— d_1^β and d_2^β are based on strike $K_1 \cdot \beta$.

Table 2 shows an example of risky-asset exposure based on a call spread, where $\beta = 1.5$ and $\alpha = 1$. When compared with the figures presented in Table 1, risky-asset exposure is considerably reduced, demonstrating that using a call-spread strategy reduces risky-asset exposure.

A call-spread strategy implies a lower initial risky-asset exposure than a call strategy, especially for low floors. The relationship between the risky-asset exposure and the investment period depends on the floor level. Table 2

Table 2. Risky-asset exposure based on a call-spread strategy

	Time horizon							
Floor = K/V	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year
90%	49%	47%	46%	45%	45%	45%	44%	44%
95%	42%	42%	42%	43%	43%	43%	43%	43%
100%	33%	36%	37%	39%	40%	40%	41%	41%
105%	22%	27%	31%	34%	36%	37%	38%	39%
110%	8%	17%	23%	28%	31%	33%	35%	37%

shows that risky-asset exposure derived from a long-term call-spread starts to converge towards the same value regardless of the floor level.

2.6 *Backtesting*

Figure 1 illustrates the historical performance of dynamic portfolio insurance between the risk-free asset and the FTSE all-shares index, using the ROBPI formula. It is possible to use historical prices inclusive of the dividend incomes of any other risky asset or equity index (e.g. DAX equity index) for this illustration.

Assumptions for backtesting are:

— 150% of the initial investment is the floor level;
— the investment period is from February 1994 to February 2004;
— 6% is the risk-free rate over the whole period;
— there is a monthly rebalancing strategy (15% minimum switch trigger for smoothed strategy); and
— there is no tax or transaction cost.

Figure 1 shows that the floor is met, despite the FTSE high volatility (23%) and a low return over this period. The objective of ROBPI was to protect the initial investment against further losses when the index performs

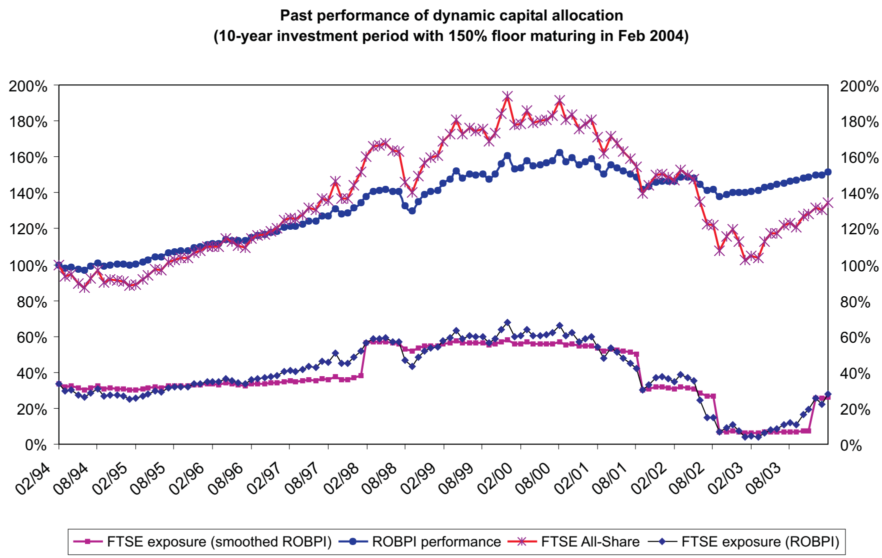

Figure 1. ROBPI targeting 150% maturity payout

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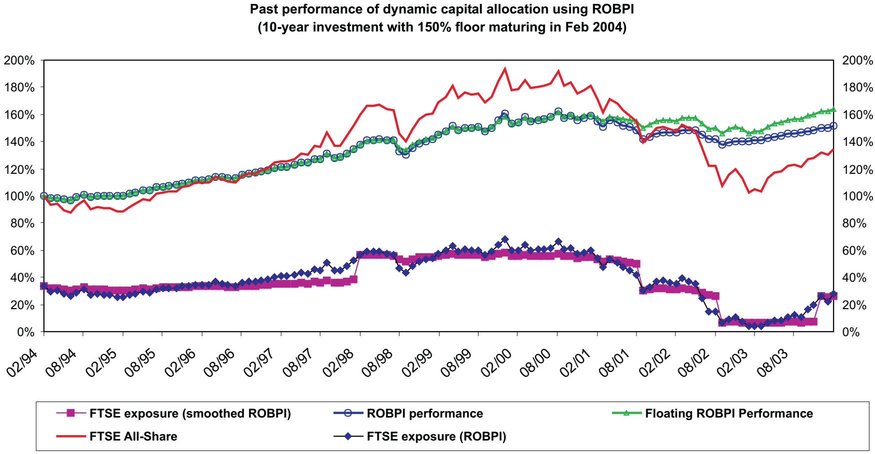

Figure 2. Floating ROBPI versus dynamic ROBPI

poorly. ROBPI has met its objective with only 9% delivered volatility of the non-smoothed strategy. Financial futures are usually used to implement transactions implied by portfolio insurance in order to minimise their costs. A smoothed risk-asset strategy is designed to reduce the cost and number of transactions.

Figure 2 shows the performance of a floating strategy versus dynamic ROBPI, starting from the same initial risky-asset exposure.

In Figure 2, dynamic ROBPI underperforms the floating strategy, reflecting the implicit cost of downside protection. In this example, dynamic strategy performance suffered from reversing asset allocation switches, which reduces the performance of all systematic portfolio insurances. It will be shown in the case study (Section 4) that a floating strategy outperforms a dynamic one in the middle percentiles, while the latter produces higher returns in other parts of the distribution. This implies that, with a floating strategy, the downside risk is higher than with a dynamic strategy.

Figure 3 shows a further example of ROBPI performance based on the FTSE all-share index, targeting a 100% floor over a six-year period starting in February 2001.

The FTSE all-share index's historical volatility over this period was 21%. Over the same time, ROBPI produced a portfolio with 6% delivered volatility. Figure 3 shows that exposure to the FTSE all-share index was reduced by 30% due to extreme market falls during this period, representing the worst equity performance for decades. Nevertheless, the performance of ROBPI shows that downside risk was under control.

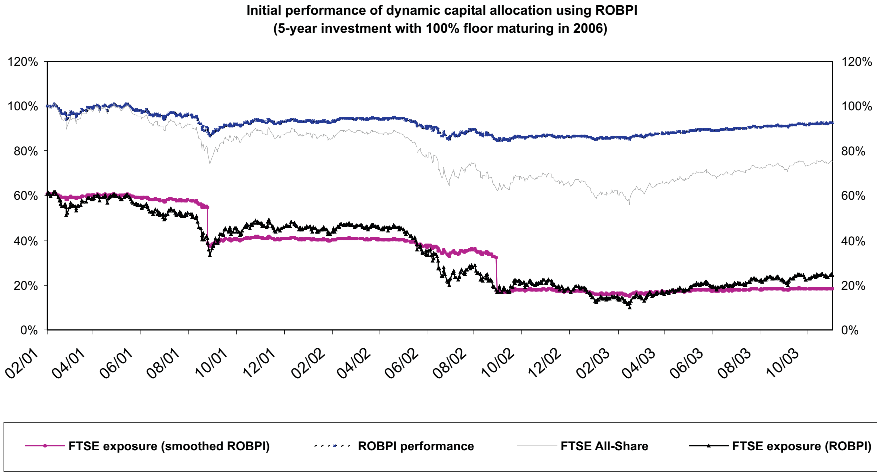

Figure 3. ROBPI targeting 100% floor over five-year period maturing in February 2007

2.7 *Comparison between ROBPI and CPPI*

CPPI is a widely used form of portfolio insurance because it is self-funded and has a simplistic formulation. CPPI has complete freedom in setting the initial risky-asset exposure. Bertrand & Prigent (2001) compared OBPI and CPPI in terms of stochastic dominance. They concluded that while no method outperformed the other in terms of stochastic dominance, OBPI could be treated a generalised form of CPPI. A comparison between ROBPI and CPPI could be more instructive here, since both methods have been used in practice and can be set to start with the same level of risky-asset exposure. Figure 4 shows a comparison between the performance of CPPI and ROBPI before transaction costs, using the following risky-asset exposure formula for CPPI:

$$w_M = (V_t - ZC \cdot B_t) \cdot \text{Multiplier}.$$

Multipliers in both CPPI strategies were set at the start to give the initial FTSE exposures shown in Figure 4. The figure shows that, starting with the same initial risky-asset exposure, the performances of CPPI and ROPBI were similar, and their implied asset allocations move in the same direction. However, exposure to the FTSE implied by CPPI can be very volatile, and even more volatile when it starts with a high initial risky-asset exposure. Leveraged, structured, protected equity funds using the CPPI technique with more than 100% initial risky-asset exposure have been widely available on the market. The comparison in Figure 4 has shown that ROBPI could be a

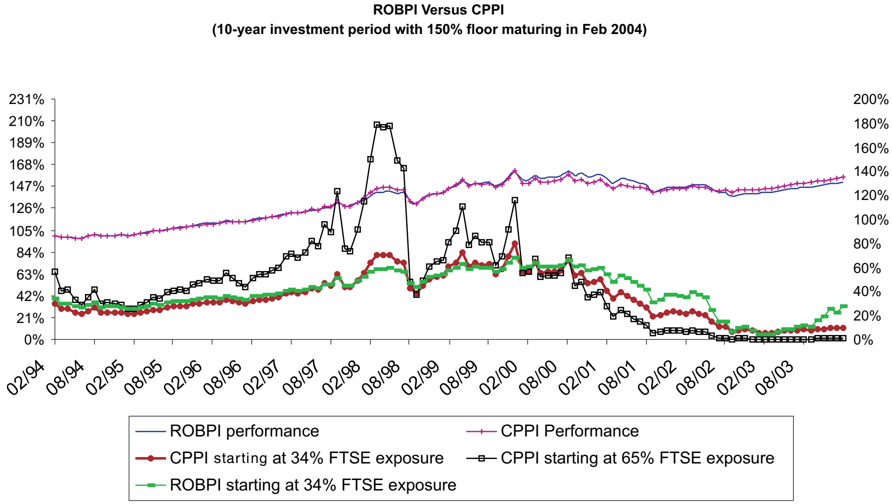

Figure 4. ROBPI versus CPPI

more efficient form of portfolio insurance, since CPPI is not a forward looking technique and has the following drawbacks:

— no analytical formula for the initial risky-asset exposure;
— insensitive to changes in risky-asset volatility;
— high probability of switching to nil risky-asset exposure and achieving no upside; and
— increased transaction costs and market impact.

On the other hand, risky-asset exposure implied by ROBPI could also be volatile if the portfolio is at-the-money near maturity. This volatility could be smoothed significantly by fixing the call proportion, as suggested in Bouchaib (2004, Section 5.4).

3. Application to With-Profits Funds

3.1 *Investment Strategy for Participating Funds*

With-profits funds provide smoothed equity exposure with certain levels of guarantees and maturity benefits. A major challenge to life offices and pension funds in the U.K. over recent years has been maintaining a balance between meeting these guarantees and offering a reasonable level of equity exposure to maximise expected returns. In the past, these companies focused on adopting high equity exposure to attract new investors, under the assumption that smoothing payouts to policyholders would be sufficient to sustain equity volatility. High volatility, combined with high equity exposure, defeated the resilience of this practice, since assets were much more volatile than liabilities. Linking the level of equity exposure to the ratio between assets and liabilities should provide a better asset and liability management (ALM) framework to balance risks and rewards in these funds. ROBPI provides an ALM tool to help benchmark equity exposure for with-profits funds and manage the cost of guarantees.

3.2 *Liability Modelling*

The ROBPI concept can be used as an ALM tool to manage equity exposure of the asset shares in participating funds. As seen in previous sections, setting the level of downside risk protection is one of the key parameters leading to the level of risky-asset exposure. Applying ROBPI to asset shares implies that an appropriate minimum liability measure has been identified to represent the present value of the minimum benefits. In the U.K. context, a bonus reserve valuation (BRV) is a suitable liability measure for this purpose. More discussion about with-profits funds' liabilities can be found in Dullaway & Needleman (2004).

The BRV measure usually includes the following items:

— future expected bonuses;
— the tax on shareholder transfers, charges and investment expenses;
— future contractual premiums;
— the intrinsic value of no-MVR guarantees;
— the intrinsic value of guarantees (minimum bonuses, guaranteed minimum pension); and
— the intrinsic value of glidepath and smoothing costs.

The intrinsic value of guaranteed annuity options (GAOs) has not been included in the BRV calculation because the value of these options depends on the value of the asset shares. Therefore, maximising returns on asset shares increases the GAO costs. These options should be managed and hedged in the estate, as they cannot be hedged by maximising asset share returns. Applying ROBPI to participating funds ensures that asset shares will be invested to meet the minimum liabilities and guarantees, and maximise the potential value of future bonuses. By replacing the present value of the floor by the BRV measure, the proportion to be invested in the risk-free asset is as follows:

$$W_f = \frac{N(d_1) - N(d_2)}{\frac{A/S}{BRV} \cdot N(d_1) - N(d_2)}$$

where:

Table 3. Risky asset exposure

Inputs: ρ	-100%	-50%	0%	50%	100%	$\sigma_{BRV} = 0$
Calculated: $\sigma*$	30%	26%	22%	17%	10%	20%
BRV/assets						
75%	43%	46%	50%	58%	77%	54%
77%	40%	43%	47%	54%	74%	50%
79%	36%	39%	43%	51%	69%	46%
81%	33%	36%	40%	47%	65%	43%
83%	30%	32%	36%	42%	60%	39%
85%	27%	29%	32%	38%	55%	34%
87%	23%	25%	28%	33%	49%	30%
89%	20%	21%	24%	29%	42%	26%
91%	16%	18%	20%	24%	36%	21%
93%	13%	14%	16%	19%	29%	17%
95%	9%	10%	11%	14%	21%	12%
99%	2%	2%	2%	3%	4%	3%

— $d_1 = \dfrac{\ln\left(\dfrac{A/S}{BRV}\right) + (\sigma^2/2)\cdot T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$; and

— A/S is asset shares.

If the BRV is assumed to follow a lognormal process with a volatility of σ_{BRV} and a correlation with the risky asset ρ, the Margrabe (1978) exchange option pricing formula should be used instead of the B&S call option formula, replacing volatility σ by:

$$\sigma* = \sqrt{\sigma^2 + \sigma_{BRV}^2 - 2\rho\cdot\sigma\cdot\sigma_{BRV}}.$$

Table 3 shows the risky-asset exposure for different ratios of the BRV over asset shares with different levels of correlation for 20% risky-asset volatility, 10% BRV volatility, 0% BRV volatility and $T = 10$.

The level of risky-asset exposure depends on the level of volatilities and correlation. Risky-asset exposure increases with the level of positive correlation.

3.3 *Additional Guidance for Financial Management*

Setting an appropriate level of risky-asset exposure, combined with the following risk reduction measures, would provide the company with a useful tool to minimise ruin probability:

— Adopt an appropriate risky-asset exposure for different cohorts.

— Consider tranche-based with-profits funds.

— Apply a guarantee charge to asset shares to finance claims on the estate.

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— Allocate returns on bonds where duration matches guarantees to asset shares.
— Adopt a dynamic regular bonus policy with a consistent grouping policy.

A focus of debate in the U.K. in recent years has been on hedging and managing the economic costs within with-profits funds. For example, Hibbert & Turnbull (2003) focused on hedging guarantees by holding replicated put options in the estate and assuming a static equity exposure in the asset shares. This is a theoretically sound hedging strategy to offset equity exposure in the asset shares. However, in practice, this strategy implies that the estate is holding negative equity exposure. This is not a popular solution, since life offices are usually reluctant to hold negative equity in the estate to offset high equity exposure in the asset shares. Experience in recent years has shown that some companies setting a high level of equity exposure in their asset shares had difficulties in maintaining the required regulatory capital to support the commensurately high expected guarantees costs. It is now understood that setting an appropriate level of equity exposure in the asset shares is the primary investment decision. ROBPI is an appropriate tool to achieve this objective (see Bouchaib's comment in Hibbert & Turnbull, 2003). Other approaches, like that of Wilkie (1985), looked at the portfolio selection process using a mean-variance optimisation technique. These techniques cannot be classified as portfolio insurance, as they focus on maximising the expected return and minimising the surplus variance.

4. Case Study: Structured Product

Stochastic projections have been used to illustrate the distribution of ROBPI performance as an investment strategy. A structured equity product representing an actively managed fund backing a single premium is used as a case study to assess the projected performance.

Liability description

Simplistic liability assumptions for the case study are:
— a single premium of \$100m;
— \$100m is the required minimum value at maturity (net of charges);
— a ten-year investment period;
— a 1% annual management charge; and
— lapse and death benefits are ignored, since the guarantee is only applicable at maturity.

4.1 *Investment Strategies*

To choose an investment strategy consistent with the life office's risk

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Table 4. Investment strategies

Strategy	Asset allocation	Initial risky-asset exposure	Floor	Call-spread parameters
1	Floating ROBPI	40%	\$100m	
2	Dynamic ROBPI (call)	40%	\$100m	
3	Dynamic ROBPI (call spread)	31%	\$100m	$\beta = 1.75$ and $\alpha = 1$
4	Floating ROBPI	48%	\$95m	
5	ROBPI with call	48%	\$95m	
6	ROBPI with call spread	44%	\$95m	$\beta = 1.75$ and; $\alpha = 0.5$

appetite, different strategies have been tested. Measuring the performance of each of them by looking at projected outcomes provides guidance for strategic asset allocation. Floating strategies are useful and help in assessing the performance of dynamic strategies. Six different investment strategies are considered. The first is a floating asset allocation. The second is a dynamic strategy based on the ROBPI with a call option. The third strategy is ROBPI with a call spread option. Subsequent strategies use higher initial risky-asset exposures by assuming a lower notional floor of \$95m. The investment strategies are summarised in Table 4.

It is understood that portfolio insurance techniques do not protect funds against gap risk due to adverse risky-asset price movements, market disturbance and price discontinuity which cannot be predicted by B&S asset models. This risk could be mitigated by the following actions:

— combining the strategy with an out-of-the-money traded put option (or reinsurance);
— implementing a conservative portfolio insurance by assuming higher floor and volatility; and
— holding a dedicated capital to back the gap risk.

4.2 *Asset Model*

As in the B&S asset model, a risky asset is assumed to follow a lognormal process, but with a term structure volatility. The cash bond price is assumed to be a deterministic function. Rebalancing the asset allocation is assumed to take place on a quarterly basis with no transaction cost or market impact. Parameters for the asset model are:

— 3% fixed risk-free rate;
— 8% risky asset expected return; and
— the term structure of risky-asset volatility is:

Strike	90%	95%	100%	105%	110%
Volatility	22%	21%	20%	19%	18%

A linear interpolation has been used to estimate the volatility for strikes between 75% and 125%. Volatility is assumed to be constant beyond these boundaries.

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4.3 *Projected Outcomes*

The Monte Carlo technique was used to generate 10,000 simulations with quarterly time steps. Key results and distributions derived from the projections are set out in Tables 5 to 13, which show the following outcomes:

— the risky-asset performance;
— the present value of shortfalls;
— the fund performance; and
— the projected risky-asset exposure.

Table 5 shows the yearly distribution of the risky-asset performance over the ten-year period.

The risky asset has a high potential reward, but also a significant downside risk. Percentiles in the rest of this section refer to risky-asset performance. This helps to illustrate the distribution of call-spread strategy performances.

Table 6 shows the distribution of the present value of shortfalls for each strategy at maturity.

Shortfalls for higher percentiles are nil. The expected shortfalls are moderate in dynamic and floating strategies based on initial risky-asset exposure below 48%.

Table 5. Distribution of risky-asset performance

Percentile	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$	$t = 10$
1st	−41%	−51%	−56%	−62%	−65%	−67%	−69%	−71%	−73%	−73%
5th	−30%	−39%	−43%	−46%	−49%	−51%	−53%	−54%	−55%	−56%
25th	−12%	−14%	−14%	−14%	−13%	−13%	−12%	−10%	−10%	−8%
40th	−2%	0%	3%	6%	10%	12%	16%	21%	25%	30%
50th	4%	10%	15%	20%	25%	31%	38%	45%	52%	60%
60th	11%	20%	27%	37%	45%	54%	62%	75%	84%	96%
75th	24%	39%	53%	69%	83%	98%	115%	135%	154%	174%
95th	58%	98%	137%	174%	218%	264%	321%	378%	437%	498%
99th	88%	150%	221%	288%	367%	445%	526%	658%	775%	897%
Mean	8%	17%	26%	37%	47%	59%	72%	88%	103%	120%

Table 6. Distribution of shortfalls

Percentile	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1st	−13%	0%	0%	−19%	−4%	−4%
5th	−8%	0%	0%	−13%	−4%	−4%
15th	−1%	0%	0%	−5%	−3%	−3%
30th	0%	0%	0%	0%	−2%	−1%
Mean	−1%	0%	0%	−2%	−1%	−1%

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Table 7. Distribution of fund performance

Percentile	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	Floating 1	Call 1	Call-spread 1	Floating 2	Call 2	Call-spread 2
1st	83%	100%	100%	75%	95%	95%
5th	89%	100%	100%	82%	95%	95%
10th	94%	100%	101%	88%	95%	96%
25th	106%	100%	101%	103%	95%	96%
40th	120%	109%	121%	119%	107%	112%
50th	130%	112%	119%	132%	114%	118%
60th	144%	112%	133%	148%	118%	131%
75th	172%	162%	172%	182%	175%	182%
90th	234%	263%	237%	256%	283%	263%
95th	287%	341%	240%	320%	368%	319%
99th	432%	584%	254%	494%	633%	487%
Mean	152%	157%	144%	158%	162%	156%

Table 7 shows the distribution of the performance of the six investment strategies.

Discounting these performances after deducting the initial investment at the risk-free rate gives the shortfalls shown in Table 6. Table 7 shows that floating strategies outperform dynamic strategies between the 25th and 60th percentiles, while call strategies outperform in both tails of the distributions. Relative to call strategies, call-spread sacrifices a significant potential upside, but improves the performance in the middle percentiles.

Figure 5 shows a comparison between the cumulative distributions of the

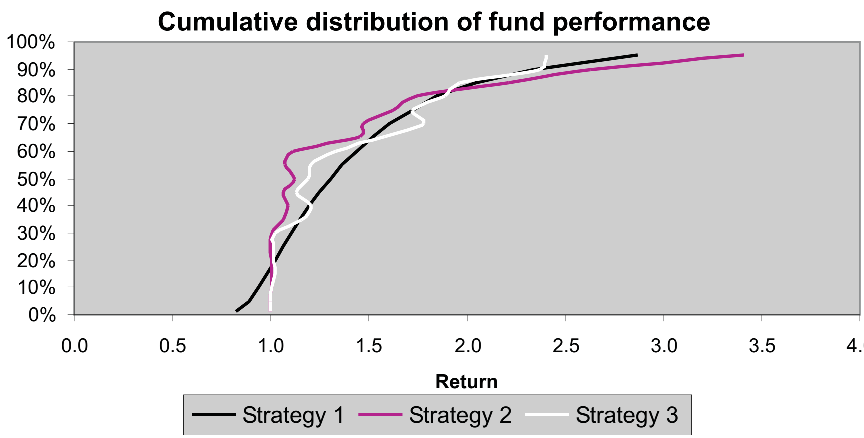

Figure 5. Cumulative distributions of fund performance under strategies 1, 2 and 3

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performance of strategies 1, 2 and 3. The relative performance between floating and dynamic strategies depends on the expected return and the volatility term structure of the risky asset.

Figure 5 suggests that the fund performance can be improved by combining floating and dynamic strategies. This will depend on the fund managers' skill in anticipating the movements in volatility and choosing the appropriate strategy at any one time.

4.4 *Projected Risky-Asset Exposure*

Tables 8 to 10 show the distribution of projected risky-asset exposures for all six strategies. Table 8 shows that the floating strategy has moderate changes to risky-asset exposure each year. After ten years, however, risky-asset exposure has fallen by more than half in the worst case scenarios.

In Table 8, risky-asset exposure is reasonably stable with a moderate increase at the 50th percentile due to the risk premium factors.

Table 9 shows the movement of risky-asset exposures for strategy 2, given the volatility term structure.

To limit the risky-asset exposure moving to extreme values (such as 100% and 0%) it is possible to introduce upper and lower boundaries which will have an impact on the performance and shortfalls.

Table 8. Projected risky-asset exposure based on strategy 1 (floating 1)

Percentile	$t=0$	$t=1y$	$t=2y$	$t=3y$	$t=4y$	$t=5y$	$t=6y$	$t=7y$	$t=8y$	$t=9y$	$t=10y$
5th	40%	31%	28%	26%	24%	23%	21%	20%	19%	19%	18%
25th	40%	36%	35%	34%	34%	33%	33%	32%	32%	32%	31%
40th	40%	39%	39%	39%	38%	39%	39%	39%	39%	39%	39%
50th	40%	40%	41%	41%	41%	42%	42%	43%	43%	44%	44%
60th	40%	42%	43%	44%	45%	45%	46%	47%	48%	48%	49%
75th	40%	44%	46%	48%	50%	51%	52%	54%	55%	56%	58%
95th	40%	51%	55%	59%	62%	65%	67%	69%	71%	73%	75%
Mean	40%	40%	41%	42%	42%	43%	43%	43%	44%	44%	45%

Table 9. Projected risky-asset exposure based on strategy 2 (call option 1)

Percentile	$t=0$	$t=1y$	$t=2y$	$t=3y$	$t=4y$	$t=5y$	$t=6y$	$t=7y$	$t=8y$	$t=9y$	$t=10y$
5th	40%	17%	11%	10%	8%	4%	2%	3%	1%	1%	0%
25th	40%	27%	26%	21%	22%	13%	13%	16%	8%	18%	1%
40th	40%	30%	32%	31%	29%	38%	32%	34%	38%	27%	40%
50th	40%	37%	36%	40%	41%	40%	39%	47%	53%	57%	50%
60th	40%	40%	38%	47%	47%	50%	48%	57%	69%	70%	48%
75th	40%	46%	52%	53%	60%	65%	77%	82%	95%	96%	97%
95th	40%	60%	73%	81%	87%	94%	98%	99%	100%	100%	100%
Mean	40%	37%	38%	40%	42%	43%	45%	47%	49%	52%	49%

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Table 10 shows that with call-spread 1, risky-asset exposure is reduced in the best scenarios.

If the risky asset performs strongly year on year, its proportion is increased initially and starts to reduce from year six. This strategy could be suitable for fund managers who believe in the mean-reversion feature of the risky-asset model.

Tables 11 to 13 show the distribution of risky-asset exposure of the three strategies, but starting with higher initial exposures. In Table 11, the initial risky asset was set higher than strategy 1, which implies a higher downside risk.

In Table 12, despite starting with a higher risky-asset exposure, the dynamic strategy moves to extreme values at maturity in the 25th and 75th percentiles, which is also a feature of strategy 2. Risky-asset exposure is higher in other scenarios.

Strategy 5 starts with 17% additional risky-asset exposure than strategy 2, which gives more volatility to the asset allocation. This extra volatility was also observed in CPPI relative to OBPI, which is not a desirable feature of an investment strategy.

In strategy 6 (Table 13), after an initial period of four years, risky-asset exposures in the worse scenarios are similar to those observed in strategy 5, but they are much lower in the best scenarios.

Table 10. Projected risky-asset exposure based on strategy 3 (call-spread 1)

Percentile	$t = 0$	$t = 1y$	$t = 2y$	$t = 3y$	$t = 4y$	$t = 5y$	$t = 6y$	$t = 7y$	$t = 8y$	$t = 9y$	$t = 10y$
5th	31%	17%	13%	12%	11%	8%	6%	7%	3%	5%	1%
25th	31%	22%	23%	22%	22%	20%	19%	24%	17%	39%	6%
40th	31%	25%	26%	27%	28%	32%	34%	40%	45%	56%	64%
50th	31%	27%	29%	31%	33%	36%	38%	45%	54%	68%	62%
60th	31%	29%	30%	35%	37%	42%	44%	52%	61%	76%	75%
75th	31%	32%	36%	39%	42%	47%	52%	57%	61%	67%	59%
95th	31%	38%	43%	47%	50%	51%	49%	45%	32%	11%	16%
Mean	31%	27%	28%	30%	32%	33%	35%	37%	39%	40%	35%

Table 11. Projected risky-asset exposures based on strategy 4 (floating 2)

Percentile	$t = 0$	$t = 1y$	$t = 2y$	$t = 3y$	$t = 4y$	$t = 5y$	$t = 6y$	$t = 7y$	$t = 8y$	$t = 9y$	$t = 10y$
5th	48%	38%	35%	32%	31%	29%	27%	26%	25%	24%	23%
25th	48%	44%	43%	42%	41%	41%	40%	40%	40%	39%	39%
40th	48%	47%	47%	46%	46%	46%	46%	47%	47%	47%	47%
50th	48%	48%	49%	49%	50%	50%	50%	51%	51%	52%	52%
60th	48%	50%	51%	52%	53%	54%	54%	55%	56%	56%	57%
75th	48%	53%	55%	56%	58%	59%	60%	62%	63%	64%	65%
95th	48%	59%	63%	66%	69%	72%	74%	76%	78%	79%	80%
Mean	48%	48%	49%	49%	50%	50%	50%	51%	51%	52%	52%

Table 12. Projected risky-asset exposure based on strategy 5 (call option 2)

Percentile	$t = 0$	$t = 1y$	$t = 2y$	$t = 3y$	$t = 4y$	$t = 5y$	$t = 6y$	$t = 7y$	$t = 8y$	$t = 9y$	$t = 10y$
5th	48%	21%	14%	13%	10%	5%	4%	4%	2%	2%	0%
25th	48%	33%	32%	27%	28%	18%	19%	22%	13%	25%	2%
40th	48%	37%	39%	39%	37%	46%	40%	43%	48%	37%	52%
50th	48%	44%	44%	48%	49%	49%	49%	57%	64%	70%	67%
60th	48%	48%	46%	55%	56%	59%	60%	68%	79%	84%	75%
75th	48%	54%	60%	62%	69%	74%	84%	89%	97%	99%	99%
95th	48%	68%	80%	87%	91%	96%	99%	100%	100%	100%	100%
Mean	48%	44%	45%	47%	48%	50%	51%	53%	55%	57%	54%

Table 13. Projected risky-asset exposure based on strategy 6 (call-spread 2)

Percentile	$t = 0$	$t = 1y$	$t = 2y$	$t = 3y$	$t = 4y$	$t = 5y$	$t = 6y$	$t = 7y$	$t = 8y$	$t = 9y$	$t = 10y$
5th	44%	22%	16%	14%	12%	7%	5%	6%	3%	3%	0%
25th	44%	31%	31%	27%	28%	21%	22%	25%	18%	34%	5%
40th	44%	35%	36%	37%	36%	43%	40%	45%	51%	50%	62%
50th	44%	40%	41%	44%	45%	47%	48%	55%	64%	73%	73%
60th	44%	43%	43%	50%	51%	55%	57%	64%	73%	83%	83%
75th	44%	48%	53%	56%	61%	65%	72%	76%	79%	80%	77%
95th	44%	59%	68%	73%	77%	79%	80%	79%	77%	76%	76%
Mean	44%	40%	41%	43%	44%	45%	47%	48%	50%	52%	49%

5. Conclusion

This paper considered a revised and general formulation of OBPI as described in the literature. The distinction of the revised approach is that it has been used in practice to underpin investment strategies for life and pension funds. To date, CPPI has been the most popular form of portfolio insurance among investment banks; this paper has shown that ROBPI is a forward looking formulation of portfolio insurance and that it has some advantages over CPPI.

However, systematic portfolio insurance techniques in general have been criticised for contributing to the 1987 market crash, despite the tempering message of Rubinstein's analysis (1988). The author's own experience in developing and implementing ROBPI methodology in volatile market conditions has proved that systematic portfolio insurance can be a very useful tool, but it is not necessarily the best approach for all investment periods and risk preferences. To achieve a wider application, ROBPI could be extended and made more flexible by introducing a risk-aversion measure to allow fund managers to implement their market views and risk preferences within portfolio insurance, as explored in Bouchaib (2006).

In recent years, the demand for CPPI has increased dramatically in

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structured finance, and it is now also applied to credit risk exposure. The demand for portfolio insurance as an asset allocation and ALM tool will also increase for life and pension funds. An opportunity for future research would be to look at how enhancements to asset modelling which redress some of the shortfalls of the B&S model could be implemented in ROBPI, which has some additional benefits relative to CPPI.

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