

Bi-level Game Model for Interaction between Arctic and Traditional Routes

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This paper proposes a bi-level model from the perspective of game theory to describe the effect of the rise of Arctic shipping routes on traditional routes and their response. The upper-level model demonstrates the competition between shipping companies that maximise their own profits via speed adjustment, which can be presented as a generalised Nash equilibrium problem and is solved by the generalised reduced-gradient method. The lower-level model illustrates the response of customers who reassign their demands with an elastic total demand, which is presented as a logit-type multi-path assignment problem and is solved by the iterative balancing method. A case study is used to examine the rationality of the proposed model and algorithm.

KEYWORDS

1. Bi-level game model.
2. Arctic routes.
3. Generalised Nash equilibrium.
4. Elastic demand.

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1. INTRODUCTION. The shipping industry has always been at the forefront of the world trade market, which is the barometer of international economic trends. Recently, due to a multitude of pressures such as the impact of natural disasters, geopolitical conflict, pirate attacks, and so on, shipping companies have started to seek alternate routes, in an attempt to reduce costs and increase profitability. In the late Twentieth Century, Arctic sea ice began to melt more quickly as a result of global warming. The Intergovernmental Panel on Climate Change (IPCC) Fifth Assessment Report showed that Arctic sea ice may completely melt during summer in the middle of the Twenty-First Century and in extreme cases during winter by the end of 2070 (Stocker, 2014), which makes Arctic navigation no longer just a dream. The Arctic sea routes considered in this paper are the Northern Sea Route (NSR), the Northwest Passage (NWP) and the Central Passage (see Figure 1). Compared with the traditional sea routes, for many destinations, the Arctic routes have a shorter shipping distance, wider routes and fewer pirates, which will lead to more common usage.

The question posed by the Arctic Council in the Arctic Marine Shipping Assessment 2009 Report aims to find the optimal shipping routes considering both the Arctic and non-Arctic routes. In terms of sailing time, the distance from Asia (China or Japan) to the east coast of North America through the NWP for general types of ships can save 25% to 44%

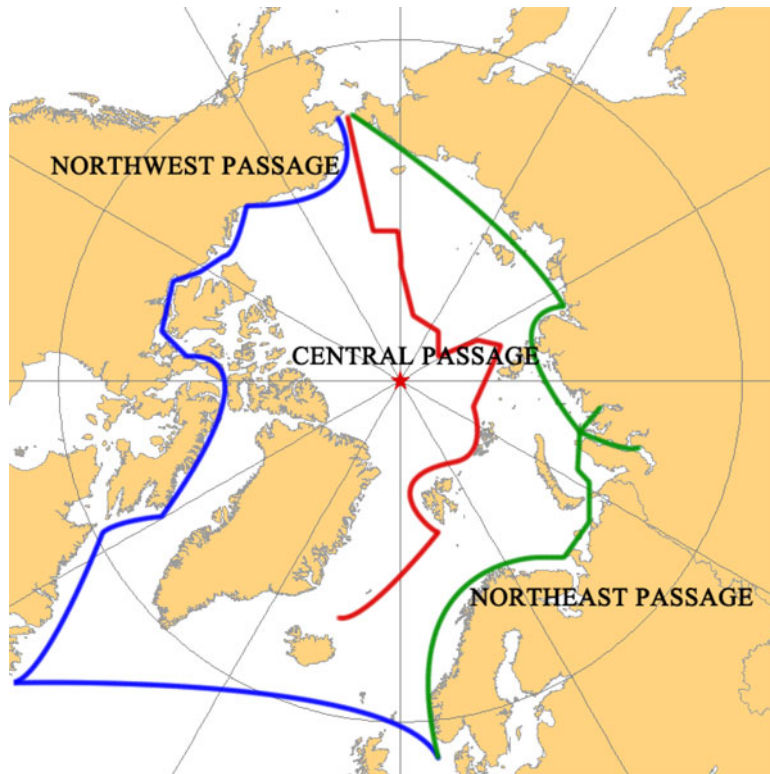


Figure 1. The Arctic marine area.

of total distance compared to routes using the Panama Canal, while navigation through the NSR from Asia to Europe can save 25% to 55% of total distance compared to using the Suez Canal (Andersen et al., 1995; Ragner, 2000; Zhang et al., 2009). In view of economic benefits, Somanathan et al. (2009) simulated the NWP and the Panama Canal by computer and the result showed the economic potential of the NWP would gradually rise with the decline in the extent of sea ice. The NSR will become an important alternative route to the Suez Canal (Verny and Grigentin, 2009; Chernova and Volkov, 2010; Srinath, 2010; Liu and Kronbak, 2010). Way et al. (2015) presented a probability simulation method to compare the optimal sailing speed of container ships on the NSR or on the Suez Canal based on the model presented by Liu and Kronbak (2010). When considering both sailing time and distance, a comparison of shipping costs was made between the NSR and the Suez Canal and results showed that the rise of the NSR will promote the agility and adaptability of the global shipping supply chain (Schøyen and Bråthen, 2011).

In order to further enhance the competitiveness of Arctic routes, route optimisation issues have been studied by some researchers. The safe navigation speed was determined by the Ice Number (IN) index (McCallum, 1996) which became the criteria to plan the optimal route (Smith et al., 2013; Melia et al., 2016). Kotovirta et al. (2009) took sea ice thickness, ice ridge thickness and sea ice concentration into consideration to analyse their impact on ship speed. Choi et al. (2015) presented a route planning model with a heuristic algorithm in which both extreme conditions and model uncertainty were considered. Other

environmental factors (for example, wind, waves, current and topography) and economic factors were also taken as variables in some integrated models (Motte et al., 1988; Mulherin et al., 1996; Choi et al., 2010; Hogben and Lumb, 1967; Ha et al., 2011; Nam et al., 2013; Aksenov et al., 2016).

In the literature mentioned above, both Arctic and non-Arctic routes were assumed to be independent of each other and no interaction between them is considered in route optimisation. However, an obvious competitive relationship actually exists between the routes relative to any Origin-Destination (OD) pair including the competition among shipping companies and the interaction between shipping companies and customers. Therefore, when the potential competitiveness of the Arctic routes is considered, the impact of the non-Arctic routes should also be considered.

For the purpose of finding a quantitative model to illustrate the interaction between Arctic routes and the traditional routes, game theory is a powerful tool to characterise the elements of dynamic competition. This paper proposes a bi-level model to demonstrate the effect of the rise of Arctic routes on the traditional routes and the response of traditional routes. The model illustrates the entire competition process among shipping companies over different sea routes and the customers during two equilibrium stages, which can be presented in a Stackelberg form (Lam and Zhou, 2000). The upper-level model illustrates the competition among shipping companies who cut their own cost by speed optimisation and maximise their own profits, which is presented as a Generalised Nash Equilibrium Problem (GNEP) and can be transformed into a type of Quasi-Variational Inequality (QVI) problem. Harker (1991) pointed out that solving this QVI problem is equivalent to finding a Variational Inequality (VI) solution, which can be solved by the generalised reduced-gradient method (Lasdon et al., 1978). The lower-level model illustrates the responses of customers who reassign their demands on each shipping route with an elastic demand, which is presented as a logit-type multi-path assignment problem and can be solved using the balancing iteration method.

In the case study, the proposed model used to simulate a practical problem illustrates the interaction between these two types of routes. Two stages (before and after the rise of the Arctic routes) of one Origin to Destination (OD) pair (no stop on the way) are compared to demonstrate the effect of the rise of the Arctic routes on the traditional routes and the response of the traditional routes. The harsh weather conditions in the Arctic region are considered to modify the results of the bi-level model so as to make the simulation more realistic. The structure of this paper is as follows: Section 2 introduces some basic concepts and notations. In Section 3 this improved bi-level game model is introduced and its mathematical characteristics are analysed. The algorithm is introduced in Section 4. Section 5 presents a case study using the proposed model and practical climatic and hydrological data to reveal the interaction between the Arctic and traditional routes. Finally, the conclusion is presented in Section 6.

2. BASIC CONCEPTS.

2.1. *Total travel cost.* The total travel cost Q_s in sailing route s generally includes four components: cost of ships, voyage cost, operating cost, and fuel-consumption cost, which can be expressed as:

$$Q_s = Pcc_s + Pvc_s + Poc_s + Pfc_s \quad \forall s \in S_w \quad (1)$$

where S_w represents the set of all sailing routes, Pcc_s , Pvc_s , Poc_s , Pfc_s represent the cost of ships, the voyage cost (including the route toll, ice-breaking fee, and pilotage, etc), the operating cost and the fuel consumption cost in sailing route s , respectively, whereas the cost of ships in sailing route s can be expressed as:

$$Pcc_s = Pc_s \times \frac{T_s}{T_a}, \quad \forall s \in S_w \quad (2)$$

where Pc_s represents the price of one ship in sailing route s , T_s represents the travel time of a voyage in sailing route s and T_a represents the total travel time in sailing route s .

The operating cost (including manning, hull and machinery insurance, protection and indemnity insurance, repairs and maintenance, administration, and others) in sailing route s can be expressed as:

$$Poc_s = Po_s \times T_s, \quad \forall s \in S_w \quad (3)$$

where Po_s represents the operating cost per day in sailing route s .

The fuel consumption is determined by the sailing speed. Dykstra (2001) defined the relationship as:

$$F_s = \frac{F}{V^2} V_s^2, \quad \forall s \in S_w \quad (4)$$

where F and V represent the standard fuel consumption and standard sailing speed while F_s and V_s represent the fuel consumption and sailing speed in sailing route s .

On the other hand, the fuel-consumption cost in sailing route s can be obtained as:

$$Pfc_s = \frac{F \times V_s^2}{V^2} \times SD_s \times Pf, \quad \forall s \in S_w \quad (5)$$

where SD_s represents the distance of sailing route s and Pf represents the fuel price.

Meanwhile, the total travel time of a voyage can be expressed as

$$T_s = Tw_s + Tv_s, \quad \forall s \in S_w \quad (6)$$

$$Tv_s = \frac{SD_s}{V_s}, \quad \forall s \in S_w \quad (7)$$

where Tw_s and Tv_s represent the waiting time of a voyage and the travel time of a voyage in sailing route s .

2.2. Generalised Nash Equilibrium Problem GNEP. GNEP, which was originally presented by Debreu (1952), is a generalisation of the traditional Nash Equilibrium Problem (NEP). Three factors, players, their strategies and their utility functions, are usually considered in a standard game model. In the NEP, only the utility function of one player can be affected by the other players' strategies; the strategies of one player are independent of the other players' strategies. In the GNEP, the utility function and the set of strategies of one player both depend on the other players' strategies.

Definition : Let us consider a non-cooperative game with n players involved. Each player $s \in S_w$ is represented by a set of strategies $Y_s \in \Xi_s \subseteq \mathbb{R}^n$, a point to set mapping $\emptyset_s : \Xi \rightarrow \Xi_s$, and a utility function $U_s : Y \rightarrow \mathbb{R}$, where $\emptyset = \emptyset_s$. The generalised Nash equilibrium

$Y^* \in \Xi$ of this game is defined as a point where no one can increase their utility by changing their strategies.

$$U_s(Y_s^*, Y_{-s}^*) \geq U_s(Y_s, Y_{-s}^*), \quad \forall Y_s \in \Xi_s(Y^*) \tag{8}$$

Thus, Ξ is a constraint set for all players.

Proposition 1: Let us suppose that each player $s \in S_w$ satisfies the conditions that:

- $\Xi_s \subseteq \mathbb{R}^n$ is a non-empty, compact, and convex set,
- $\emptyset_s : \Xi \rightarrow \Xi_s$ is a non-empty and closed map, and
- $U_s : Y \rightarrow \mathbb{R}$ is continuously differentiable and pseudo-concave.

Then, $Y^* \in \Xi(Y^*)$ is an optimal solution to the GNEP if and only if $Y^* \in \Xi(Y^*)$ is an optimal solution to the following QVI problem:

$$\sum_{s \in S_w} F_s(Y^*)^T (Y_s - Y_s^*) \geq 0, \quad \forall Y_s \in \Xi_s(Y^*) \tag{9}$$

Thus, $F_s(Y)$ can be expressed as:

$$F_s(Y) = -\nabla_{Y_s} U_s(Y) \tag{10}$$

3. BI-LEVEL MODEL WITH SPEED OPTIMISATION. To better clarify the proposed model, it is illustrated from two aspects. At the upper-level, shipping companies would obviously aim at maximising their own profits, which are mainly determined by the freight volume and unit cargo profits linked to unit freight revenue and travel costs. In recent years, the shipping industry has continued to slump; it is currently facing the pressure of weak trade growth and excess capacity. Therefore, an increasing number of scholars have started to study shipping-cost optimisation, which includes route selection, fleet management, and network design, to reduce freight costs and enhance economic competitiveness (Bijlsma, 2008; Tran and Haasis, 2015; Zhou et al., 2016). Among these factors, the network design, which is concerned with the choice of ports and related infrastructure, is not considered in this study. This study focuses on the optimisation of route selection and fleet management. In general, the fuel consumption cost of shipping companies accounts for more than 50% of the total travel cost (Golias et al., 2009; Notteboom, 2006), whereas fuel consumption and sailing speed have a significant relationship (Ronen, 1982; Wang and Meng, 2012; Psaraftis and Kontovas, 2013). Therefore, to cut the total cost, the fuel consumption needs to be reduced by adjusting the sailing speed or choosing a shorter route (Corbett et al., 2009; Lang and Veenstra, 2010; Norstad et al., 2011). At the lower-level, the volume of freight in each route depends on the choice of the customers, which is a response to the results of the competition between the shipping companies. In addition, if the external environment (such as world economy, politics, law and technology) is assumed to remain unchanged, the total customer demand is related not only to the revenue but also to the travel time (Wang et al., 2013).

To clarify this issue, this paper proposes a bi-level model with speed optimisation in the Stackelberg form in which the shipping companies are presented as leaders while the customers are presented as followers. The upper-level model studies the competition among

the shipping companies, whereas the lower-level model studies the change in the total customer demand with the change in their shipping cost and the reassignment of customer demand in each route.

3.1. *Lower-level model.* For the customers, the actual shipping cost in sailing route s is composed of the charges imposed by shipping companies and the time cost of navigation, which can be obtained as:

$$C_s = \lambda P + T_s, \quad \forall s \in S_w \tag{11}$$

where λ is the weight coefficient that reflects the relationship between the time and economic costs in the actual shipping cost in sailing route s .

It is assumed that the customer demand monotonically decreases with the increase in the actual shipping cost, which depends on the distance of the routes and sailing speed of the ships when other factors are fixed. For the shipping companies, they can reduce their time cost by speed optimisation in any route. Therefore, the total customer demand can be obtained as follows:

$$D = G(\bar{E}) = D^0 \exp(-\beta \bar{E}) \tag{12}$$

D^0 is the baseline customer demand before the rise of the Arctic routes, β is a dispersion coefficient, and \bar{E} is the expected minimum shipping cost for the customers, which can be expressed as (see Ben-Akiva and Lerman, 1985):

$$\bar{E} = E(\min \{C_s\}) = -\frac{1}{\theta} \ln \left(\sum_{s \in S_w} \exp(-\bar{\theta} C_s) \right), \quad \forall s \in S_w. \tag{13}$$

$\bar{\theta}$ represents the sensitivity of the different types of customers relative to their actual shipping cost.

In fact, the lower-level model describes the change in the total customer demand with the change in the shipping cost in response to the speed adjustment made by the companies.

3.2. *Upper-level model.* It is assumed that the shipping companies that use the traditional routes have reached equilibrium during competition before the rise of the Arctic routes, which we designate as Stage 1. Due to the rise of the Arctic routes, some companies change their shipping routes from the traditional ones to the Arctic routes. A new equilibrium among shipping companies will be finally reached, which is designated as Stage 2. For convenience, the shipping companies on the same route are classified as a type of player. Then, the problem of studying the competition among n players is simplified, which is equal to the number of elements in S_w . Therefore, the utility function of the shipping companies in sailing route s can be obtained by:

$$U_s = (P - Q_s) \times \frac{D_s}{CC_s}, \quad \forall s \in S_w \tag{14}$$

From Equations (1)–(7), the total cost function in sailing route s can be obtained as follows:

$$Q_s = \left(TW_s + \frac{SD_s}{V_s} \right) \left(\frac{PC_s}{T_a} + P_{O_s} \right) + P_{VC_s} + \frac{F \times V_s^2}{V^2} \times SD_s \times P_f, \quad \forall s \in S_w \tag{15}$$

For any $s \in S_w$, $P_{VC_s} > 0$ and $0 \leq V_s \leq V_{smax}$ is obtained.

The number of shipping events in each route is determined by the total customer demand, which is related to the actual shipping cost of the customers in each route. Generally, the number of shipping events in a route is small if the actual shipping cost to the customers in this route is high, which can be expressed according to the following logit-type multi-path assignment model:

$$D_s = \frac{e^{-\bar{\theta}(\lambda P + T_s)}}{\sum_{k \in S_w} e^{-\bar{\theta}(\lambda P + T_k)}} \times D, \quad \forall s \in S_w \tag{16}$$

Let:

$$X_s = e^{-\bar{\theta}(\lambda P + T_s)}, \quad \forall s \in S_w \tag{17}$$

$$X_{-s} = \sum_{k \in S_w, k \neq s} e^{-\bar{\theta}(\lambda P + T_k)}, \quad \forall s \in S_w \tag{18}$$

Then, Equation (15) can be rewritten as:

$$V_s = \frac{\bar{\theta} S D_s}{\ln(D - D_s) - \ln(D_s) - \ln(X_{-s}) - \bar{\theta} \lambda P}, \quad \forall s \in S_w. \tag{19}$$

From Equation (14):

$$Q_s = a_s (\ln(D - D_s) - \ln(D_s) - \ln(X_{-s}) - \bar{\theta} \lambda P) + \frac{b_s}{(\ln(D - D_s) - \ln(D_s) - \ln(X_{-s}) - \bar{\theta} \lambda P)^2} + e_s \tag{20}$$

Thus, $a_s = \frac{1}{\bar{\theta}} \left(\frac{P_{c_s}}{T_a} + P_{o_s} \right)$, $b_s = \bar{\theta}^2 \frac{F}{V^2} S D_s^3 P f$, and $e_s = \left(\frac{P_{c_s}}{T_a} + P_{o_s} \right) T w_s + P v c_s$.

Then, the utility function of the shipping companies in sailing route s can be rewritten as:

$$U_s = (P - Q_s) \times \frac{D_s}{CC_s} = \left(P - a_s (\ln(D - D_s) - \ln(D_s) - \ln(X_{-s}) - \bar{\theta} \lambda P) - \frac{b_s}{(\ln(D - D_s) - \ln(D_s) - \ln(X_{-s}) - \bar{\theta} \lambda P)^2} - e_s \right) \times \frac{D_s}{CC_s} \tag{21}$$

It is assumed that each type of company can optimise its total travel by speed adjustment to maximise its profit. Then, the utility maximisation model in sailing route s can be expressed as:

$$\max_{V_s, D_s} (U_s) = U_s(V_s, D_s, D_{-s}) \tag{22a}$$

subject to:

$$D_s + D_{-s} = D, \quad \forall s \in S_w \tag{22b}$$

$$0 \leq D_s \leq D, \quad \forall s \in S_w \tag{22c}$$

Let Ξ_s be a set that satisfies Equations (22b) and (22c). Then U_s can be expressed as:

$$\max_{Y_s \in \Xi_s} (U_s(Y_s, Y_{-s})) \tag{23}$$

where $U = (U_s, U_{-s})$, $Y_s = (D_s, V_s)$, and $Y_{-s} = (D_{-s}, V_{-s})$.

3.3. *Concavity of the utility function.* The upper-level model can be presented as a multi-player non-cooperative game model. The existence of the generalised Nash equilibrium solution is analysed using the mathematical properties of its utility function.

Proposition 2: Utility function U_s in Equation (22a) is strictly concave with respect to Y_s .

Proof. From $s \in S_w$:

$$\begin{aligned} \frac{\partial U_s}{\partial D_s} = & \left(\frac{a_s}{CC_s} - \frac{2b_s}{CC_s} \frac{1}{(\ln(D - D_s) - \ln D_s - \ln(X_{-s}) - \bar{\theta}\lambda P)^3} \right) \left(\frac{D}{D - D_s} \right) \\ & + \left(P - a_s ((\ln(D - D_s) - \ln(D_s) - \ln(X_{-s}) - \bar{\theta}\lambda P)) \right. \\ & \left. - \frac{b_s}{(\ln(D - D_s) - \ln(D_s) - \ln(X_{-s}) - \bar{\theta}\lambda P)^2} - e_s \right) \frac{1}{CC_s} \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{\partial^2 U_s}{\partial D_s^2} = & \left(\frac{a_s}{CC_s} - \frac{2b_s}{CC_s} \frac{1}{(\ln(D - D_s) - \ln D_s - \ln(X_{-s}) - \bar{\theta}\lambda P)^3} \right. \\ & \left. - \frac{6b_s}{CC_s} \frac{1}{(\ln(D - D_s) - \ln D_s - \ln(X_{-s}) - \bar{\theta}\lambda P)^6} \right) \times \left(\frac{D^2}{(D - D_s)^2 D_s} \right) \end{aligned} \tag{25}$$

As mentioned, fuel consumption accounts for 50% of the total travel cost; thus:

$$\left(\frac{Pc_s}{T_a} + Po_s \right) T_s < \frac{FV_s^2}{V^2} SD_s Pf_s \tag{26}$$

and:

$$\left(\frac{a_s}{CC_s} - \frac{2b_s}{CC_s} \frac{1}{(\ln(D - D_s) - \ln D_s - \ln(X_{-s}) - \bar{\theta}\lambda P)^3} \right) < 0 \tag{27}$$

From Equation (19):

$$\frac{\partial^2 U_s}{\partial D_s^2} < 0 \tag{28}$$

The utility function is proven to be strictly concave.

As utility function U_s is strictly concave with respect to Y_s , \emptyset_s is composed of the linear equality in Equation (22b) and the inequality in Equation (22c). Then, according to the

conclusion proven by Harker (1991), the QVI formulation of Equation (9) can be equivalent to the following VI problem: finding $Y^* \in \Xi$ such that:

$$F(Y^*)^T(Y - Y^*) \geq 0, \quad \forall Y \in \Xi_s(Y^*) \tag{29}$$

where $Y = (Y_1, \dots, Y_n)$ is the optimal solution of the GNEP in Equation (23).

4. SOLUTION ALGORITHM. To summarise, the whole bi-level model proposed in this paper is aimed at studying the game process between the customers and shipping companies whose profits are based on both total travel cost and total demand of customers, which is equivalent to solving the following optimisation problem:

$$\max_{V_s, D_s} (U_s) = U_s(V_s, D) \tag{30}$$

As mentioned earlier, solving Equation (27) is equivalent to finding $V^* = (V_s^*, V_{-s}^*)$ such that:

$$F(D^*, V^*)^T(V^* - V) \geq 0, \quad \forall V \in \Xi_s(Y^*) \tag{31a}$$

With equilibrium point $V^* = (V_s^*, V_{-s}^*)$, the total customer demand can be expressed as:

$$D^* = D(V^*) \tag{31b}$$

In this paper the generalised reduced-gradient method has been used to solve the upper-level model, which is presented as a GNEP of Equation (31a). The iterative balancing method (Bell, 1995) is used to solve the lower-level problem, which can be described as a logit-type multi-path assignment problem. The formulation of the algorithm to solve this bi-level model with speed optimisation is proposed as follows:

- Step 1 Initialise the sailing speed on each route $v^j, j = 1$
- Step 2 Find v^j [solve Equation (31a) using the generalised reduced-gradient method, see Figure 2] such that:

$$\sum_k F(v_s^j, v_{-s}^j, D(v_s^j, v_{-s}^j)) \geq 0, \quad \forall v = (v_s^j, v_{-s}^j) \in V \tag{32}$$

Thus, the gradient of the utility function can be expressed as:

$$F = \frac{dU_s}{dV_s} = \frac{\partial U_s}{\partial V_s} + \frac{\partial U_s}{\partial D} \frac{\partial D}{\partial V_s} \tag{33}$$

Moving steps α^j can be solved by:

$$\min f(v^j + \alpha^j(v^j - z)) \tag{34a}$$

subject to:

$$0 < \alpha^j < 1, \quad \forall j \in J. \tag{34b}$$

The total customer demand in Equation (31b) can be solved using the iterative balancing method.

- Step 3 Output optimal speed $v_{bes} = (v_{bes1}, \dots, v_{besn})$ and optimal utility function $U_{max} = (U_{max1}, \dots, U_{maxn})$.

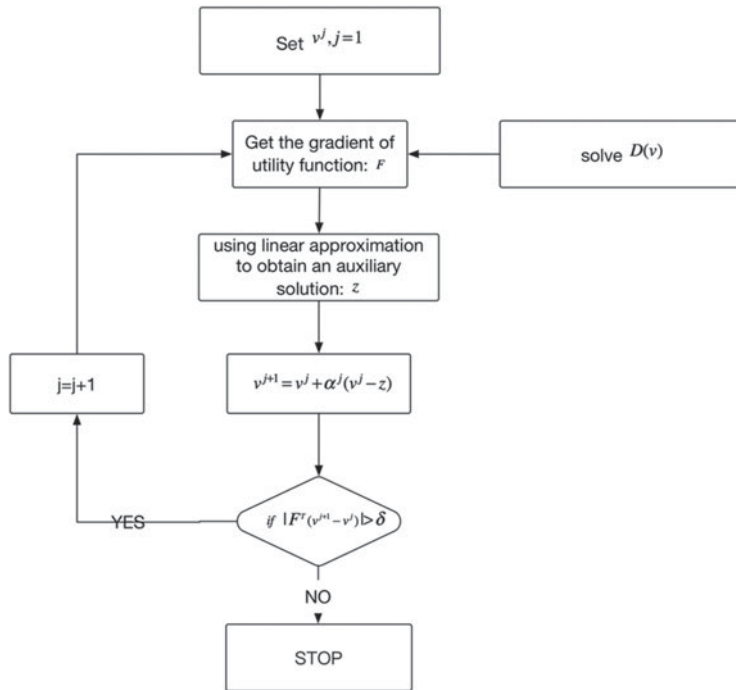


Figure 2. Algorithm flow of the generalised reduced-gradient method.

5. CASE STUDY.

5.1. *Study area and work flow.* In this paper, a case study is used to investigate the effectiveness of the bi-level model. The OD pair is from Dalian to Rotterdam, two routes are chosen to connect these two ports, one is the Northern Sea Route (NSR), while the other is the traditional route via Suez Canal (SUE) (Figure 3). For shipping companies, two kinds of companies can be selected for transport.

Company 1: Using the traditional route all the year round.

Company 2: Using the NSR when it is navigable and choosing the traditional route in the rest of the year

Work flow of the case can be seen in Figure 4. Firstly, the bi-level model has been proposed to find the optimal speed and demand change of each company when the equilibrium between two companies is reached. After considering the harsh climatic and hydrological conditions in the Arctic, the vessel speed should be adjusted which leads to the adjustment of elastic demand. Finally, the economic potential of each company in one year can be analysed with the adjusted speed and demand.

5.2. *Bi-level model for optimal speed and elastic demand.* The case presented is based on a scenario originally used by Liu and Kronbak (2010) (with partial data from Way et al. (2015)) so as to demonstrate the application of the proposed model and to test the convergence of the solution algorithm. The simulated scenario is presented as follows:

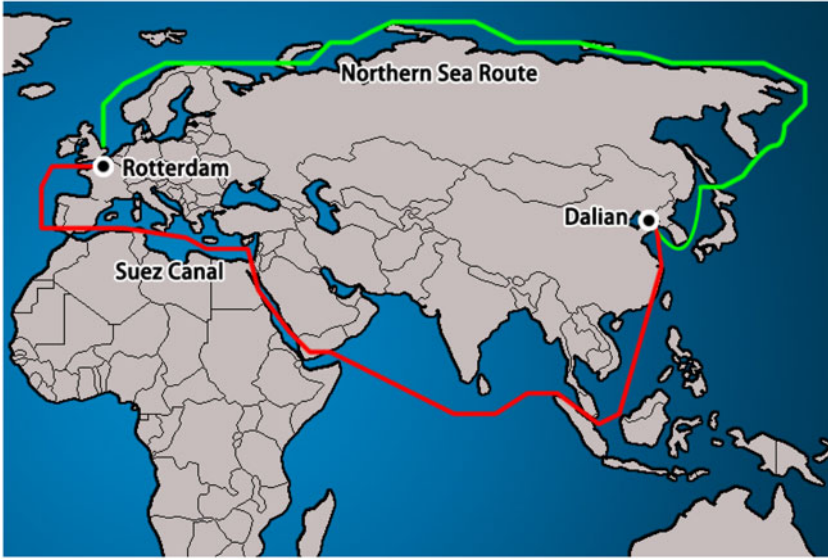


Figure 3. Study area.

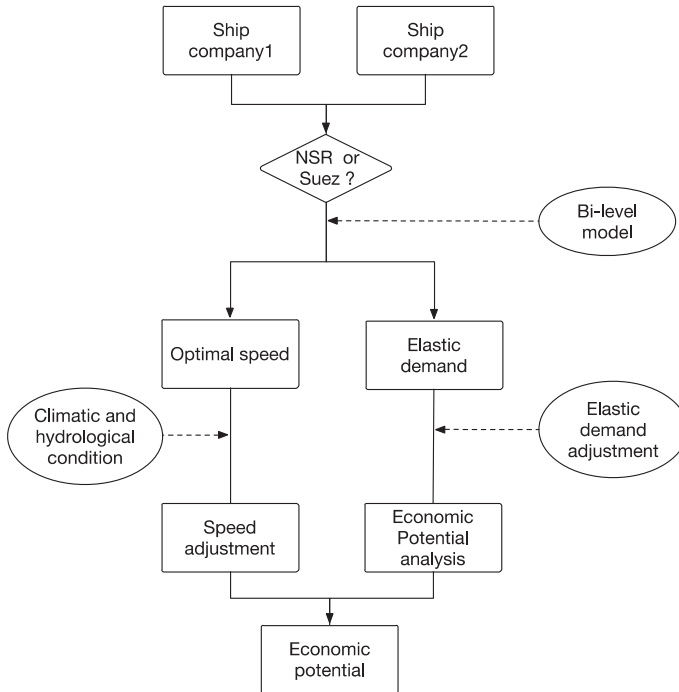


Figure 4. Flow of case study.

- The sailing distance on the NSR from Dalian to Rotterdam is 7,931 nautical miles, whereas that on the traditional route via the Suez Canal (SUE) is 10,907 nautical miles.
- A 4,300 Twenty-foot Equivalent Unit (TEU) container ship has been chosen on the traditional route while a CSC3 ice class vessel on the NSR is assumed to have the same main dimensions except the lightship weight. The maximum speed of a ship on each route is no more than 25 knots.
- Three fuel prices are assumed for comparison purposes: USD 350, USD 700, and USD 900 per ton.
- The average pilotage and ice-breaking fee on the NSR is USD 446,000 per transit.
- The voyage fee on the SUE is USD 240,800 per transit.
- The price of a 4,300 TEU ship on the NSR is USD 5.28 M per year, whereas that of a vessel on the SUE route is USD 4.4 M per year.
- The operating cost (including manning, hull and machinery insurance, protection and indemnity insurance, repairs and maintenance, administration, etc) on the NSR is USD 8,925 per day, whereas that on the SUE route is USD 6,100 per day.
- The expected revenues of all shipping companies are assumed to be the same, that is USD 3 M per transit.
- The total customer demand in the first equilibrium state is 4.3 M TEU per year.

The above scenario is introduced into the proposed bi-level model and the first equilibrium is considered, which is the competition result among shipping companies before the Arctic sea route rise, as Stage 1. Similarly, Stage 2 represents another equilibrium, which is the result of the competition among shipping companies with the rise of the Arctic sea route. The game begins at Stage 1. Each company type maximises its profits through speed adjustment, which can reduce its total travel cost in the upper-level and attract more customer demand in the lower-level. Finally, the new equilibrium in Stage 2 is reached. Throughout this game, three factors are used to compare Stage 1 with Stage 2: speed, maximum profits and total customer demand.

First, the selection of model parameters is analysed. The relationships between the sailing speed and dispersion parameter $\bar{\theta}$ are shown in [Figure 5](#). It can be seen that under different fuel-price scenarios, the sailing speed increases, and the difference in the sailing speed between Stages 1 and 2 becomes more obvious as the value of $\bar{\theta}$ increases. This result can be interpreted to mean that a larger $\bar{\theta}$ implies a better understanding of the shipping companies that can more markedly change their sailing speed according to the variation in the customer demand. [Table 1](#) lists the relationship between sensitivity parameter β and the total customer demand. It can be seen that the value of the total customer demand decreases as the value of β increases under different fuel-price scenarios both in Stages 1 and 2. Meanwhile, the difference in the total customer demand among different fuel-price scenarios becomes more definite with the increase in parameter β , which implies that the total customer demand is more sensitive to a larger value of β . In addition, if the value of β exceeds 0.05, the results in both Stages 1 and 2 greatly deviate from the original data (4.3 M TEU).

In this numerical experiment, the dispersion parameter is given as 0.1, whereas the sensitivity parameter is given as 0.01. The convergence of the proposed solution algorithm is shown in [Figures 6](#) and [7](#). It can be seen that stable solutions are obtained by the solution algorithm. Finally, the optimal speed and customer demand in each stage under different fuel-price scenarios are listed in [Table 2](#).

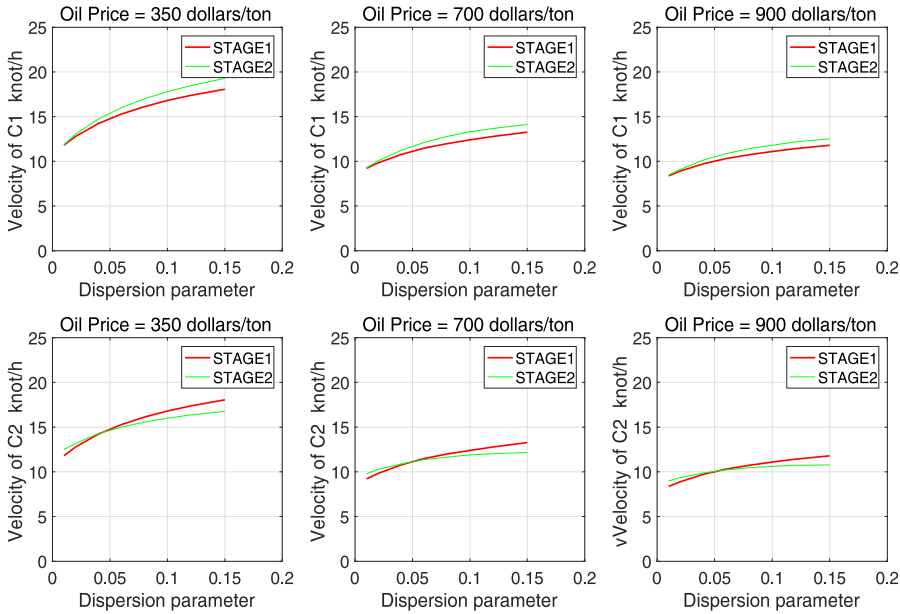


Figure 5. Dispersion parameter $\bar{\theta}$ and sailing speed.

Table 1. Sensitivity parameter and total customer demand. (The values in the brackets are the fuel-price scenarios: USD 350, USD 700, and USD 900 per ton).

Parameter β	Total Customer Demand (10^8 TEU)	
	Stage 1	Stage 2
0.001	(4.08,4.08,4.03)	(4.11,4.08,4.07)
0.005	(3.33,3.17,3.09)	(3.44,3.32,3.26)
0.01	(2.57,2.33,2.23)	(2.75,2.56,2.47)
0.05	(0.33,0.20,0.16)	(0.46,0.32,0.27)
0.1	/	/

By comparing the data (see Table 2) in Stages 1 and 2, some interesting phenomena are observed. First, shipping companies on different sea routes would take different measures to join the competition in the event the equilibrium in Stage 1 is broken with the rise of the NSR. The shipping companies on the traditional route would decide to minimise their delivery time cost by increasing speed, aiming at attracting more customer demand, which is their optimal strategy given the competition with other shipping companies on the NSR, although the profits themselves actually decrease in Stage 2. Meanwhile, for the NSR, shipping companies take the opposite measure, that is, lowering the shipping speed because the shorter shipping distance on the NSR guarantees that shipping companies can cut their travel cost by appropriately lowering their shipping speed as well as maintaining competitiveness, although the shipbuilding cost, daily operating cost, and even the voyage fee on the NSR are all higher than those for the SUE route. Therefore, the profits of the shipping companies in using the NSR increase in Stage 2. However, the equilibrium in

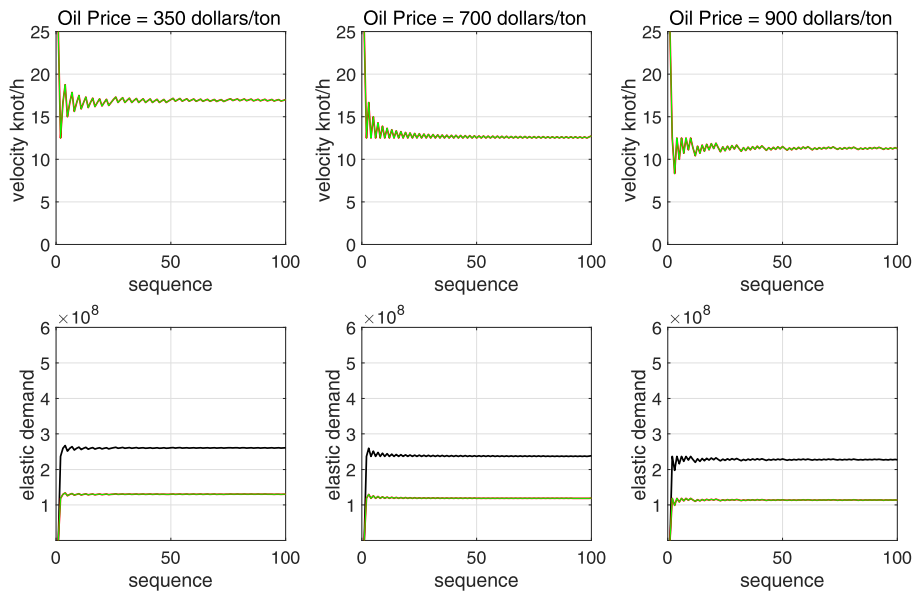


Figure 6. Convergence of proposed model in Stage 1. (Red lines: optimal speed and demand of Company 1. Green lines: optimal speed and demand of Company 2. Black lines: total customer demand).

Stage 2 is fragile. In fact, only in the case where no communication exists between these two types of shipping companies can this equilibrium be maintained. Otherwise, the SUE shipping companies will transform into NSR shipping companies when they realise that the profits in using the NSR are much higher than those in the SUE route, irrespective of the strategies they choose to maximise the profits on their current sea route. As a conclusion, a relative decline in the SUE may be a result of a rise of the use of the NSR.

Secondly, the total customer demand increases according to the competition among the companies that use the SUE and NSR routes. If these two company types are considered as a whole, it can be seen that the total profits rise in Stage 2 compared with those in Stage 1. Therefore, this Stackelberg game becomes a win–win game in which both customers and shipping companies optimise their operating patterns.

The conclusion that traditional routes may decline in face of the rise of the Arctic routes and the total volume of freight can be increased as the Arctic routes rise have been proved in previous works (Bekkers et al., 2016; Countryman et al., 2016). Therefore, according to this numerical case, this model can well reflect the interaction mechanism between the Arctic and more traditional routes.

5.3. *Economic potential based on climatic and hydrological conditions.* The bi-level model is an ideal mathematic model, and it does not consider the impact of climatic and hydrological conditions. For the foreseeable future, vessel speeds will be affected by harsh Arctic weather (Nam et al., 2013). Data related to water depth comes from a product called ETOPO1 provided by the National Geophysical Data Center (NGDC), with a resolution of 1 arc-minute. Wave height data comes from the 1979–2017 daily global atmospheric reanalysis product ERA-Interim daily data provided by the European Centre for Medium-Range Weather Forecasts (ECMWF), with an accuracy of $0.125^\circ \times 0.125^\circ$. Sea

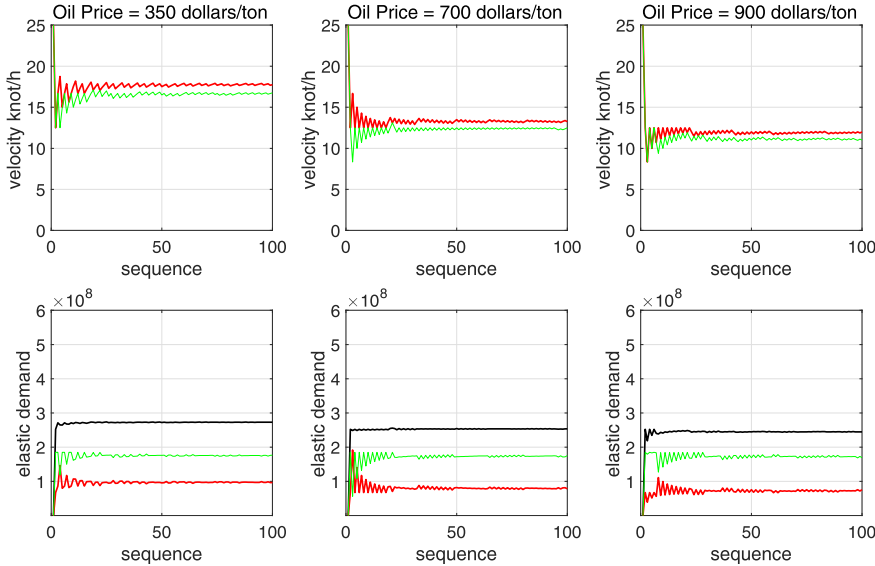


Figure 7. Convergence of the proposed model in Stage 2. (Red lines: optimal speed and demand of Company 1. Green lines: optimal speed and demand of Company 2. Black lines: total customer demand).

Table 2. Optimal speed and profits of each shipping company and total customer demand. (The values in the brackets are the fuel-price scenarios: USD 350, USD 700, and USD 900 per ton. Company 1 is defined as the shipping company on the SUE. Company 2 is defined as the shipping company on the NSR.)

	Optimal Speed (knot/h)		Total Customer Demand (10^8 TEU)	Profits (10^{10} USD)	
	Company 1	Company 2		Company 1	Company 2
STAGE1	(17,12.75,11.25)	(2.61,2.39,2.27)	(2.61,2.39,2.27)	(3.51,2.46,2.01)	(3.51,2.46,2.01)
STAGE2	(17.75,13.25,12)	(2.73,2.54,2.45)	(2.73,2.54,2.40)	(2.48,1.49,1.14)	(5.61,4.58,4.19)

ice concentration and thickness data comes from the 2006-2100 daily data of the Community Climate System Model version 4 (CCSM4) model under the medium Representative Concentration Pathway (RCP45) provided by the National Oceanic and Atmospheric Administration/ National Centre for Atmospheric Research (NOAA/NCAR), with a lattice precision of $1^\circ \times 0.5^\circ$. Wind field data comes from the 2006–2010 monthly data from the GFDL-ESM2G model provided by NOAA under the RCP45, with an accuracy $1^\circ \times 0.43^\circ$. Current field data is derived from the 1993–2014 AVISO+ multi-satellite fusion daily data provided by the French National Space Research Center (CNRS), with an accuracy of $0.75^\circ \times 0.37^\circ$. Due to different spatial accuracy and time scales of the climatic and hydrological data, all the data should be interpolated into the same time and space scales. In this paper, time scale is assumed to be annual average, and the spatial scale is $1^\circ \times 0.5^\circ$.

5.3.1. *Optimal speed adjustment.*

5.3.1.1. *Navigable time on the NSR.* The Arctic Ice Regime Shipping System (AIRSS) is used to evaluate the navigable days for a CSC3 ice class vessel on the NSR. In this system, the Ice Number (IN) index reflecting the navigable situation under specific ice

Table 3. Ice type (Transport Canada, 1998).

Ice type	Characteristics
New	Newly formed ice, which is consist of ice edges, grease ice, crushed ice clusters, etc. These types of ice are just loosely frozen together, only to be seen in the floating process;
Grey	The young ice has a thickness of 10–15 cm, which is lower than that of nilas and is easy to expand and break;
Grey white	The young ice has a thickness of 15–30 cm;
Thin first year	One year ice does not exceed one winter formation and the thickness is between 30–70cm;
Medium first year	One year ice has a thickness of 70–120 cm;
Thick first year	One year ice has a thickness of 120–200 cm;
Second year	Adult ice has gone through at least one summer melt and the thickness is between 200–300 cm;
Multi year	Multi-year ice has gone through at least two summer melts and the thickness is larger than 300 cm.

Table 4. Ice multiplier (knots) (Transport Canada, 1998).

	Open water	Grey	Grey white	Thin first year 1st stage	Thin first year 2nd stage	Medium Medium year	Thick Thick year	Second year	Multi year
CAC 3	2	2	2	2	2	2	2	1	–1

conditions can be calculated as:

$$IN = (C_a IM_a) + (C_b IM_b) + \dots + (C_n IM_n) \quad (35)$$

where C_a represents the ice concentration of ice type a , IM_a is the multiplier which is used to calculate the impact weight of ice type a on vessel navigation. When the IN is larger than zero, it means the CSC3 ice class vessel can safely sail in this area, otherwise, it cannot navigate there. The details of all kinds of ice and ice multiplier are presented in Tables 3 and 4. If the IN of all girds on the NSR are larger than zero, that day can be calculated as a navigable day.

5.3.1.2. *Ice condition to speed adjustment.* Sea ice concentration and thickness are the key factors to Arctic vessel speed. Generally, the thicker and more concentrated the sea ice becomes, the greater the reduction of the ship's sailing speed. In some harsh ice condition areas, an icebreaker has to be hired to help navigate, which will undoubtedly increase the shipping cost for the whole route. The relationship between sea ice thickness and concentration can be seen in Table 5. DS is the designed vessel speed. In an area where sea ice concentration is less than 30%, without considering other meteorological and hydrological factors, the CSC3 vessel can sail at the DS speed. Additionally, the italic part refers to the vessel speed with the help of an icebreaker.

5.3.1.3. *Wind, wave and current to speed adjustment.* In open water areas, vessel speed is also affected by the wind, wave and current conditions. The impact of these factors on vessels depends on their value and direction and is presented in Table 6. For example, if the sailing direction is consistent with the direction of wind and flow, vessel speed will be increased, otherwise, it will hinder the navigation of vessels. In addition, when the sea ice concentration is larger than 30%, these factors can be neglected.

Table 5. Relationship between sea ice and vessel speed (Nam et al., 2013).

Sea ice thickness (cm)	<30	30–120	120–180	180–240	>240
Sea ice concentration (%)					
Open water			DS		
<30	DS	10	8	8	6
30–60	DS*0.8	8	8	7	6
60–80	DS*0.6	6	10	10	8
80–100	DS*0.5	8	6	6	4

Table 6. Relationship between vessel speed and current, wave, wind (knots) (Nam et al., 2013).

Sea ice concentration (%)	Wave height (m)	The direction of wind and current		
		Head sea	Beam sea	Following sea
Open water	<3	DS-1-VC	DS	DS+VC
	3–5	DS-2-VC	DS	DS+1+VC
	>5	DS-6-VC	DS-3	DS-3+VC
<30	<1		10	
	1–2		8	
	2–3		6	
	3–5		5	
	5–7		4	
	>7		3	
>30	0		DS	

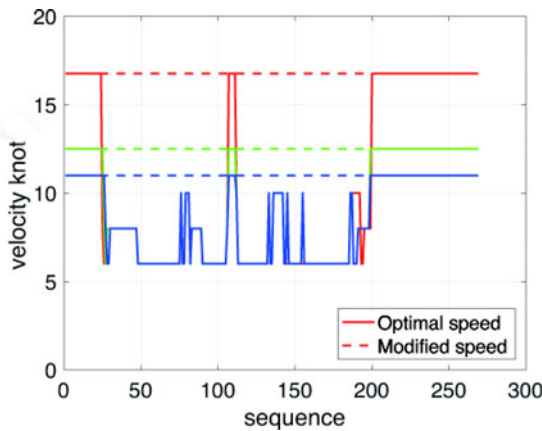


Figure 8. Adjustment of vessel speed on the NSR.

Firstly, the navigable time has been calculated, and then the optimal speed derived from the bi-level model is incorporated into the above models and the results can be seen in Figure 8. The navigable days in the year 2050 can reach to 250 and the average adjusted speed \bar{V}_s^A on the NSR is 10.54 (Oil price = 350), 8.97 (Oil price = 700), 8.41 (Oil price = 900) while the speed on the traditional route is unchanged.

5.3.2. Elastic demand adjustment. According to Equations (11)–(13), the total demand can be changed according to the variation of time cost manipulated by the change

Table 7. Demand and speed adjustment and annual profits. (The values in the brackets are the fuel-price scenarios: USD 350, USD 700, and USD 900 per ton. Company 1 is defined as the shipping company on SUE. Company 2 is defined as the shipping company on NSR).

	Adjusted speed (knot/h)	Adjusted demand (10 ⁸ TEU)	Adjusted total demand (10 ⁸ TEU)	Annual profits (10 ¹⁰ USD)
Company 1	(17, 12.75, 11.25)	(1.32, 1.03, 0.94)	(2.53, 2.35, 2.28)	(3.16, 1.77, 1.32)
Company 2	(10.54, 8.97, 8.41)	(1.21, 1.32, 1.34)		(2.88, 2.36, 2.1)

of vessel speed, the equation can be rewritten as:

$$D^A = D^0 \exp \left\{ \min \left[\frac{\beta}{\theta} \ln \left(\sum_{s \in S_w} \exp(-\bar{\theta}(\lambda P + T_s^d)) \right) \right] \right\}, \quad \forall s \in S_w \tag{36}$$

where T^d is the navigation time according to the adjusted vessel speed and can be presented as:

$$T_s^d = T_w + \sum_{1:n} \frac{SD_{sn}}{V_{sn}^A}, \quad \forall s \in S_w \tag{37}$$

When the total demand is decided, the elastic demand can be derived for each company according to Equation (16):

$$D_s^A = \frac{e^{-\bar{\theta}(\lambda P + T_s^d)}}{\sum_{k \in S_w} e^{-\bar{\theta}(\lambda P + T_k^d)}} \times D, \quad \forall s \in S_w. \tag{38}$$

Based on Equations (1)–(7) and Equation (14), the annual profits of these two companies can be derived with adjusted speed and demand. Results are shown in Table 7. Ships on the NSR cannot navigate all year round even in the middle of this century and cannot reach the optimal speed due to the harsh weather. Using the bi-level model, the actual demand of Company 2 will be less, as will the annual profits, while the demand and annual profits of Company 1 will increase. Additionally, the actual total demand will have a tiny decrease. Overall, in this practical case, the NSR has potential economic competitiveness with the SUE route and will be a viable alternative route for container shipping.

6. CONCLUSION. Due to climate change, Arctic sea ice has been decreasing, which has started to open Arctic sea routes. Compared with traditional routes, Arctic routes are shorter, have fewer pirates, less congested waters and the potential for more sea room. However, the worse weather conditions, more fragile ecosystem, and lack of infrastructure, ports and emergency rescue make this comparison complex and multi-dimensional. The most direct method to compare the economic potential of these two routes is to calculate the average annual profits on each route by taking all the main factors into consideration and by assuming that all routes are independent of each other. In fact, an obvious interaction exists among the routes between any OD pair. Therefore, when the potential competitiveness of Arctic routes is considered, the impact of non-Arctic routes needs to be taken into account. Thus, a bi-level model from the perspective of game theory is proposed to solve this issue.

In this bi-level game model, two levels are presented to depict the dynamic variation of the world liner shipping market linking shipping companies (their strategies and

profits) and customers (their cost and demand). The upper-level model demonstrates the competition among shipping companies who maximise their own profits through speed optimisation. The lower-level model illustrates the response of customers who reassign their demand with elastic total demand. A case study is presented to demonstrate the application of the proposed model and to test the convergence of the solution algorithm. The bi-level model that has been described in this paper is an ideal model that assumes the Arctic ice is completely melted and vessels can navigate on that route all the year round. To make the study more practical, the impact of harsh weather conditions have been examined in Arctic regions on vessel speed and elastic demand of each company in the case study section. With the adjusted vessel speed and demand, annual profits of each company are analysed. The results reveal that in 2050, the NSR has potential economic competitiveness compared to the traditional route and will be an alternative route for container shipping. Furthermore, this model can be applied to simulate the variation of the world shipping market demand and provide decision support for shipping companies to develop future development strategies (including route selection, new shipbuilding amount and type, sailing speed setting, etc).

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