BULL. AUSTRAL. MATH. SOC. 05B05
VOL. 61 (2000) [463–472]

TWO-BLOCK CONFIGURATION FORMULAE FOR BTD'S WITH PARAMETERS $(V, B, R, 3, \Lambda)$

M.A. FRANCEL

In this paper, we count two-block configurations in a general setting. In particular, no restriction is put on the pair repetition factor (that is, the parameter Λ) of the block designs being considered. Besides giving formulae for the counts of the two-block $BTD(V, B, R, 3, \Lambda)$ configuration classes, a basis for the formulae is given and shown to have size four. Details of the special cases where Λ equals two and three are also presented, along with 2-block $BIBD(v, b, r, 3, \lambda)$ configuration results.

1. INTRODUCTION

In recent years, researchers have shown an interest in n-block configuration formulae related to a variety of block designs [**2, 3, 4, 5, 7**]. For all known formulae, both K, the design block size, and Λ, the design pair repetition factor, are restricted to single values. The purpose of this paper is to demonstrate that limiting both the block size and the pair repetition factor is unnecessary. To do this, the paper develops reasonable two-block configuration formulae in a more general setting. In particular, the restriction on the pair repetition factor is lifted. In the work that follows, the block size is restricted to three, but the pair repitition factor can be any acceptable positive value. In Section 2, we present the two-block configuration classes and the formulae in this general setting. In Section 3, we give a basis for the set of formulae. In Section 4, we discuss the special cases where Λ is two or three. We close by showing how the BTD configuration formulae can be used to generate related $BIBD$ configuration formulae.

A *balanced ternary design*, BTD, with parameters (V, B, R, K, Λ) is a collection of B blocks on V elements such that each element occurs R times in the design; each block contains K elements, where an element may occur $0, 1$, or 2 times in a block (that is, a block is a collection of elements rather than a set of elements); and each pair of distinct elements occurs Λ times in the design. BTD's are *regular* in the sense that every design element occurs singly in ρ_1 blocks and doubly in ρ_2 blocks where $R = \rho_1 + 2\rho_2$ [**1**]. Because of this regularity, BTD parameters are given here as $(V; B; R, \rho_1, \rho_2; K; \Lambda)$.

Received 1st September, 1999

An *n-block BTD configuration* is a collection of any n distinct blocks in the BTD. The set of all n-block configurations in a BTD can be partitioned, in a natural way, into *configuration classes.* Each class can be represented by a *template* that reflects the underlying relationships between elements and blocks in the configurations of that class. For example, the 2-block configurations of the $BTD(3;4;4,2,1;3;3)$ with blocks $\{\mathbf{112},\ \mathbf{223},\ \mathbf{331},\ \mathbf{123}\}$ partition into two configuration classes:

$$\{\{\mathbf{112},\ \mathbf{223}\},\ \{\mathbf{223},\ \mathbf{331}\},\ \{\mathbf{331},\ \mathbf{112}\}\} \text{ and}$$
$$\{\{\mathbf{123},\ \mathbf{112}\},\ \{\mathbf{123},\ \mathbf{223}\},\ \{\mathbf{123},\ \mathbf{331}\}\}.$$

These are represented by the templabes **aab**, **bbc** and **abc**, **aab** respectively.

Given an n-configuration class in a $BTD(V;B;R,\rho_1,\rho_2;K;\Lambda)$, an *$n$-configuration formula* for the class is a formula that counts the number of configurations in the class. A set of m-configurations, $m \leqslant n$, is said to be a *generating set* for the n-configuration formulae, if every n-configuration formulae can be expressed as a linear combination of the counts of the configurations in the generating set plus a rational expression in the design parameters. We are assuming configuration coefficients are also rational expressions in the design parametrs. If the generating set is minimal, it is called a *basis.*

The simplest example of configuration formulae are the 1-configuration formulae: $|\mathbf{aab}| = V\rho_2$, and $|\mathbf{abc}| = B - V\rho_2$. Note each formula is expressed solely as a rational expression in the design parameters. When this is the case, we say the configuration is *constant.* Configurations that are not constant are said to be *variable.* Variable n-configuration formulae first appear when $n = 2$.

As illustrated above, throughout the paper we shall use bolded triples to denote BTD blocks. Unless otherwise noted distinct letters will be used to indicate distinct elements. For example, the block pair **aab, bbc** contains three distinct elements; **a** appearing twice in the first block, **b** appearing once in block one and twice in block two, and **c** appearing once in block two.

2. Two-block configurations

There are fourteen distinct classes of two-block BTD configurations for designs with block size three. The fourteen classes are represented by the following templates: {**abc**, **abc**}, {**abc**, **abd**}, {**abc**, **ade**}, {**abc**, **def**}, {**aab**, **aab**}, {**aab**, **aac**}, {**aab**, **bba**}, {**aab**, **bbc**}, {**aab**, **ccb**}, {**aab**, **ccd**}, {**aab**, **abc**}, {**aab**, **acd**}, {**aab**, **bcd**}, {**aab**, **cde**}. Table 2.1 lists these two-block configuration templates along with the formulae that count the number of configurations in the classes and the formulae dependencies. In the table, each class is labelled C_i for some i. Throughout the paper, we shall use this label to refer to the configuration class. The symbol c_i will be used to denote $|C_i|$, the number of 2-configurations of type C_i in the BTD.

In this section, we shall show that $B = \{\{\mathbf{abc}, \mathbf{abc}\}, \{\mathbf{aab}, \mathbf{aab}\}, \{\mathbf{aab}, \mathbf{bba}\}, \{\mathbf{aab}, \mathbf{ccb}\}\} = \{C_1, C_5, C_7, C_9\}$ is a generating set for the 2-configuration $BTD(V; B; R, \rho_1, \rho_2; 3; \Lambda)$ formulae. In Section 3, we shall prove that B is a basis.

Below we generate a system of ten linear equations involving the c_i's that will give us the desired formulae.

There are $V\rho_2$ blocks of type **aab** in a $BTD(V; B; R, \rho_1, \rho_2; 3; \Lambda)$. If we match each block **aab** with every other block where **a** is double, we get each of the $\{\mathbf{aab}, \mathbf{aab}\}$ and $\{\mathbf{aab}, \mathbf{aac}\}$ configurations twice. Thus,

$$(1) \qquad c_5 + c_6 = V\rho_2(\rho_2 - 1)/2.$$

If we match each block **aab** with everh block $\mathbf{bb}x$, x will be **a** or **c**, and we get each $\{\mathbf{aab}, \mathbf{bba}\}$ twice, and each $\{\mathbf{aab}, \mathbf{bbc}\}$ once. Thus,

$$(2) \qquad 2c_7 + c_8 = V\rho_2^2.$$

If we match each block **aab** with every block $\mathbf{cc}x$ where **c** is neither **a** nor **b**, we get each $\{\mathbf{aab}, \mathbf{ccb}\}$ and $\{\mathbf{aab}, \mathbf{ccd}\}$ twice, and each $\{\mathbf{aab}, \mathbf{bbc}\}$ once. Thus,

$$(3) \qquad c_8 + 2c_9 + 2c_{10} = V\rho_2(V - 2)\rho_2.$$

We next look at counts that involve one block of form **xyz** and one of form **wwt**. Since there are $V\rho_2$ blocks of form **wwt**, there are $B - V\rho_2$ blocks of form **xyz**. If we match each block **abc** with each of the blocks where **a**, **b**, or **c** is double, we get each $\{\mathbf{abc}, \mathbf{aab}\}$ and $\{\mathbf{abc}, \mathbf{aad}\}$ once. Thus,

$$(4) \qquad c_{11} + c_{12} = (B - V\rho_2)3\rho_2.$$

If we match each block **abc** with each of the blocks that contains a double element different from **a**, **b**, or **c**, we get each $\{\mathbf{abc}, \mathbf{dda}\}$ and $\{\mathbf{abc}\ \mathbf{dde}\}$ once. Thus,

$$(5) \qquad c_{13} + c_{14} = (B - V\rho_2)(V - 3)\rho_2.$$

If we match each block **aab** with each of the other blocks that contain a pair **ab**, we get each $\{\mathbf{aab}, \mathbf{aab}\}$ and $\{\mathbf{aab}, \mathbf{abb}\}$ four times and each $\{\mathbf{aab}, \mathbf{abc}\}$ once. Thus,

$$(6) \qquad c_{11} + 4(c_5 + c_7) = V\rho_2(\Lambda - 2).$$

The last match we make with the blocks of form **aab** are with those blocks that contain **b** singly. The remaining two elements in the second block can be **aa**, **cc**, **ac**, or **cd**.

https://doi.org/10.1017/S0004972700022498 Published online by Cambridge University Press

The blocks where the pair is **aa** or **cc** each appear twice. The ones where the pair is **ac** or **cd** each appear once. Thus,

$$(7) \qquad 2c_5 + 2c_9 + c_{11} + c_{13} = V\rho_2(\rho_1 - 1).$$

Lastly we match two blocks each with form **abc**. We begin by matching the block **abc** with all blocks that contain a pair of elements from **abc**. The generated list will include each **abc**, **abc** six times, and each **abc**, **abd** and **abc**, **abb** twice. Thus,

$$(8) \qquad 2c_{11} + 6c_1 + 2c_2 = (B - V\rho_2)3(\Lambda - 1).$$

Next match each block **abc** with all blocks that contain **a** singly. The remaining two elements in the second block can be **bb**, **bc**, **bd**, **de**, or **dd**. The blocks that contain **bb** or **dd** will each appear once in the list. The block pairs **abc**, **ade** and **abc**, **abd** will each apear four times, and the block pair **abc**, **abc** six times. Thus,

$$(9) \qquad c_{11} + 6c_1 + 4c_2 + 2c_3 + c_{13} = (B - V\rho_2)3(\rho_1 - 1).$$

Lastly, to count **abc**, **cde** configurations; we match each **abc** block with all other blocks that contain no duplicated element. This gives us

$$(10) \qquad c_1 + c_2 + c_3 + c_4 = (B - V\rho_2)(B - V\rho_2 - 1)/2.$$

The above set of ten linear equations can be solved to give the formulae:

$$\begin{aligned}
\mathbf{c}_2 &= -3c_1 + 4c_5 + 4c_7 - 5V\rho_2\Lambda/2 + 7V\rho_2/2 + 3B\Lambda/2 - 3B/2,\\
\mathbf{c}_3 &= 3c_1 - 7c_5 - 8c_7 + c_9 + 5V\rho_2\Lambda - 5V\rho_2 - 3B\Lambda + 3B/2 + 3B\rho_1/2 - 2V\rho_1\rho_2,\\
\mathbf{c}_4 &= -c_1 + 3c_5 + 4c_7 - c_9 - 5V\rho_2\Lambda/2 + 2V\rho_2 + 3B\Lambda/2 - B/2 + 2V\rho_1\rho_2\\
&\qquad - 3B\rho_1/2 + B^2/2 - BV\rho_2 + V^2\rho_2^2,\\
\mathbf{c}_6 &= -c_5 + V\rho_2^2/2 - V\rho_2/2,\\
\mathbf{c}_8 &= -2c_7 + V\rho_2^2,\\
\mathbf{c}_{10} &= c_7 - 3V\rho_2^2/2 + V^2\rho_2^2/2,\\
\mathbf{c}_{11} &= -4c_5 - 4c_7 + V\rho_2\Lambda - 2V\rho_2,\\
\mathbf{c}_{12} &= 4c_5 + 4c_7 - V\rho_2\Lambda + 2V\rho_2 + 3\rho_2 B - 3V\rho_2^2,\\
\mathbf{c}_{13} &= 2c_5 + 4c_7 - 2c_9 - V\rho_2\Lambda + V\rho_2 + V\rho_1\rho_2,\\
\mathbf{c}_{14} &= -2c_5 - 4c_7 + 2c_9 + V\rho_2\Lambda - V\rho_2 - V\rho_1\rho_2 + BV\rho_2 - 3B\rho_2 - V^2\rho_2^2 + 3\rho_2^2.
\end{aligned}$$

We note these formulae have as independent variables c_1, c_5, c_7, and c_9. Thus $\{C_1, C_5, C_7, C_9\}$ is a generating set for the set of 2-block $BTD(V; B; R, \rho_1, \rho_2; 3; \Lambda)$ configuration formulae.

Configuration Classes	Configuration Count	Count Dependencies
$C_1 = \{\mathbf{abc}, \mathbf{abc}\}$	c_1	c_1
$C_2 = \{\mathbf{abc}, \mathbf{abd}\}$	$-3c_1 + 4c_5 + 4c_7 - 5V\rho_2\Lambda/2$ $+7V\rho_2/2 + 3B\Lambda/2 - 3B/2$	c_1, c_5, c_7
$C_3 = \{\mathbf{abc}, \mathbf{ade}\}$	$3c_1 - 7c_5 - 8c_7 + c_9 + 5V\rho_2\Lambda - 5V\rho_2$ $-3B\Lambda + 3B/2 + 3B\rho_1/2 - 2V\rho_1\rho_2$	c_1, c_5, c_7, c_9
$C_4 = \{\mathbf{abc}, \mathbf{def}\}$	$-c_1 + 3c_5 + 4c_7 - c_9 - 5V\rho_2\Lambda/2$ $+2V\rho_2 + 3B\Lambda/2 - B/2 + 2V\rho_1\rho_2$ $-3B\rho_1/2 + B^2/2 - BV\rho_2 + V^2\rho_2^2$	c_1, c_5, c_7, c_9
$C_5 = \{\mathbf{aab}, \mathbf{aab}\}$	c_5	c_5
$C_6 = \{\mathbf{aab}, \mathbf{aac}\}$	$-c_5 + V\rho_2^2/2 - V\rho_2/2$	c_5
$C_7 = \{\mathbf{aab}, \mathbf{bba}\}$	c_7	c_7
$C_8 = \{\mathbf{aab}, \mathbf{bbc}\}$	$-2c_7 + V\rho_2^2$	c_7
$C_9 = \{\mathbf{aab}, \mathbf{ccb}\}$	c_9	c_9
$C_{10} = \{\mathbf{aab}, \mathbf{ccd}\}$	$c_7 - 3V\rho_2^2/2 + V^2\rho_2^2/2$	c_7, c_9
$C_{11} = \{\mathbf{aab}, \mathbf{abc}\}$	$-4c_5 - 4c_7 + V\rho_2\Lambda - 2V\rho_2$	c_5, c_7
$C_{12} = \{\mathbf{aab}, \mathbf{acd}\}$	$4c_5 + 4c_7 - V\rho_2\Lambda + 2V\rho_2 + 3\rho_2 B$ $-3V\rho_2^2$	c_5, c_7
$C_{13} = \{\mathbf{aab}\ \mathbf{bcd}\}$	$2c_5 + 4c_7 - 2c_9 - V\rho_2\Lambda + V\rho_2$ $+V\rho_1\rho_2$	c_5, c_7, c_9
$C_{14} = \{\mathbf{aab}, \mathbf{cde}\}$	$-2c_5 - 4c_7 + 2c_9 + V\rho_2\Lambda - V\rho_2$ $-V\rho_1\rho_2 + BV\rho_2 - 3B\rho_2 - V^2\rho_2^2 + 3\rho_2^2$	c_5, c_7, c_9

Table 2.1. Two-block $BTD(V; B; R, \rho_1, \rho_2; 3; \Lambda)$ Configurations

3. A basis for the 2-block configuration formulae

Each of the formuale in Table 2.1 is written in terms of the design parameters and counts of the configurations in the set $\{\{\mathbf{abc}, \mathbf{abc}\}, \{\mathbf{aab}, \mathbf{aab}\}, \{\mathbf{aab}, \mathbf{abb}\}, \{\mathbf{abb}, \mathbf{ccb}\}\} = \{C_1, C_5, C_7, C_9\}$. To show that $B = \{C_1, C_5, C_7, C_9\}$ is a basis, we must show that the formulae for the configurations in B are independent of the design parameters and each other. In each of the following examples, three of c_1, c_5, c_7, and c_9 are equal with the fourth being different.

Example 1. [6] Let D_{11} and D_{12} contain the blocks:

112, 113, 114, 115, 116, 117, 221, 223, 224, 225, 226, 227,
331, 332, 334, 335, 336, 337, 441, 442, 443, 445, 446, 447,
551, 552, 553, 554, 556, 557, 661, 662, 663, 664, 665, 667,
771, 772, 773, 774, 775, 776.

Besides the already mentioned blocks, assume D_{11} also contains the blocks:

137, 124, 235, 346, 457, 156, 267,

137, 124, 235, 346, 457, 156, 267.

Besides the already mentioned blocks, assume D_{12} also contains the blocks:

137, 124, 235, 346, 457, 156, 267,

123, 145, 167, 246, 257, 347, 356.

D_{11} and D_{12} are both BTD's with parameters $(7;56;24,12,6;3;6)$. In D_{11}, $c_1 = 7$, $c_5 = 0$, $c_7 = 21$, and $c_9 = 105$. In D_{12}, $c_1 = 0$, $c_5 = 0$, $c_7 = 21$, and $c_9 = 105$. Thus, c_1 is independent of the design parameters and c_5, c_7, and c_9.

Example 2. [4] Let D_{21} contain the blocks

112, 114, 116, 133, 159, 159, 177, 188, 223, 224, 225, 267, 267, 288, 299, 335, 336, 348, 348, 377, 399, 445, 447, 449, 466, 556, 557, 558, 668, 669, 778, 799, 889.

Let D_{22} contain the blocks

112, 113, 114, 155, 166, 177, 188, 199, 225, 226, 227, 233, 244, 288, 299, 338, 339, 344, 355, 366, 377, 445, 466, 478, 478, 499, 599, 568, 568, 577, 679, 679, 889.

D_{21} and D_{22} are both BTD's with parameters $(9;33;11,5,3;3;2)$. In D_{11}, $c_1 = 3$, $c_5 = 0$, $c_7 = 0$, and $c_9 = 27$. In D_{12}, $c_1 = 3$, $c_5 = 0$, $c_7 = 0$, and $c_9 = 39$. Thus, c_9 is independent of the design parameters and c_1, c_5 and c_7.

Example 3. Let D_{31} contain the blocks

112, 223, 331, 448, 889, 994, 556, 667, 775,

158, 346, 279, 147, 268, 359, 169, 245, 378,

158, 346, 279, 147, 268, 359, 169, 245, 378,

221, 332, 113, 884, 998, 449, 665, 776, 557,

259, 346, 178, 247, 169, 358, 268, 145, 379,

259, 346, 178, 247, 169, 358, 268, 145, 379.

Let D_{32} contain the blocks

112, 223, 331, 448, 889, 994, 556, 667, 775,
259, 346, 178, 247, 169, 358, 268, 145, 379,
259, 346, 178, 247, 169, 358, 268, 145, 379,
225, 559, 992, 334, 446, 663, 117, 778, 881,
237, 458, 169, 124, 567, 389, 268, 479, 135,
237, 458, 169, 124, 567, 389, 268, 479, 135.

D_{31} and D_{32} are both BTD's with parameters $(9; 54; 18, 14, 2; 3; 4)$. In D_{31}, $c_1 = 18$, $c_5 = 0$, $c_7 = 9$, and $c_9 = 9$. In D_{32}, $c_1 = 18$, $c_5 = 0$, $c_7 = 0$, and $c_9 = 9$. Thus, c_7 is independent of the design parameters and c_1, c_5 and c_9.

Using the above three examples, it is straighforward to show that c_5 must be independent of the design parameters and c_1, c_7, and c_9. Thus $\{C_1, C_5, C_7, C_9\}$ is a basis for the desired formulae.

4. A pair repetition factor of two or three

Configuration Class	Number of Configurations when $\Lambda = 3$	Number of Configurations when $\Lambda = 2$
$C_1 = \{\text{abc}, \text{abc}\}$	m	m
$C_2 = \{\text{abc}, \text{abd}\}$	$3\big(V(V-1)/2 - V\rho_2\big) - 3m$	$V(\rho_1 - \rho_2)/2 - 3m$
$C_3 = \{\text{abc}, \text{ade}\}$	$V\rho_1(\rho_1 - 1)/2 - 3V(V-1)$ $-V\rho_2(\rho_1 - 7) + 3m + n$	$\big[V(\rho_1 - \rho_2)(\rho_1 - 3)$ $+6m - n\big]/2$
$C_4 = \{\text{abc}, \text{def}\}$	$V^2(\rho_1 - \rho_2)^2/18 + 3V(V-1)/2 - m$ $-n - 23V\rho_2/6 + V\rho_1(3\rho_2 + 1)/3$ $-V\rho_1^2/2$	$V(\rho_1 - \rho_2)$ $\big[V(\rho_1 - \rho_2) - 9\rho_1 + 15\big]/18$ $-n + m/2$
$C_5 = \{\text{aab}, \text{aab}\}$	none	none
$C_6 = \{\text{aab}, \text{aac}\}$	$V\rho_2(\rho_2 - 1)/2$	$V\rho_2(\rho_2 - 1)/2$
$C_7 = \{\text{aab}, \text{bba}\}$	none	none
$C_8 = \{\text{aab}, \text{bbc}\}$	$V\rho_2^2$	$V\rho_2^2$
$C_9 = \{\text{aab}, \text{ccb}\}$	n	n
$C_{10} = \{\text{aab}, \text{ccd}\}$	$V(V-3)\rho_2^2/2 - n$	$V(V-3)\rho_2^2/2 - n$
$C_{11} = \{\text{aab}, \text{abc}\}$	$V\rho_2$	none
$C_{12} = \{\text{aab}, \text{acd}\}$	$V\rho_2(\rho_1 - \rho_2 - 1)$	$V\rho_2(\rho_1 - \rho_2)$
$C_{13} = \{\text{aab bcd}\}$	$V\rho_2(\rho_1 - 2) - 2n$	$\big(V\rho_2(\rho_1 - 1) - m\big)/2$
$C_{14} = \{\text{aab}, \text{cde}\}$	$V\rho_2\big(V(\rho_1 - \rho_2)/3 - 2\rho_1 + \rho_2 + 2\big) + 2n$	$V\rho_2(\rho_1 - \rho_2)(V-3)/3 - m$

Table 3.1. Two-block BTD Configurations

https://doi.org/10.1017/S0004972700022498 Published online by Cambridge University Press

When Λ is small, namely has value two or three, it affects the configuration types that can occur (that is, the classes that are non-empty). When Λ is two, no configurations of type C_5, C_7, nor C_{11} can exist. When Λ is three, no configurations of type C_5, nor C_7 can exist. In both cases, the size of the basis is reduced to two.

In these special cases, the formulae can be simplified by replacing all occurrences of c_5 and c_7 (and c_{11} in the case $\Lambda = 2$) with zero, and by replacing each occurrence of Λ with its known value. Table 3.1, shown above, reflects these actions.

Note in the two special cases where Λ, the pair repetition factor of the designs, is limited to two or three, the configuration classes C_6, C_8 and C_{12} are constants. Recall in the general setting, where Λ can be any positive integer, all configuration types are variable.

5. When ρ_2 is zero

A $BIBD(V, B, R, 3, \Lambda)$ can be thought of as a $BTD(V; B; R, \rho_1, 0; 3; \Lambda)$. Thus, all formulae for n-block $BIBD(V, B, R, 3, \Lambda)$ configuration counts can be derived from the corresponding formulae for n-block $BTD(V; B; R, \rho_1, 0; 3; \Lambda)$ configuration counts. One simply needs to substitute zero for ρ_2 and ignore those configurations that contain a block with a repeated element. Applying these actions to the set of formulae given in Section 2, we get the following results:

There are four distinct classes of two-block $BIBD(v, b, r, 3, \lambda)$ configurations. They are {**abc, abd**}, {**abc, abd**}, {**abc, ade**}, {**abc, def**}.

The following formulae represents the counts for these configurations:

$$\mathbf{c}_2 = -3c_1 + 3b\lambda/2 - 3b/2,$$
$$\mathbf{c}_3 = 3c_1 - 3b\lambda + 3b/2 + 3br/2,$$
$$\mathbf{c}_4 = -c_1 + 3b\lambda/2 - b/2 - 3br/2 + b^2/2.$$

All of the above configurations formulae are variable, with the set $\{C_1\}$ acting as a basis. But if we further restrict the $BIBD$ to designs with parameters $(v, b, r, 3, 1)$, then $c_1 = 0$ and the four formulae reduce to the two formulae, and all counts become constant. This latter result was first presented by Grannell, Griggs and Mendelsohn in the original paper on configuration counts [5].

References

[1] E.J. Billington, 'Balanced n-ary designs: A combinatorial survey and some new results', *Ars Combin.* **17** (1984), 37–72.

[2] P. Danziger, M.J. Grannell, T.S. Griggs and E. Mendelsohn, 'Five line configurations in Steiner triple systems', *Utilitas Math.* **49** (1996), 153–59.

[3] M.A. Francel and D.J. John, 'Counting balanced ternary designs', *Ars Combin.* **49** (1998), 161–184.

[4] M.A. Francel and D. Sarvate, 'One and two-block configurations in balanced ternary desings', *Ars Combin.* **49** (1998), 217–225.

[5] M.J. Grannell, T.S. Griggs and E. Mendelsohn, 'A small basis for four-line configurations in Steiner triple systems', *J. Combin. Des.* **3** (1995), 51–59.

[6] D. Hoffman (private communication).

[7] P. Horak, N. Phillips, W.D. Wallis and J. Yucas, 'Counting frequencies of configurations in Steiner triple systems', *Ars Combin.* **46** (1997), 65–75.

Department of Mathematics and Computer Science
The Citadel
Charleston SC 29409
United States of America