SUNSPOT MAGNETIC FIELDS AND UMBRAL DOTS

P. R. WILSON
Dept. of Applied Mathematics, University of Sydney, Sydney, Australia

Abstract. The fine structure features of the umbral magnetic fields (i.e. large field gradients and changes of polarity in local regions) are considered as evidence of strong non-linear interactions between magnetic and thermal or mechanical forces in the umbra. It is suggested that the umbral dots observed in white light are the optical manifestation of these interactions. A three-dimensional radiative transfer analysis of possible models for these bright features is discussed and this enables one to place limits on the geometry of these features and on the non-radiative energy requirements of the models. Of the several models considered those which were compatible with convection as the source of this energy were found to be quite inconsistent with the available data. The most likely model was found to have a diameter of 200 km, a height of 50 km and an average emission of non-radiative energy of 4×10^3 erg $\mathrm{s}^{-1}/\mathrm{cm}^{-3}$ throughout this region. It is shown that this is two orders of magnitude greater than the energy available from Joule heating by locally twisted magnetic fields. However, if the energy flux transported through the umbra by Alfvén waves is partially dissipated in regions of locally twisted fields it is shown that the emission into a volume of the above dimensions is of the right order of magnitude.

1. Introduction

One of the problems of doing theoretical work in this field is the difficulty of obtaining reliable observations. If one takes a set of observations at any given time and attempts to develop a theory or model to account for them, it may be that by the time the theory is presented, no one believes the observations any more. The observational data which form the basis of the present study are the following:

(i) Umbral dots are of approximately photospheric brightness and have diameters of order 200 km. This has been deduced by Beckers and Schröter (1968) from colour intensity ratios. It is generally agreed that they have lifetimes of between 30 and 60 min.

(ii) In some local regions of the umbra the magnetic field may have a large horizontal gradient, and even change polarity within distances of order 500 km. Mogilevsky (1967) and others suggest that these variations are associated with the umbral dots. Zwaan and Buurman (1971) has studied one sunspot umbra in which a particularly dark region showed no evidence of umbral dots or field inhomogeneities, while outside this region, umbral dots were observed together with indications of small scale polarity reversals.

(iii) An umbral dot is an isolated structure; i.e. the distance between neighboring dots is large compared with their diameters (Danielson, 1964; Beckers and Schröter, 1969).

Although there are several observers who would reject some of these 'observations', there is sufficient evidence for them to warrant an investigation of some of their theoretical consequences and these turn out to be rather interesting. However, it should be emphasized that this is essentially a model atmosphere investigation and that the results must be treated with some caution. They, do however, suggest a new mechanism for small scale atmospheric heating which appears to be worth further investigation.

Howard (ed.), Solar Magnetic Fields, 475–479. *All Rights Reserved.*

2. The Umbral Dots

Some attempts have already been made to investigate multicomponent umbrae. In 1963, Makita postulated a two-component umbra in order to explain the observation of both Fe I and Fe II lines. Obridko later considered the effect on the mean umbral opacity of patches of photospheric matter occupying less than 10% of the spot area.

Both Makita and Obridko appear to envisage hot vertical columns extending through a height of order 1000 km. However, it is probably intuitively obvious that a hot column of diameter 200 km, height 1000 km, and effective temperature 5800 K cannot exist in a region of temperature 4000 K for periods of up to one hour without having a considerable warming effect on its surroundings. This is a problem in three-dimensional transfer theory and I recently carried out an analysis of some possible models (Wilson, 1969). It was shown that, because of these three-dimensional effects, the postulated surface area of a bright feature in the umbra places a constraint on the depth of the feature. By considering various models, it was possible to show that those with surface diameters 300 km or less could not extend through depths greater than 300 km below the surface.

This is of some significance because, if convection (i.e. the upward motion of hot plasma along the field lines) is to be considered as the explanation of the dots, they must extend to considerable depths in the umbra since most umbral models do not show a sharp temperature increase until depths of order 800 km are reached. Because models with surface diameters of 300 km or less cannot extend through depths greater than 300 km, it would appear that they are inconsistent with convection as the energy source. Although a model of diameter 500 km could not be limited in this way, the computed emergent intensities (when integrated over a seeing function) were inconsistent with the direct optical data of Beckers and Schröter. Of the smaller models considered, one, with diameter 200 km and depth 250 km, was found to be consistent with these data. Although the data are too uncertain to warrant any strong claims for it, it will serve as a working model for this discussion.

3. Departures from Radiative Equilibrium

The method of solving the transfer equation involves postulating models for the emission of non-radiative energy into the region of the dot. A similar method was used in investigating granulation models.

For model 1 it was shown that a maximum emission of 2.5×10^4 erg $\mathrm{s}^{-1}/\mathrm{cm}^{-3}$ is required. If convection is ruled out as the source of this energy, the magnetic data, which indicate large horizontal field gradients in the dot region, suggest Joule heating as a likely source and this may be estimated from $E = (c/4\pi \operatorname{curl} H)^2/\sigma$. Mogilevsky's observations suggest that $\operatorname{curl} H = \partial H_z/\partial x$ might be of order 20 G/km and, taking the conductivity $\sigma = 10^{10}$ esu, this yields only 20 erg $\mathrm{cm}^{-3}\mathrm{s}^{-1}$, a value consistent with estimates of 0.4–40 erg $\mathrm{cm}^{-3}/\mathrm{s}^{-1}$ given by Kopecký and Obridko (1968).

Beckers (private communication) suggested that it was perhaps unfair to consider the maximum energy emission required. However, the model derived from the transfer analysis gives the dimensions of the region of abnormal emission (i.e. diameter 200 km and depth 50 km) and if the actual non-radiative emission is averaged over this volume, the energy required is 4×10^3 erg cm^{-3} s^{-1}, which is still two orders of magnitude greater than the best which might be obtained from Joule heating. Having made considerable but unsuccessful efforts to bridge this gap, I feel that it is necessary to look for another source of non-radiant energy.

4. Alfvén Waves

Both Musman (1967) and Savage (1969) have investigated the possibility of energy transport through the umbra by hydromagnetic waves. Both use linear stability analysis which must be somewhat suspected in view of the complex nature of sunspot fields revealed by these data. However, the boundary conditions also appeared to present considerable difficulties. Musman, using open boundary conditions at the upper boundary of the superadiabatic layer, found that so much energy was carried away by the hydromagnetic waves that all the oscillatory perturbations were quickly damped. Savage, however, found that allowing for the density decrease in the stable layer permits sufficient reflection of these waves to reduce the threshold for the emission of hydromagnetic waves from these layers. Using a sunspot model of Yun, Savage suggests that hydromagnetic waves can be maintained and, calculating the energy flux from

$$F = \tfrac{1}{2}\varrho v_{\max}^2 v_a$$

(v_a is the Alfvén velocity), he finds a flux of 2×10^{10} erg cm^{-2} s^{-1}. Thus he is able to account for perhaps $\frac{1}{7}$ of the entire sunspot flux deficit (for umbral area 3×10^{18} cm^2 this gives 6×10^{28} cf 4×10^{29} erg/s). Savage considers the possibility that umbral dots are due to hydromagnetic waves being absorbed in these layers but discards it because the frictional damping length, as given by the formula $\zeta_{\text{frict}} = (v_A/\omega^2)\,(\varrho_n + \varrho_i/\varrho_n\tau_n)$, is of order 10^7 km which is far too large.

However, these calculations assume vertical parallel fields whereas the evidence of the large gradients in these regions suggest that the field lines are folded over and under these conditions the frictional damping length must be greatly reduced.

Consider an umbral model in which field lines are vertical outside a region of diameter 200 km and carry a hydromagnetic wave flux of 2×10^{10} erg cm^{-2} s^{-1}. Within this region the field lines are folded over a height which, according to our working model, is of order 50 km. Consider the case in which all the hydromagnetic energy flux is converted into thermal energy by frictional damping throughout the volume. Then the energy released per unit volume and second is

$$E = F/h_c .$$

If $h_c = 50$ km, then $E = 4 \times 10^3$ erg cm^{-3}/s^{-1}.

This agreement with the energy required is too good to be true. However, it does suggest that this mechanism might be worth further investigation.

5. Discussion

It is important to point out that a field reversal in the neighborhood of the dot is not a necessary condition for the mechanism, although it would probably be sufficient. Any large local gradient in the field would probably be adequate to disrupt the flow of Alfvén energy but whether it would be reflected back downwards or dissipated locally would depend on the particular configuration considered.

Zwaan's observation of a particularly dark region of uniform field without dots and a more normal umbral region in which dots and field reversals can be detected is of considerable significance. If a sunspot is cool because the deep magnetic field inhibits the convection, then undoubtedly there will be some equipartition of magnetic and thermal energy in this region which will enhance the field strength. Gough (1966) has shown that the most unstable mode of convection in an umbral region is a deep narrow cell. When fully established, eddy viscosity changes the most likely configuration to a broader eddy structure, but where the magnetic field prevents the flow around the eddy, the field may be deformed into a shape typical of the long thin mode rather than those which exist in established convection. In these regions the field would exhibit a pronounced fine structure and even a reversal of polarity over a small horizontal scale without altering the average overall field strength.

If this takes place well below the visible region, one would expect that the umbra should be dark and that the field should be uniform, with no evidence of reversal. However, if in parts of the umbra this interaction occurs closer to the visible region, then some folding (and hence reversal) of the field may still exist in the visible layers and thus give rise to the umbral dots.

These considerations suggest that it would be an observational point of some interest to know whether dots are always seen provided the umbra is sufficiently over-exposed, or whether there are some spots in which no umbral structure is observable. If over-exposure increasingly reveals the presence of more dots, one would like to know whether they become everywhere dense or whether at a given depth in the umbra they can be regarded as individually isolated features.

References

Beckers, J. M. and Schröter, E. H.: 1968, *Solar Phys.* **4**, 303.
Gough, D. O.: 1966, thesis, University of Cambridge.
Kopecký, M. and Obridko, V.: 1968, *Solar Phys.* **5**, 354.
Makita, M.: 1963, *Publ. Astron. Soc. Japan* **15**, 145.
Mogilevski, E. I., Demkina, L. B., Ioshpa, B. A., and Obridko, V. N.: 1968, in K. O. Kiepenheuer (ed.), 'Structure and Development of Solar Active Regions', *IAU Symp.* **35**, 215.
Musman, S.: 1967, *Astrophys. J.* **149**, 201.
Obridko, V. N.: 1968, *Bull. Astron. Inst. Czech.* **19**, 186.
Savage, B. D.: 1969, *Astrophys. J.* **156**, 707.
Wilson, P. R.: 1969, *Solar Phys.* **10**, 404.
Zwaan, C. and Buurman, J.: 1971, this volume, p. 220.

Discussion

Rösch: On the photographs which have been shown, more penumbral filaments appear, less bright, on the inner side of the penumbra, when the aperture increases, and finally, umbral dots appear to fill the entire umbra. So it seems more realistic to imagine that the limit between penumbra and umbra is not a vertical cylinder, but a dish, through the content of which one sees less and less bright penumbral filaments on the edges, and umbral dots in the center; these umbral dots could be just the tips of filamentary features of the same nature as the so-called penumbral ones. The smaller the diameter of the dish, the more transparent its central part, so that umbral dots appear brighter in small spots than in large ones. Of course, one should try to explain how such structures could fit with all that is known about the three dimensional magnetic velocity fields.

Wilson: You have raised several interesting questions of umbral and penumbral structure. However it would require much more time than we have now to reply to them adequately. I do think that the question of the intrusion of penumbral filaments into the umbra and the form of the umbral-penumbra boundary is of great importance.

Kuperus: It seems to me that you treat the convective motions and the Alfvén waves rather separately. There is not really a need for doing so since it seems that both types of motions can be strongly coupled with a high degree of efficiency. For rotational waves (internal gravity waves) in stable layers this coupling has been demonstrated by M. J. Lighthill.

Wilson: I did not intend to imply that they should be treated separately – far from it. I think that the inhibition of convection and the generation of Alfvén waves at some depth below the surface of the penumbra must be intimately connected. The equipartition of energy between convective and magnetic modes may give rise to the enhancement and bending of the field in these deep regions. Some of these bends in the field may also occur in the visible layers and give rise to the dots. In umbral regions where no dots are seen the magnetic-convective interaction may occur at great depths; where many dots are observed it may be much closer to the surface.

Athay: I find it very difficult to have any degree of confidence in quantitative estimates of mechanical energy requirements in sunspots. Such estimates imply that you have both a good radiative equilibrium model and a good actual model for the spot, and I doubt that we have either.

Wilson: The model does not involve a discussion of the mechanical energy requirements of sunspots as a whole. The calculations concern only the non-radiative energy required to produce this particular model of a bright dot isolated in a surrounding medium which has temperature density and pressure parameters of a typical umbra.

Musman: I do not believe that the amplitude of Alfvén waves would be large enough to cause reversals in the sign of the magnetic field. My theory predicted that disturbances would most easily escape at small amplitudes.

Wilson: I do not suggest that the Alfvén wave motions cause the field reversals. Rather that the 'observed' reversals, caused possibly by 'would be' convective motions will cause dissipation of energy carried by the Alfvén waves. However, I agree that there is no quantitative theory for this process as yet.