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A NOTE ON THE ANNUITY ROLE OF ESTATE TAX

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The previous conclusion that a uniform lump-sum estate tax could implicitly provide annuity income was reached by ignoring the inheritance that agents receive. However, when the agents leave a bequest, they should also receive an inheritance from their parents. Thus, we make the inheritance received—bequests left cycle complete and fully endogenous. Interestingly, the differential timing and sizes of inheritance then generate unequal wealth effects even with actuarially fair annuity markets. To restore the first best, the government has to adopt an estate tax regime that is no longer uniform. Thus, once bequest is fully endogenized, a uniform estate tax no longer bears the annuity role. Further, the differential timing in receiving inheritance creates an unequal wealth distribution, which is also nonstationary. The paper manifests the importance of accounting for and tracing the inheritance received by agents for any crucial policy recommendation.

Keywords: Bequests, Estate Tax, Annuity

1. INTRODUCTION

Proper taxation of inherited wealth is a highly debated issue. There is a recent surge of interest toward this subject to have a better understanding of it so that relevant policies can be conducted. From the children's perspective, inheritances are pure luck since they cannot choose their parents and hence, estate taxes play an important redistribution role. As to parents, some may claim that estate tax is not fair since it penalizes parents who save for their children. There is no doubt that the redistribution role of the estate tax is important and the estate tax literature takes this seriously in various model settings.[1] Another and often overlooked role of the estate tax is its role as an annuity, especially when private actuarially fair annuity markets are thin.[2] Actuarially fair annuities provide insurance against

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longevity risks. By pooling premiums and paying only survivors, an annuity regime provides higher than market returns for the survivors and improve ex ante welfare.

Although several studies analyze the taxation of inheritance from normative and positive angles (see, for instance, Piketty and Saez (2013) and Cagetti and De Nardi (2009)), the possible annuity role of the estate taxation has remained somewhat unexplored. One exception is Kopczuk (2003), who showed that the first best allocations, the allocations when annuity market is available and agents can choose it optimally, can be implemented by imposing uniform estate taxes when the annuity markets are missing and agents have strong bequest motives. More precisely, Kopczuk (2003) studies the annuity role of estate taxation in a delicate model and claims that if there are no actuarially fair annuity markets, it might be a good idea to raise all or part of tax revenue in the form of an estate tax. Estate taxation can bring about a transfer from the short-lived to the other individuals. A risk-neutral government, then, can transfer resources between different states of the world at actuarially fair rates without loss in revenue. Hence, estate taxation can substitute for private annuity markets and even social security. However, noticeably, this result was established by not accounting for inheritance received by agents.

In this paper, we critically analyze the annuity role of estate taxation when both inheritance received and bequests leaving are included (therefore, the bequests cycle is complete) and they are endogenously determined in the model. In this fully endogenous setup, the differential timing in receiving inheritance creates an unequal wealth distribution among the receivers and ignoring this leads to inaccurate policy recommendations. The paper manifests the importance of accounting for and tracing the bequests in one's lifetime in policy recommendation.

We use a two-period overlapping generations economy where each agent is subject to a survival risk[3] and can live up to two periods.[4] Agents work and receive labor income in the first period, and retire and live on savings in the second period.

We first generate the results in Kopczuk (2003) ignoring the inheritance and show that estate taxes can generate the first best solution (the Laissez-Faire allocations).

Afterward, we proceed to account for the inheritance that an agent is supposed to receive. We call this model as our main model. This simple but full endogenization of inheritance received and bequest left leads to the conclusion that a lump-sum uniform estate taxation cannot have an annuity role, a clear deviation from the conclusion of the existing literature. Precisely, we show that the estate tax regime can implement the first best allocations by imposing taxes that are contingent on the level and timing of inheritance received by agents. Actually, when agents have strong bequest motives, they leave bequests, and thus they should also receive an inheritance from their parents. The timing and size of inheritance generate unequal wealth effects even when there are actuarially fair annuity markets. We show that to restore the first best allocations, defined as those when annuities are available, the government has to adopt an estate tax regime that is no longer uniform. Instead, the estate taxes should be contingent on the timing and level of inheritance received by each agent. Thus, this paper once again manifests the importance of accounting for and tracing the inheritance received by agents in a model that aims to discuss intergenerational transfers and related policies. We then replace actuarially fair annuities by actuarially not-fair annuities following Lockwood (2012) and show that our results do not change at all.

In the online supplement, we provide extensions and robustness analysis (see Bishnu and Kumru (2020)). More precisely, we extend the main model by incorporating fully funded social security program, consumption tax, and endogenous labor supply one at a time and show that estate tax does not provide meaningful annuities in all these extensions. We also conduct the robustness check and show that our main result is robust to changes in bequest specifications.

The rest of the paper is structured as follows. Section 2 introduces the model. Section 3 concludes. We delegate the derivation of inequalities created by the received inheritance to Appendix.

2. MODEL

We begin by reproducing the claim of Kopczuk (2003), by ignoring the inheritance received by the agents in the model. Then, we proceed to account for inheritance that agents also receive. Thus, we make the entire inheritance received—bequests leaving process in one's lifetime consistent and complete in the model. Due to this logical and simple generalization, we observe that a key result in the literature that a lump-sum tax on inheritance can implement the first best allocation when the annuity market is missing is no longer valid. At the same time, we show how important it is to track the timing of the inheritance received in one's lifetime when designing for the optimal estate tax to serve its annuity role.

2.1. Model without Inheritance Received (Kopczuk (2003))

2.1.1. With annuities. Suppose, an agent can live up to two periods. The survival probability to the second period is $p \in (0, 1)$, whereas for the first period it is unity. The government requires a per capita tax revenue R. Here, we assume that annuities are actuarially fair. The agent receives labor income y when young, pays a lump-sum tax T, and solves the following expected utility maximization problem over the lifetime:

$$\max_{c_1,c_2,B_1,B_2,a,k} EU = u(c_1) + (1-p)v(B_1) + pu(c_2) + pv(B_2), \tag{1}$$

subject to:

$$c_1 + a + k = y - T; \quad B_1 = k; \quad c_2 + B_2 = \frac{a}{p} + k. \tag{2}$$

The utility function u and v are both strictly increasing and strictly concave as in Lockwood (2012). The constraint set (2) above implies that the agent can choose both annuity (a) and storage (k), with the latter for the purpose of bequest. The net

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return on storage is assumed to be zero. The individual probability of surviving in the second period is p. This implies that the fraction of agents alive in the second period is p too. The rate of return on per capita annuity is therefore $1/p$. B_1 and B_2 represent the bequests left by the agents. Rewriting the lifetime budget constraint gives us the following:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T.$$

It is obvious that the government wants to implement tax $T = R$ in order to satisfy the revenue requirement. The optimal allocations satisfy $u'(c_1) = u'(c_2) = v'(B_1) = v'(B_2)$. We denote the optimal allocations by the quadruplet $(c_1^*, c_2^*, B_1^*, B_2^*)$. Given u and v are strictly concave, we can characterize the first best allocations in the economy when actuarially fair annuity markets exist as follows: $c_1^* = c_2^* = c^*$; $B_1 = B_2 = B^*$; and $(1+p)c^* + B^* = y - R$.

2.1.2. Absence of annuities but presence of estate tax. Now consider the situation where the annuity markets are completely absent. Instead, the agent only has an access to a storage technology. Without government intervention, it is known that the first best allocations cannot be achieved. The following tax regime is supposed to implement the first best: the agent pays a lump-sum tax T when young, and estate tax E if she dies in the first period. The estate tax is equal to zero in the second period. We still assume away the inheritance.

In this case, the budget constraints that the agent faces are as follows:

$$c_1 + k = y - T; \quad B_1 = k - E; \quad c_2 + B_2 = k. \tag{3}$$

To implement the first best allocation, we need to set[5]

$$E = c^* \quad \text{and} \quad T = y - 2c^* - B^*.$$

In order to demonstrate how this tax scheme works, we need to show that the first best allocations are not only feasible but also satisfy the first-order conditions under such tax arrangement. Inserting the taxes T and E to the budget constraints, we can have the following equations: $c_1 = y - (y - 2c^* - B^*) - k$; $B_1 = k - c^*$; and $c_2 = k - B_2$. It is straight forward to show that as long as $k = c^* + B^*$, the first best allocations ($c_1 = c_2 = c^*$ and $B_1 = B_2 = B^*$) satisfy the budget.

The optimization problem can be rewritten as:

$$u(y - T - k) + (1-p)v(k - E) + pu(k - B_2) + pv(B_2).$$

The first-order conditions are as follows:

$$-u'(c_1) + (1-p)v'(B_1) + pu'(c_2) = 0,$$
$$-pu'(c_2) + pv'(B_2) = 0.$$

As in above, the first best allocations satisfy them once again.

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To verify the taxes, one should notice that the balanced budget condition needs to be satisfied:

$$T + (1 - p)E = R.$$

By inserting the amount of taxes, we get $y - R = (1 + p)c^* + B^*$. This is the same as the one shown in the first best allocations. In this economy when annuity markets are missing, the allocations demonstrate not only smoothness across time but also equality across agents thanks to the lump-sum estate taxation. Thus, the estate tax can play the same role as annuities do. This is precisely the understanding from the existing literature. We have to emphasize that this trait, however, is achieved when inheritance received is not considered.

2.2. Model with Received Inheritance—Main Model

We depart from the above analysis (the existing literature) where inheritance that an agent can receive over her lifetime is not accounted for. Since agents do leave behind bequests to their offsprings, inheritance should be included instead of being ignored in their lifetime assets calculation. The following analysis shows that once inheritance received is included and the entire process of inheritance received—bequest leaving becomes complete and endogenous, the timing of inheritance received by the agent creates a crucial differential wealth effect.[6] Since timing and size of inheritance received are different for different individuals, a wealth heterogeneity is automatically created in the population. There will be inequalities in both consumption and bequest left among the agents. We observe that this creates enough ground to claim that in this scenario, a uniform estate tax no longer bears the annuity role. We also focus on the wealth distribution under this scenario. At the end, in the online supplement, we verify that the results we find are quite robust to extensions and various changes in the model environment (see Bishnu and Kumru (2020)). The formal derivations are presented below.

2.2.1. Uniform estate tax lacks the annuity role. In order to establish our results, we restart with an economy where actuarially fair annuities are present. Now, there are at least two types of agents[7] in the economy: those who receive inheritance when young and otherwise. For type I (denoted by n_1) who receive inheritance when they are young, the budget constraints are

$$c_1(n_1) + a + k(n_1) = y - T + H_1(n_1); \quad B_1(n_1) = k(n_1);$$
$$c_2(n_1) + B_2(n_1) = \frac{a}{p} + k(n_1).$$

For type II (denoted by n_2) who receive inheritance in the second period, the budget constraints are

$$c_1(n_2) + a + k(n_2) = y - T; \quad B_1(n_2) = k(n_2); \quad c_2(n_2) + B_2(n_2) = \frac{a}{p} + k + H_2(n_2).$$

In these constraints, $H_1(n_1)$ and $H_2(n_2)$ denote inheritance received in period one (in this case, parents die early) and period two, respectively. The lifetime budget constraints for type I and II agents are as follows:

$$c_1(n_1) + pc_2(n_1) + (1-p)B_1(n_1) + pB_2(n_1) = y - T + H_1(n_1), \quad (4)$$

$$c_1(n_2) + pc_2(n_2) + (1-p)B_1(n_2) + pB_2(n_2) = y - T + pH_2(n_2), \quad (5)$$

respectively.

We can now show the main result of this paper: once inheritance is accounted for instead of being ignored, we cannot generate the allocations where $c_1 = c_2 = c^*$ and $B_1 = B_2 = B^*$ that hold for different types of agents (here n_1 and n_2) even when there are actuarially fair annuities. The proof is simple. By contradiction, suppose instead we do have $c_1 = c_2 = c^*$ and $B_1 = B_2 = B^*$ for both the groups n_1 and n_2, and there is no inequality across all agents. Then, we must also have $H_1 = H_2 = B^*$. However, this contradicts the fact both budget constraints in (4) and (5) have to hold at the same time. A similar analysis of estate tax presented in Section 2.1 then confirms that the same E can never be optimal inheritance tax for both the groups. The simple idea behind this results is that once inheritance is incorporated along with bequests leaving, a natural phenomenon that differential timing of receiving inheritance creates a wealth heterogeneity in the population. In this scenario, same consumption c^* and same bequests leaving B^* cannot be the optimal choice for the all heterogeneous groups, and therefore, one uniform tax on inheritance cannot work for all. This proposition is summarized as follows.

PROPOSITION 1. *In the absence of inheritance received, with actuarially fair annuities and bequest motive, each agent chooses $c_1 = c_2 = c^*$ and $B_1 = B_2 = B^*$. However, in the presence of it,*

(a) the differential timing and amount of inheritance received affect the lifetime wealth and this, in turn, generates the inequalities in consumption and bequest in the economy, that is,

$$c_1(n_1) = c_2(n_1) = c^*(n_1) \neq c^*(n_2) = c_2(n_2) = c_1(n_2) \text{ and } B^*(n_1) \neq B^*(n_2).$$

(b) a "uniform" estate tax no longer bears the annuity role.

We now examine the possibility that estate tax regime can reproduce the distribution of allocations, including the inequality, in the economy with annuities. We find out that in order to implement the first best allocations where inequality exists, the estate tax needs to be contingent on the timing (and also other factors if any such as size) of the inheritance that received by agents. Suppose at the steady state, an agent receives inheritance $H(n)$ when young and another agent receives the same level of inheritance when old. Their budget constraints are as follows::

$$c_1(n_1) + k(n_1) = y - T(n_1) + H_1(n_1),\ B_1(n_1) = k(n_1) - E_1(n_1),\\ c_2(n_1) + B_2(n_1) = k(n_1);$$
$$c_1(n_2) + k(n_2) = y - T(n_2),\ B_1(n_2) = k(n_2),\\ c_2(n_2) + B_2(n_2) = k(n_2) - E_2(n_2) + H_2(n_2),$$

respectively. Instead of a single tax E, we need to bring two different estate taxes for two different types. More specifically, E_1 is the estate tax recommended for type I agent and similarly E_2 for type II. To implement the first best allocations, we want

$$E_1(n_1) = c^*(n_1),\quad T(n_1) = y - 2c^*(n_1) - B^*(n_1) + H_1(n_1);$$
$$E_2(n_2) = c^*(n_2),\quad T(n_2) = y - 2c^*(n_2) - B^*(n_2) + pH_2(n_2).$$

The levels of estate and lump-sum taxes are both dependent on the size and the timing of the inheritance received by the agent. We want to emphasize that although this implementation can restore the first best allocations, it is not similar to any estate tax regimes prevailing in the world. Hence, the following proposition.

PROPOSITION 2. *When the inequality exists, uniform estate tax and lump-sum tax cannot generate the first best allocations. Instead, both taxes need to be contingent on timing and size*[8] *of inheritance received by agents.*

The actual distribution of inheritance, and hence the induced level of consumption and bequest left, is indeed not stationary in an economy with actuarially fair annuities and without any government intervention, where the only uncertainty is survival risks. The distribution of inheritance tends to be fanning out over time. As a result, the unequal inheritances that parents leave for their children, on top of survival risks, generate inequality behind the veil of ignorance. It is noticeable that even with the same level of inheritance, the wealth effect differs with the timing of inheritance. Additionally, inheritance does not alter inter-temporal choices (Euler equations). If we assume that both u and v are strictly increasing and strictly concave functions, then we have $c_1 = c_2$ and $B_1 = B_2$ for all agents.[9]

Now, we prove that in this Laissez-Faire economy, the distribution of inheritance is not stationary; hence, the distributions on the levels of consumption and bequests left are also nonstationary. The proof is *by contradiction*. Suppose, there is a stationary distribution after n generations. Assume that the lowest level of bequest is H_1, and the mass of agents leaving H_1 is b_1. The mass of agents that leaving H_1 early is $(1 - S)b_1$, while those who leave H_1 late has mass Sb_1. Suppose, H_1 is also the lowest level of bequest left among generation n+1. In order to have a stationary distribution, the mass of agents who leave H_1 must be the same as the previous generation. However, notice that the Sb_1 agents who receive H_1 late will have less lifetime wealth than $(1 - S)b_1$ agents who receive them early. Hence, the only possible level of mass to leave H_1 is Sb_1 which is strictly less than b_1, the mass of agents leaving H_1 in the previous generation. Hence, the proof of nonstationarity.

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Next, we argue that after many generations, the lowest level of inheritance is strictly above zero. This is also easy to prove *by using contradiction*. With strong bequest motive, even those who receive zero inheritance will leave some bequest. By combining the above arguments, we can show that the distribution of inheritance is nonstationary. Neither are the distribution of level of consumption. The above discussions have been summarized in the following proposition:

PROPOSITION 3. *Due to the differential timing of the bequest received, the distribution of inheritance is nonstationary in nature with the lowest level of bequest strictly positive.*

Appendix demonstrates the inequality generated by differential timing in inheritance received in a detailed manner.

2.2.2. With actuarially not-fair annuity. Assume that the annuity guarantees higher return R_a than the market return R_m. Following Lockwood (2012), we assume that $R_a = (1-\lambda)\,\frac{R_m}{p}$ where $\lambda \geqslant 0$ is the load, that is, the percentage by which premiums exceed expected discounted benefits. Notice that $\lambda = 0$ represents the actuarially fair case. In line with the assumption in the main model, without any loss of generality, we set $R_m = 1$. Both groups have the following expected utility maximization problem:

$$\max_{c_1, c_2, B_1, B_2, a, k} EU = u(c_1) + (1-p)v(B_1) + pu(c_2) + pv(B_2), \tag{6}$$

where the budget constraint of type I and type II agents are

$$\begin{aligned} c_1(n_1) + a + k(n_1) &= y - T + H_1(n_1),\; B_1(n_1) = k(n_1), \\ c_2(n_1) + B_2(n_1) &= a\,(1-\lambda)\,\frac{1}{p} + k(n_1); \\ c_1(n_2) + a + k(n_2) &= y - T,\; B_1(n_2) = k(n_2), \\ c_2(n_2) + B_2(n_2) &= a\,(1-\lambda)\,\frac{1}{p} + k(n_2) + H_2(n_2), \end{aligned}$$

respectively. If we rewrite the above budget constraints, we get

$$\begin{aligned} &(1-\lambda)\,c_1(n_1) + pc_2(n_1) + \left[(1-\lambda) - p\right] B_1(n_1) + pB_2(n_1) \\ &\quad = (1-\lambda)\,(y - T + H_1(n_1)); \\ &(1-\lambda)\,c_1(n_2) + pc_2(n_2) + \left[(1-\lambda) - p\right] B_1(n_2) + pB_2(n_2) \\ &\quad = (1-\lambda)\,(y - T) + pH_2(n_2), \end{aligned}$$

for type I and II agents, respectively. In order to have the coefficient of B_1 strictly positive, we need a condition $R_m\,(1-\lambda) - p > 0$ which imposes a further restriction on the inequality $R_a > R_m$. Note that with the above assumption $R_m = 1$ which implies that the loading should be less than the probability of death, we have $R_a > 1$.

Now we show that a uniform tax rate does not work for an economy with annuity that is not actually fair. Note that if we want to have $c_1 = c_2 = c^*$ and

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$B_1 = B_2 = B^*$, and there is no inequality across all agents, we must have $H_1(n_1) = H_2(n_2) = B^*$. It can be verified from the above two budget constraints that it is possible only when $R_a = 1$. This can be shown easily by comparing the above two budget constraints. The left-hand sides are the same. If we equate the right-hand sides, we get $H_2(n_2)/H_1(n_1) = (1 - \lambda)/p$ which, in fact, is exactly equal to R_a by construction. Since we need $H_1(n_1) = H_2(n_2) = B^*$ at the steady state, the requirement $1 = B^*/B^* = (1 - \lambda)/p = R_a$ should hold. However, this contradicts with the fact that $R_a = (1 - \lambda)/p > 1$ for any load λ as shown above. Therefore, there does not exist any load λ for which a flat tax rate can be justified. The above discussion is presented below as a proposition.

PROPOSITION 4. *Both the Propositions 3 and 2 hold when annuity market is not actuarially fair.*

3. CONCLUSION

This paper emphasizes on natural inequality that is induced through differential timing of bequests received by agents because of differential timing of death of their parents. In this light, we examine the annuity role of estate tax, a role that is immensely important but it is yet to attract due attention. Our paper clearly reveals that previous conclusions that a uniform lump-sum estate tax could implicitly provide annuity income were reached by ignoring inheritance received by agents. Technically, using an incomplete model of bequest transfers resulted in recommending a uniform estate tax if it has to serve the annuity role. Noticing while agents leave bequest, they also receive inheritance from their parents, we simply complete the bequest cycle. When the model is complete, interestingly, we observe that the differential timing and sizes of inheritance generate unequal wealth effects even with the actuarially fair annuity markets. Moreover, the distributions of wealth and consumption are not stationary over time even in the first best allocations. To restore the first best allocations, the government has to adopt an estate tax regime that is no longer uniform. Thus, once the inheritance is determined by a uniform estate tax, it no longer bears the annuity role. As we showed in the Online Supplement (Bishnu and Kumru (2020)), our results are robust to extensions and many different specifications of the model. Our paper once again manifests the importance of accounting for and tracing the inheritance received by agents in any model that aims to discuss intergenerational transfers and related policies.

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit https://doi.org/10.1017/S1365100520000292.

NOTES

1. Blumkin and Sadka (2003) examine the estate taxation with intended and accidental bequest motives showing the estate tax is highly sensitive to the relative importance of two motives. While,

Cremer and Pestieau (2001) show that marginal tax rates may be regressive and positive under some circumstances, Farhi and Werning (2010) show that estate tax should be progressive. Piketty and Saez (2013) derive optimal inheritance tax formulas that capture the key equality–efficiency trade-off. Cagetti and De Nardi (2009) conduct a positive analysis of estate taxation using a large-scale quantitative model.

2. There is also interest toward annuity role of social security. For instance, Caliendo et al. (2014) show that unlike a private annuity market actuarially fair fully funded social security fails to provide any welfare gains even when agents have no other way to insure against longevity risks.

3. Survival risk and bequests have been extensively studied in the literature. See Yaari (1965), Davies (1981), Abel (1985), Hurd (1989), Bernheim (1991), Cardia and Michel (2004), and Davidoff et al. (2005).

4. Instead of a two period, a multi-period model can easily be constructed but we do not see any change in the arguments that we have provided above. Also it is straightforward to present our results in a continuous time setup.

5. We consider only the interior solution.

6. There are many external factors that can create differential wealth effects in the population. In our case, the model itself generates an inequality not the external factors. Due to the survival risk, while a fraction of agents die early and leave early bequests, the rest of agents survive and leave late bequests. In other words, wealth differential in the model is the natural consequence of the inheritance received at different time points.

7. As mentioned in the introduction, restricting to only a two-period model is for simplicity since we only need two different types. A multi-period extension or a continuous setup can be constructed easily. Notice also that given the structure of the generations, it is ensured that the agents cannot die strictly prior to their parents.

8. Note that the variation in size that we are particularly interested in this paper is the one that the same amount left at a different time point which results in differential present value of income through inheritance. Of course other types of heterogeneity such as differential amount of bequests left definitely strengthen our results.

9. In the Appendix, we have explicitly shown the derivation of bequests for a particular economy.

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APPENDIX

To illustrate an inequality that a particular economy can generate, we assume that $u(.) = v(.)$. With this specification, it is straightforward to show that $c_1 = c_2 = B_1 = B_2$ at the optimum. Thus, there exists smooth consumption and bequest. The level and the timing of the inheritance agents receive, however, are different.

Let i ($i = n_1,\ n_2$) denote the agent's types. Agents who receive inheritance when young denoted by n_1, agents who receive inheritance when old denoted by n_2. A type i agent of generation t who makes decision in the period t is represented by the pair (G_t, i).

Suppose we start with a hypothetical situation where the agent who receives $H_2(n_2)$ when old leaves $H_2(n_2)$. The agent's budget constraint is as follows:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T + pH_2(n_2).$$

Combined with the first-order conditions that $c_1 = c_2 = B_1 = B_2$, we have the following allocations for the type II agent of generation t: $c_1(G_t, n_2) = c_2(G_t, n_2) = B_1(G_t, n_2) = B_2(G_t, n_2) = H_2(n_2) = \frac{y-R}{2}$.

Agents from generation $t+1$ who receive $\frac{y-R}{2}$ when young have the following lifetime budget constraint and allocations:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - R + \frac{y-R}{2};$$
$$c_1(G_{t+1}, n_1) = c_2(G_{t+1}, n_1) = B_1(G_{t+1}, n_1) = B_2(G_{t+1}, n_1) = \frac{3}{2(2+p)}(y-R).$$

Similarly, agents from generation $t+2$ who receive $\frac{3}{2(2+p)}(y-R)$ when young have the following lifetime budget constraint and allocations:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T + \frac{3(y-R)}{2(2+p)};$$
$$c_1(G_{t+2}, n_1) = c_{2(G_{t+2}, n_1)} = B_1(G_{t+2}, n_1) = B_2(G_{t+2}, n_1) = \frac{7+2p}{2(2+p)^2}(y-R).$$

Those who receive inheritance $\frac{3(y-R)}{2(2+p)}$ when old have the followings:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - R + p\frac{3(y-R)}{2(2+p)};$$
$$c_1(G_{t+2}, n_2) = c_2(G_{t+2}, n_2) = B_1(G_{t+2}, n_2) = B_2(G_{t+2}, n_2) = \frac{4+5p}{2(2+p)^2}(y-R).$$

From above, it is clear that type n_1 agents who receive inheritance earlier are better off than type n_2 agents who receive inheritance later. Since our main goal is to illustrate the existence of inequality created by the timing of inheritance received, the above example is enough.

As we showed in the diagram below, along the extreme path of survival (S) where all the agents survive for two periods till infinity, we have $c_1 = c_2 = B_1 = B_2 = (y-R)/2$ when $t \to \infty$. Similarly, the path which has no survival (NS) throughout right after survival (S) will converge to the allocation $(y-R)/(2+p)(1-p)$.

Now we start with the other possibility where agents receive inheritance when they are young (the right-hand side right after any generation node in the diagram below). If $H_1(n_1)$ be the inheritance received when young, the agent's budget constraint is as follows:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T + H_1(n_1).$$

Combined with the first-order conditions that $c_1 = c_2 = B_1 = B_2$ we have

$$c_1(G_t, n_1) = c_2(G_t, n_1) = B_1(G_t, n_1) = B_2(G_t, n_1) = H(n_1) = a(y-R)$$
$$\text{where } a = (1+p)^{-1}.$$

Let us now check the allocations in period $t+1$ for G_{t+1} generations. If the agent receives inheritance of $(y-R)\,/\,(1+p)$ when old, she will have the following lifetime budget and resource allocations:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - R + pa(y-R),$$
$$c_1(G_{t+1}, n_2) = c_2(G_{t+1}, n_2) = B_1(G_{t+1}, n_2) = B_2(G_{t+1}, n_2) = \frac{1+ap}{2+p}(y-R).$$

If the agent in generation $t+1$ receives the same inheritance $(y-R)\,/\,(1+p)$ when young, she will have the following lifetime budget and resource allocations:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T + a(y-R),$$
$$c_1(G_{t+1}, n_1) = c_2(G_{t+1}, n_1) = B_1(G_{t+1}, n_1) = B_2(G_{t+1}, n_1)$$
$$= \frac{1+a}{2+p}(y-R) = a(y-R).$$

One can notice that the allocations of generations t and $t+1$ type n_1 agents are the same. Now check out the case where (G_{t+1}, n_2) agents leave $\frac{1+ap}{2+p}(y-R)$. If an agent in generation $t+2$ receives this inheritance when young, the allocations will not be different from the above:

$$c_1(G_{t+2}, n_1) = c_2(G_{t+2}, n_1) = B_1(G_{t+2}, n_1) = B_2(G_{t+2}, n_1) = \frac{1+ap}{2+p}(y-R).$$

However, if inheritance is received when old, the agent's lifetime budget constraint and allocations will be as follows:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T + p\frac{1+ap}{2+p}(y-R),$$
$$c_1(G_{t+2}, n_2) = c_2(G_{t+2}, n_2) = B_1(G_{t+2}, n_2) = B_2(G_{t+2}, n_2)$$
$$= \left(1 + \frac{p}{2+p} + \left(\frac{p}{2+p}\right)^2\right)\frac{y-R}{2+p}.$$

We continue with this exercise for one more generation to derive the values when $t \to \infty$. We can now safely write that if an agent in generation $t+3$ receives $\left(1 + \frac{(1+ap)p}{2+p}\right)(y-R)$ when young, the allocations would be the same as their parental generations and therefore,

$$c_1(G_{t+3}, n_1) = c_2(G_{t+3}, n_1) = B_1(G_{t+3}, n_1) = B_2(G_{t+3}, n_1)$$
$$= \left(1 + \frac{(1+ap)p}{2+p}\right)\frac{y-R}{2+p}.$$

However, if bequest is received when old, we can show that the agent's lifetime budget constraint and allocations will be

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T + p\left(1 + \frac{(1+ap)\,p}{2+p}\right)(y-R),$$
$$c_1(G_{t+3}, n_2) = c_2(G_{t+3}, n_2) = B_1(G_{t+3}, n_2) = B_2(G_{t+3}, n_2)$$
$$= \left(1 + \frac{p}{2+p} + \left(\frac{p}{2+p}\right)^2 + \left(\frac{p}{2+p}\right)^3\right)\frac{y-R}{2+p}.$$

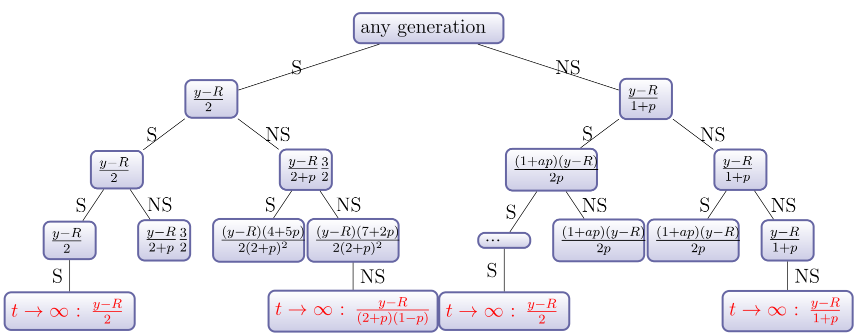
From here, it is easy to see the allocations when $t \to \infty$. The two extreme values of the distributions are given by:

$$c_1(G_\infty, S) = c_2(G_\infty, S) = B_1(G_\infty, S) = B_2(G_\infty, S)$$
$$= \left(1 + \frac{p}{2+p} + \left(\frac{p}{2+p}\right)^2 + \left(\frac{p}{2+p}\right)^3 + ...\right)\frac{y-R}{2+p} = \frac{y-R}{2}$$

and

$$c_1(G_\infty, NS) = c_2(G_\infty, NS) = B_1(G_\infty, NS) = B_2(G_\infty, NS) = \frac{y-R}{1+p},$$

respectively. (G_∞, S) and (G_∞, NS) denote the two extremes cases—survival (S) for two periods and survival for only one period (NS) for all generations. It is easy to check that given $0 < p < 1$, $\frac{y-R}{1+p} > \frac{y-R}{2}$. This particular path has been presented as the right path (starting with NS) in the diagram.



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