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Sheared flow-driven vortices and solitary waves in a non-uniform plasma with negative ions and non-thermal distributed electrons

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Abstract. The coupled drift-ion acoustic (DIA) waves in an inhomogeneous magnetoplasma having negative and positive ions can be driven by the parallel sheared flows in the presence of Cairns distributed non-thermal electrons. The coupled DIA waves can become unstable due to shear flows. The conditions of modes instability are discussed with effects of non-thermal electrons. These are the excited modes and start interactions among themselves. The interaction is governed by the Hasegawa–Mima equations with analytical solutions in the form of a vortex chain and dipolar vortex. On the other hand, for scalar nonlinearity the Kortweg deVries-type equation is obtained with solitary wave solution. Possible application of the work to the space and laboratory plasmas are highlighted.

1. Introduction

Both theoretically and experimentally, the low-frequency waves in plasmas with negative ions have been studied for many years. Ion-acoustic waves in plasmas with negative ions have been investigated by D'Angelo et al. (1966), Wong et al. (1975), and Song et al. (1991a, 1991b). Electrostatic ion–cyclotron waves have been examined by D'Angelo and Merlino (1986) and experimentally investigated by Song et al. (1989). Similarly, the Kelvin–Helmholtz (K-H) instability and the lower hybrid waves in plasmas with negative ions were respectively investigated by D'Angelo and Song (1991) and D'Angelo (1992). The electron–ion plasmas with negative ions (sometime called bi-ion–electron plasmas) are found enriched in space plasmas. Such a plasma with different composition systems, such as Ar^+plasma with F^- ions, H^+ plasma with O_2^- ions, H^+ plasma with H^- ions, etc., occurs in the D-region of the Ionosphere (Mishra and Chhabra 1996) and also in the Martian magnetosphere (Sauer et al. 1998). The existence of a second ion population leads to an additional coupling between ions and electrons through the Lorentz force (in magnetized plasmas) and charge neutrality. Even a small component of heavy ions in a proton–electron plasma may substantially alter the plasma wave and flow properties. The multi-ion nature of space plasmas gives rise to interesting effects and supports strong low-frequency (and hence typically long wavelength) electrostatic perturbations. Recently, Kim and Merlino (2007) studied experimentally the formation of negative ions in a thermally ionized potassium plasma and observed the electrostatic ion–cyclotron wave spectrum in a plasma containing K^+ positive ions (39 amu), electrons, and $C_7F_{14}^-$ (350 amu) negative ions. The electron–proton plasmas with negative ions have applications in the earth's ionosphere, mesosphere, solar atmosphere, and micro-electronic plasma-processing reactors (Peterson et al. 1993; Elifomov et al. 1997; Maslennikov et al. 1997). Cometary tails and magnetospheres of unmagnetized planets, such as Venus and Mars, comprise multi-ion plasmas, which are the result of the interaction of solar wind with extended sources of heavy ions. The (Ar^+, F^-) plasma was used to study the ion-acoustic wave propagation (Nakamura and Tsukabayashi 1984) in laboratory experiment. At a critical concentration of negative ions, both compressive and rarefactive solitons are observed. Large-amplitude solitary waves in a multi-component plasma with negative ions by using the experimental results of Nakamura and Tsukabayashi (1984) are studied by Nakamura et. al. (1985). Besides the plasmas with negative ions at laboratory level, a pair plasma has been successfully created with sufficiently dense pair-ion (PI) plasma consisting of equal mass, positive, and negative fullerene (C_{60}^+ and C_{60}^-) ions (Oohara and Hatakeyama 2003, 2007; Oohara et al. 2005).

The sheared flow has significant effect on both linear and nonlinear electrostatic oscillations in non-uniform multi-species magnetoplasmas and have been discussed by several authors (Shukla et al. 2000; Mirza et al. 2001; Haque et al. 2005; Vranjes and Poedts 2005). Shukla et al. (2000) investigated the nonlinear propagation of low-frequency electrostatic waves in a strongly magnetized electron–positron plasma in the presence of parallel and perpendicular (Mirza et al. 2001) sheared plasma flows with massive charged dust grains. Detailed studies of electrostatic wave instabilities in a current-carrying magnetoplasma with equilibrium density and ion velocity gradients are studied by Shukla et al. (2002) by using a non-Boltzmann electron response and ion density perturbation, which includes the ion-neutral drag. The nonlinear theory of large-amplitude magnetosonic waves in high-β space plasmas has been studied by Pokhotelov et al. (2007). Also, Pokhotelov et al. (1996) studied the nonlinear structures in the Earth's magnetosphere and atmosphere.

Sheared flow-driven drift waves and the counter-rotating vortices in electron–positron–ion plasmas have also been studied (Haque et al. 2005). Vranjes et al. (2005) studied the analysis of waves and instabilities in pair-ions fullerene ($C_{60}^{\pm}$) plasmas. They have studied parallel shear flow instability for obliquely propagating perturbations in cold pair-ion plasmas. In view of some experiments either performed (Oohara and Hatakeyama 2003, 2007; Oohara et al. 2005) or to be performed in future, in either a Q-machine negative ion plasma (e.g., Song et al. 1989) or in a discharge plasma with SF_6^+ and SF_6^- ions (D'Angelo 1992), the mode is briefly reanalyzed here in inhomogeneous magnetoplasmas with some fraction of electrons.

The objective of this study is to investigate the behavior of the drift wave (K-H) instability arising from shear in the flow parallel to the magnetic field in electron–ion plasmas containing an appreciable fraction of negative ions. In the presence of sheared flow, the electrostatic modes interact nonlinearly among themselves. The corresponding mode coupling equations are governed by a set of nonlinear equations in which we incorporate the transverse two-dimensional vector nonlinearity (because of polarization drift) and nonlinearity because of the coupling between the $\mathbf{E}\times\mathbf{B}$ and parallel pair ion flows. It is shown that stationary solutions of the nonlinear mode coupling equations can be represented in the form of a vortex street and a dipolar vortex. Moreover, in the case of weak dispersion (long wavelength) and scalar nonlinearity, a low-frequency drift wave interacting with a shear flow and a Kortweg deVries (KdV) type of equation is obtained. The coherent solution of the nonlinear equation is presented in the form of solitary structure.

2. Formulation of the problem

Consider an inhomogeneous plasma that has positive and negative ions (either SF_6^+ and SF_6^- proposed by D''Angelo (1992) or C_{60}^+ and C_{60}^- or $H^+$and $H^-$of Oohara and Hatakeyama (2003, 2007) and Oohara et al. (2005)) with electrons embedded in an external magnetic field $\mathbf{B} = B_0\ \hat{z}$, where B_0 is the strength of the magnetic field and $\hat{z}$ is a unit vector along the z-axis. At equilibrium we have $n_{+0}(x) = n_{-0}(x) + n_{e0}(x)$, where $n_{j0}(x)$ is the unperturbed non-uniform number density of jth species ($j = +$ for positive ions, $-$ for negative ions, and e for electron). The density of plasma components is assumed to vary in the x-direction. In the presence of an inhomogeneous flow of species j along the magnetic field lines $\hat{z}V_{j0}(x)$ having gradients along the x-axis for obliquely propagating perturbation, the plasma may become unstable due to parallel shear flow instability. This instability may arise because of the fact that adjacent layers of the streaming fluid have different velocities, and it develops provided that the change in perpendicular velocity exceeds some critical value. In case of low-frequency perturbations (in comparison with ion gyrofrequency $\Omega_c = eB_o/m_i c$) the parallel components of positive and negative ions fluid velocities are described by

$$D_t v_{jz} - \frac{c\acute{V}_{j0}}{B_0}\partial_y\phi = \frac{qe}{m_i}\partial_z\phi, \tag{1}$$

where $q = -1$ for positive ion and $+1$ for negative ion, also $\acute{V}_{j0} = \partial_x V_{j0}$. The perpendicular component of positive and negative ion fluid velocities can be written as

$$\mathbf{v}_{j\perp} \approx \frac{c}{B_0}\hat{z}\times\nabla\phi + \frac{qc}{B_0\Omega_j}\left[\partial_t + \frac{c}{B_0}\hat{z}\times\nabla\phi\cdot\nabla + \left(V_{j0}+v_{jz}\right)\partial_z\right]\nabla_\perp\phi \approx \mathbf{v}_E + \mathbf{v}_{pj}, \tag{2}$$

where $\mathbf{v}_E$ and $\mathbf{v}_{pj}$ are the electric and polarization drifts of positive and negative ions, respectively, ϕ is the electrostatic potential, and $|D_t| = \left|\partial_t + \frac{c}{B_0}(\hat{z}\times\nabla\phi)\cdot\nabla\right|$ with assumption that $|D_t| \geqslant \left(V_{j0}+v_{jz}\right)\partial_z$. For simplicity, both positive and negative ions temperatures are taken to be zero. On the other hand, the non-thermal distribution function for electrons is (Cairns et al. 1995) $f_e(v) = \frac{n_{eo}}{(3\alpha+1)\sqrt{2\pi v_{th}^2}}(1+\frac{\mu v^4}{v_{th}^4})\exp(-\frac{v^2}{v_{th}^2})$, where n_{eo} is the electron density, v_{th} is the thermal speed of electrons, and μ is a parameter which determines the percentage of fast non-thermal electrons. We ignore the effect of streaming velocity on electrons. Replacing $\frac{v^2}{v_{th}^2}$ with $(\frac{v^2}{v_{th}^2}-2\phi)$ in the presence of non-zero potential and after integration of the resulting distribution function over all microscopic velocities, gives the following expression for electron density (Cairns et al. 1995):

$$n_e = n_{eo}\left[1-\beta\frac{e\phi}{T_e}+\beta\left(\frac{e\phi}{T_e}\right)^2\right]\exp\left(\frac{e\phi}{T_e}\right), \tag{3}$$

where $\beta = \frac{4\mu}{1+3\mu}$ with $\mu \geqslant 0$. It can be easily checked that $0 \leqslant \beta > 4/3$. The continuity equations for negative and positive ions are given by

$$D_t n_j + \mathbf{v}_E\cdot\nabla_\perp n_{jo} + n_{jo}\left(\nabla_\perp\cdot\mathbf{v}_{j\perp}\right) + n_{jo}\partial_z v_{jz} = 0. \tag{4}$$

Using (2) and (4) we get the following outcome:

$$D_t\,(n_- - n_+) + \frac{c\,(n_{+o} + n_{-o})\,\kappa_n}{B_o}\partial_y\phi + \frac{c\,(n_{+o} + n_{-o})}{B_o\Omega_c} D_t\nabla_\perp^2\phi + (n_{+o}\partial_z v_{+z} - n_{-o}\partial_z v_{-z}) = 0, \tag{5}$$

where $\kappa_n = (\frac{1}{n_{+o}+n_{-o}})\frac{\partial n_{eo}}{\partial x}$ is the inverse density inhomogeneity scale length. Using the Poisson's equation,

$$\nabla^2\phi = 4\pi\sum_j q_j n_j \tag{6}$$

with (5) and linearized form of (3), we get

$$D_t\left(\nabla^2 + \frac{\omega_{p\pm}^2}{\Omega_c^2}\nabla_\perp^2\right)\phi - \frac{1-\beta}{\lambda_{De}^2}\partial_t\phi + \frac{\omega_{p\pm}^2}{\Omega_c}\kappa_n\partial_y\phi + 4\pi e[n_{-o}\partial_z v_{-z} - n_{+o}\partial_z v_{+z}] = 0. \tag{7}$$

Here $\omega_{p\pm} = \sqrt{\frac{4\pi e^2}{m_i}(n_{+o} + n_{-o})}$ is the effective ion plasma frequency and $\lambda_{De} = \sqrt{\frac{T_e}{4\pi e^2 n_{eo}}}$ is the electron Debye Length. Equations (1) and (7) are the governing equations for nonlinearly coupled electrostatic waves in a non-uniform bi-ion magnetoplasmas with parallel shear flows. In the linear limit, the local dispersion relations can be derived from (1) and (7) by assuming that perturbation is proportional to $\exp[i(k_y y + k_z z - \omega t)]$, and by using the long wavelength approximation ($k^2\lambda_{De}^2 \ll 1$), we get the following dispersion relation:

$$\omega^2\left[1 + \eta_p\rho_{sn}^2 k_y^2\right] - \eta_p\omega\omega_{*n} - \eta_p c_{sn}^2 k_z^2 + c_{sn}^2 p^{-1} A_\pm k_y k_z = 0, \tag{8}$$

where $\rho_{sn} = \frac{c_{sn}}{\Omega_c}$ is the modified ion Larmor radius and $c_{sn} = \frac{c_s}{\sqrt{1-\beta}}$ is the modified ion acoustic speed due to the non-thermal Cairns distributed electrons whereas $c_s = \sqrt{\frac{T_e}{m_i}}$. Similarly, $\omega_{*n} = \frac{\omega_*}{1-\beta}$ is the modified drift frequency with $\omega_* = -v_* k_y$ (where $v_* = \frac{cT_e}{eB_0}\kappa_n$ is the drift velocity), $A_\pm = \frac{(\acute{V}_{+o} - \delta\acute{V}_{-o})}{\Omega_c}$, $\eta_p = \frac{2-p}{p}$, $\delta = (1-p)$, and $p = \frac{n_{eo}}{n_{+o}}$. In the absence of negative ions, non-thermal electrons and sheared flows (8) will then give the dispersion relation of Mikhailovskii (1974) and Mushtaq (2008) for coupled drift-ion-acoustic (DIA) waves in a two-component e-i plasma as $\omega^2[1 + \rho_s^2 k_y^2] - \omega\omega_* - c_s^2 k_z^2 = 0$; for pure drift waves we have $\omega[1 + \rho_s^2 k_y^2] + v_* k_y = 0$, and similarly for pure ion-acoustic waves in magnetized plasma, $\omega = c_s k_z/\sqrt{1 + \rho_s^2 k_y^2}$. Equation (8) can be simplified to give

$$\omega^2 - \omega\acute{\omega}_* - a\left(1 - bA_\pm\right) = 0, \tag{9}$$

where $\acute{\omega}_* = \frac{\eta_p\omega_{*n}}{1+\eta_p\rho_{sn}^2 k_y^2}$ is the effective/modified drift frequency $k^2 = k_y^2 + k_z^2$, $a = \frac{\eta_p c_{sn}^2 k_z^2}{1+\eta_p\rho_{sn}^2 k_y^2}$, and $b = \frac{1}{p\eta_p}\frac{k_y}{k_z}$. The roots of (9) are

$$\omega = \frac{\acute{\omega}_*}{2} \pm \frac{1}{2}\left[\acute{\omega}_*^2 + 4a\left(1 - bA_\pm\right)\right]^{1/2}, \tag{10}$$

which is the dispersion relation for coupled DIA waves with effects of non-thermal electrons and parallel shear flows of negative and positive ions. Equation (10) yields the shear flow instability of coupled DIA wave if the argument in the square root is negative, i.e.

$$\frac{\acute{V}_{+o}}{\Omega_c} > \left(\frac{\acute{\omega}_*^2}{4ab} + \frac{1}{b} + \frac{\delta\acute{V}_{-o}}{\Omega_c}\right). \tag{11}$$

For homogeneous plasma relation (10) will be $\omega = c_{sn}k_z\sqrt{\frac{\eta_p}{1+\eta_p\rho_{sn}^2 k_y^2}}(1 - \frac{1}{\eta_p p}\frac{k_y}{k_z}A_\pm)^{1/2}$, and the instability of ion-acoustic waves will occur if

$$A_\pm > (2-p)\left(\frac{k_z}{k_y}\right). \tag{12}$$

Again, it is obvious that increasing concentration of negative ions decreases p and hence increases $(2-p)$, which decreases instability and this is exactly the same result as observed in Ichiki et al. (2009). The graphical analysis of the two modes of relation (10) are further discussed in Sec. 5.

3. Sheared flow-driven drift vortices

We now discuss the quasi-stationary nonlinear solutions of (1) and (2) as well as (7). In the quasi-stationary frame, we let $\xi = y + \gamma z - ut$, where γ and u are constants giving the angle and speed of nonlinear structure. Thus, in the stationary frame, (1) for positive and negative ions can be written as

$$D_{\xi\phi}v_{jz} = -\frac{e}{mu}\left(\frac{\acute{V}_{jo}}{\Omega_c} \mp \gamma\right)\partial_\xi\phi, \tag{13}$$

where $D_{\xi\phi} = \{\partial_\xi - \frac{c}{B_0 u}\left(\partial_x\phi\partial_\xi - \partial_\xi\phi\partial_x\right)\}$. It can be shown that (13) is exactly satisfied by $v_{jz} = -\frac{e}{mu}(\frac{\acute{V}_{jo}}{\Omega_c} \mp \gamma)\phi$. Transforming (7) into the stationary frame and eliminating v_{jz}, we obtain

$$D_{\xi\phi}\left(\nabla_\perp^2\phi - \Gamma\phi\right) = 0. \tag{14}$$

Equation (14) is a modified Hasegawa–Mima (HM) equation affected by the Cairns distributed energetic electrons and ions parallel sheared flows. Here $\Gamma = \frac{1}{\eta_p\rho_{sn}^2(1+\frac{\Omega_c^2}{\omega_{p\pm}^2})}[1 - \frac{v_{*n}}{u} - \frac{\gamma c_{sn}^2}{pu^2}(\gamma + \delta\gamma - A_\pm)]$, with $v_{*n} = -\frac{\eta_p v_*}{1-\beta}$ is a modified drift speed. For the derivation of (14) it is assumed that $\nabla_\perp^2 = \partial_x^2 + \partial_\xi^2 \gg \gamma^2\partial_\xi^2$. Equation (14) admits both vortex street and dipolar vortex solution, i.e. for $\Gamma = 0$ and $\Gamma \neq 0$ respectively; we will discuss these one by one.

3.1. Vortex street solution

To find the analytical vortex street solution of (14) we put $\Gamma = 0$, then (14) is satisfied by the ansatz

$$\nabla_\perp^2\phi = \frac{L_1 L_2}{L_3}\exp\left[-\frac{2}{L_1}\left(\phi - \frac{uB_o}{c}x\right)\right], \tag{15}$$

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where L_1 and L_2 are arbitrary constants and L_3 measures the size of the vortex street. The analytical solution of (15) is given by Mikhailovskii (1974) and Petviashvili and Pokhotelov (1992),

$$\phi = \frac{uB_o}{c}x + L_1 \ln\left[2\cosh(L_2x) + 2\left(1 - L_3^{-2}\right)\cos(L_2\xi)\right]. \tag{16}$$

For $L_3 > 0$, the vortex profile given by (16) resembles the Kelvin–Stuart 'cat's eye', which represents a row of identical travelling vortices for $L_3^2 > 1$. Parameter L_1 characterizes the amplitude of vortex street. The vortex chain speed in this case is $u = \frac{v_{*n}}{2}[1 \pm \sqrt{1 + \frac{4\gamma c_{sn}^2}{pv_{*n}^2}(\gamma + \delta\gamma - A_\pm)}]$. For $L_3 = 1$, (16) becomes a solution in the form of a zonal flow (Shukla et al. 2000),

$$\phi = \frac{uB_ox}{c} + L_1 \ln\left[2\cosh(L_2x)\right]. \tag{17}$$

3.2. Double vortex solution

When $\Gamma \neq 0$ we get a double vortex solution, then (14) is satisfied by the ansatz

$$\nabla_\perp^2\phi = C_1\phi + C_2x, \tag{18}$$

where $C_1(= \Gamma - \frac{c}{uB_o}C_2)$ and C_2 are constants. To get the double vortex solution of (18) we transformed it into polar coordinates (r, θ) such that $x = r\cos\theta$, $\xi = r\sin\theta$, where $r = \sqrt{x^2 + \xi^2}$ and $\theta = \tan^{-1}(\xi/x)$. We divide the (r, θ) plane into an outer region $(r > R)$ and an inner region $(r < R)$ of an arbitrary circle of radius R, also called the vortex radius. In the outer region, to maintain boundedness of the solution, we must have $C_2 = 0$ to avoid the direct dependence of the space variable x. Thus, the solution of (18) for $r > R$ becomes (Liu and Horton 1986)

$$\phi_{\text{out}}(r, \theta) = Q_1K_1(\lambda_1 r)\cos\theta, \tag{19}$$

where Q_1 is a constant, K_1 is the first-order MacDonald function, and $\lambda_1 = \sqrt{\Gamma}$. Since λ_1 ought to be positive, thus for well-behaved outer solution it is necessary that Γ should be positive. Now for the inner region solution $(r < R)$ we have $C_2 \neq 0$ and assuming that $(\Gamma - \frac{c}{uB_o}C_2) = -\lambda_2^2$, (18) has both homogeneous and non-homogeneous parts. The homogeneous part is of ordinary Bessel type and is treated as for the outer region. Thus, the total solution for inner region turns out to be

$$\phi_{\text{in}}(r, \theta) = \left(Q_2J_1(\lambda_2 r) + \frac{Q_3}{\lambda_2^2}r\right)\cos\theta,$$

where $Q_3 = \frac{uB_o(\lambda_1^2 + \lambda_2^2)}{c}$, Q_2 and λ_2 are constants, and J_1 is the Bessel function of order one. The constants of integration Q_1, Q_2, and λ_2 can be found from the physically justified continuity conditions of ϕ, $\partial_r\phi$, and $\nabla^2\phi$ at the boundary of the circle, i.e. at $r = R$. For a given λ_1, the constant λ_2 can be determined by using the transcendental equation $\frac{K_2(\lambda_1 R)}{\lambda_1 K_1(\lambda_1 R)} = -\frac{J_2(\lambda_2 R)}{\lambda_2 J_1(\lambda_2 R)}$, which may be obtained from the continuation of $\nabla_\perp^2\phi$ at vortex interface; here J_2 and K_2 are the Bessel and modified Bessel functions of the second order. The constants Q_1 and Q_2 are determined from the matching of electrostatic potential and electric field at the vortex interface and their respective values are

$$Q_1 = \frac{uB_o}{c}\frac{R}{K_1(\lambda_1 R)},$$

$$Q_2 = -\frac{uB_o}{c}\frac{\lambda_1^2}{\lambda_2^2}\frac{R}{J_1(\lambda_2 R)}.$$

4. Drift soliton in the presence of sheared flows

We now present the possibility of formation of drift solitary structure in the presence of sheared flows. For low-frequency perturbations (with weak dispersion) and scalar nonlinearity we suppose that $\left(\mathbf{v}_E \cdot \nabla + V_{j0}\partial_z\right) \ll v_{jz}\partial_z$, which makes parallel components with sheared flow as

$$\partial_t v_{jz} + v_{jz}\partial_z v_{jz} - \frac{c\acute{V}_{j0}}{B_0}\partial_y\phi = \frac{qe}{m_i}\partial_z\phi. \tag{20}$$

Equation (20) after transformation into quasi-stationary frame, $\xi = y + \gamma z - ut$, yields

$$v_{\pm z} = \pm\frac{\gamma}{u}\left(\frac{c\Omega_c}{B_o}\sigma_\pm\right)\phi + \frac{1}{2}\left(\frac{\gamma}{u}\right)^3\left(\frac{c\Omega_c}{B_o}\sigma_\pm\right)^2\phi^2, \tag{21}$$

with $\sigma_\pm = (1 \mp \frac{\acute{V}_{\pm 0}}{\gamma\Omega_c})$. Also, (3) under the assumption $\frac{e\phi}{T_e} \ll 1$ can be expanded as

$$n_e = n_{eo}\left[1 + (1 - \beta)\frac{e\phi}{T_e} + \left(\frac{e\phi}{T_e}\right)^2 + \ldots\right]. \tag{22}$$

After transformation into ξ by using (5) and (6) along with (21) and (22), and by employing the assumptions of weak dispersion and scalar nonlinearity we get the following nonlinear equation:

$$\frac{d^2\phi}{d\xi^2} - A\phi + B\phi^2 = 0. \tag{23}$$

This is a normalized transformed modified KdV equation for drift wave coupled with ion-acoustic wave in inhomogeneous pair-ion plasma with sheared flow and in the presence of non-thermal Cairns distributed electrons. The coefficients A and B are given as

$$A = \frac{1 - \eta_p F + 2H^2\Psi_\pm}{\eta_p\left[1 + \frac{(1 + \gamma^2)G^2}{\eta_p}\right]}, \tag{24}$$

$$B = \frac{2H^4(1 - \beta)^2\epsilon_\pm - 1}{2(1 - \beta)\eta_p\left[1 + \frac{(1 + \gamma^2)G^2}{\eta_p}\right]},$$

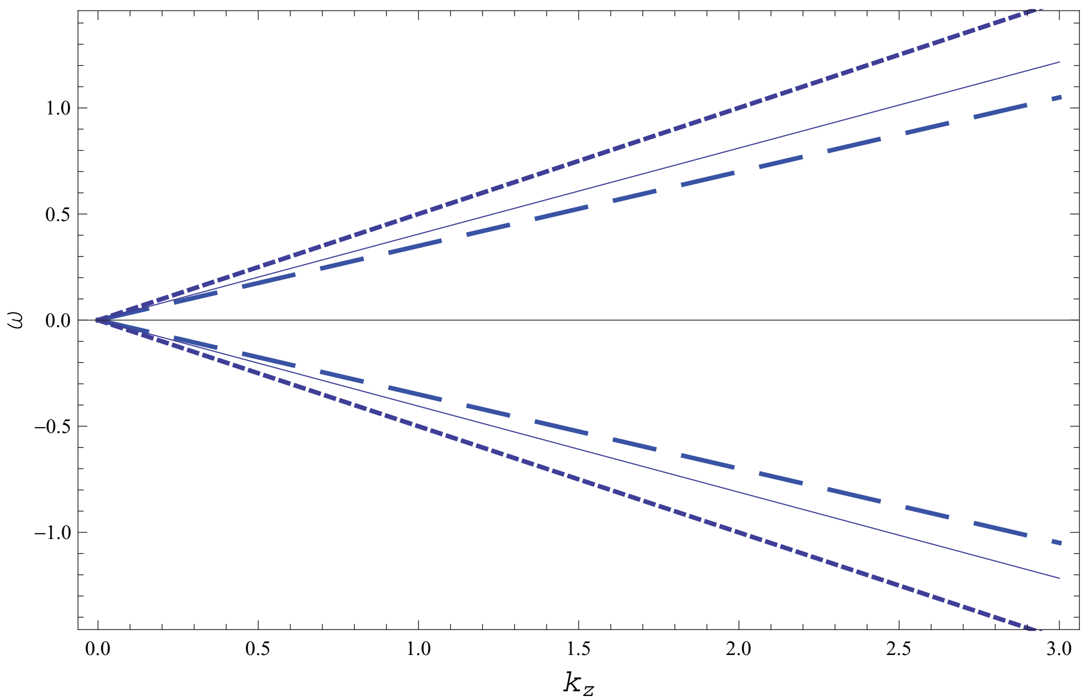

Figure 1. (Colour online) The frequency of the drift-ion-acoustic wave ω against k_z with effect of non-thermal Cairns distributed electrons (β) such that $\beta = 0$ (dashed line) 0.4 (solid line), and 0.8 (dotted line). The wave number k_z is scaled to $k_z/10^{-7}$ and frequency ω is $\omega/10^{13}$. All other parameters are the same as mentioned in Sec. 5.

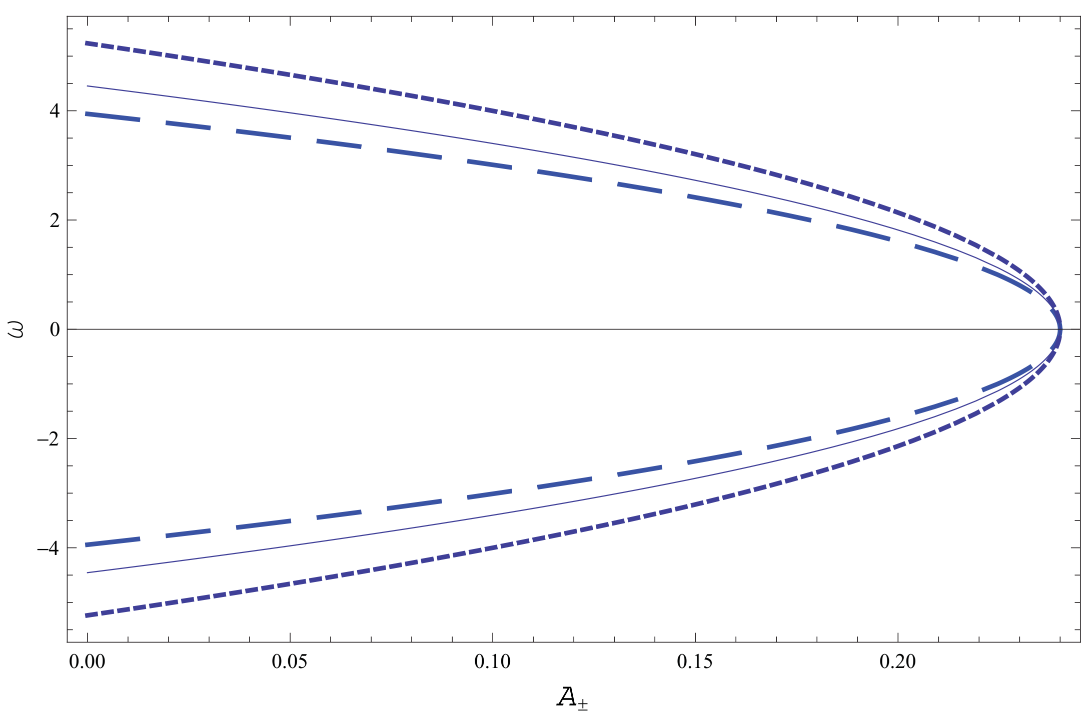

Figure 2. (Colour online) The effect of sheared flow $A_\pm$ on the frequency of the drift-ion-acoustic wave ω by varying the values of non-thermal electrons (β) such that $\beta = 0$ (dashed line), 0.4 (solid line), and 0.8 (dotted line). The frequency ω is scaled to $\omega/10^{13}$. The wave numbers in this case are taken as $k_y = 50$ and $k_z = 10$. All other parameters are the same as men tioned in Sec. 5.

where $H = \frac{\gamma c_{sn}}{\sqrt{2u}}$, $F = -\frac{v_{*n}}{u}$ (with $v_{*n} = \frac{v_*}{1-\beta}$), $G = \frac{\lambda_{De}}{\rho_s}$, $\Psi_\pm = (\frac{\sigma_- n_{-o} + \sigma_+ n_{+o}}{n_{eo}})$, and $\epsilon_\pm = (\frac{\sigma_+^2 n_{+o} - \sigma_-^2 n_{-o}}{n_{eo}})$. It should be noted that parameters are normalized in (23) in the following way: $\xi = \xi/\rho_{sn}$ and $\phi = e\phi/T_e$. By multiplying (23) by $\frac{d\phi}{d\xi}$ and integrating with respect to ξ by employing the boundary conditions (BCs), the energy integral equation is obtained as

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + S(\phi) = 0, \tag{25}$$

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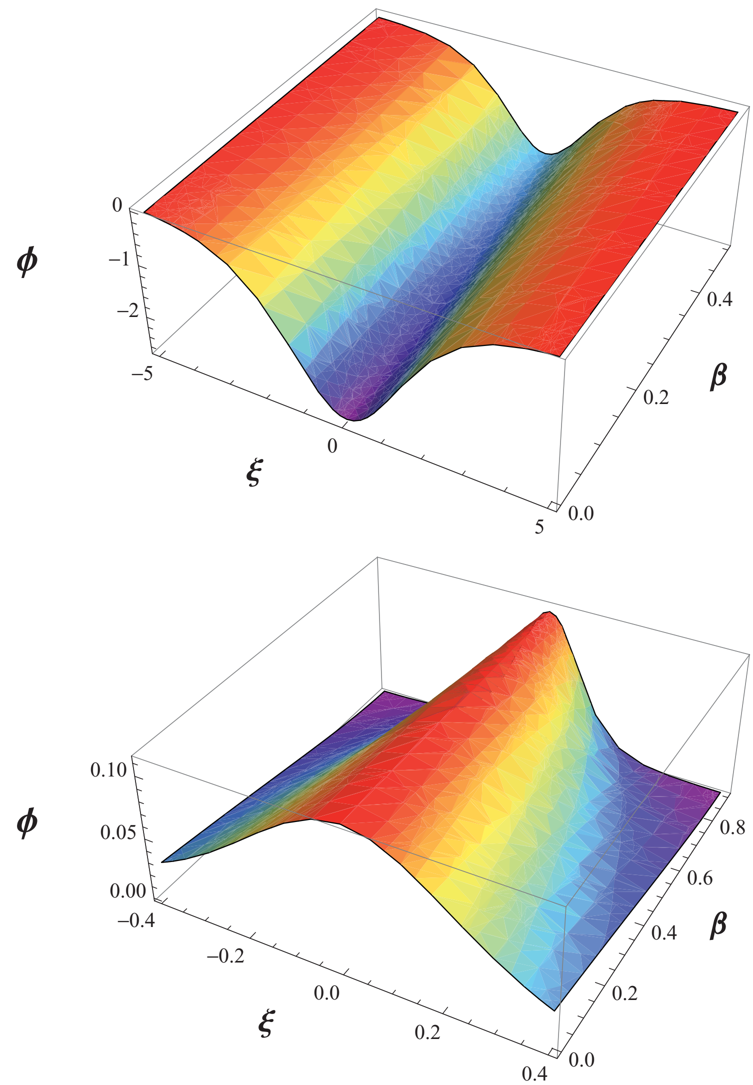

Figure 3. (Colour online) Bird's eye view of negative (upper panel) and positive (lower panel) electrostatic potential ϕ as a function of ξ and non-thermal Cairns distributed electrons β for fixed value of $|\frac{\acute{V}_{\pm 0}}{\Omega_c}| \sim 0.4$. Other parameters are $\gamma = 15°$, $G = 0.005$, $v_{*n} = -0.008$, $u \simeq (10^6 - 10^7)$, $n_{eo} = 0.8 n_{+o}$, and the remaining parameters are taken according to the numerics shown in Sec. 5.

which suggests that the evolution of solitary excitation is analogous to the problem of motion of a unit mass in a pseudopotential given by

$$S(\phi) = -\frac{A\phi^2}{2} + \frac{B\phi^3}{3}. \tag{26}$$

The condition for the existence of solitary wave is that $\frac{d^2 S}{d\phi^2} < 0$ at $\phi = 0$, which suggests that the formation of solitary structure in PI electron plasma in the presence of sheared flow is possible only if $\frac{d^2 S}{d\phi^2} = -A < 0$. It means that A should always be positive for the existence of solitary wave pulse; otherwise it will give collapse solution or shock-like solution. Hence, the necessary condition for the existence of solitary wave depends on β, v_*, Ω_c, $\sigma_\pm$, and η_p. Again, using appropriate BCs we obtain a solitary wave solution of (23) as

$$\phi = \left(\frac{3A}{2B}\right) \sec h^2 \left(\frac{y + \gamma z - ut}{\sqrt{4/A}}\right), \tag{27}$$

where $(\frac{3A}{2B})$ represents the maximum amplitude and $\sqrt{4/A}$ shows the width of drift solitary waves in a plasma with negative ions and in the presence of sheared flows and non-thermal effects. Similar type of result was derived in Pokhotelov et al. (2007) for magnetosonic solitons in a high-β space plasma, taking into account both non-Maxwellian distributions and finite amplitude effects.

5. Results and discussion

We have plotted the dispersion relations and the solitary wave dynamics by using some typical parameters of e-i plasma with negative ions, as used in Q-machine (Ichiki et al. 2009), as $T_e = 0.2$ eV, $B_o = 0.3$ T, $n_{+o} = 10^9/\text{cm}^3$, $k_y \sim$ few centimeters, let it be $\simeq$10 cm, $\beta \sim 0.7$ (optimized value), $(m_+/m_-) \sim 1$, and $|\frac{\acute{V}_{\pm 0}}{\Omega_c}| \sim 0.4$. The graphical analysis of the two modes of relation (10)

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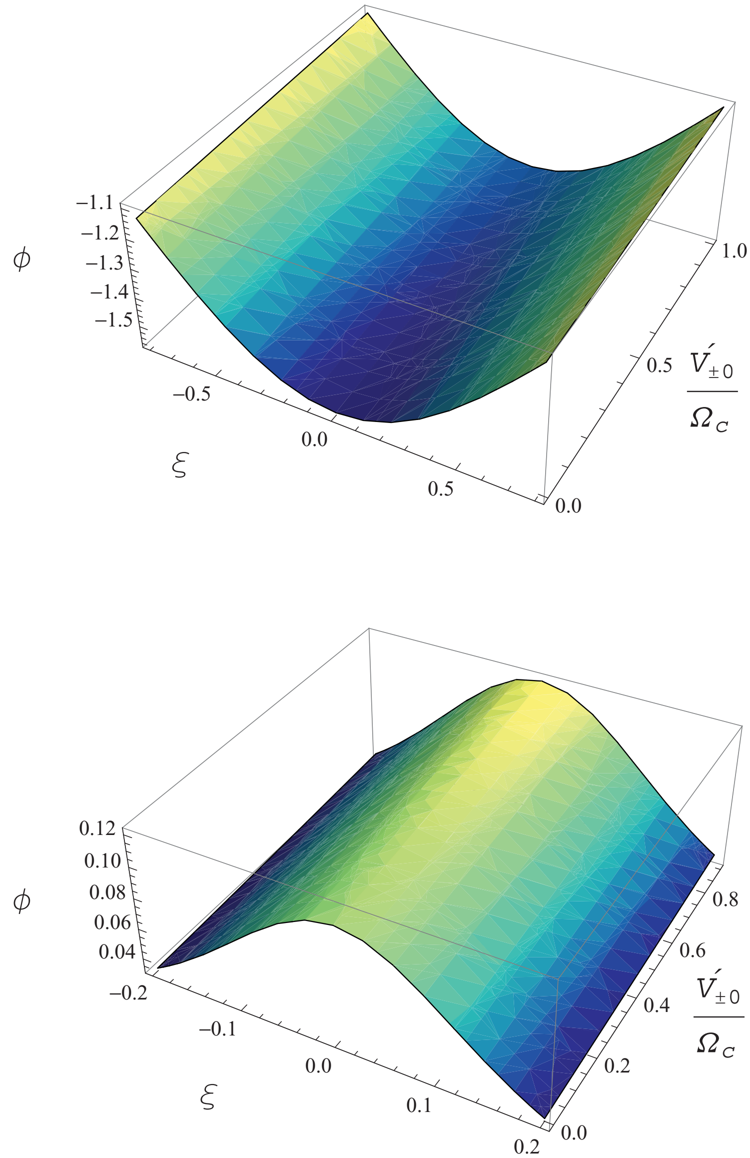

Figure 4. (Colour online) Bird's eye view of negative (upper panel) and positive (lower panel) electrostatic potential ϕ as a function of ξ and sheared flow $\frac{\acute{V}_{\pm 0}}{\Omega_c}$ for fixed value of $\beta \sim 0.7$. Other parameters are $\gamma = 15^{\circ}$, $G = 0.005$, $v_{*n} = -0.008$, $u \simeq (10^6 - 10^7)$, $n_{eo} = 0.8 n_{+o}$, and the remaining parameters are taken according to the numerics shown in Sec. 5.

are displayed in Figs. 1 and 2 (by using the above-mentioned numerics) for coupled DIA waves with effects of non-thermal electrons and parallel shear flows of negative and positive ions. Figure 1 shows the effect of non-thermal electrons (β) on the wave frequency against the parallel wave number k_z for fixed perpendicular wave number k_y. The condition $k_z < k_y$ is used while calculating the quantities to plot the dispersion relations. The plots of the two modes in Fig. 1 are almost straight lines and show that the frequencies of the forward and backward propagating DIA waves are enhancing with the increasing values of β. Similarly, the plots of the frequency as a function of $A_{\pm}$ for both forward and backward propagating modes are plotted in Fig. 2. The plots show a decreasing trend with respect to $A_{\pm}$. However, it is observed that for increased values of β the frequencies of the modes are increasing.

We now parametrically investigate the effects of sheared flows and non-thermal electrons on the behavior of drift solitary waves by using the above-mentioned parameters. It is clear from (27) that the solitary potential profile is positive (negative) if $B > 0$ ($B < 0$) for the given positive values of A. Therefore, B ($H = H_c$) $= 0$, where $H_c = (\frac{1}{2(1-\beta)^2\epsilon_{\pm}})^{1/4}$ is the critical value of H such that $H_c \gtrless \frac{1}{(2(1-\beta)^2\epsilon_{\pm})^{1/4}}$ gives the solitary waves with a positive and negative potential respectively. Consequently H depends on the ratio of $\sim \frac{c_{sn}}{u}$, and it is found numerically that at fixed c_{sn} the critical value of H occurs at $u_c \simeq 5.4 \times 10^6$, below which,

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i.e. ($u \leqslant u_c$), gives positive (hump) solitary waves and from $u > u_c$ we get negative (dip) solitary waves. Using the parametric values we have numerically analyzed the solitary wave structure with effect of non-thermal electrons and sheared flows as shown in Figs. 3 and 4. The effect of Cairns distributed non-thermal electrons on the drift solitary waves is shown in Fig. 3. The corresponding negative and positive solitary waves are shown in the upper and lower panels of Fig. 3. It is found that the amplitude of both types of solitary waves increases with increased values of β. Similarly, the effect of sheared flow $\frac{\acute{V}_{\pm 0}}{\Omega_c}$ on the dynamics of drift solitary waves is shown in Fig. 4. It is observed that amplitude of the corresponding negative and positive solitary waves increases with sheared flow. Thus, the effect of sheared flow is such that it affects both amplitude and width of the solitary structure under the assumption of weak dispersion and large-scale (scalar) nonlinearity. Sheared flow can distort the solitary structure and at large values there is possibility that it will lead to collapse of these coherent structures. Thus, the present studies of solitary waves in the presence of sheared flow are possible under certain prevailing assumptions and beyond those there is possibility that we will not get such solutions.

Summing up, we have studied both linear and nonlinear propagation of low-frequency electrostatic waves in a non-uniform electron–ion magneto plasma with negative ions with effect of non-thermal electrons and background velocity gradients. It is shown that sheared plasma flow can generate instability in such plasma system even when both positive and negative ions velocity gradients are equal and in the same direction. The differential flow gradient, however, can be produced because of non-uniformity of density gradient. The instability mechanism is similar to the K-H instability, where the slippage has been invoked by non-uniform flows among adjacent layers. Because of sheared flow and density inhomogeneity, the electrostatic perturbations interact nonlinearly and transfer energy within the wave spectra. These fluctuations are self-organized in the form of either a vortex street or a double vortex. The nonlinear vortical structures have been presented by mode coupling equations that contain vector as well as scalar nonlinearity. The solutions were obtained in the form of vortex street and double vortices. On the other hand, by incorporating scalar nonlinearity at larger scale length by employing weak dispersion assumption a KdV-type nonlinear equation was derived, which admits a solitary wave solution for DIA waves. The effects of non-thermal electrons and sheared flow on the dynamics of DIA solitary waves were highlighted. The present results can help to understand the origin of non-thermal electrostatic waves in laboratory and space plasmas (like Earth's magnetosphere and atmosphere (Pokhotelov et al. 1996)) where there are free energy sources due to ion velocity gradients and streaming particle motions. Our results may also be useful (specially the votex structures) in the high-latitude ionospheric region, by comparing these with observations made by Satellite (IC-B-130) in Chmyrev (1988), where plasma density gradient and sheared flows are the key factors to drive the vertical and solitary structures.

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