Models and the Semantic View

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I begin by distinguishing two notions of model, the notion of a *truth-making structure* and the notion of a *mathematical model* (in one specific sense). I then argue that although the models of the semantic view have often been taken to be both truth-making structures and mathematical models, this is in part due to a failure to distinguish between two ways of truth-making; in fact, the talk of truth-making is best excised from the view altogether. The result is a version of the semantic view which is better supported by the direct evidence offered for it, better equipped to achieve its avowed aims, and, I think, closer to the intentions of the original proponents of the view in many ways, despite some of their own declarations to the contrary.

1. Introduction. Talk of models, though certainly not a novel feature of the philosophy of science of recent years, has become more and more central to the discussion of a good many topics in the field. Unfortunately, however, the word 'model' has become multiply ambiguous along the way. It is easy to be led astray in such a situation, by moving in too carefree a way between importantly different notions of model, and the aim of this article is to describe one way in which I think that we have indeed gone astray. More specifically, I want to describe a way in which I think the semantic view of scientific theory structure has often been misunderstood, or perhaps misformulated, and lay out what I take to be the right way of understanding it—or, at any rate, the best way.¹

I should make it plain that I am playing for the other side here. Al-

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1. At the conference I concluded with a consideration of the implications of my main thesis for structuralism about scientific representation, for the partial structures approach, and for structural realism. Space limitations have forced me to excise that material here.

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though I will be describing the way in which, it seems to me, the semantic view of scientific theories can be made strongest, I argue elsewhere that the best philosophical account of such central scientific representations as the Bohr model of the hydrogen atom, and the nuclear model of the cell, involves thinking of such representations as *propositional* models (propositional, note, not sentential). And, relatedly, I think it is at least worth exploring the idea that scientific theories might fruitfully be regarded as collections of propositional models (Thomson-Jones, forthcoming a). Ultimately, then, the point of this article is not to defend the semantic view, but rather to get the view right, as part of the job of assessing it.

2. Truth-making and Representation. Let me begin by presenting brief characterizations of two distinct notions of model.

Truth-making structure: A model as a (generally) nonlinguistic structure that provides an interpretation for, and makes true, some set of sentences.

Mathematical model: A model as a mathematical structure used to represent a (type of) system under study.

(I articulate a taxonomy of models containing these two notions in Thomson-Jones [forthcoming a, forthcoming b].) The first thing to notice is that to call something a model in either sense is, in part, to ascribe a certain role to it—interpreting and making true in one case, representing in the other. The distinction between those two roles is fundamental to the points I want to make about the semantic view (see also Frisch 2005, 5–6). First, though, it will help to examine the two stronger notions, of truth-making structure and of mathematical model, a little more closely.

2.1. Truth-making Structures. Consider the sets

 $S_1 = \{$ Posh, Ginger, Baby, Sporty, Scary $\},$

 $S_2 = \{ \text{Posh} \},\$

$$S_3 = \{ \text{Posh, Ginger} \},\$$

the ordered triple

$$< S_1, S_2, S_3 >,$$

and the sentence

 $(\forall x)(Px \rightarrow Qx).$

On one standard logical notion of model, $\langle S_1, S_2, S_3 \rangle$ is a model of $(\forall x)(Px \rightarrow Qx)'$ (or, strictly speaking, of its singleton), because if we provide an interpretation for the language in question by mapping ' \forall ' onto the first element of the tuple, and 'P' and 'Q' onto the second and third, respectively, then ' $(\forall x)(Px \rightarrow Qx)$ ' comes out true on the standard definition of truth on an interpretation for a first-order language. (That the mapping in question is the relevant one is fixed, on this approach, by a preestablished ordering of the terms in the language.) The triple is thus a nonlinguistic structure that provides an interpretation for, and makes true, some set of sentences—a truth-making structure. I will call models of this sort "Tarskian models," as the notion has its roots in Tarski's work on the semantics of first-order languages. (For a standard presentation, see Shoenfield [1967, 18–22].)

Tarskian models are a very familiar kind of truth-making structure, but there are a number of other ways in which the truth-making structure notion of model is employed in the literature, and one in particular deserves special attention in the present context: a system is sometimes counted as a truth-making structure, and so as a model in the present sense, because it makes true the members of a set of *fully interpreted* sentences of a language such as ordinary scientific English. Consider, for example, the sentence

R. Every resistor heats up when a current passes through it.

We can say that some particular electrical circuit is a model of this sentence, or even of the rather minimal "theory" it expresses, if, indeed, every resistor in that circuit heats up when a current passes through it. Bas van Fraassen is using the notion of model as truth-making structure in this sort of way when he writes, in *Laws and Symmetry* (1989, 218): "A model is called a model of a theory exactly if the theory is entirely true if considered with respect to this model alone. (Figuratively: the theory would be true if this model was the whole world.)"

There is an apparent puzzle here, however: if a truth-making structure is a thing which provides an interpretation for a set of sentences (and does so in such a way that the sentences come out true on the interpretation), then how can anything be a truth-making structure for a set of *fully interpreted* sentences? The solution to this puzzle is to notice that its formulation trades on an ambiguity in the notion of interpretation. To see this, think carefully about what is going on when we regard a certain electrical circuit as a structure that makes R come out true: as van Fraassen's formulation suggests, we are regarding the quantifier 'every' as implicitly restricted in its scope to the little universe of entities which comprise that circuit. As we move from one circuit to another, asking whether each is a model of R, we change the domain of discourse, the set of entities

over which the term 'every' quantifies. There are some delicate issues in the semantics of natural language here, but one way of putting the point is as follows. As we move from one circuit to another, the meaning of 'resistor', of 'current', and even of 'every' is held fixed, and it is in that sense that R appears here as a fully interpreted sentence. Equally, however, as we move from one circuit to another, the domain of discourse changes, and so the proposition which R expresses changes; in that respect, each individual circuit can be regarded as providing a different interpretation for R. In this latter sense of 'interpretation', some circuits are truth-making structures for R, and so they are models of it in the present sense.

This distinction between ways of truth-making is worth spelling out in detail, I think, because it helps us to see something which will prove important in thinking about the best way of understanding the semantic view. We can say that $\langle S_1, S_2, S_3 \rangle$ is a truth-making structure for (and so a model of) $(\forall x)(Px \rightarrow Qx)$, and that a certain electrical circuit is a truth-making structure for (and so a model of) R, but we should not lose sight of the important differences between the two cases. The tuple $\langle S_1, S_2 \rangle$ S_2 , S_3 > plays a role in a reasonably elaborate semantic theory for the language in which $(\forall x)(Px \rightarrow Qx)$ is written. It provides interpretations (in one sense, at least) for uninterpreted terms in the sentence in question, and when we say that $\langle S_1, S_2, S_3 \rangle$ is a model of the sentence, we are implicitly invoking the semantic theory, with its notion of interpretation and its definition of truth in an interpretation. The electrical circuit we are imagining, however, does not provide an interpretation for uninterpreted terms in the language of R (except in the very limited sense that it provides a domain of discourse for the 'every' quantifier it contains), and when we say that the circuit is a model of R, or a truth-making structure for it, we are not invoking any semantic theory. In fact, what we are saying when we say that the circuit is a truth-making structure for R could also be said simply by saying that the circuit fits a description given by R—in that system (and here comes R...), every resistor heats up when a current passes through it. The same maneuver will not work, note, in the Tarskian case: it makes no sense to say " $<S_1$, S_2 , S_3 is an object such that $(\forall x)(Px \rightarrow Qx)$," because the sentence one will have uttered in saying *that* is itself not fully interpreted.

So, at the risk of overstating the difference, we might speak of truthmaking structures that are *serious interpreters*, like $\langle S_1, S_2, S_3 \rangle$, and of those which, like the electrical circuit, are *mere description fitters*. The distinction will be important when, in Section 3, we turn to consider the best way of understanding the semantic view.

2.2. *Mathematical Models*. Consider the following excerpt from van Fraassen's paper, "The Semantic Approach to Scientific Theories":

The systems [in the domain of inquiry] are physical entities developing in time. They have accordingly a space of possible states, which they take on and change during this development. This introduces the idea of a cluster of models united by a common *state-space*; each [model] has in addition . . . a 'history function' which assigns to [the modeled system] a history, i.e., a trajectory in that space. (1987, 109)

The idea here, then, is that the possible states of a given type of system are represented by points in a mathematical space which in this context we call a *state space*. To take the most obvious example, in classical particle mechanics the state space for a system of *n* particles will be a 6*n*-dimensional vector space, the phase space, each point of which corresponds to an assignment of three spatial coordinates (x, y, z) and three components of momentum (p_x, p_y, p_z) to each of the *n* particles. We can define a *trajectory* through the state space to be a function which maps points in some interval of the real line to points in the state space. The points in the domain of such a function are taken to represent times, and so a trajectory represents a particular evolution of the state space with a trajectory defined on it.

The notion of model I wish to fix upon is obtained by generalizing from the characterization just presented in two simple ways: we will not insist that the systems under study be physical entities, thus making room for an uncontentious application of the notion in the context of economics, for example; and we will not insist that a mathematical object represent the evolution of a system over time in order to count as a model in this third sense. We are left with a notion of model on which a model is simply a mathematical structure used to represent the structure and/or behavior of a system, or kind of system, from the domain of inquiry corresponding to a given discipline—the notion of a mathematical model.

Keeping the notions of mathematical model and of truth-making structure apart will prove vital in the next part of the discussion, on the proper understanding of the semantic view. The important thing to remember is that although a mathematical model is, and a truth-making structure can be, a mathematical entity, the associated functions are quite distinct. Truth-making structures interpret sentences and make them true; mathematical models represent systems from some given domain of inquiry. A truth-making structure might *also* function as a representation—nothing excludes that—but it equally well may not, and it is no part of the notion of a mathematical model as I have defined it that a mathematical model play any role in interpreting or making true any sentence. The conceptual distinction here is clean enough, even if there are objects to which both concepts apply.

3. Making the Most of the Semantic View. Although there are a number of variant versions of the semantic view on the market, the majority of them centrally involve at least one of two claims. First there is:

I. A scientific theory is a collection of models.

The 'T here is for 'Identification', as this claim identifies scientific theories with objects of a certain sort. The second claim is perhaps a less ambitious one, although one would not want to say that it is entailed by I. It is a methodological recommendation directed primarily to those who are engaged in philosophical work on the sciences:

M. A scientific theory is best thought of as a collection of models.

My question, rather unsurprisingly, is now: What notion of model is being invoked when a proponent of the semantic view utters *I* or *M*? We will also be addressing the distinct and less hermeneutical question: Given the avowed aims of the semantic view, and the reasons we are offered for believing it, what notion of model *should* a proponent of the semantic view be employing in making one or both of the claims in question?

My main thesis can be simply stated: Although the semantic view has often been taken to be the view that theories are, or are best thought of as, collections of entities which are *both* truth-making structures *and* mathematical models—what we might call "double aspect" models—it should in fact be understood as the view that theories are, or are best thought of as, simply collections of mathematical models. The truth-making structure notion of model is best excised from the semantic view altogether (insofar as it is truly present in the first place). The result is a version of the semantic view which is, I think, closer to the intentions of the original proponents of the view in many ways, despite some of their own declarations to the contrary, which is better supported by some of their own direct arguments for it, and which stands a better chance of achieving the avowed aims of the view.²

Two caveats: First, I am focusing here on the versions of the semantic view to be found in the work of Patrick Suppes and Bas van Fraassen. A number of the points I make here carry over to other well-known versions of the view, but there are also differences (see Thomson-Jones [forthcoming b] for further discussion). Second, given the space limita-

^{2.} Stephen Downes (1992) also argues that proponents of the semantic view have overextended the logician's notion of model. The details of our respective analyses are quite different, however, and conflict in some important ways.

tions, I will assume without argument that, for Suppes and van Fraassen, the models referred to in I or M are mathematical models in my sense, and that the only remaining question is whether they are also intended to be functioning as truth-making structures.

Now it is true that both Suppes and van Fraassen make explicit statements to the effect that the notion they wish to employ is, or is very closely related to, a logical notion (Suppes 1960, 289; 1967, 57; van Fraassen 1980, 44), and van Fraassen begins his presentations of the semantic view in both *The Scientific Image* (1980, 41–44) and *Laws and Symmetry* (1989, 217– 220) by introducing a notion of model on which models are clearly functioning as truth-making structures in one way or another. When it comes to showing the naturalness and plausibility with which theories in the empirical sciences can be viewed as collections of models, however, it is quite unclear that the models in question are, as constituents of those theories, functioning as truth-making structures in any substantial way.

To see this, let us focus on an early example of the way Suppes makes the case that theories can be thought of as collections of models, namely, his 1957 "axiomatization" of Newtonian particle mechanics (1957, 294):³

Definition 1. A system $\beta = \langle P, T, s, m, f, g \rangle$ is a [model] of particle mechanics if and only if the following seven axioms are satisfied:

Kinematical Axioms.

Axiom P1. The set P is finite and nonempty.

Axiom P2. The set T is an interval of real numbers.

Axiom P3. For p in P, s_p is twice differentiable on T. Dynamical Axioms.

Axiom P4. For p in P, m(p) is a positive real number. **Axiom P5.** For p and q in P and t in T,

$$f(p,q,t) = -f(q,p,t).$$

Axiom P6. For p and q in P and t in T,

$$s(p,t) \times f(p,q,t) = -s(q,t) \times f(q,p,t).$$

Axiom P7. For p in P and t in T,

$$m(p)D^{2}s_{p}(t) = \sum_{q \in P} f(p,q,t) + g(p,t).$$

3. The word 'model' replaces the word 'system'. Suppes himself does not use the word 'model' much in his 1957 discussion, but some of his remarks, combined with his references back to the 1957 discussion in later work, make it clear that he wishes to apply the term 'model' to the "systems of particle mechanics" defined in the quoted passage.

In presenting a theory this way, we begin by defining what Suppes calls a "set-theoretical predicate"—in this case, the set-theoretical predicate 'is a [model] of particle mechanics'. We then pick out a class of set-theoretical entities simply by instructing our audience to consider the class of objects to which the predicate applies. And the fundamental idea behind the semantic approach to scientific theories, in Suppes's version of that approach, is a particular filling out of the idea expressed by M: it is that philosophers of science will gain new insight into the structure of scientific theories if they think of them as the sorts of things that can be presented in just this way—that is, by way of the definition of a set-theoretical predicate.

So in what sense, if any, are the models picked out by the set-theoretical predicate 'is a model of particle mechanics' functioning as truth-making structures? My answer is that they are doing so only in the trivial, description-fitting sense, and that there is no reason to regard them as truth-making structures in *any* sense for a linguistic formulation of the theory being presented.

Consider some model, S, drawn from the collection of structures picked out by the predicate. Then in the terminology I introduced in Section 2, the first point to note here is that S is not a serious interpreter of the predicate or the "axioms" that compose it but is a mere description fitter for them. In other words, the sense in which S is a truth-making structure for the axioms in question is just the relatively thin sense in which some given electrical circuit can be a truth-making structure for R. The axioms-or, less misleadingly, the clauses of the predicate-are not uninterpreted sentences for which S provides an interpretation, in the way in which a Tarskian model interprets (and makes true) some sentence of a first-order language; they are fully interpreted sentences of mathematical English such as 'The set P [or: The first set in the tuple] is finite and nonempty' and 'The set T [or: The second set in the tuple] is an interval of real numbers'. All that S does is to provide a domain of discourse for the quantifiers involved in these sentences. And as we have seen, in the end this is simply to say that S fits the description we have given in our attempt to pick out a certain kind of mathematical structure.

Given this, we can now see that the fact that S is, in a thin sense, a truth-making structure for *the sentences being used to present the theory* (namely, the various clauses of the set-theoretical predicate) does not make S a truth-making structure for *a linguistic formulation of the theory being presented* (namely, Newtonian particle mechanics), even in the thin sense. As the clauses of the predicate are fully interpreted sentences, they would have to make claims about forces, masses, accelerations, and the like to amount to a formulation of Newtonian particle mechanics; but they do not. Axiom P2, for example, is that T, the second element of any tuple

which is to satisfy the set-theoretical predicate in question (and so count as a model of the theory), must be an interval of real numbers, and so it is about the features a certain mathematical sort of entity must have; more generally, axioms in Suppes's sense are simply components of a characterization of a class of mathematical structures. (van Fraassen [1980, 65] makes essentially this point.)⁴

Finally, note that we have been given no reason to think that *S* is playing a more substantial truth-making role (or, indeed, any truth-making role at all) with regard to any *other* set of sentences that might amount to a linguistic formulation of Newtonian particle mechanics. (Certainly there is no explicit mention of any formal language, or any set of sentences in such a language; indeed it is proclaimed to be one of the advantages of this approach that no such mention need be made.) Importantly, this is in no way a defect of the definition, as a means of presenting Newtonian particle mechanics, *if* the point is only to carve out a class of mathematical structures (the models of the theory, in our present sense) which, according to the theory, are adequate in one way or another to the job of representing actual or possible collections of particles as they move under the influence of various forces. The set-theoretical predicate is a perfectly good tool for picking out a collection of mathematical models, in other words.

The conclusion I wish to draw is that, insofar as claims I or M are supported by the ease with which we can give presentations of empirical theories on which they look like collections of models, the appropriate notion of model is simply that of the mathematical model. The mathematical structures such presentations pick out are clearly meant to be used for the purposes of representation; but there is no special reason for supposing that they play a role as truth-making structures for any linguistic formulation of the relevant theory, or any *substantial* truth-making role at all. Given all this, it is hard to resist the conclusion that the label 'semantic view' (and the label 'model-theoretic view', for that matter) is as much a misnomer as the name 'received view' has become.

I have laid out my point by focusing on the details of Suppes's version of the semantic view, but it seems clear that van Fraassen agrees with Suppes on all the relevant points. In characterizing Suppes's proposal, van Fraassen writes approvingly: "to present a theory, we define the class of its models directly, without paying any attention to questions of ax-

4. Note thus one disanalogy between this case and the case of R and the electrical circuit: if R is taken to express a simple and partial theory of currents in circuits, then the fact that the circuit is a truth-making structure for the sentence in question, albeit in the mere description-fitting sense, *does* mean that the circuit is a truth-making structure for a linguistic formulation of the theory being presented. But the (concrete, physical) circuit, note, is not plausibly thought of as a component of a scientific theory.

iomatizability, in any special language" (1987, 109), and elsewhere van Fraassen describes his own picture of theory structure as closely allied to Suppes's:

To present classical mechanics, for instance, [Suppes] would give the definition: 'A [model] of classical mechanics is a mathematical structure of the following sort . . .' where the dots are replaced by a settheoretic predicate. Although I do not wish to favour any mathematical presentation as the canonical one, I am clearly following here his general conception of how, say, the theory of classical mechanics is to be identified. (1980, 66)

So far I have argued that reading I and M, the defining theses of the semantic view, as involving only the notion of the mathematical model, so that the talk of truth-making falls away, thus makes those theses seem better supported by the direct evidence that is offered for them than otherwise. But note that such a reading also fits nicely with Suppes's slogan that "philosophy of science should use mathematics, and not meta-mathematics" (quoted in van Fraassen 1980, 65), for model theory is surely meta-mathematics. It makes the most sense, too, of van Fraassen's claims that the semantic view is "a view of theories which makes language largely irrelevant to the subject" (1987, 108) and that, on the semantic view, "models are mathematical structures, called models of a given theory only by virtue of belonging to the class defined to be the models of that theory" (1987, 109 n. 2).

Finally, it would seem that employing only the notion of a mathematical model in advancing I or M would offer the best hope of securing certain of the avowed goals of the semantic view. An account of theory structure, I take it, derives its primary philosophical value from the accounts it facilitates of confirmation, explanation, theory testing, and the like, and from the light it thus sheds on debates over realism, rationality, and other such abiding concerns. Proponents of the semantic view have repeatedly claimed that their view of theory structure has at least two advantages when it comes to this sort of philosophical work: that by construing theories nonlinguistically it allows us to sidestep a slew of familiar problems about the meaning, reference, and classification of various terms in the language scientists employ, and that it leads to a way of thinking about theories which maintains greater closeness to scientific practice than the older, syntactic view. Given this, it seems that a proponent of the semantic view ought to prefer the notion of a mathematical model to either the notion of a model as a truth-making structure for a linguistic formulation of the theory, or the "double aspect" notion on which models play both truth-making and representational roles. For if the models to which I and M refer are to be thought of as functioning crucially as truth-

makers for sentences making up some linguistic formulation of the theory, there is surely a significant danger that we will begin to focus once more on the sentences in question and the language in which they are written;⁵ as we do so, we are likely to drift further and further away from scientific practice.

4. The Implications for the Semantic View. Understanding the semantic view in the way I am recommending leaves us with a considerably more flexible approach to understanding theory structure than we must take, at least in practice, if the models of the semantic view are to function as truth-making structures of the "serious interpreter" variety. There are many kinds of mathematical structure, and many ways in which those many kinds can, and do, serve representational ends. We will thus have, as philosophers of science, a rich palette at our disposal when we turn to understanding the details of theory structure, the relationship of theories to the phenomena, intertheory relations, and related topics such as confirmation, explanation, and the rest, if we allow ourselves to draw on the great variety of mathematical structures there are in constructing our accounts. But if we insist on regarding scientific theories as collections of objects which, in addition to functioning as representations, also play a substantial role in some semantic theory-objects which are truth-making structures nontrivially—we will, in practice, end up limiting ourselves to thinking in terms of the sorts of structures which have been employed in the semantic theories we have managed to construct (and typically with the effect of forcing ourselves into relying on the tools of set theory). Such an approach, I would claim, carries with it extraneous baggage, makes the task more difficult than it need be, and lessens the chances of ultimate success.

Overall, then, it seems to me that the semantic view has little to lose, and something to gain, by employing in its central theses a concept of model shorn of the trappings of model-theoretic semantics and the role of truth-making. We should set aside the "double aspect" notion of model and drop altogether the ideas that the models which compose a theory play some nontrivial truth-making role or that regarding them as doing so is an important part of understanding scientific theories. The notion of a mathematical model—that is, of a mathematical structure that functions as a representation of systems from the domain of inquiry—is all we need to obtain the most attractive and useful version of the semantic view.

^{5.} Accordingly, it will also be difficult to get philosophers of science to stop thinking of laws as fundamental to theories, something van Fraassen, for one, wants very much to do (see van Fraassen 1989).

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