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On stabilization of bipedal robots during disturbed standing using the concept of Lyapunov exponents

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SUMMARY

Design of balancing control and the stability analysis of a biped during disturbed standing are investigated. A PD-based switching state feedback control is used to stabilize the biped at the upright position while satisfying the constraints between the feet and the ground. The concept of Lyapunov exponents is used for the stability analysis, and a stability region is determined. Furthermore, the stability region is compared and agrees well with the one from the previous work that predicts the feasible movement during which balance of human standing can be maintained. This agreement shows the potential of the concept of Lyapunov exponents to be used as a measure of balancing of human standing. The work contributes to bipedal balancing control, which is important in the development of bipedal robots.

KEYWORDS: Balancing control; Bipedal robots; Stability analysis; Stability region; Lyapunov exponents.

I. INTRODUCTION

The balancing control of disturbed bipedal standing is important for preventing falls of humans and bipedal robots. The controller should be designed for motion regulation while satisfying the constraints between the feet and the ground.[1,2] Such a design task is extremely challenging.

Another challenge for balancing bipedal standing is the lack of a single quantitative criterion and an effective tool for stability analysis. Pai and Patton[3] investigated the balancing of human standing with the consideration of the constraints between the feet and the ground. They determined the feasible stability boundaries using the computer simulations of movement termination with the aid of an optimization routine. The limitation, from a viewpoint of stability, comes from their definition of the stability, *i.e*. the center of mass of the biped can be moved into a region between the heel and the toe within a short time period and with a zero angular velocity. However, the biped satisfying their stability criteria may still fall from the upright position. Thus, the long-term behavior of dynamical systems should be considered.

Lyapunov's second method is a powerful method for stability analysis (Wu *et al.*[4]), but due to the lack of a constructive method, it is difficult to derive a Lyapunov function for highly nonlinear bipedal systems. Sekhavat *et al.*[5] employed the concept of Lyapunov exponents to analyze the stability of nonlinear dynamical systems and showed that the method is constructive and powerful.

The balancing control and stability analysis of a biped during disturbed standing are studied here. The biped is simplified as a two-dimensional inverted pendulum with one link for both feet. The foot-link is in contact with the ground, but is not fixed. A PD-based state switching feedback controller is used to stabilize the biped to the upright position while satisfying three constraints between the foot-link and the ground, *i.e*. no lifting, no slippage, and the center of pressure (COP) remaining within the contact region between the feet and the ground. The stability of the control system is analyzed using the concept of Lyapunov exponents and a stability region is determined and compared with the previous work.[3]

II. METHOD

The simplified bipedal model is shown in Fig. 1. The dynamic equations are developed using the Euler-Lagrangian Equation and are shown below:

$$\tau = mgr\sin\theta - (I + mr^2)\ddot{\theta} \tag{1a}$$

$$F_{gx} = mr\ddot{\theta}\cos\theta - mr\dot{\theta}^2\sin\theta \tag{1b}$$

$$F_{gy} = (m_f + m)g - mr\ddot{\theta}\sin\theta - mr\dot{\theta}^2\cos\theta \tag{1c}$$

$$x_{cop} = L_f - a - \frac{bF_{gx} - \tau + cm_f g}{F_{gy}} \tag{1d}$$

Three constraints can be written as:

$$F_{gy} \geq 0 \tag{2a}$$

$$|F_{gx}| \leq \mu F_{gy} \tag{2b}$$

$$0 \leq x_{cop} \leq L_f \tag{2c}$$

A PD-based switching state feedback control law is designed to stabilize the biped at the upright position while keeping the foot-link stationary. The controller considers each constraint, shown in (2), and determines the control torque bounds. The controller is shown as:

$$\tau = \begin{cases} \tau_{PD} & \text{if } \tau_{lower} \leq \tau_{PD} \leq \tau_{upper} \\ \tau_{upper} & \text{if } \tau_{PD} \geq \tau_{upper} \\ \tau_{lower} & \text{if } \tau_{PD} \leq \tau_{lower} \end{cases} \tag{3}$$

where $\tau_{PD} = k_p\theta + k_d\dot{\theta}$, τ_{upper} and τ_{lower} depend on the state space, *i.e.* θ and $\dot{\theta}$, which have been determined from our previous work.[1,2] The state space model of the

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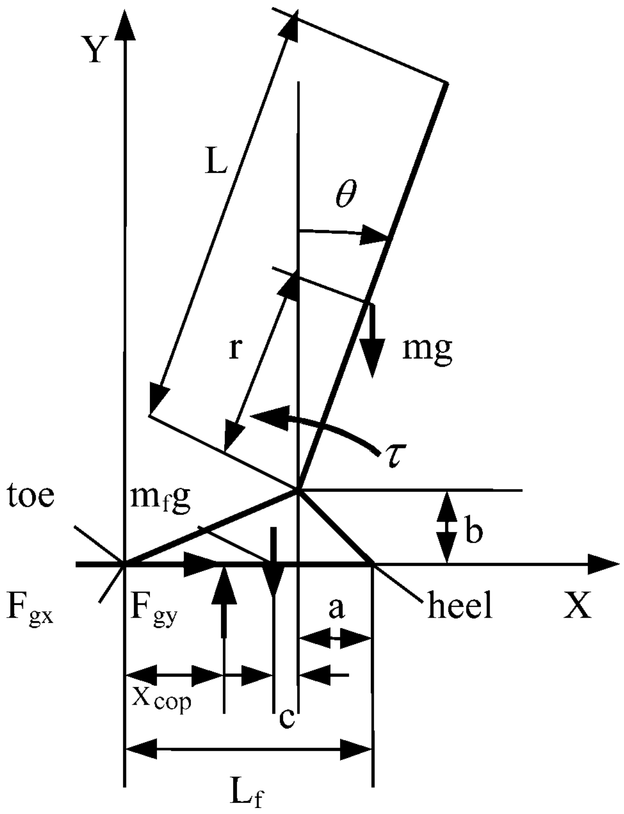

Fig. 1. Simplified bipedal model.

system is shown as:

$$\dot{x} = \begin{cases} \begin{cases} x_2 \\ \dfrac{mgr\sin x_1 - \tau_{PD}}{I + mr^2} \end{cases} & \text{if } \tau_{lower} \le \tau_{PD} \le \tau_{upper} \\ \begin{cases} x_2 \\ \dfrac{mgr\sin x_1 - \tau_{upper}}{I + mr^2} \end{cases} & \text{if } \tau_{PD} \ge \tau_{upper} \\ \begin{cases} x_2 \\ \dfrac{mgr\sin x_1 - \tau_{lower}}{I + mr^2} \end{cases} & \text{if } \tau_{PD} \le \tau_{lower} \end{cases} \quad (4)$$

Lyapunov's stability analysis investigates the long-term behavior of motion under the influence of disturbance in the initial states. Lyapunov exponents are the average exponential rates of divergence or convergence of nearby orbits in the state space, and can infer the system stability (see Sekhavat *et al.*[5] for detailed discussions). If the largest Lyapunov exponent is convergent and negative, the dynamic system is asymptotically stable about the equilibrium point, which is the upright position. Note that the controller is overall continuous, but the non-differentiable points occur at the instants of switching, where the derivative of the right-hand side of the state space model (4) does not exist. Thus, linearization of the nonlinear equations at these points, required by the calculation of Lyapunov exponents for stability analysis, is addressed by resorting to the work of Muller[6] in which the conventional calculation procedure has been extended to systems with non-differential points. The non-standard finite difference scheme proposed by Mikens[5] is used to suppress the numerical instabilities and to improve calculation efficiency.

III. SIMULATION RESULTS

To demonstrate the importance of considering the constraints between the feet and the ground for the control design, a conventional PD control, designed without the consideration of the constraints, is compared with our state-switching controller. Figure 2a shows the angular displacements using a conventional PD feedback controller and our PD-based switching state feedback controller to stabilize the biped from the initial states $\theta_0 = 0$ rad and $\dot{\theta}_0 = -0.6$ rad/sec to the upright position. Same control gains were used for both controllers. From Fig. 2a, we can see that using the conventional PD controller, the biped is stabilized to the upright position within 0.5 second, while using our PD-based switching state controller, the biped oscillates approximately 3 seconds, and then settles down at the upright position. It is expected that the transient period from the switching state control system is longer due to the control bounds determined by the constraints. Figure 2b shows the control torques from our PD-based switching state controller (the solid line), the conventional PD controller (the dash line) and the control bounds (dotted lines) satisfying the constraints. Figure 2b shows that the control torque, determined from our switching state feedback controller, is always within the control bounds, indicating that the constraints between the foot-link and the ground are satisfied. Together with the angular displacement shown by the solid line in Fig. 2a, it can be concluded that our PD-based switching state feedback controller can stabilize the biped at the upright position meanwhile satisfying the constraints shown in (2). Figure 2b also shows that the control torque from the conventional PD controller is below the lower bound of the control torque. This indicates that if the foot-link is not fixed on the ground, the constraints, shown in (2), will be violated, and stabilization of the biped is out of the question. The numerical results, shown in Fig. 2, indicate the importance to consider the constraints between the foot-link and the ground when the balancing control law is designed.

Two Lyapunov exponents for the control system, shown in (4), were calculated. After 100 seconds, the largest Lyapunov exponent converges to -9.47 and the second Lyapunov exponent converges to -22.11. Thus, the control system shown in (4) is exponentially stable about the upright position. The stability region was determined based on the largest Lyapunov exponent. We only considered the biped leaning posteriorly, *i.e.* the angular displacement ranged from 0° to positive 63° since leaning posteriorly is considered more dangerous.[3] The determined stability region was compared with the previous work[3] based on a different stability definition. Both results are shown in Fig. 3. The region surrounded by the solid curve represents the stability region obtained in the previous work[3] where neither forward falls nor backward falls of a human subject would be initiated. Dots are the initial states that convergent and negative Lyapunov exponents were obtained. The region surrounded by the dots is a stability region, *i.e.* the bipedal model can be stabilized at the upright position with the

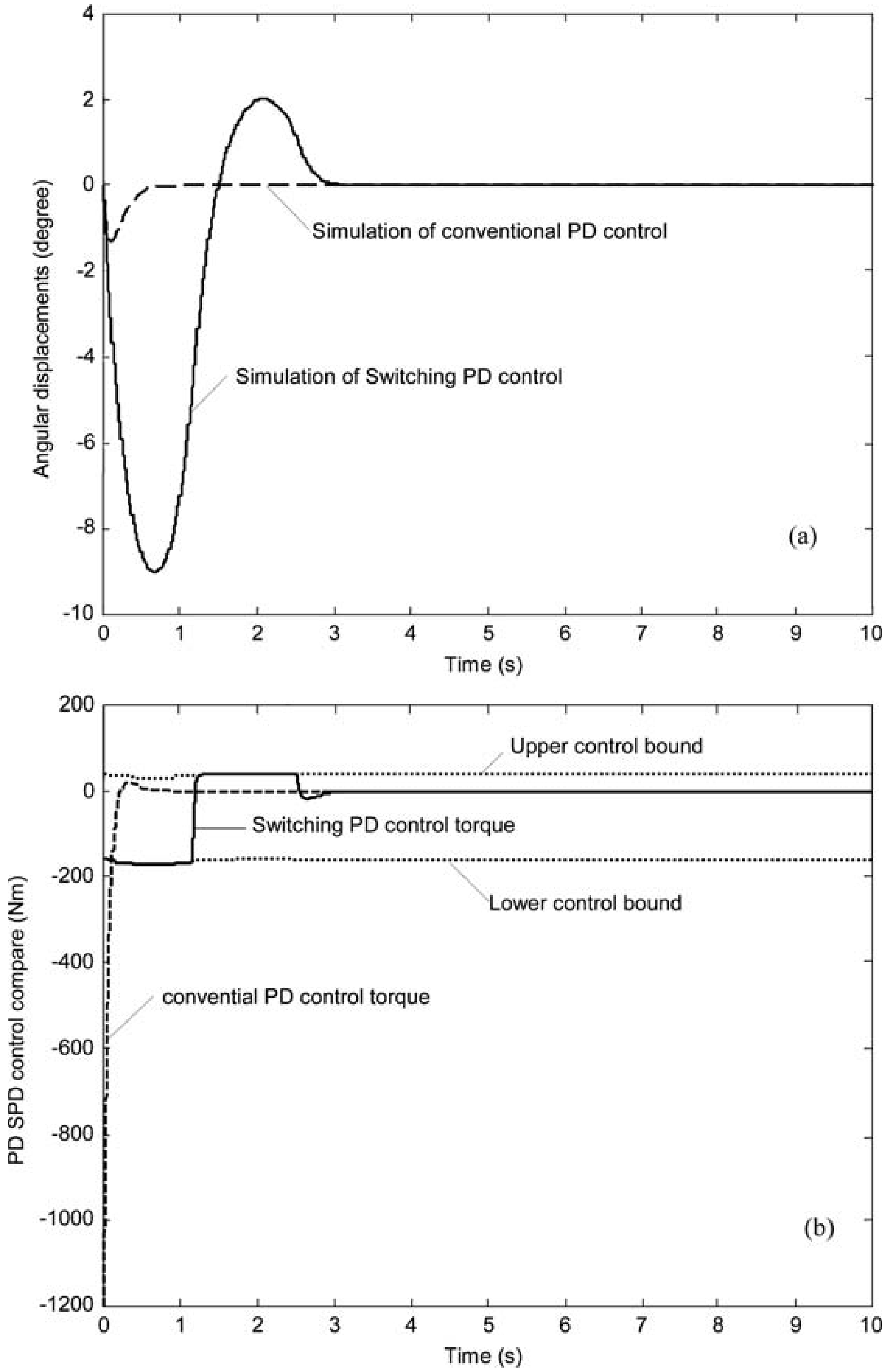

Fig. 2. Simulation results using a conventional PD controller and the PD-based switching state feedback control (a) simulated angular displacements (b) control torques.

foot-link remaining stationary. Stars are initial states such that convergent Lyapunov exponents can not be obtained. In the region outside the stars, convergent Lyapunov exponents can not be obtained due to the violation of the constraints between the foot-link and the ground. With the constraints violated, the biped collapses, which terminates the calculation.

Although the stability criteria are significantly different, Fig. 3 shows that both stability regions agree overall reasonably well. Especially as the angular displacement below 30° and the angular velocity lower than 1 rad/s, the stability region from our work is almost identical to the one from the previous work.[3] The definition of the stability used in the previous work was based on the clinical observations on balancing of human subjects and was intended to developing a clinical tool to assess a person's ability to maintain standing posture. Such a definition only concerns the system performance within a short time period, while Lyapunov stability deals with long term dynamic behaviour. The agreement between the stability regions suggests that the two stability concepts are related and, to certain extent, are equivalent. This indicates that the concept of Lyapunov exponents has great potential to be used as a measure for

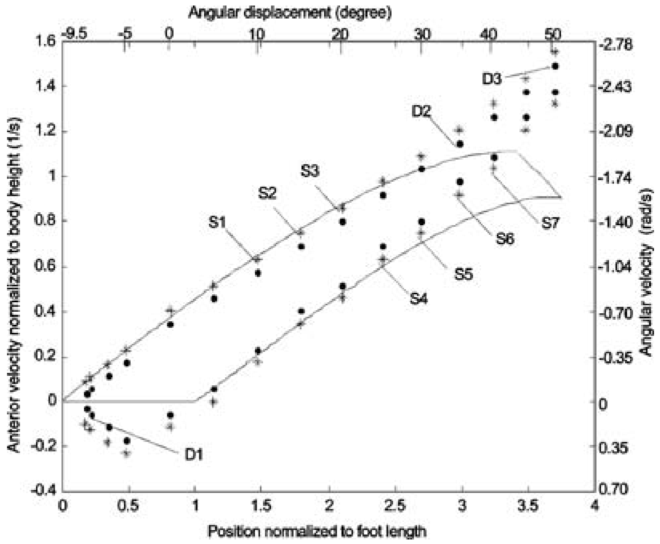

Fig. 3. Stability regions determined based on two different stability criteria.

assessing a person's ability to maintain the upright posture during standing.

IV. CONCLUSIONS

A PD-based switching state feedback controller was used to stabilize the biped at the upright position, while satisfying three constraints between the foot-link and the ground. It shows that the consideration of constraints is imperative and has significant impact on the control design. The stability of the constrained control system was analyzed using the concept of Lyapunov exponents and the stability region was determined. It demonstrated that the concept of Lyapunov exponents is a constructive and powerful tool for stability of highly nonlinear systems. It is especially powerful in determining stability regions, which is crucial information for balancing control of bipedal movement. The stability region, determined in this work, agrees well with the one from previous work,[3] which predicts the feasible movement for balancing human standing. This agreement indicates that the concept of Lyapunov exponents has great potential to be used as a measure of balancing of human locomotion. Although the bipedal model and proposed PD switching state controller are simple, this work set up a framework for developing more advanced balancing controllers for bipedal standing using more realistic models.

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