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INDETERMINACY IN CASH-IN-ADVANCE MODELS AND THE ROLE OF FRICTIONS

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A monetary cash-in-advance model is known to be prone to real indeterminacy if the intertemporal elasticity of substitution in consumption is sufficiently low. Moreover, if the model features habit formation in consumption, the scope of indeterminacy increases substantially. This paper shows that many of the nominal frictions and real rigidities commonly used in the New Keynesian paradigm act to decrease the scope of this indeterminacy. These frictions include stickiness in prices and wages, adjustment costs in investment, and variable capacity utilization. When they are all used together in the model, the problem of indeterminacy nearly vanishes, even when habit formation in consumption is allowed.

Keywords: Indeterminacy, Sunspots, Habit Formation, Cash in Advance

1. INTRODUCTION

After Ball and Romer (1990) and Chari et al. (2000) showed that standard sticky-price New Keynesian models cannot generate large output effects of monetary shocks, economists tried to face the challenge by incorporating an increasing number of additional frictions into their models. These frictions include habit formation in preferences, adjustment costs in investment, variable capacity utilization, and nominal wage stickiness. In a great majority of those models, money is introduced via a money-in-the-utility (MIU) specification [e.g., Blanchard and Kiyotaki (1987) and Christiano et al. (2005)]. In some others, it is introduced in an ad hoc fashion [e.g., Ball and Romer (1990) and Dotsey and King (2006)] or according to a transaction cost specification [e.g., Sims (1994), Schmitt-Grohe and Uribe (2004), and Altig et al. (2005)], and in a very few models a cash-in-advance (CIA) specification in which both consumption and investment spending are subject to the CIA constraint is used [e.g., Danthine and Kurmann (2004) and Wang and Wen (2006)]. Models in which only consumption is subject to the CIA constraint (to which I will refer as "standard CIA models") have hardly been used in the literature [for two rare examples see Yun (1996) and Ellison and Scott (2000)].

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One reason for the rare usage of standard CIA models in the New Keynesian literature might be that they are prone to local real indeterminacy in the form of sunspot equilibria if the intertemporal elasticity of substitution (IES) in consumption is sufficiently low [see Woodford (1994) and Carlstrom and Fuerst (2003)]. Sunspot equilibria describe a situation where extrinsic uncertainty causes multiple stationary equilibria even though the fundamentals do not change. In other words, the model may lead to a continuum of convergent equilibrium trajectories indexed by the initial conditions of the variables that are not predetermined. To see how indeterminacy arises in a standard monetary CIA model, consider a sunspot-driven increase in current consumption. This leads to an increase in expected inflation, which, in turn, causes the nominal interest rate to rise. Because the nominal interest rate acts like an inflation tax on money balances, the CIA restriction brings about a fall in future consumption. If the intertemporal elasticity of substitution is high enough, the future consumption falls sharply and offsets the initial increase in expected inflation. If the IES is not high enough, initial beliefs are fulfilled and the sunspot mechanism is supported.[1]

Moreover, Auray et al. (2005) show that, if a standard CIA model with exogenous monetary growth is modified to include habit formation in preferences, the scope of equilibrium real indeterminacy substantially increases because habit persistence weakens the intertemporal substitution motives. As the authors note, when there is habit formation in preferences, the marginal utility of future consumption is an increasing function of current consumption. Therefore, a rise in current consumption triggers a rise in future consumption. If the habit persistence parameter is large enough, this effect can dominate the intertemporal substitution mechanism and cause future consumption to increase.[2] In other words, prophecies are fulfilled.

The fact that habit formation increases the scope of indeterminacy implies a serious setback for the standard CIA model because the assumption of habit formation in consumption is considered essential by many economists in accounting for the dynamics of consumption and asset pricing [see Fuhrer (1998), Campbell and Cochrane (1999), and Christiano et al. (2005)]. So it may seem that in modern New Keynesian models featuring habit formation in preferences, a standard CIA specification is not a good choice for introducing money.

This paper shows that this is not necessarily true. If a standard CIA model featuring habit persistence in consumption incorporates some other frictions widely used in the literature as well, the problem of indeterminacy may nearly vanish over a wide range of empirically plausible parameter values. These frictions are sticky prices, sticky wages (which, of course, require imperfect competition in product and labor markets, respectively), and two sources of real rigidities, namely, adjustment costs in investment and variable capacity utilization. Despite their ad hoc specifications, these frictions are considered by many economists to be indispensable features of a state-of-the-art New Keynesian model.

In a discussion paper, Weder (2006) analytically shows that sluggish price adjustment reduces the scope of sunspot equilibria in a monetary CIA model without capital. The intuition behind this result is the following. When prices are sticky, the sunspot-related increase in inflation becomes smaller and, as a result, a smaller degree of IES can generate the sufficient fall in future consumption that will offset the initial increase in expected inflation. In this paper, I show that this result can be generalized to other frictions as well: If a friction in a monetary CIA model causes a fall in the sunspot-related inflation, either by itself or in combination with others, it decreases the scope of indeterminacy, as is the case with price stickiness.[3]

The main findings of the paper can be summarized as follows. When introduced alone, none of these frictions can help a plausibly calibrated standard CIA model featuring habit persistence to avoid indeterminacy. However, when there are adjustment costs in investment, adding either price or wage stickiness into the model reduces the scope of indeterminacy substantially. Furthermore, if all the frictions are present together, the standard CIA model becomes immune to indeterminacy for almost any empirically plausible parameter set. Throughout the paper, the CIA model incorporating all frictions will be referred to as "the full-fledged CIA model."

These findings imply that, from the determinacy point of view, a standard CIA specification is as good a choice as any to introduce money into a New Keynesian model featuring habit persistence, provided that the model incorporates the other widely used frictions. However, this result does not guarantee that it will perform equally well in business cycle analysis. To shed some light on this point, I calibrate and simulate the full-fledged CIA model to estimate its response to an exogenous monetary shock. I find that the results are very similar to those of a standard MIU model incorporating the same frictions.

The rest of the paper is organized as follows. The full-fledged model is presented in Section 2. Section 3 explores the single and combined effects of the above-mentioned frictions on the scope of indeterminacy. Section 4 presents the qualitative responses of the full-fledged CIA model to a positive monetary shock and compares these results with those of a MIU model. Section 5 concludes.

2. THE FULL-FLEDGED CASH-IN-ADVANCE MODEL

The real side of the model is essentially the same as in Christiano et al. (2005). The economy is populated by a continuum of households indexed by $j \in [0, 1]$ and each household is a monopolistic supplier of a differentiated labor service. There are two sectors in the economy: one producing intermediate goods and the other producing final goods. The intermediate goods sector consists of a continuum of monopolistically competitive firms indexed by $z \in [0, 1]$, each selling a differentiated good. The final goods sector consists of a single representative firm producing a homogeneous final good for a competitive market.

2.1. Households and Wage Setting

Household j is a monopolistic supplier of a differentiated labor service, N_t^j. It sells this labor service to a representative, competitive agency that transforms it into an aggregate labor input, N_t, using the technology

$$N_t = \left[\int_0^1 \left(N_t^j\right)^{(\theta_w-1)/\theta_w} dj\right]^{\theta_w/(\theta_w-1)}, \tag{1}$$

where $\theta_w > 1$ denotes the intratemporal elasticity of substitution across different labor types. The agency, in turn, sells the composite labor N_t to firms producing intermediate goods. The agency's cost minimization problem yields

$$N_t^j = \left[\frac{W_t^j}{W_t}\right]^{-\theta_w} N_t \tag{2}$$

and

$$W_t = \left[\int_0^1 \left(W_t^j\right)^{1-\theta_w} dj\right]^{1/(1-\theta_w)}. \tag{3}$$

Equation (2) gives the total demand for household j's labor by all firms producing intermediate goods. W_t is the aggregate nominal wage index. The household takes W_t and N_t as given.

Households set wages in staggered contracts à la Calvo. The duration of each contract is randomly determined so that in any given period, the household is allowed to reoptimize its wage with probability $(1-\gamma_w)$. Whenever the household is not allowed to reset its wage contract, its wage rate is given by

$$W_t^j = \Pi_{t-1} W_{t-1}^j, \tag{4}$$

where $\Pi_t = P_t/P_{t-1}$ is the gross rate of inflation.

Because each household is a monopolistic supplier of a differentiated labor service and reoptimizes his wage under conditions peculiar to him, households are heterogeneous with respect to the wage rate, labor supply, consumption and asset holdings. However, as shown by Erceg et al. (2000), under the assumption that households have access to a complete set of state-contingent contracts whose payoffs are contingent on whether they can reset their wage contracts, they become homogeneous with respect to consumption and asset holdings. The notation used below reflects this result.

The preferences of household j are characterized by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t - hC_{t-1}\right)^{1-\sigma}}{1-\sigma} - \Psi \frac{\left(N_t^j\right)^{1+\phi}}{1+\phi}\right], \tag{5}$$

where $0 < \beta < 1$ is the subjective discount factor, $0 \leq h < 1$ is the degree of habit persistence, $\sigma > 0$ is the IES, $\phi > 0$ is the inverse of the elasticity of labor supply, Ψ is any positive constant, C is consumption, and N is the number of hours worked.

Household j's budget constraint in period t is given by

$$\frac{B_t + M_t}{P_t} = \frac{R_{t-1}B_{t-1} + M_{t-1} + T_t}{P_t} + q_t s_t K_t + \frac{W_t^j}{P_t} N_t^j + \Phi_t - A(s_t)K_t - C_t - I_t, \tag{6}$$

where B_t is the stock of nominal bonds with net supply $B_t = 0$ in equilibrium, M_t is the stock of money, R_t is the gross nominal rate of interest, T_t is a lump-sum payment made by the monetary authority, q_t is real rental rate of capital, s_t is the utilization rate of capital assumed to be set by the household, K_t is physical stock of capital, W_t^j is nominal wage rate, Φ_t is firms' profits, and I_t is investment. The $A(s_t)K_t$ term denotes the cost of setting the utilization rate to s_t where $A(\cdot)$ is an increasing convex function. Following Christiano et al. (2005), I assume that $s_t = 1$ in the steady state and $A(1) = 0$. Under these assumptions, the steady state of the model is independent of the elasticity parameter $\kappa = A''(1)/A'(1)$.

The stock of installed capital, which is assumed to be owned by the household, evolves according to

$$K_{t+1} = (1-\delta)\, K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{7}$$

where δ denotes the physical rate of depreciation and I_t denotes investment. $S(\cdot)$ is an increasing and convex function determining adjustment costs in investment. I assume that $S(1) = S'(1) = 0$ and $S''(1) > 0$, so that the steady state of the model is not affected by adjustment costs.

Consumption purchases have to be made in cash; therefore the household is subject to the following CIA constraint:

$$C_t \le \frac{M_{t-1} + T_t}{P_t}. \tag{8}$$

In every period t, the household maximizes (5) with respect to C_t, M_t, B_t, K_{t+1}, I_t, and s_t subject to the budget constraint (6), the capital accumulation equation (7), and the CIA constraint (8).

In addition, in any period t in which household j is able to reset its wage contract, the household maximizes (5) with respect to the wage rate W_t^j, subject to the budget constraint (6) and the labor demand equation (2).

2.2. The Final Goods Producer

The representative firm in the final goods sector produces a homogeneous good Y_t using intermediate goods $Y_t(z)$. It ensembles the intermediate goods according

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to the production function

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \qquad \textbf{(9)}$$

where $\theta > 1$ is the intratemporal elasticity of substitution across different varieties of intermediate goods.

The final goods producer operates in a competitive output market and its profit maximization problem yields the inputs demand function

$$Y_t(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\theta} Y_t \qquad \textbf{(10)}$$

and the price index

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \qquad \textbf{(11)}$$

2.3. Intermediate Goods Sector

Each firm $z \in [0, 1]$ hires a composite labor service N_t from the labor agent and capital services $s_t K_t$ from the households to produce a differentiated intermediate good of type z using the technology

$$Y_t(z) = K_t^s(z)^{\alpha} N_t(z)^{1-\alpha} - \psi, \quad 0 < \alpha < 1, \qquad \textbf{(12)}$$

where $K_t^s = s_t K_t$ and $\psi > 0$ denotes fixed costs of operating the firm in each period.

As there is perfect competition in the input markets, cost minimization implies that

$$q_t = \alpha \mathrm{mc}_t \frac{Y_t(z)}{K_t^s(z)} \qquad \textbf{(13)}$$

$$\frac{W_t}{P_t} = (1-\alpha)\mathrm{mc}_t \frac{Y_t(z)}{N_t(z)}, \qquad \textbf{(14)}$$

where mc_t is real marginal cost.

Intermediate goods producers set their prices in a staggered fashion under assumptions symmetric to those stated above for wage contracts. In any given period, the firm is allowed to reset its price with probability $(1 - \gamma)$. Whenever the firm is not allowed to optimize its price, it simply indexes to lagged inflation,

$$P_t(z) = \Pi_{t-1} P_{t-1}(z). \qquad \textbf{(15)}$$

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If firm z gets a chance to reset its price in period t, it does so to maximize the expected discounted profit flow this new price will generate,

$$E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} Y_{t+i}(z) \left[P_t(z) - P_{t+i} \mathrm{mc}_{t+i}(z) \right] \tag{16}$$

subject to (10). The variable $\Lambda_{t,t+i}$ denotes a stochastic discount factor defined as $\Lambda_{t,t+i} = (\beta\gamma)^i \, (\lambda_{t+i}/\lambda_t) \, \Pi_{t,t+i}^{-1}$, with $\Pi_{t,t+i} \equiv P_{t+i}/P_t$.

2.4. The Monetary Authority and the Resource Constraint

Money is exogenously supplied by the central bank according to the rule

$$M_t = \mu_t M_{t-1}, \tag{17}$$

where μ_t is the exogenous gross rate of money growth, such that

$$T_t = M_t - M_{t-1}. \tag{18}$$

The growth rate of money is assumed to follow a stationary AR(1) process of the form

$$\log\mu_t = \rho\log\mu_{t-1} + \varepsilon_t \tag{19}$$

and the steady state inflation is assumed to be zero.

Finally, the aggregate resource constraint is given by

$$Y_t = C_t + I_t + A(s_t)K_t \tag{20}$$

3. THE SCOPE OF INDETERMINACY AND THE ROLE OF FRICTIONS

In this section, I show that even though the introduction of habit formation in consumption preferences into the CIA model increases the scope of indeterminacy to a great extent, other frictions have opposite effects, so that the full-fledged model becomes immune to indeterminacy under any empirically reasonable parameter set.

As explained before, besides the CIA constraint, the full-fledged CIA model features two nominal frictions, sticky prices and sticky wages, and four sources of real rigidities, imperfect competition in product and labor markets, habit formation, adjustment costs in investment, and variable capacity utilization. Because imperfect competition in product markets and in labor markets are prerequisites for price and wage stickiness, respectively, the analysis below is centered around the following five frictions: habit formation, price stickiness, wage stickiness, investment adjustment costs, and variable capacity utilization.

Because the model employed in this study features endogenous investment, it is impossible to obtain analytical solutions. Therefore, the exposition in this section is based entirely on numerical simulations. To this end, I solve and calibrate the

TABLE 1. Baseline parameter values

Parameter	Symbol	Value
Subjective discount factor	β	0.99
Intertemporal elasticity of substitution	σ	1
Inverse labor supply elasticity	ϕ	1
Depreciation rate	δ	0.025
Capital share of output	α	0.3
Price elasticity of demand	θ	6
Wage elasticity of demand	θ_w	21
Nonadjustment rate for prices	γ	0.75
Nonadjustment rate for wages	γ_w	0.64
Habit persistence	h	0.65
Variable capacity utilization parameter	κ	0.01
Investment adjustment cost parameter	S''	3.2
Elasticity of money demand	σ_m	10.62
Persistence of money growth shock	ρ	0.55

model by log-linearization around a zero-inflation steady state. Table 1 gives a list of the baseline parameter values I use in the simulations.[4]

Panel A of Figure 1 depicts the determinacy regions for the baseline CIA model (which features no extra frictions) and its several variants formed by adding one or more frictions other than variable capacity utilization. The results for capacity utilization are not included in Panel A for two reasons. First, as a single friction, it has negligible effects on determinacy and second, its exclusion simplifies the exposition substantially because the direction and magnitude of its effect on determinacy radically change, depending on the type of other frictions present in the model. This is explained in more detail below.

When interpreting the results portrayed in Panel A, we need a yardstick value for σ. Unfortunately, there is no consensus in the literature on the magnitude of IES. Following a seminal paper by Hall (1988), which asserts that IES in consumption is very small and possibly not different from zero, economists have come up with many different estimates; some of which go well beyond unity [see Pedersen (1991), Hu (1993), and Attanasio and Weber (1995)]. As to be explained below, the choice of σ is not crucial for the main result of the paper, because even when $h = 0.7$, the indeterminacy threshold for the full-fledged model falls to such a low level as $\sigma \leq 0.0023$. However, in the baseline parameter set, I used $\sigma = 1$, as it is very widely used in the literature.

As Panel A of Figure 1 shows, the determinacy region (the area above the curve) for the baseline model rapidly shrinks as habit persistence increases. This is perfectly in line with the main theoretical result of Auray et al. (2005). When there is no habit formation, the model avoids indeterminacy for IES greater than 0.50. If the degree of habit formation is $h = 0.65$—a typical value considered to be plausible in the literature—the threshold for indeterminacy jumps to 10.87.

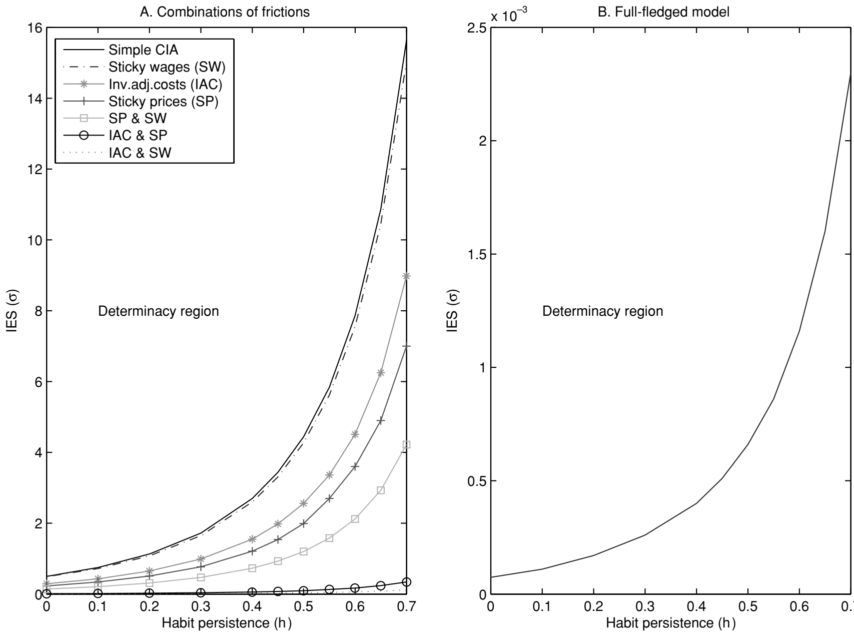

FIGURE 1. Determinacy region.

Obviously, a model that would require such a degree of IES would have no empirical appeal.

Panel A also shows that each remaining friction, even when applied alone, acts to reduce the scope of indeterminacy. Neither of them, however, can reduce it to levels that would help the model avoid indeterminacy under reasonable parameter values. Consider the effect of introducing price stickiness into the model. As a single friction, it has the greatest reducing effect on the scope of indeterminacy. However, if we assume $h = 0.65$ as before, a plausible degree of price stickiness (say, $\gamma = 0.75$) decreases the indeterminacy threshold for IES to 4.86, which is still very high. So it is clear that no single friction can help the standard CIA model avoid indeterminacy for plausible parameter values.

However, when there is more than one friction in the model, and one of the frictions is adjustment costs in investment, the threshold falls to reasonable levels even when $h = 0.65$. For example, when there are adjustment costs ($S'' = 3.2$) and prices are sticky ($\gamma = 0.75$), the threshold falls to 0.24. If prices are flexible but wages are sticky ($\gamma_w = 0.64$), for the same level of adjustment costs the threshold is 0.09. When all the frictions are employed simultaneously (including variable capacity utilization), the problem of indeterminacy arising from low IES nearly vanishes as the threshold falls to very low values. This is shown in Panel B. The figure makes it clear that the full-fledged CIA model can be considered to be quite immune to indeterminacy even for values of IES very close to zero.

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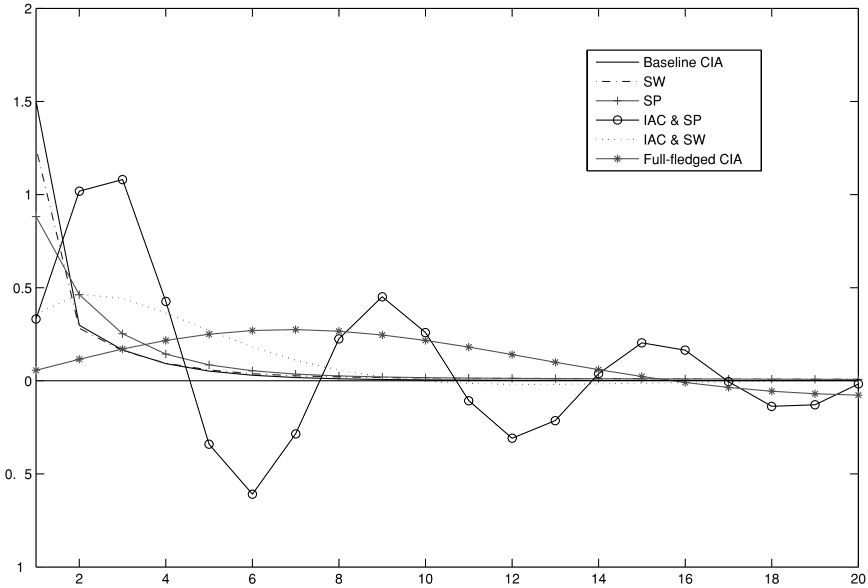

FIGURE 2. Response of inflation to a monetary shock (for $\sigma = 1$ and $h = 0.1$).

As briefly explained in Section 1, these frictions reduce the scope of indeterminacy, mainly due to the fact that they all act to lower the sunspot-related increase in inflation. Even though the sunspot mechanism in the model employed in this paper is too complex to be analytically traced, Figure 2 may be helpful in demonstrating this result. Figure 2 depicts the response of inflation in some of the models considered in Figure 1 to a 1% money growth shock. The vertical axis shows absolute deviation of inflation from its steady state value, which is zero. There is a very clear correlation between Figures 1 and 2: The smaller the inflation on impact, the wider is the determinacy region. As expected, adding only variable capacity utilization to the baseline model has a negligibly small effect on inflation (not shown in the figure). However, when it is combined with the other frictions, it has a considerable effect, as is the case in the determinacy region analysis.

The results in Figure 1 have been obtained by changing the value of σ against constant friction parameters. Figure 3 shows the results pertaining to a similar set of experiments, in which the friction parameters are allowed to change against a fixed value of σ. During these simulations I have assumed $\sigma = 1$ as before. The figure portrays determinacy regions drawn for different combinations of frictions. The transparent surface with edges shows the upper bound of the determinacy region for cases in which there is no capacity utilization. The semi-transparent gray surface without edges corresponds to cases with variable capacity utilization.

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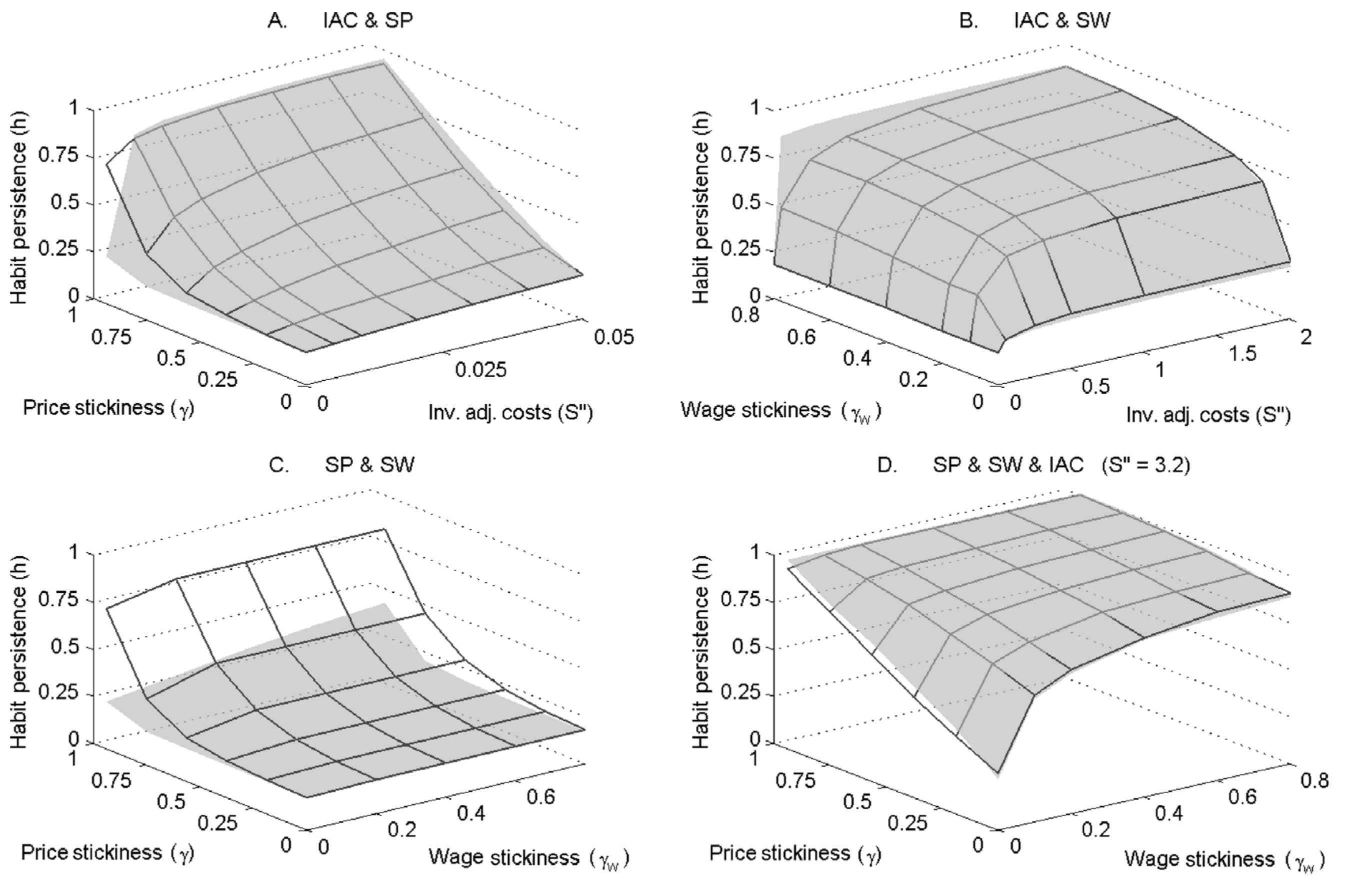

FIGURE 3. Determinacy region (below the surface) for $\sigma = 1$

Considering that the estimates in the literature for the habit persistence parameter span a wide range from 0.63 to 1 [see Dennis (2005)], the numerical results displayed in Figure 3 can be summarized in four points.

First, no additional friction can, by itself, help the model avoid indeterminacy under a plausible parameter set. For example, when there is only price stickiness ($\gamma = 0.75$) or wage stickiness ($\gamma_w = 0.64$) present in the model, determinacy requires $h \leq 0.35$ and $h \leq 0.18$, respectively (as can be seen from Panels A and B). Similarly, introducing investment adjustment costs to a CIA model by itself cannot help avoiding indeterminacy since the threshold approaches $h = 0.30$ even as adjustment cost parameter S'' goes to infinity (Panels A and B). In another simulation not reported in the figure, I found that when the only friction was variable capacity utilization, determinacy required $h \leq 0.17$.

Second, even a very low level of adjustment costs in investment is sufficient for the CIA model to avoid indeterminacy if prices are reasonably sticky. For example, when only prices are sticky, with the price stickiness parameter γ being equal to 0.75, a very low level of adjustment costs ($S'' = 0.05$) ensures determinacy for values of $h \leq 0.72$ (see Panel A). For the baseline adjustment cost level $S'' = 3.2$, the threshold rises to 0.81. The combination of adjustment costs and sticky wages, however, cannot produce determinacy for the same parameter set.

Third, the magnitude and the direction of the effect of variable capacity utilization on determinacy depends on the existence of other frictions. When all the frictions are present (Panel D), its effect is negligibly small (but still favorable). When there are no adjustment costs in the model, capacity utilization acts to lower the positive effect of price and wage stickiness on determinacy substantially. This is most easily seen in Panel B. However, when adjustment costs are present, variable capacity utilization acts to enhance the effects of price and wage stickiness.

Forth, when the model incorporates all frictions (Panel D), the issue of indeterminacy almost vanishes since the threshold for h quickly rises to 0.98 for the baseline parameter values.

One important implication of the last point is that, from the determinacy point of view, the full-fledged CIA model is an eligible workhorse, as is its MIU counterpart, to be used in the New Keynesian paradigm. Panel B of Figure 1 shows that this result is quite robust to changes in σ. When all the frictions are incorporated into the model, even very small values of σ are sufficient to avoid indeterminacy for very high values of h.

4. PERFORMANCE OF THE FULL-FLEDGED CASH-IN-ADVANCE MODEL

Once it is established that the full-fledged model can be safely used in the New Keynesian business cycle analysis, a natural step forward would be to investigate how well it preforms when compared with other models with different monetary specifications. Since this is beyond the scope of this paper, here I try to provide some tentative results. To this end, I calibrate and simulate both the full-fledged model and its MIU counterpart with the baseline parameter set and present their impulse–response functions after a monetary shock.[5]

As mentioned in Section 1, Yun (1996) and Ellison and Scott (2000) provide two rare examples in which a standard CIA specification is employed in a standard sticky price New Keynesian model.[6] Both papers conclude that such a model is unable to produce output persistence. Moreover, Ellison and Scott (2000) find that the model also generates an extremely volatile output at very high frequencies. These results are not surprising because, when investment is not subject to the CIA constraint and the only other friction is price stickiness, investment, acting like a buffer for consumption, becomes very volatile and dictates the dynamics of output. Wang and Wen (2006) point out that under these conditions, output persistence requires that investment be subject to the CIA constraint as well. Below I show that, as expected, the full-fledged CIA model overcomes these shortcomings of the standard CIA model and that it produces results very similar to those of its MIU counterpart.[7]

The model is calibrated at quarterly frequency and the parameter values used are listed in Table 1. For the parameters β, σ, ϕ, δ, and α, I assume values considered to be standard in the literature: the subjective discount factor is $\beta = 0.99$, the intertemporal elasticity of substitution for consumption is assumed to be $\sigma = 1$, the inverse of the labor supply elasticity with respect to wages is, $\phi = 1$, the capital depreciation rate is $\delta = 0.025$, and, finally, the capital share of output is $\alpha = 0.3$.

For the parameters θ, θ_w, h, γ_w, and κ, I use estimated parameter values reported in Christiano et al. (2005). I set the price elasticity of demand for intermediate goods, θ, to 6 and the wage elasticity of demand for labor variety, θ_w, to 21. The habit persistence parameter is $h = 0.65$. The wage stickiness parameter (nonadjustment rate) γ_w is 0.64. The steady-state elasticity of the marginal capacity utilization cost, $\kappa = A''(1)/A'(1)$, is 0.01. For the price stickiness parameter (nonadjustment rate), I assume the standard value $\gamma = 0.75$, which is also in accord with the recent findings in Nakamura and Steinsson (2007). I set the parameter of investment adjustment cost, S'', to 3.2 so that investment is twice as volatile as output.

For the MIU model I use a felicity function of the form

$$\frac{(C_t - hC_{t-1})^{1-1/\sigma}}{1-1/\sigma} + \frac{\chi}{1-\sigma_m}\left(\frac{M_t}{P_t}\right)^{1-\sigma_m} - \Psi\frac{(N_t)^{1+\phi}}{1+\phi}, \qquad \textbf{(21)}$$

and for the elasticity of money demand parameter σ_m I use the point estimate in Christiano et al. (2005), which is 10.62.

Figure 4 displays impulse response functions of output, consumption, investment, and nominal interest rate over 20 quarters with respect to a 1% shock in money growth. Except for the nominal interest rate and inflation, the vertical axis measures percentage deviations from the steady state. The responses of the

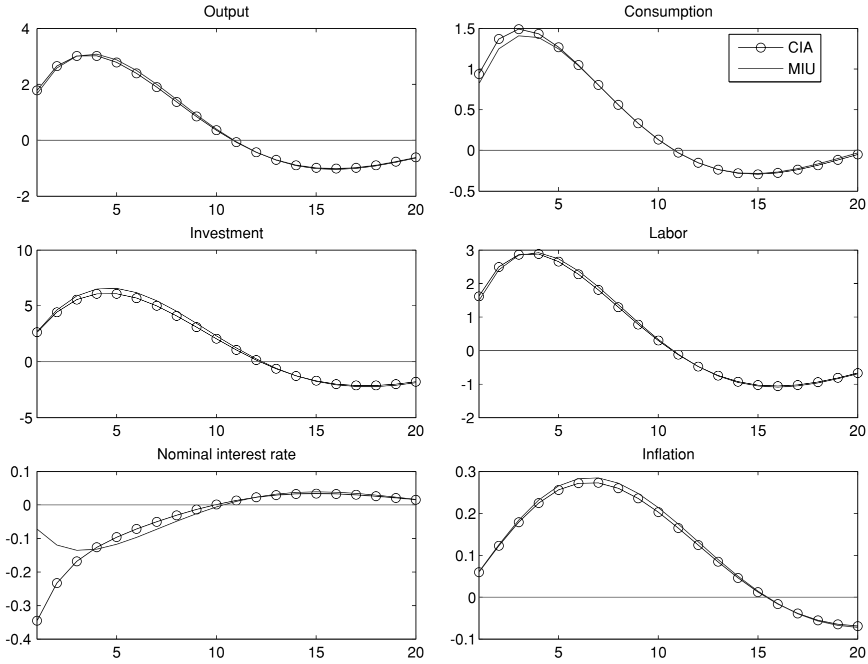

FIGURE 4. IRFs for the full-pledged CIA and MIU models.

nominal interest rate and inflation are expressed in absolute deviations from the steady state.

The impulse response functions displayed in Figure 4 shows that the full-fledged CIA model produces results very similar to those of its MIU counterpart. These results are in accord with most of the stylized facts regarding how major macroeconomic variables respond to a monetary shock: output, consumption, investment and inflation all rise in a hump-shaped fashion; investment is more volatile than output, and output is more volatile than consumption; employment closely tracks output; and the nominal interest rate falls after an expansionary monetary shock (liquidity effect). In contrast to a standard sticky-price CIA model as employed in Yun (1996) and Ellison and Scott (2000), the full-fledged model also generates reasonable degrees of persistence in output and inertia in inflation.

The only variable for which the impulse response functions of both models differ significantly is the nominal interest rate. On impact, the CIA model produces a fall in the nominal interest rate of approximately 35 basis points, whereas the fall for the MIU model is approximately 8 basis points. However, because both models have been simulated with the same parameter set, this result does not mean that the CIA model is decisively more successful in generating the liquidity effect than the MIU model. This is a result that needs to be investigated further. However, the fact that both models produce a liquidity effect is in line with the results in

Edge (2000) and Keen (2004), who show that to produce appropriate responses of output, consumption, investment, and price level, along with a liquidity effect, a model with price stickiness must incorporate either financial frictions or habit persistence in consumption.

5. CONCLUSIONS

Monetary CIA models are known to be prone to sunspot equilibria when money is created according to an exogenous money growth rule. Furthermore, the presence of habit formation in consumption increases the likelihood of indeterminacy by weakening the intertemporal substitution motives that act to counter the sunspot mechanism. In this paper I have shown that many frictions widely adopted in the New Keynesian paradigm, either alone or in combination with others, narrow down the scope of indeterminacy in a monetary CIA model. These frictions are price stickiness, wage stickiness, adjustment costs in investment, and variable capacity utilization. They decrease the possibility of sunspot equilibria because they all act to lower the increase in the expected inflation caused by a sunspot-driven increase in consumption. Therefore, a smaller fall in future consumption (hence, a smaller IES) becomes sufficient to offset the initial rise in inflation.

I have also shown that no single friction can reduce the scope of indeterminacy sufficiently to help a plausibly calibrated monetary CIA model to avoid indeterminacy. An acceptable fall in the scope of indeterminacy requires that the model feature investment adjustment costs along with either sticky prices or sticky wages. Moreover, if all the frictions are present in the model, the scope of indeterminacy shrinks to such a low degree that the model becomes immune to sunspot equilibria over a very wide range of parameters.

Finally, I have tentatively shown that, when subjected to a monetary shock, a standard CIA model that incorporates all these frictions produces results very similar to those of a standard New Keynesian model in which money is introduced via a MIU specification.

These results imply that a standard CIA specification can safely be used in the modern New Keynesian framework in which the above-mentioned frictions have come to be standard features.

NOTES

1. As clearly demonstrated by Auray et al. (2005), this result is robust against the introduction of capital as an alternative means of escaping from the inflation tax.
2. An elegant treatment of this issue with analytical results can be found in Auray et al. (2005).
3. Although in a different context, the effect of investment adjustment costs on indeterminacy is analyzed by Kim (2003), who shows that in a neoclassical growth model without money, investment adjustment costs increase the required degree of increasing returns for indeterminacy to arise.
4. The choice of these parameter values is discussed in more detail in Section 4.
5. A more decisive comparison would, of course, require that both models be estimated separately and simulated with multiple shocks. This could be the subject of another paper.

6. In both papers, there is no habit formation in consumption and IES is assumed to be unity. These assumptions, when combined with the assumption of sticky prices, guarantee determinacy for almost any plausible set of parameters [see panel (A) in Figure 1].

7. It must be noted that there is no functional equivalence between the CIA constraint (8) and the particular MIU function used (21), as the latter has a zero cross derivative between consumption and real money balances [see Feenstra (1986)].

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