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Electrostatic rogue waves in a plasma with a relativistic electron beam

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Abstract. The properties of nonlinear electrostatic acoustic rogue waves in a three-component plasma composed of electron, positron, and relativistic electron beam are investigated. The reductive perturbation method is used to obtain a Korteweg–de Vries equation. The dynamics of the modulationally unstable wave packets described by the Korteweg–de Vries equation gives rise to the formation of rogue pulses that is described by a nonlinear Schrödinger equation for small wave number. The effects of physical parameters on the profile of rogue waves are investigated numerically. The electrostatic rogue waves, as predicted here, may be associated with the nonlinear structures caused by the interaction of relativistic jets with plasma medium, such as in the active galactic nuclei and in the magnetosphere of collapsing stars.

1. Introduction

Many types of nonlinear electrostatic structures could propagate in electron–positron plasmas, such as solitary, shock, blow-up, and rogue waves (Mahmood and Ur-Rehman 2009; Moslem et al. 2009; El-Tantawy et al. 2011; Moslem 2011; El-Tantawy et al. 2012). The rogue waves are observed experimentally in plasma physics (Bailung et al. 2011), in ocean (Kharif et al. 2009), in optical waves (Solli et al. 2007), in superfluid helium (Ganshin et al. 2008), in Bose–Einstein condensates (Bludov et al. 2009; Bludov et al. 2010), in atmosphere (Stenflo and Marklund 2010), and in plasmonics (Maier 2007). In plasma physics, the investigation of rogue waves has been conducted by solving the nonlinear Schrödinger (NLS) equation. El-Labany et al. (2011) studied the nonlinear ion-acoustic (IA) rogue waves in a plasma composed of positive ions, negative ions, and isothermal electrons as predicted in Titan's atmosphere. They found that the waves exist only for a specific range of the negative ion masses. Lareder et al. (2011) investigated the rogue waves in the Alfvén wave (AW) turbulence regime that is described by the randomly driven derivative NLS equation in the presence of a weak dissipation. They found that as the dissipation is reduced, rogue waves form less frequently but reach larger amplitudes. The surface plasma rogue waves arise due to a complied electromagnetic and electrostatic field near the plasma–vacuum interface (Moslem et al. 2011b). Moreover, dusty plasma is expected to support rogue waves (Moslem et al. 2011a).

Recently, the interaction of an electron beam with plasmas has been the subject of many investigations, because of their importance in various applications of laboratory experiment and technology. For example, ranging from non-destructive testing of materials to the acceleration of charged particles and in an inertial confinement fusion scheme: fast ignition. Moreover, the applications of an electron jet/beam plasma are considered as promising tools for producing products and materials with unique biological properties that can be used for biomedical applications (Vasilieva et al. 2010). Several theoretical attempts have been made to explain the observed electron beam in different regions of Earth's magnetosphere, as reported by satellite missions, e.g. the FAST mission at the auroral region, the S3-3, Viking, and the GEOTAIL and POLAR missions (Moslem et al. 2012). The presence of an electron beam in plasmas is known to be associated with various interesting effects, including nonlinear wave amplification, and it could modify the properties and the existence conditions of the nonlinear excitations. The problem of generating relativistic and subrelativistic electron beams is among most challenging problems. An important scientific problem in future accelerators is the generation of extremely short relativistic electron bunches. Recently, linear accelerators that produce relativistic electron beams are down to a size that allows them to be flown on spacecraft and sounding rockets. This opens the window to new opportunities for atmospheric/ionospheric modification experiments where the mesosphere and lower thermosphere regions can be perturbed down to 40 km altitude (Neubert and Gilchrist 2004). Recently, Moslem et al. (2012) studied the propagation of electrostatic solitary waves (ESWs) in three-dimensional (3D) plasma composed of an electron–positron plasma with the relativistic electron beam. For this purpose, the Kadomtsev–Petviashvili (KP) equation is derived by employing the perturbation method and they studied the effects of relevant physical parameters on the solitary wave profile. In view of the crucial importance of the electron–positron plasmas with the relativistic electron beam, there is a need for investigating the nonlinear electrostatic rogue/rational waves that may propagate in a 1D electron–positron–electron beam (*e–p–b*) plasmas. To the best of authors' knowledge, the effect of the relativistic electron beam on the rogue waves in electron–positron plasmas has not been discussed yet. Here we are mainly interested in the propagation of the rogue waves in 1D plasma with the relativistic electron beam.

This paper is organized as follows. In Sec. 2, we present the basic set of fluid equations describing the dynamics of the nonlinear electrostatic structures in the *e–p–b* plasma. In Sec. 3, we use the reductive perturbation method to derive the Korteweg–de Vries (KdV) equation. When the frequency of the carrier wave is much smaller than the electron plasma frequency, then the KdV equation can be transformed to the NLS equation. An analytical solution of the NLS equation along with the numerical analysis is also presented. Finally, the results are summarized in Sec. 4.

2. Basic equations

Let us consider a 1D, weakly relativistic, unmagnetized, collisionless three-component plasma consisting of electron beams, electrons, and positrons. The nonlinear dynamics of the ESWs are governed by the electron beam fluid equations

$$\frac{\partial n_b}{\partial t}+\frac{\partial (n_b u_b)}{\partial x}=0, \tag{1}$$

$$\frac{\partial \gamma u_b}{\partial t}+u_b\frac{\partial \gamma u_b}{\partial x}=\frac{\partial \phi}{\partial x}-3\sigma n_b\frac{\partial n_b}{\partial x}, \tag{2}$$

and the electron/positron fluid equations

$$\frac{\partial n_{e,p}}{\partial t}+\frac{\partial (n_{e,p} u_{e,p})}{\partial x}=0, \tag{3}$$

$$\frac{\partial u_{e,p}}{\partial t}+u_{e,p}\frac{\partial u_{e,p}}{\partial x}\mp\frac{\partial \phi}{\partial x}+3\rho n_{e,p}\frac{\partial n_{e,p}}{\partial x}=0. \tag{4}$$

The system is closed by the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2}=n_e+n_b-n_p, \tag{5}$$

where $n_j (j=b,e,$ and $p)$ is the electron beam/electron/positron number density, $u_j (j=b,e$ and $p)$ is the electron beam/electron/positron fluid velocity, and ϕ is the electrostatic potential. Here, $\gamma=(1-u_b^2/c^2)^{-1/2}\backsimeq 1+u_b^2/2c^2$ is the relativistic factor for the electron beam, $\sigma=T_b/T_e$ is the electron beam-to-electron temperature ratio, and $\rho=T_p/T_e$ is the positron-to-electron temperature ratio. The variables appearing in (1)–(5) have been appropriately normalized. Thus, n_j is normalized by the unperturbed electron density n_{e0}, u_j is normalized by the electron thermal speed $V_{te}=(T_e/m)^{1/2}$, and ϕ is normalized by T_e/e. The space and time variables are in units of the electron Debye radius $\lambda_{De}=(T_e/4\pi e^2 n_{e0})^{1/2}$ and the inverse of the plasma frequency $\omega_{pe}^{-1}=(m/4\pi e^2 n_{e0})^{1/2}$, respectively, e the magnitude of the electron charge, m the electron mass, and c the speed of light in vacuum. The quasi-neutrality condition at equilibrium $\beta=1+\delta$, where $\beta=n_{p0}/n_{e0}$ and $\delta=n_{b0}/n_{e0}$ (the index 0 denotes the unperturbed density states).

3. Derivation of the evolution equations and numerical analysis

To investigate the nonlinear propagation of the ESWs, we shall employ the reductive perturbation method (Washimi and Taniuti 1966). According to this method, the independent variables can be detailed as

$$X=\epsilon^{1/2}(x-\lambda t)\quad \text{and}\quad T=\epsilon^{3/2}t, \tag{6}$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity and λ the phase velocity of the wave and normalized by V_{te}. The dependent variables are expanded as

$$F=F_0+\sum_{m=1}^{\infty}\varepsilon^m F^m, \tag{7}$$

where $F=[n_b,n_e,n_p,u_b,u_e,u_p,\phi]^T$ and $F_0=[\delta,1,\beta,u_{b0},0,0,0]^T$. Employing the stretching (6) and the expansions (7) into (1)–(5), we can isolate distinct orders in ε. The lowest order in ε yields

$$n_b^{(1)}=-\frac{\delta}{\tilde{\lambda}^2\tilde{\gamma}-3\sigma\delta^2}\phi^{(1)},\quad u_b^{(1)}=-\frac{\tilde{\lambda}}{\tilde{\lambda}^2\tilde{\gamma}-3\sigma\delta^2}\phi^{(1)}, \tag{8}$$

$$n_e^{(1)}=-\frac{1}{\lambda^2-3}\phi^{(1)},\quad u_e^{(1)}=-\frac{\lambda}{\lambda^2-3}\phi^{(1)}, \tag{9}$$

$$n_p^{(1)}=\frac{\beta}{\lambda^2-3\rho\beta^2}\phi^{(1)},\quad u_p^{(1)}=\frac{\lambda}{\lambda^2-3\rho\beta^2}\phi^{(1)}, \tag{10}$$

where

$$\tilde{\lambda}=(\lambda-u_{b0})\text{ and }\tilde{\gamma}=\left(1+\frac{3u_{b0}^2}{2c^2}\right).$$

The Poisson equation gives the compatibility condition as

$$\frac{1}{\lambda^2-3}+\frac{\delta}{\tilde{\lambda}^2\tilde{\gamma}-3\sigma\delta^2}+\frac{\beta}{\lambda^2-3\rho\beta^2}=0. \tag{11}$$

The next order in ε gives a system of equations in the second-order perturbed quantities. Eliminating the latter and making use of the first-order results, we finally obtain the KdV equation

$$\frac{\partial \phi^{(1)}}{\partial T}+A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial X}+B\frac{\partial^3 \phi^{(1)}}{\partial X^3}=0, \tag{12}$$

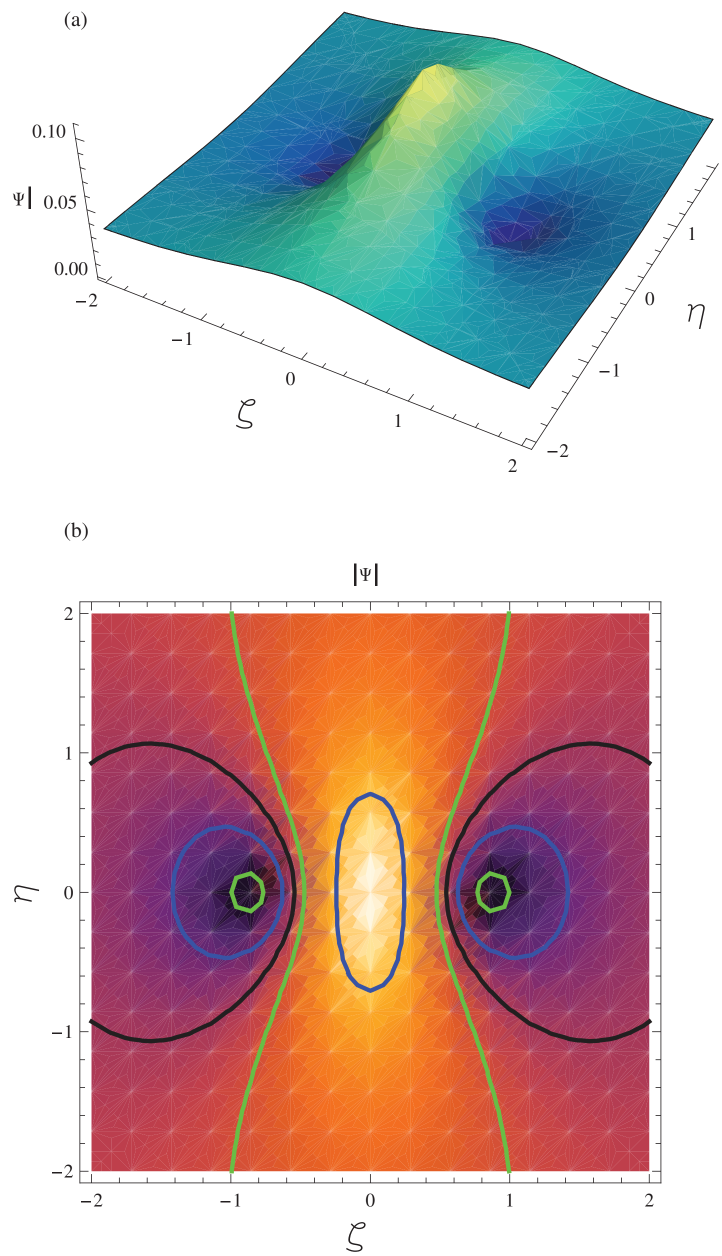

Figure 1. (Colour online) (a) The absolute value of the rogue wave profile $|\Psi|$ and (b) the contour plot of the rogue wave potential $|\Psi|$ are depicted against ζ and η, with $\rho = k = 0.9$, $\sigma = 1$, $\delta = 0.1$, and $u_{b0}/c = 0.06$.

where

$$A = -3B\left[\frac{(\lambda^2+1)}{(\lambda^2-3)^3} + \frac{\delta\left(\tilde{\lambda}^2\left(\tilde{\gamma} - \frac{u_{b0}}{c^2}\tilde{\lambda}\right) + \sigma\delta^2\right)}{\left(\tilde{\gamma}\tilde{\lambda}^2 - 3\sigma\delta^2\right)^3} + \frac{\beta\left(\lambda^2+\rho\beta^2\right)}{\left(\lambda^2-3\rho\beta^2\right)^3}\right],$$

and

$$B = \frac{1}{2}\left[\frac{\lambda}{(\lambda^2-3)^2} + \frac{\delta\tilde{\gamma}\tilde{\lambda}}{\left(\tilde{\gamma}\tilde{\lambda}^2 - 3\sigma\delta^2\right)^2} + \frac{\beta\lambda}{\left(\lambda^2-3\rho\beta^2\right)^2}\right]^{-1}.$$

Now, we will study the modulational instability of a weakly nonlinear wave packet described by the KdV equation (12). Note that when the frequency of the carrier wave is much smaller than the ion plasma frequency, then the KdV equation is also used to study the nonlinear evolution of modulationally unstable modified ion-acoustic wave packets through the derivation of the NLS equation. Therefore, we consider the solution of (12) in the form of a weakly modulated sinusoidal wave by expanding ϕ as (El-Labany et al. 2007; Abdelsalam et al. 2011)

$$\phi = \sum_{n=1}^{\infty} \varepsilon^n \sum_{l=-n}^{n} \phi_l^{(n)}(\zeta,\eta)\exp[il(kX-\omega T)]. \tag{14}$$

Here, k and ω are real variables representing the fundamental (carrier) wave number and frequency, respectively, of the nonlinear electrostatic waves. The stretched

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variables ζ and η given by

$$\zeta = \varepsilon(X + v_g T) \quad \text{and} \quad \eta = \varepsilon^2 T, \tag{15}$$

where v_g is the envelope group velocity to be determined later.

Assume that all perturbed states depend on the fast scales via the phase $(kX - \omega T)$ only, while the slow scales (ζ, η) enter the arguments of the lth harmonic amplitude $\phi_l^{(n)}$. Since $\phi_l^{(n)}$ must be real, the coefficients in (14) have to satisfy the condition $\phi_{-l}^{(n)} = \phi_l^{(n)*}$, where the asterisk indicates the complex conjugate. The first-order approximation $(n = 1)$ with $(l = 1)$ provides the electrostatic waves dispersion relation $\omega = -Bk^3$. The second-order approximation $(n = 2)$ with the first harmonic $(l = 1)$ yields $v_g = 3Bk^2$. The annihilation of secular terms at the third harmonic modes ($n = 3$ and $l = 1$) yields the NLS equation

$$i\frac{\partial \Psi}{\partial \eta} + \frac{1}{2}P\frac{\partial^2 \Psi}{\partial \zeta^2} + Q\Psi \mid \Psi \mid^2 = 0. \tag{16}$$

For simplicity, we have assumed $\phi_1^{(1)} \equiv \Psi$. The dispersion and nonlinear coefficients, respectively, are given by

$$P = 6Bk \tag{17}$$

and

$$Q = \frac{A^2}{P}. \tag{18}$$

The NLS equation (16) describes the nonlinear evolution of a modulated amplitude electrostatic acoustic wave carrier. It can also be derived directly from the system of (1)–(5) by using the derivative expansion method. In this case, the derivation of the NLS equation has been carried out for an arbitrary frequency of the carrier wave. If we use a similar approach to derive the NLS equation from (1)–(5), then we obtain after long derivations the NLS equation with very complicated expressions for the nonlinear and dispersion coefficients Q and P. The derivation of the NLS equation should, in principle, reduce to (16) in the limit of the low wave frequency, i.e. when the frequency of the carrier wave is much smaller than the ion plasma frequency. Actually, the derivation of the NLS equation for an arbitrary frequency is more general and gives us information about the stability (instability) of the propagating carrier wave, but in our case we have three fluid equations, which are very difficult to be combined to derive the NLS equation for an arbitrary frequency. So, we used the limit of the low wave frequency as a special case to study the rogue waves.

Equation (16) has a rational solution that is located on a non-zero background and localized both in the ζ and η directions as (Kibler et al. 2010; Abdelsalam et al. 2011)

$$\Psi(\eta, \zeta) = \frac{P}{A}\left[\frac{4(1 + 2iP\eta)}{1 + 4P^2\eta^2 + 4\zeta^2} - 1\right]\exp(iP\eta). \tag{19}$$

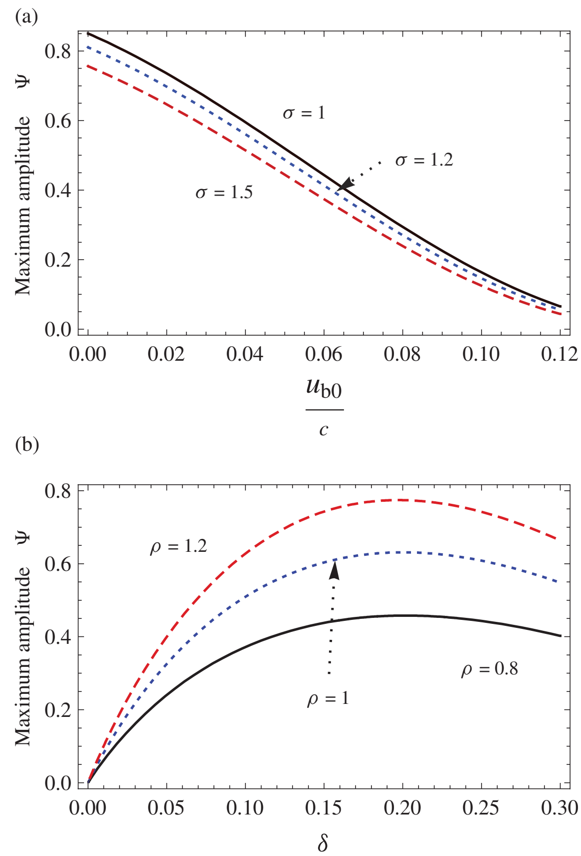

Figure 2. (Colour online) The maximum value of the rogue wave amplitude Ψ_0 is depicted against (a) u_{b0}/c for different values of σ with $\rho = k = 0.9$ and $\delta = 0.1$ and (b) δ for different values of ρ with $\sigma = k = 0.9$ and $u_{b0}/c = 0.06$.

The solution (19) represents the profile of the rogue wave within the modulational unstable region, as depicted in Fig. 1. Note that the light color corresponds to the high amplitude region, which concentrates a significant amount of energy into a relatively small area in space, and therefore the rogue waves generate in our plasma. This property of the nonlinear solution may serve as the basis for the explanation of the electrostatic rogue wave (ERW) in electron beam–electron–positron plasmas. The rogue wave is usually an envelope of a carrier wave with a wavelength smaller than the central region of the envelope. It is straightforward to see that a negative sign for PQ is required for wave amplitude modulational stability. On the other hand, a positive sign of PQ allows for a random perturbation of the amplitude to grow and thus the ERWs could be created. It is clear from (17) and (18) that P and Q have always the positive polarity.

Now, we numerically analyze the wave envelope Ψ and investigate how the electron beam-to-electron temperature ratio σ, the positron-to-electron temperature ratio ρ, the electron beam-to-electron density ratio δ, and electron beam velocity relativistic u_{b0}/c change the profile of the rogue wave envelope Ψ. It is seen from Fig. 2(a) that increasing σ and u_{b0}/c would lead to decrease the amplitude of the rogue waves. Moreover, the rogue pulses amplitude enhances with the increase

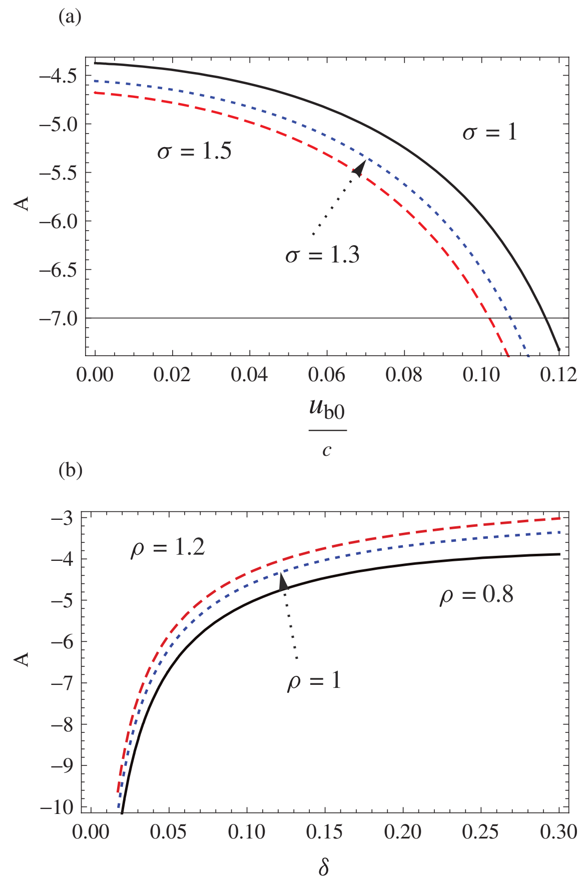

Figure 3. (Colour online) The nonlinear coefficient A is depicted against (a) $\sigma - u_{b0}/c$, where $\rho = k = 0.9$ and $\delta = 0.1$, and (b) $\rho - \delta$, where $\sigma = k = 0.9$ and $u_{b0}/c = 0.06$.

of ρ, while the amplitude increases with δ for small values, but it reduces lingeringly for larger values of δ, as depicted in Fig. 2(b). Physically, it is evident from Fig. 3 that increasing σ and u_{b0}/c would lead to enhance the nonlinear coefficient A and then dissipating the energy from the system would make the pulses shorter, but the increase of ρ leads to reduce A, concentrating a significant amount of energy which makes the pulses taller (because $\Psi(0,0) \propto 1/A$).

4. Summary

To summarize, the properties of the nonlinear electrostatic acoustic rogue waves in a three-component plasma composed of electron, positron, and relativistic electron beam are investigated. It is found that in certain conditions modulated electrostatic acoustic wave packets appear in the form of ERWs. The effects of physical parameters on the rogue wave profiles are examined numerically. It is found that the electron beam velocity reduces the rogue waves amplitude. Finally, the present results may help in understanding the electrostatic acoustic rogue waves associated with the electron–positron–electron beam plasmas that could be at the active galactic nuclei and the collapsing stars magnetosphere.

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