Fractional Arithmetic in the *Tabula Alimentaria* of Veleia*

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ABSTRACT

The Tabula Alimentaria *of Veleia records the details of two second-century* A.D. *imperial alimentary schemes at the northern Italian town of Veleia, providing a rare insight into the workings of these schemes. Imperial loans are made to local landowners in exchange for pledges of specified property. Interest paid by landowners is used to fund cash subsidies for the upbringing of selected local children. In the early twentieth century, the French scholar Félix de Pachtere came close to demonstrating a consistent arithmetical relationship between a landowner's declared property value and the loan received. However, anomalies remained. This article proposes a revised formula which establishes a precise and consistent linkage between loan amounts and property declarations. Based on this arithmetical dataset, the paper proposes some hypotheses about how these fractional computations might have been performed in second-century Rome.*

Keywords: *alimenta*; de Pachtere; Roman arithmetic; Roman fractions; Veleia

I ROMAN ARITHMETICAL PRACTICE

We know very little about the practice of everyday arithmetic in the Roman world. We have no surviving text describing everyday computational processes. The archaeological evidence is extremely limited.[1] We are primarily reliant on inferences drawn from incidental literary allusions, and reconstructions of likely procedures based on figures in business and other records which lack the underlying computations. We know still less about the handling of fractions in such computations, although some conjectures have been made. For example, a passage from Horace tells us that schoolboys were taught how, by long calculations, to divide into a hundred parts.[2] Maher and Makowski have suggested a possible formula for those calculations by considering an example from literature.[3] Columella, writing on investment in viticulture, provides us with a pair of annual interest figures, the second being the interest on the total after the first has been added to the principal.[4] This pair of figures is useful for computational hypothesis-testing since the first corresponds precisely with that obtained by decimal calculation,

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[1] We know from literary references that *calculi* (pebbles or small counters) were widely used (see Schärlig 2001: 122–3), but no counting-tables or counting-boards from the Roman world have been found: if made of wood they generally would not have survived. In addition to Schärlig, a survey of the evidence can be found in Pullan 1968. An early but still valuable extensive analysis of Roman arithmetical practice can be found in Friedlein 1869.

[2] Hor., *Ars P.* 325–30.

[3] Maher and Makowski 2001: 379–82.

[4] Columella, *Rust.* 3.3.8–9.

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while the second appears to be slightly rounded. A hypothesis that explains both should, in principle, carry enhanced plausibility.[5]

The *Tabula Alimentaria* of Veleia (*TAV*), an inscription that sets out in detail the financial arrangements for two Trajanic alimentary schemes, provides an opportunity to examine not just two, but dozens of comparable calculations, if it can be shown that those calculations were computed on a like basis. This article argues that this is the case, and considers the implications for our understanding of the Roman arithmetical practices of the time.

II THE IMPERIAL ALIMENTARY SCHEMES AND THE *TABULA ALIMENTARIA* OF VELEIA

In the late first or early second century A.D., imperial schemes providing cash payments to support the upbringing of children (*alimenta*) began to appear in Italian towns.[6] The contemporary evidence for the *alimenta* is almost entirely epigraphic, comprising some sixty or seventy inscriptions whose findspots are widely distributed across Italy.[7] Most of the inscriptions are dedicatory, but two large bronze tablets provide significant detail about the financial structure of the *alimenta*: one is from Ligures Baebiani, some 20 km north of Benevento;[8] the other, longer inscription is from Veleia, situated about 30 km south of Piacenza in northern Italy.[9] In each case, the imperial endowment was provided through loans of varying amounts made to local landowners and secured by pledges of specified property. The landowners paid monthly interest to the schemes, thereby ensuring a regular supply of funds for *alimenta* payments.

The *TAV*, currently displayed in the Museo Archeologico in Parma, Italy, is approximately 1.37 m in height and 2.85 m wide. The inscription comprises a full-width *praescriptio* of three lines, followed by 671 lines of text in seven columns. The style of engraving is freehand. It describes two successive alimentary schemes endowed by the emperor Trajan, which provided cash support to the families of 300 children at Veleia.[10] The two schemes have been dated, based on evidence within the text, to *c.* A.D. 101 and 106/114 respectively.[11] This arithmetical analysis is concerned with the later, much larger scheme.[12] Documented first in the *TAV*, it occupies six and a half of the

[5] Unfortunately that potential dual-fit plausibility is lacking in Maher and Makowski's hypothesis, in my view. The first of the two problems is to find 6 per cent of HS 29,000. It is extremely unlikely that Columella would have resorted to a complicated formula involving seven fractional terms to perform the simple task of finding the hundredth part of this round figure.

[6] Scholarship on the *alimenta* includes the following, which offer a variety of opinions on its purpose: Duncan-Jones 1982; Woolf 1990; Lo Cascio 2000; Jongman 2002.

[7] Woolf 1990: 197.

[8] *CIL* 9.1455 = Dessau, *ILS* 6509 *cum add.*

[9] *CIL* 11.1147 cf. vol. 11, p. 1252 = Dessau, *ILS* 6675. An early critical edition is that of Pietro de Lama 1819. De Lama, Prefect of the Ducale Museo at Parma, was responsible for the reassembly of the eleven fragments of the *TAV*. The critical edition of Criniti 1991 provides a detailed history, description and analysis. An updated online only edition can be found in Criniti 2016. An extensive online bibliography for the *TAV* can be found in Criniti 2018.

[10] We know from references within the text that there had been at least one other alimentary scheme at Veleia. Two landowners' declarations refer to land pledged to an otherwise unattested previous scheme managed by Titus Pomponius Bassus (col. III 13, 53). The sparse evidence relating to Pomponius Bassus' scheme is treated in detail by Soricelli 2001.

[11] Criniti 1991: 68.

[12] The relationship between loan amounts and property pledges in the first scheme is straightforward. Seven properties owned by five proprietors are pledged. Each property attracts a loan of 10 per cent of its declared value.

seven columns, and accounts for forty-six of the fifty-one loans recorded.[13] The *praescriptio* sets out its purpose and basic parameters:

> Property pledge in consideration of HS 1,044,000 so that by the *indulgentia* of the greatest and best *princeps*, the Emperor Caesar Nerva Trajan Augustus Germanicus Dacicus, male and female children may receive *alimenta:*
> Legitimate boys – 245 in number – HS 16 (per month) making HS 47,040 (annually)
> Legitimate girls – 34 in number – HS 12 (per month) making HS 4,896 (annually)
> Illegitimate boy – HS 144 (annually)
> Illegitimate girl – HS 120 (annually) [14]
> Total HS 52,200 (per annum), which is equal to interest at 5 per cent on the above capital sum.[15]

Each of the forty-six landowners' pledges follows a similar, though not identical, formulation. First, the landowner's name is given. In the ten cases where a single property is involved, that property is identified by name and location, and by adjacent properties or features. The landowner declares an aggregate value for the property after, in some cases, deduction of an unspecified amount for land tax (*vectigal*). The landowner pledges (*obligare*) the property and agrees to receive a loan in a stated amount from the emperor. Transactions involving more than one property take a slightly different form, in which first a landowner makes a declaration of aggregate property value, after deduction, if any, for *vectigal* and properties pledged to earlier alimentary schemes, and commits to receive an overall loan amount. Then individual properties or groups of properties (numbering twenty-six in the largest case, Loan 13) are identified, and a value for each property or group declared, with a portion of the aggregate loan being allocated to each declaration. Thus Loan 11 (col. II, 18–26) takes the following form:

> L. Sulpicius Verus has declared *praedia rustica* (to the value of) HS 71,522. He commits to receive a loan of HS 5,714 and pledges the *fundus Lubuatini Obsidianus Arrianus*, which is in Veleia, Valerian district, bounded by the public land of Lucensius, and by (the property of) Minicius Verus and Vicrius Sabinus and L Atilius; and the *fundus Marianus* in Veleia, Salvian district, bounded by (the property of) Granius Piscus and Tarsinius and Sulpicius Bacchus; which two properties combined he has declared at HS 28,600 for HS 2,214; also the *fundi Luciliani Didiani*, which are in Veleia, Valerian district, bounded by (the properties of) the Lucenses and Valeria Polla, which he has declared at HS 40,000 for HS 3,500.

The Loan 11 sub-loan amounts, HS 2,214 and HS 3,500, sum to the stated overall loan, HS 5,714. This is true for all but one of the thirty-six multi-property loans, the exception being Loan 31, which will be considered shortly. By contrast, the property sub-declarations, HS 28,600 and HS 40,000, sum to HS 68,600, whereas the overall declaration is HS 71,522. Across the multi-property loans, some overall declarations are more than the sum of the parts, some are equal, and some are less. Further, the ratio of sub-loan amount to sub-declaration value appears to follow no discernible pattern. Although this investigation is concerned with the relationship between the overall declarations and the overall loans, these observations are relevant to that analysis.

[13] The obligations are conventionally numbered for identification in the order they appear in the *TAV*. In the tables in this paper they are shown for convenience in order of ascending value of property declaration.
[14] The monthly amounts for the illegitimate boy and girl are unstated but are clearly HS 12 and HS 10 respectively per month.
[15] All translations are by the author.

III THE ANALYSIS OF FÉLIX DE PACHTERE AND A REVISED FORMULA

In the early twentieth century, French scholar Félix de Pachtere made a strong case for the existence of a consistent arithmetical relationship between each landowner's overall property declaration and the overall loan received. He determined that loan amounts were calculated using property declarations rounded down to the nearest HS 1,000, and concluded that a common multiplier of 8.05 per cent was applied to each rounded-down declaration to arrive at the related loan amount. He applied this insight to identify around a dozen emendations to the inscribed figures, nearly all of which can be attributed to simple inscribing errors. De Pachtere's emendations are shown in Table 1.

De Pachtere came very close to providing a full explanation of the arithmetical procedure applied to derive overall loan amounts from overall property declarations. However, an 8.05 per cent multiplier works precisely for only four of the loans, and de Pachtere observes:

> Il est remarquable que, sauf exception, la différence soit croissante des plus petites aux plus fortes obligations. Il y a ici et là des interruptions et des régressions dont on pourra apprécier la valeur en consultant la colonne 6 de la table qui suit. Mais, s'il est impossible d'expliquer ces anomalies, elles sont si peu importantes qu'on peut négliger d'en tenir compte.[16]

> It is notable that, discounting exceptions, the deviation increases from the smaller to the larger pledges. There are, here and there, some interruptions and regressions, the value of which can be seen by consulting column 6 of the following table. But, if these anomalies are impossible to explain, they are so insignificant that one need not take them into account.

De Pachtere's column 6 is reproduced in column H of Table 2. His claim that the anomalies are insignificant has not been universally accepted. Duncan-Jones says of de Pachtere's analysis:

> … it is not clear that his more drastic substitutions are necessarily correct. There is too much residual error in the [*TAV*] to make it easy to say that the loans were always accurately related to the declared valuation.[17]

As will be seen, the residual error lies not in the *TAV*, but in de Pachtere's arithmetical proposition.

A Revised Arithmetical Formula

De Pachtere inferred from the simple relationships 4,025 to 50,000 (Loans 8, 23 and 29) and 8,050 to 100,000 (Loan 39) that the underlying multiplier must be 8.05 per cent.[18] But this yields inaccurate and rather puzzling results for the other loans. The problem is resolved through minor adjustments to de Pachtere's formula: the overall declaration, rounded down to the nearest HS 1,000 as in de Pachtere's formula, is multiplied not by 8.05 per cent but 8 $^{7}/_{144}$ per cent; and the non-integer remainder for the resulting loan amount is ignored (so that, for example, the Loan 4 calculated figure HS 12,233.89 is rounded down to HS 12,233 and not up to the nearer HS 12,234).[19]

16 De Pachtere 1920: 99 n. 2.
17 Duncan-Jones 1982: 311 n. 2.
18 De Pachtere 1920: 99.
19 An 8 $^{7}/_{144}$ per cent multiplier is far more consistent with Roman fractional arithmetic than 8 $^{1}/_{20}$ (8.05) per cent. One-twentieth is neither a canonical Roman fraction nor precisely expressible using the Roman duodecimal

TABLE 1 Summary of de Pachtere's emendations. Roman numerals for sub-HS 1,000 elements of the figures are omitted except where relevant to the emendation. In Loans 14 and 29, marked with an asterisk, the emendation applies to the loan figure; all others apply to the property declaration (de Pachtere 1920: 98–100).

LOAN NO.	INSCRIBED FIGURE (HS)	EMENDATION (HS)	AND EXPLANATION OF DE PACHTERE
29*	3,075 $\overline{\text{III}}$ LXXV	4,025 $\overline{\text{IIII}}$ XXV	Error L for I
14*	12,104 $\overline{\text{XII}}$ CIIII	12,153 $\overline{\text{XII}}$ CLIII	Equals sum of sub-loans
4	107,400 $\overline{\text{CVII}}$	152,400 $\overline{\text{CLII}}$	Error V for L
5	233,080 $\overline{\text{CCXXLIII}}$	224, 080 $\overline{\text{CCXXIIII}}$	Error L for I
21	233,530 $\overline{\text{CCXXXIII}}$ DXXX	203,530 $\overline{\text{CCIII}}$ DXXX	Doubling XXX (inattention)
44	246,842 $\overline{\text{CCXLVI}}$	236,842 Roman figure not shown	None
2	310,545 $\overline{\text{CCCX}}$	315,545 Roman figure not shown	Omission of V
22	418,250 $\overline{\text{CDXIIX}}$	417,250 Roman figure not shown	Error extra I
9	490,000 $\overline{\text{CCCCLXXXX}}$	480,000 Roman figure not shown	Error extra X
30	673,660 $\overline{\text{DCLXXIII}}$	663,660 Roman figure not shown	Error extra X
17	1,014,090 $\lvert\overline{\text{X}}\rvert$ $\overline{\text{XIIII}}$	1,043,090 $\lvert\overline{\text{X}}\rvert$ $\overline{\text{XLIII}}$	Error I for L
31	1,158,150 VNDECIENS $\overline{\text{LVIII}}$	1,132,150 Roman figure not shown	None
13	1,180,600 $\lvert\overline{\text{X}}\rvert$ $\overline{\text{CLXXX}}$	1,177,600 Roman figure not shown	Nearest fit to loan at 8.05% rate

The results of the revised formula are compared with those of de Pachtere's formula in columns H and I of Table 2. After de Pachtere's emendations, the revised formula yields precisely the inscribed loan figure for thirty-five of the forty-six loans. Of the remainder, the HS 1 deviation in Loan 19 is unexplained.[20] All others but Loan 13 can be

fractional system. A relevant discussion of Roman fractional arithmetic can be found in Maher and Makowski 2001.

[20] The Loan 19 deviation cannot be a simple inscribing error, since the sub-loan total is equal to the inscribed figure.

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TABLE 2 De Pachtere's formula and the new formula compared. De Pachtere's emendations are marked with an asterisk (HS).

LOAN NO.	INSCRIBED LAND DECLARATION	AFTER DE PACHTERE'S EMENDATION	ROUND DOWN TO 1,000	INSCRIBED LOAN	DE PACHTERE'S CALCULATION	REVISED CALCULATION	INSCRIPTION DEVIATION FROM DE PACHTERE'S FORMULA	INSCRIPTION DEVIATION FROM NEW FORMULA
A	B	C	D C ROUNDED	E	F D x 8.05%	G D x 8 $^{7}/_{144}$%	H E–F	I E–G
8	50,000	50,000	50,000	4,025	4,025.00	4,024	–	1
29*	50,000	50,000	50,000	4,025*	4,025.00	4,024	–	1
23	50,350	50,350	50,000	4,025	4,025.00	4,024	–	1
7	51,000	51,000	51,000	4,104	4,105.50	4,104	(1.5)	–
33	53,900	53,900	53,000	4,265	4,266.50	4,265	(1.5)	–
36	55,800	55,800	55,000	4,426	4,427.50	4,426	(1.5)	–
27	58,350	58,350	58,000	4,668	4,669.00	4,668	(1.0)	–
12	58,800	58,800	58,000	4,668	4,669.00	4,668	(1.0)	–
34	62,920	62,920	62,000	4,990	4,991.00	4,990	(1.0)	–
32	65,400	65,400	65,000	5,231	5,232.50	5,231	(1.5)	–
35	69,260	69,260	69,000	5,553	5,554.50	5,553	(1.5)	–
40	71,256	71,256	71,000	5,714	5,715.50	5,714	(1.5)	–
11	71,522	71,522	71,000	5,714	5,715.50	5,714	(1.5)	–
18	75,975	75,975	75,000	6,036	6,037.50	6,036	(1.5)	–
3	77,192	77,192	77,000	6,197	6,198.50	6,197	(1.5)	–
10	80,000	80,000	80,000	6,438	6,440.00	6,438	(2.0)	–
38	90,200	90,200	90,000	7,243	7,245.00	7,243	(2.0)	–
37	98,000	98,000	98,000	7,887	7,889.00	7,887	(2.0)	–
39	100,000	100,000	100,000	8,050	8,050.00	8,048	–	2
4*	107,400	152,400*	152,000	12,233	12,236.00	12,233	(3.0)	–
1	108,000	108,000	108,000	8,692	8,694.00	8,692	(2.0)	–

42	113,600	113,600	113,000	9,094	9,096.50	9,094	(2.5)	–
20	132,450	132,450	132,000	10,624	10,626.00	10,624	(2.0)	–
19	148,420	148,420	148,000	11,912	11,914.00	11,911	(2.0)	1
14	151,200	151,200	151,000	12,153	12,155.50	12,153	(2.5)	–
26	155,812	155,812	155,000	12,475	12,477.50	12,475	(2.5)	–
41	158,800	158,800	158,000	12,716	12,719.00	12,716	(3.0)	–
25	210,866	210,866	210,000	16,902	16,905.00	16,902	(3.0)	–
5*	233,080	224,080*	224,000	18,028	18,032.00	18,028	(4.0)	–
21*	233,530	203,530*	203,000	16,338	16,341.50	16,338	(3.5)	–
44*	246,842	236,842*	236,000	19,000	18,998.00	18,994	2.0	6
46	269,000	269,000	269,000	21,650	21,654.50	21,650	(4.5)	–
45	271,100	271,100	271,000	21,811	21,815.50	21,811	(4.5)	–
15	292,820	292,820	292,000	23,501	23,506.00	23,501	(5.0)	–
2*	310,545	315,545*	315,000	25,353	25,357.50	25,353	(4.5)	–
28	351,633	351,633	351,000	28,250	28,255.50	28,250	(5.5)	–
22*	418,250	417,250*	417,000	33,562	33,568.50	33,562	(6.5)	–
24	420,110	420,110	420,000	33,804	33,810.00	33,804	(6.0)	–
6	425,000	425,000	425,000	34,206	34,212.50	34,206	(6.5)	–
9*	490,000	480,010*	480,000	38,630	38,640.00	38,633	(10.0)	(3)
30*	673,660	663,660*	663,000	53,362	53,371.50	53,362	(9.5)	–
16	843,879	843,879	843,000	67,850	67,861.50	67,849	(11.5)	1
17*	1,014,090	1,043,090*	1,043,000	83,950	83,961.50	83,947	(11.5)	3
31*	1,158,150	1,132,150*	1,132,000	91,110	91,126.00	91,110	(16.0)	–
13	1,180,600	1,177,600*	1,180,000(a)	94,765	94,748.50	94,973(a)	16.5	(208)(a)
43	1,600,000	1,600,000	1,600,000	128,780	128,800.00	128,777	(20.0)	3
		Total:	12,974,000	1,044,010	1,044,166.00	1,044,202		

(a) Based on inscribed declaration. For the reasons set out below, de Pachtere's emendation is rejected.

attributed to rounding to HS 5 or 10. We will return to Loan 13 and the rounded figures shortly.

Properties of the Revised Formula

Loan amounts derived from property declarations rounded down to the nearest HS 1,000 vary not continuously but stepwise, by increments of HS 80 or HS 81. A property declaration of HS 69,260 (Loan 35) yields a loan of HS 5,553 ($69{,}000 \times 8\ {}^{7}/_{144}$ per cent). Declarations of HS 71,522 and HS 71,256 (Loans 11 and 40) each yield the same HS 5,714 loan ($71{,}000 \times 8\ {}^{7}/_{144}$ per cent). It is straightforward to construct a table showing the loan amount associated with any property declaration, the relevant extract from which is shown in Table 3(a).

Since only every 80th or 81st integer appears in the loan column, the fact that thirty-five of the forty-six inscribed loan figures appear in the table cannot be a result of chance. We can be very confident that these inscribed figures are derived from the revised formula. We can also infer with high confidence that the rounded loan amounts, which, with the exception of Loan 44, are within three sesterces of a figure appearing in the loan table, are derived from the revised formula and then rounded. This is a probabilistic inference that does not depend on individual property declarations or de Pachtere's emendations. Rather, the validity of the loan figures being independently established, the formula can be applied to those figures in order to validate all of de Pachtere's emendations to property declarations except for the Loan 13 declaration, where the loan amount neither appears in the loan table nor can be attributed to rounding.[21]

Possible Objections: Loans 13 and 31

The anomalous result for Loan 13 raises, *prima facie*, an obvious objection to a claim that the revised formula fully explains the relationship between loan amounts and property declarations. De Pachtere's emendation to the Loan 13 property declaration (for which a simple explanatory inscribing error is not easily identifiable) does not resolve the discrepancy. Indeed, since the inscribed loan amount HS 94,765 does not appear in the loan table (see Table 3(b)), no such emendation can provide a resolution. The recorded loan amount cannot be the result of a simple inscribing error since the Loan 13 sub-loans also sum to HS 94,765. It must be inferred that the loan figure is correctly recorded, and thus deviates from the revised formula. Nonetheless, an explanation consistent with the revised formula may be conjectured, rooted in the limitations of Roman fractional arithmetic. It seems that the administrators who performed the *TAV* calculations were operating to a precision of no better than ${}^{1}/_{144}$ per cent.[22] At this level of precision they would not have been able to generate a loan total of exactly HS 1,044,000.[23] The identified arithmetical procedures positively require an adjustment to at least one of the loan amounts if the desired HS 1,044,000 in aggregate is to be achieved. The solution the administrators may have adopted was simply to make an adjustment to Loan 13, the largest property pledge except for Loan 43, the latter being a special case as it is a loan not to an individual but to a neighbouring civic community,

[21] The inscribed figure of 94,765 falls between loan table figures of 94,732 and 94,812. Its presence cannot be explained by rounding.

[22] The precise fit to the inscribed values breaks down if the multiplier is increased or decreased by as little as ${}^{1}/_{1728}$ per cent (${}^{1}/_{12}$ of ${}^{1}/_{144}$ per cent).

[23] In the same way that the endowment, HS 1,044,000 divided by the total rounded property pledges, HS 12,974,000, is close to but not exactly $8\ {}^{7}/_{144}$ per cent, the converse, multiplying each of the rounded property pledges by $8\ {}^{7}/_{144}$ per cent, would yield a figure close to, but not exactly, HS 1,044,000. As shown in Table 2 at the foot of column G, that figure is HS 1,044,202.

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Table 3 Extracts from table of loan amounts as a function of property declarations (all figures in HS)

3(a) Formula properties

Property declaration	Loan	Increment
65,000 to 65,999	5,231	–
66,000 to 66,999	5,312	81
67,000 to 67,999	5,392	80
68,000 to 68,999	5,473	81
69,000 to 69,999	5,553	80
70,000 to 70,999	5,634	81
71,000 to 71,999	5,714	80
72,000 to 72,999	5,795	81
73,000 to 73,999	5,875	80
74,000 to 74,999	5,955	80
75,000 to 75,999	6,036	81

3(b) Loan 13

Property declaration	Loan	Increment
1,175,000 to 1,175,999	94,571	–
1,176,000 to 1,176,999	94,651	80
1,177,000 to 1,177,999	94,732	81
1,178,000 to 1,178,999	94,812	80
1,179,000 to 1,179,999	94,893	81
1,180,000 to 1,180,999	94,973	80
1,181,000 to 1,181,999	95,054	81
1,182,000 to 1,182,999	95,134	80
1,183,000 to 1,183,999	95,215	81
1,184,000 to 1,184,999	95,295	80
1,185,000 to 1,185,999	95,376	81

3(c) Loan 31

Property declaration	Loan	Increment
The inscribed figure HS 91,110 appears in the table		
1,127,000 to 1,127,999	90,707	–
1,128,000 to 1,128,999	90,788	81
1,129,000 to 1,129,999	90,868	80
1,130,000 to 1,130,999	90,949	81
1,131,000 to 1,131,999	91,029	80
1,132,000 to 1,132,999	91,110	81
1,133,000 to 1,133,999	91,190	80
1,134,000 to 1,134,999	91,271	81
1,135,000 to 1,135,999	91,351	80
1,136,000 to 1,136,999	91,432	81
Bormann's emendation to HS 91,910 does not appear		
1,140,000 to 1,140,999	91,754	–
1,141,000 to 1,141,999	91,834	80
1,142,000 to 1,142,999	91,915	81
1,143,000 to 1,143,999	91,995	80
1,144,000 to 1,144,999	92,076	81
The sum of the sub-loans HS 95,910 does not appear		
1,190,000 to 1,190,999	95,778	–
1,191,000 to 1,191,999	95,858	80
1,192,000 to 1,192,999	95,939	81
1,193,000 to 1,193,999	96,019	80
1,194,000 to 1,194,999	96,100	81

the *coloni Lucenses*.[24] (It is unclear why the loans sum to HS 1,044,010 and not the exact figure HS 1,044,000.) If this conjecture is correct, the Loan 13 figures as inscribed are not inconsistent with the revised formula, and de Pachtere's unsuccessful attempt at resolution by emendation can be rejected as unnecessary.

24 The smallest deviation from 8 $^{7}/_{144}$ per cent is ensured by applying the adjustment to the largest (non-colony) loan.

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An objection might also be raised in relation to Loan 31, where a simple inscribing error cannot easily be identified. Further, uniquely in the *TAV*, the sum of the eighteen sub-loans (HS 95,910) differs from the overall loan amount (HS 91,110). The editor of *CIL* 11 (1888), Eugen Bormann, writing before de Pachtere and unsuspecting of any relationship between overall property declarations and overall loans, posits two emendations to reconcile the HS 4,800 discrepancy: the overall loan, inscribed (col. V, 58) HS $\overline{\text{LXXXXI}}$ CX (91,110), should read HS $\overline{\text{LXXXXI}}$ C∞X (91,910); and sub-loan 16 should read not the inscribed (col. V, 91) HS $\overline{\text{XXII}}$ (22,000) but HS $\overline{\text{XIIX}}$ (18,000).[25] But since the inscribed overall loan figure appears in the loan table whereas neither Bormann's emendation nor the sub-loan total are present (see Table 3(c)), the inscribed figure should be strongly preferred.

An alternative reconciliation which has since, it seems, been overlooked, is found in the eighteenth-century editions of Maffei and de Masdeu.[26] They accept the inscribed total HS 91,110. They read sub-loan 16 not as HS $\overline{\text{XXII}}$ (22,000) but HS $\overline{\text{XVII}}$ (17,000), thereby reducing the discrepancy between the total of sub-loans and the overall loan from HS 4,800 to HS 200. An inspection of the *TAV* supports their reading. There is a missing stroke in the second character of the sub-loan 16 figure (one of several missing strokes in the *TAV*), so that what we see is something like HS $\overline{\text{X}\backslash\text{II}}$. The characteristic form of the letter and numeral V in the surrounding text has a vertical or near-vertical right-hand stroke. A restoration of the missing stroke to render a V is no less plausible than restoration of an X.[27] It is then not difficult to conjecture simple inscribing errors that could account for the missing HS 200. De Masdeu proposed an emendation of sub-loan 14 (Col. V, 86) from HS ∞CC (1,200) to HS ∞CD (1,400), but a missing CC from one of the other sub-loans is an alternative possibility. Which is correct is not material to the analysis and probably unknowable. This reconciliation, comprising a credible reading of an incomplete figure and a single emendation, not only retains the overall loan figure of HS 91,110, but is simpler than the two emendations of Bormann. It must be accepted. De Pachtere's emendation to the Loan 31 property declaration must also be retained, since it brings the declaration into line with the loan value according to the revised formula.

IV SOME CONJECTURES ON THE ARITHMETICAL OPERATIONS

Some three or four decades after the Veleian alimentary schemes were set up, the jurist Lucius Volusius Maecianus wrote a short treatise to the future emperor Marcus Aurelius on the subdivision of the *as* (the unit of Roman currency, but also carrying the meaning 'the whole', hence *heres ex asse*, heir to the whole estate). This, Maecianus says, is necessary for inheritance arrangements and many other purposes.[28] He explains the Roman fractional system, giving names and signs for duodecimal fractions down to the *scriptulum*, the 288th part of the whole. His treatise affirms the primacy of the duodecimal system in Roman fractional calculations of the time, and ties this directly to the dividing up of money and property. Maecianus explains at length the relationships between different combinations of fractions but gives no information as to how to carry

[25] *CIL* vol. 9, p. 225 n. 1.

[26] Maffei 1749: CCCXCIV; de Masdeu 1788: 231. The critical apparatus of Criniti 1991: 178 makes no mention of a possible HS XVII reading.

[27] Both de Lama 1819: 11 n. 2 and Criniti 1991: 75 observe that the inscription has a number of missing strokes. Further, they note that the missing strokes are vertical, adding weight to the preference for a restoration of a characteristic right-vertical V over an X.

[28] Maecianus, *Distributio item vocabula ac notae partium in rebus quae constant pondere numero mensura* 1–38 (Hultsch 1866: 61–71).

out division of property in practice. The *TAV* allows us to test conjectures on how such calculations might have been performed.

The value 8 $^{7}/_{144}$ would conventionally at the time have been expressed as a sum of duodecimal unitary fractions, that is 8 $^{1}/_{24}$ $^{1}/_{144}$ or possibly 8 $^{1}/_{36}$ $^{1}/_{48}$.[29] From this it might be suspected that the computations would proceed in the same way, with each element calculated separately and the results aggregated. But this does not consistently yield the inscribed result, assuming remainders are ignored at each stage. For example, the computation for Loan 25 would be as follows:

	HS	
(a) Property declaration	210,866	
(b) Round (a) down to HS 1,000	210,000	
(c) Hundredth part of (b)[30]	2,100	
(d) Eight times (c)	16,800	
(e) $^{1}/_{24}$ of (c)	87	(remainder 12)
(f) $^{1}/_{144}$ of (c)	14	(remainder 84)
(d)+(e)+(f)	16,901	
Inscribed loan (HS)	16,902	

The same HS 16,901 results from a similar computation using 8 $^{1}/_{36}$ $^{1}/_{48}$ per cent. Further, calculating the fractional component as $^{7}/_{144}$ by first establishing the 144th part (HS 14, as above) then multiplying by 7 (HS 98) yields, after adding the 8 per cent figure of HS 16,800, a total of HS 16,898. Only calculating the fractional component by first multiplying by 7 ($2{,}100 \times 7 = 14{,}700$) and then dividing by 144 (to give HS 102, remainder 12) yields the inscribed HS 16,902. This suggests that the accountant not only used non-unitary fractions, but also multiplied by the numerator as the first step, rather than first finding the 144th part.

No procedure has been identified that would generate exactly the figures that appear to be rounded. But if we allow that the accountant might have made time-saving approximations, an alternative conjecture arises. The roundings are hard to explain in terms of a sum-of-the-parts method. For example, Loan 44 (calculated loan HS 18,994) shows both the largest rounding by some distance (HS 6) and the 'roundest' result (HS 19,000). But the 8 per cent and $^{7}/_{144}$ per cent elements of Loan 44 are HS 18,880 and HS 114 respectively. There is no obvious reason to round up either figure, and rounding after adding them together yields negligible economy of computational effort.

The rounding is more readily explained by a calculation that does not involve a summation of parts. This would be the case if the accountant had conceived the loan multiplier entirely in terms of 144th parts. Expressed in this way, 8 $^{7}/_{144}$ represents 1,159 144th parts. The necessary computation would be to multiply the hundredth part of each rounded property declaration by 1,159 and divide the result by 144. This gives the same result as multiplying by 8 and by $^{7}/_{144}$ and adding the results. We do not know how the Romans would have effected such a division, but one possible method involves repeated subtraction of the divisor from the dividend, using *calculi* and a counting-table, subtracting at each iteration the largest factor-ten multiple (or intermediate quinary) of

[29] This convention is evident within the *TAV* itself, for example: *item fund(us) Messianum p(ro) p(arte) III et XXIIII*, 'also the Messianus farm in respect of a third and a twenty-fourth part' (col. II, 55). The fractions sum to three-eighths.

[30] Because the declaration in each case is rounded down to a multiple of HS 1,000, finding the hundredth part is straightforward, by substitution of I for each C, V for each D, X for each M, etc. There will never be a fractional element.

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TABLE 4 Conjectural calculation process for Loan 44 (all figures HS)

Property declaration	$\overline{\text{CCXXXVI}}$ DCCCXXXXII	236,842
Dividend (236,000/100 x 1,159)	$\overline{\text{MMDCCXXXV}}$ CCXXXX	2,735,240
Divisor	CXXXXIIII	144

Iteration	Quotient		Quotient x Divisor		Remainder after subtracting Q x D	
					$\overline{\text{MMDCCXXXV}}$ CCXXXX	2,735,240
1	$\overline{\text{X}}$	10,000	$\overline{\text{MCCCCXXXX}}$	1,440,000	$\overline{\text{MCCLXXXXV}}$ CCXXXX	1,295,240
2	$\overline{\text{V}}$	5,000	$\overline{\text{DCCXX}}$	720,000	$\overline{\text{DLXXV}}$ CCXXXX	575,240
3	$\overline{\text{I}}$	1,000	$\overline{\text{CXXXXIIII}}$	144,000	$\overline{\text{CCCCXXXI}}$ CCXXXX	431,240
4	$\overline{\text{I}}$	1,000	$\overline{\text{CXXXXIIII}}$	144,000	$\overline{\text{CCLXXXVII}}$ CCXXXX	287,240
5	$\overline{\text{I}}$	1,000	$\overline{\text{CXXXXIIII}}$	144,000	$\overline{\text{CXXXXIII}}$ CCXXXX	143,240
"Shortcut" termination point. Remainder is close to 1,000*d*. Add final $\overline{\text{I}}$ (1,000) for shortcut quotient $\overline{\text{XVIIII}}$ (19,000).						
6	D	500	$\overline{\text{LXXII}}$	72,000	$\overline{\text{LXXI}}$ CCXXXX	71,240
7	C	100	$\overline{\text{XIIII}}$ CCCC	14,400	$\overline{\text{LVI}}$ DCCCXXXX	56,840
8	C	100	$\overline{\text{XIIII}}$ CCCC	14,400	$\overline{\text{XXXXII}}$ CCCCXXXX	42,440
9	C	100	$\overline{\text{XIIII}}$ CCCC	14,400	$\overline{\text{XXVIII}}$ XXXX	28,040
10	C	100	$\overline{\text{XIIII}}$ CCCC	14,400	$\overline{\text{XIII}}$ DCXXXX	13,640
11	L	50	$\overline{\text{VII}}$ CC	7,200	$\overline{\text{VI}}$ CCCCXXXX	6,440
12	X	10	$\overline{\text{I}}$ CCCCXXXX	1,440	$\overline{\text{V}}$	5,000
13	X	10	$\overline{\text{I}}$ CCCCXXXX	1,440	$\overline{\text{III}}$ DLX	3,560
14	X	10	$\overline{\text{I}}$ CCCCXXXX	1,440	$\overline{\text{II}}$ CXX	2,120
15	X	10	$\overline{\text{I}}$ CCCCXXXX	1,440	DCLXXX	680
16	I	1	CXXXXIIII	144	DXXXVI	536
17	I	1	CXXXXIIII	144	CCCLXXXXII	392
18	I	1	CXXXXIIII	144	CCXXXXVIII	248
19	I	1	CXXXXIIII	144	CIIII	104

↳ Final quotient $\overline{\text{XVIII}}$ DCCCCLXXXXIIII (18,994)

the divisor *d* that is smaller than the remainder.[31] Each subtraction generates one Roman numeral of the quotient. The application of this method to Loan 44 is illustrated in Table 4.

[31] Counting-table computations with Roman numerals may well have been similar to those using Arabic numerals, with each column of the counting-table representing ten times its right-hand neighbour, so the rightmost columns would be $\overline{\text{C}}$, $\overline{\text{X}}$, $\overline{\text{I}}$, C, X, I, with places at the top of or between each column for single counters representing D, L, V etc. Friedlein 1869, Turner 1951, Pullan 1968 and (to the extent Greek practices

After five subtractions (10,000*d*, 5,000*d* and 1,000*d* three times), the quotient totals $\overline{\text{XVIII}}$ (18,000) and the remainder on the counting-board stands at $\overline{\text{CXXXXIII}}$ CCXXXX (143,240).[32] A further fourteen subtractions are required to arrive at $\overline{\text{XVIII}}$ DCCCCLXXXXIIII (18,994). But perhaps an experienced accountant, seeing a remainder after the fifth subtraction that was close to, but less than, 1,000*d* ($\overline{\text{CXXXXIIII}}$ or 144,000), would have known that finishing the computation would be a lengthy exercise for little additional accuracy, even if he could not know the exact result or number of further subtractions required.[33] In this knowledge, he might instead have simply treated the remainder as 1,000*d*, adding $\overline{\text{I}}$ to the quotient for a rounded answer of $\overline{\text{XVIIII}}$ (19,000). He might have applied a similar principle, with smaller roundings, to the other rounded figures.

The time-saving incentive is not the same where the answer slightly exceeds a round number. This might explain why all but one of the roundings is upward. It does not fully explain why some numbers are rounded and other similar numbers are not, unless this is simply a matter of variations in diligence on the part of the accountant(s).[34]

The plausibility of this conjecture depends in large measure on whether, for the Roman accountant, multiplying the hundredth part by 1,159 then dividing by 144 would, for conceptual or practical reasons, have been preferred to (a) multiplying the hundredth part by eight and recording the result; (b) multiplying the hundredth part by seven and dividing the result by 144; and (c) adding the two. It is not obvious that this would be the case. Nonetheless, the *TAV* dataset, and the arithmetical conjectures it permits, provide a useful addition to the sparse evidence we have for the arithmetical processes employed at Rome in the second century A.D.

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might be read across to Rome) Schärlig 2001 all offer ideas on how Roman division might have been effected. Their ideas are necessarily speculative.

32 Expanded forms are shown, e.g. XXXX for XL. Each numeral is represented by a counter on the board.

33 Because he knows (a) the remainder is slightly less than 1,000*d*, (b) therefore the remaining quotient will be at least DCCCCLXXXX (990) and at most DCCCCLXXXXVIIII (999) and (c) generating each Roman numeral requires one subtraction. Similarly, calculating the rounded inscribed Loan 39 figure of $\overline{\text{VIII}}$ L requires five subtractions, whereas calculating the precise figure $\overline{\text{VIII}}$ XXXXVIII would require twelve.

34 Loan 3, HS 6,197, for example, has not been rounded up to HS 6,200.

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