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AN EXPRESSIVIST ANALYSIS OF THE INDICATIVE CONDITIONAL WITH A RESTRICTOR SEMANTICS

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Abstract. A globally expressivist analysis of the indicative conditional based on the Ramsey Test is presented. The analysis is a form of 'global' expressivism in that it supplies acceptance and rejection conditions for all the sentence forming connectives of propositional logic (negation, disjunction, etc.) and so allows the conditional to embed in arbitrarily complex sentences (thus avoiding the Frege–Geach problem). The expressivist framework is semantically characterized in a restrictor semantics due to Vann McGee, and is completely axiomatized in a logic dubbed ICL ('Indicative Conditional Logic'). The expressivist framework extends the AGM (after Alchourron, Gärdenfors, Makinson) framework for belief revision and so provides a categorical ('yes'–'no') epistemology for conditionals that complements McGee's probabilistic framework while drawing on the same semantics. The result is an account of the semantics and acceptability conditions of the indicative conditional that fits well with the linguistic data (as pooled by linguists and from psychological experiments) while integrating both expressivist and semanticist perspectives.

§1. Introduction. Perhaps the most influential – and controversial – thesis about the indicative conditional was first formulated by F. P. Ramsey in a famous footnote:

> If two people are arguing 'If p will q?' and both are in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q. (Ramsey 1931, p. 249)

The footnote is usually interpreted as implying that the acceptability of a conditional should depend on the acceptability of the consequent of the conditional upon hypothetically assuming the antecedent – what has become known as the *Ramsey Test* for conditionals. The Ramsey Test can be stated in both *graded* ('quantitative') and *categorical* ('qualitative') form:

RTgraded A conditional $\varphi \to \psi$ is acceptable to the degree that ψ is acceptable on the assumption that φ.
RTCategorical A conditional $\varphi \to \psi$ is acceptable if and only if ψ is acceptable on the assumption that φ.

In this study the categorical version will be the primary focus of attention.

The Ramsey Test in its various guises has been influential as many have taken it to fit well with linguistic intuition, and in the past couple of decades this fit with

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linguistic intuition has been corroborated by a growing number of empirical results drawn from linguistic experiments (e.g. Over & Evans, 2003; Evans, Over, & Handley, 2003; Oberauer, 2006; Douven & Verbrugge, 2010; Vidal & Baratgin, 2017). Neither the intuitions nor the results have gone unchallenged, but there is a wide acceptance that the Ramsey Test is a very good predictor of the acceptability conditions for the indicative conditional in a wide range of cases and for a wide range of speakers.

If there is any consensus in the area it ends roughly there. A core controversy concerns the meaning-theoretical status of the Ramsey Test. By itself it provides acceptability conditions for the conditional; it doesn't explicitly provide any truth conditions. So, does it in some direct or indirect way characterize the *semantic content* of the conditional, or is it 'merely' a principle governing the *pragmatics* of the conditional (in the guise of acceptance conditions)? If the Ramsey Test is 'merely' a pragmatic principle governing the acceptance of conditionals, how is it related to the semantic content of the conditionals?

The issue came to the fore with the famous 'impossibility/triviality theorems' of Lewis (1976) (striking against the Ramsey Test in its graded probabilistic guise) and Gärdenfors (1986) (striking against the categorical version of the Ramsey Test). The results suggest that one cannot both take indicative conditionals to express propositions like any other, *and* insist that they abide by the Ramsey Test, at least not when the test is articulated in standard ways of modeling the epistemic states of speakers (e.g. by a probability measure, or as sets of possible worlds).[1]

The reactions to the problem of squaring a propositional semantics for the conditional with its acceptability conditions have been many and varied. Some, let us call them 'traditional' expressivists', have taken the above to show that indicative conditionals do not have a propositional content in any standard sense of the word: they are not truth-evaluable and, to the extent that such constructs combine to form more complex sentences (conjunctions, negated conditionals, etc.), this has to be explained in some other way than by the usual semantic means.[2] Others, let us call them 'radical pragmaticists', have instead drawn the conclusion that indicative conditionals have a 'standard' propositional content (e.g. a content that can be represented by sets of possible worlds) but that this content cannot be used to explain the Ramsey Test (which is then explained pragmatically).[3] Still others, let us call them 'revisionist', try in various ways to have their cake and eat it by either challenging the assumptions of

[1] A related issue draws on an observation first made by Gibbard (1981): two speakers that have factually correct beliefs and so have no grounds for disagreement on ordinary factual matters apparently can still on good grounds disagree on conditional matters. In this way the indicative conditional behaves similarly to other 'epistemic' modals like 'might' (as in "It might be raining in Oslo") and 'probably' (as in "The coin did probably not land heads ten times in a row") that also, in their default use, have what appear to be epistemically grounded acceptability conditions that are difficult to reconcile with the idea that they express propositions like any other.

[2] Prominent local expressivist accounts are found in Adams (1975), Levi (1988), Edgington (1995).

[3] Lewis (1976), Jackson (1979), and Grice (1989) have defended the position that the semantic content of the indicative conditional is that of the material conditional (and so with a representational content), and have tried to find pragmatic constraints that would explain the Ramsey Test. However, the resulting discrepancy between the semantic content of conditionals and their actual use has been found to be deeply problematic. See for instance the extensive discussion in Edgington (1995) and Bennett (2003).

the impossibility results, be it by challenging assumptions about how epistemic states are to be represented, or by challenging assumptions about how the Ramsey Test is to be articulated in a given epistemic framework. This paper follows the revisionist tradition.

It will be shown how one can treat the Ramsey Test as a norm of use that serves as a determinant of the semantic content of conditionals, including the compositional contribution of their content when they appear embedded in complex sentences. As a result such sentences can stand in semantic relations to other propositions, have a nontrivial logic, and can be said to express propositions. Conditional propositions (propositions with a conditional content) are in this respect no different than descriptive or normative propositions.

The approach is two-tiered. On the one hand the analysis draws on a meaning theoretical framework that can be characterized as a form of 'global' expressivism. This deals solely with acceptance and rejection conditions of sentences and so belongs to a meaning-theoretical tradition that eschews truth-conditional semantics in favor of a more pragmatic user-oriented perspective on linguistic meaning. The idea is to model the mental state of an agent through the sentences accepted or rejected in that state, and through two dynamic operators – *hypothetical expansion* and *hypothetical revision* – that allow the agent to explore what goes beyond or is contrary to what the agent accepts and rejects. However, this 'use-based' model – an *expressivist model* – can then be given a complete semantic characterization in terms of selection functions on possible worlds that assigns propositional contents to both sentences (conditional and otherwise) and the mental states of the agent.

The epistemic framework can in many ways be viewed as an extension of the AGM-framework for belief revision. The semantic framework is, with minor differences, due to McGee (1989). Before turning to the meat of the paper, I will briefly outline the basic ideas involved.

1.1. Global expressivism: a model. Global expressivism concurs with 'local' expressivism by holding that the Ramsey Test is meaning-determining: the Ramsey Test endows the indicative conditional with any content it may have. Global expressivism goes beyond local expressivism in holding that the treatment given to conditionals in the form of the Ramsey Test – necessary and sufficient conditions of competent use– should be given to *all* sentence-forming expressions (the basic idea is outlined in Cantwell, 2014). So, for instance, in a simple propositional language, the meaning of the sentential atoms, and of negation, conjunction, and disjunction is to be (partially) determined by conditions of acceptance and rejection, just as the Ramsey Test (together with its 'negative' companion, discussed further in Appendix A) determines conditions of acceptance and rejection for the conditional. When all sentence-forming expressions are treated in this way the so called Frege–Geach problem that is endemic to local expressivism[4] disappears: one achieves a compositional account of the acceptance and

[4] The full picture is somewhat complex. Some local expressivist accounts provide partial accounts of how conditionals embed in complex constructions. For instance, Levi (1988), Hansson (1992), and Rott (2011) and others allow that conditionals iterate (so $\varphi \rightarrow (\varphi \rightarrow \chi)$ is a meaningful sentence) and introduce special clauses for negation. But this does not solve the general problem of embedding conditionals within other complex constructions. Arló-Costa & Levi (1996), Arló-Costa (1999a), Kern-Isberner (1999), and Giordano, Gliozzi, & Olivetti (2005) allow conditionals to be embedded within boolean connectives, however,

rejection conditions for sentences of arbitrary complexity, and with it a principled account of the semantic relations between sentences that make up their logic.

A fundamental feature of the Ramsey Test is that it involves *hypothetical* thought. There are many different (but related) modes of hypothetical cognition, drawing in part on the mind's capacity for *mental simulation* (or mental *emulation*). There is *pretense* (one pretends, say, to be a duck), there is *mind-read* (or 'mind-emulation', one 'puts' oneself in the mental state of others), there is spatially or temporally distal imaginings (one 'puts' oneself in another place or another time), there is 'wishful thinking' (one imagines things being as they should be), and so on. I believe these modes of hypothetical or simulated thought can form the basis for an expressivist account of a number of linguistic phenomena (tense, aspect, attitude modalities, and so on), suggesting that hypothetical thought is as much a cornerstone of language comprehension as possible worlds are a cornerstone of formal semantics (this echoes Stalnaker's comment that "a possible world is the ontological analogue of a stock of hypothetical beliefs" (1968, p. 33)). However, the focus here is limited to the indicative conditional and its sibling *disjunction*.

For disjunctions and conditionals are related. We take "If the butler didn't do it, the gardener did" to entail "Either the butler did it or the gardener did." The converse direction also seems plausible, except when acceptance of the disjunction is based on the fact that one accepts one of the disjuncts (e.g. that the butler did it) in which case the inference does not seem compelling. In the framework this is explained by taking the hypothetical mode of thought involved in acceptance of disjunctions to be (hypothetical) *expansion*, while taking the hypothetical mode of thought involved in acceptance of conditionals to be (hypothetical) *revision* – two closely related yet distinct forms of hypothetical thought. The grounding of disjunctions and conditionals in hypothetical thought have been the focus of extensive studies in cognitive psychology and cognitive linguistics,[5] but the ambition here is not to provide a psychologically realistic account. Rather, the ambition is to show that by invoking only structures that can be said to have a realistic psychological basis, one can *in principle* provide a full fledged semantic theory of the conditional with properties that accord with the experimental and empirical data acquired by psychologists and linguists.

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except for negated conditionals, they do not provide any interpretation of such constructions over and above the requirement that the boolean connectives obey the laws of sentential logic (thus we are told what the logic *is*, but not *why* it is; no account is given of the semantic properties that would explain how the logic arises); furthermore, conditionals are not allowed to iterate. These accounts are thus incomplete. In addition – and more importantly – the accounts all introduce distinct clauses for the case when boolean constructions contain conditionals, which means that on these accounts the boolean connectives do not simply inherit their semantic properties in a compositional manner when they contain conditionals, they need to be interpreted anew in a piecemeal way: a seemingly inherent weakness of the local expressivist paradigm.

[5] For instance, the mental models framework of Johnson-Laird (1983, 2006) and Johnson-Laird & Byrne (2002), built on the hypothesis that speakers construct hypothetical scenarios ('mental models', in Johnson-Laird's terminology) when reasoning with disjunctions and conditionals, has considerable empirical support, explaining both successful and unsuccessful reasoning with such constructs. See also Barrouillet, Grosset, & Lecas (2000), Oberauer (2006), Barrouillet & Gauffroy (2008) for a more detailed discussion of the actual cognitive underpinnings of the hypothetical thought involved in the Ramsey Test. The treatment in the present paper is, by contrast, heavily idealized.

As indicated, *belief revision theory* in the *AGM*-tradition (after the authors in Alchourrón, Gärdenfors, & Makinson, 1985), provides much of the formal structure needed. In particular, its relatively rich epistemic framework supplies the underpinnings for the two modes of hypothetical thought exploited here: hypothetical expansion (for disjunctions) and hypothetical revision (for conditionals).

The similarities between the present global expressivist framework and AGM are many but I will here stress the main differences (it is well known – see references in footnotes 2 and 4 – that the AGM-framework needs to be extended or revised in order to accommodate the Ramsey Test). One difference is that the hypothetical expansion and revision in the present framework apply to attitudes in general, not only to beliefs. Another difference is that the AGM-framework is most naturally thought of as a framework for dealing with 'real' revision, as opposed to the 'hypothetical' or 'simulated' belief change involved in the Ramsey Test (hypothetical revision is materially different from real revision and also subject to somewhat different structural constraints).[6] An important feature is that while the correlates of the AGM-postulates for revision will hold for factual sentences, several of them will not hold for hypothetical revision with conditionals (these are the source of the 'triviality' result for the categorical Ramsey Test).

A fundamental difference is that the AGM-framework presupposes a consequence relation on the language. Thus in the AGM-framework the semantic relations among sentences are taken as given. By contrast, it is a central feature of the present framework that the semantic relations among sentences are to be explained by means of the framework: the present framework is as much a meaning theoretical framework as an epistemic.

The ambition to characterize semantic content by appeal to the dynamics of mental states has a close cousin in *dynamic semantics* (see e.g. Kamp, 1981; Gärdenfors, 1988; Groenendijk & Stockhof, 1991; Veltman, 1996; Vermuelen, 1993, see also Kolodny & MacFarlane, 2010; Yalcin, 2012; Starr, 2014b for more recent accounts). Dynamic semantics also strives to characterize linguistic meaning in terms of the dynamics of mental states or, more generally, *information states*. Very briefly: in dynamic semantics the semantic content of a sentence is taken to reside in its potential to change information states. Stressing the differences between the frameworks rather than the similarities one should highlight the fact that the information states employed in dynamic semantics are best thought of as characterizing the *content* of a mental state (e.g. as a set of possibilities) so the dynamics becomes a dynamic of contents. By contrast, global expressivism in the present guise instead takes the primary elements to be mental structures involved in the mental manipulation of sentences and its dynamics is a dynamics of such structures; the *contents* of these mental structures are not given in advance, and to the extent that they have a content, this is taken to be in need of explanation. (Once the semantics is in place, however, the expressivist framework can be viewed as providing a species of dynamic semantics, see Section 4.)

[6] See, however, Levi (1996) who holds that AGM is best seen as a framework for hypothetical changes of belief.

Global expressivism has another close meaning theoretical cousin in *inferentialism* (as represented by thinkers like Dummett, Prawitz, and Brandom).[7] Again stressing the differences rather than the similarities one should note that in the inferentialist tradition the content of a sentence is supposed to be *implicitly* characterized by its role in inference, rather than *explicitly* by necessary and sufficient norms of use as the present approach requires. The form of global expressivism developed here provides necessary and sufficient conditions (not for *truth*, but) for *linguistically competent judgment* of truth (acceptance) and falsity (rejection) for a given language. That is, it provides necessary and sufficient conditions for how a given language is to be competently used to express one's mental state (regardless of whether the mental state provides an accurate reflection of the state of the world). In contrast to inferentialism, *valid inference* here becomes a derivative notion: if any ideally rational linguistically competent agent that accepts φ will also accept ψ, then φ entails ψ.

1.2. A semantic framework: the restrictor analysis. The main semantic idea adopted in this study goes as follows: in making an assumption one excludes the possibility that the assumption is false and one thereby constrains the space of possibilities relative to which any subsequent claim is to be evaluated. Coupled with the Ramsey Test it thus provides a version of a *restrictor* semantics for conditionals. The basic idea is not new, nor is the specific implementation utilised here.

Much of current linguistic theorizing builds on (or is an reaction to) the seminal work by Kratzer (e.g. 1977; 1979; 1991, see collection in 2012) who developed the view that the antecedent of a conditional $\varphi \to \psi$ is semantically to be viewed as a restricted modality. On this view the conditional is given the logical form $\Box_\varphi \psi$, where $\Box_\varphi$ can be interpreted as a modality that universally quantifies over some contextually given domain of possibilities *restricted* to those possibilities where φ holds. So, for instance "If φ, then ought ψ" gets the logical form $\text{OUGHT}_\varphi \psi$ which can be interpreted, roughly, as stating that in all the best worlds where φ holds, ψ holds. Kratzer's analysis (while differing in the details) thus echoes Hansson's (1968) semantic analysis of conditional ought sentences but generalizes it to arbitrary conditionals. There are, however, standard objections to treating the antecedent of a conditional as a modality. First, a 'bare' conditional like "If Jim came late again, he was fired" does not involve any explicit modality at all. Second, the consequent of a conditional can contain several different modalities, as in:

1. If Jane kills Jim she should – and probably will – kill him gently.

When we have two or more modalities in the consequent it becomes difficult to take the modal force as emanating from the antecedent.

Some more recent developments of the restrictor analysis deviate from Kratzer's treatment by treating the antecedent of a conditional not as a modality itself, but solely as a restrictor of the domain of possibilities relative to which the consequent of

[7] Brandom (1994) idea that the logical constants (such as conditional) 'makes explicit' underlying inferential dispositions is closely related to the present approach, although the focus here is not on inference. One should also note that the present framework is closely related to *bilateral* inferentialism (e.g. Bendall, 1979; Smiley, 1996; Rumfitt, 2000) which treats the attitudes of acceptance and rejection on par. This connection is discussed in more detail in Cantwell (2015).

the conditional (which may or may not contain any modalities) is to be semantically evaluated. The antecedent of a conditional thus serves only to restrict the domain of possibilities without in any way quantifying over the domain. When subsequent modalities are evaluated they are semantically taken to quantify over this restricted domain. This relieves us of forcing the antecedent to function as an implicit modality and provides a natural logical form for (1):

$$(\text{Jane kills Jim}) \rightarrow (\text{OUGHT Kill Gently} \wedge \text{PROBABLY Kill Gently}).$$

We find such restrictor treatments in Cantwell (2008), Kolodny & MacFarlane (2010), Thomason (2012) and Yalcin (2012), but the basic semantic idea was, as far as I can tell, first formulated by McGee (1989) (who only applied it to conditional sentences: not to other constructs – such as modalities – that also are semantically sensitive to the domain of possibilities).[8]

To make sense of restrictor semantics one must distinguish between the space of what *is* possible (in a more absolute sense), and the space of what in a given context is to *count as* possible. The latter will be called the *modal background* (the term comes from Kratzer who takes the modal background to be the set of states occurring in each proposition in what she calls the 'modal base'). The restrictor framework provides a natural semantic analysis of what it means to make an assumption: to make an assumption is to restrict the domain of possibilities, that is, to restrict what is to count as possible. McGee used the idea to semantically characterize probability measures that allow a conditional to satisfy (a restricted version of) the graded/probabilistic Ramsey Test while avoiding triviality (more on this below). Here it will be used to semantically characterize qualitative representations of mental states that allow a conditional to satisfy the categorical Ramsey Test while avoiding triviality.

The paper is organized as follows. Section 2 presents the formal structure in which the global expressivist framework is developed. Section 3 presents the logic of the indicative conditional that results from this framework. Appendix A provides a fuller discussion on the 'negative Ramsey Test' and Appendix B collects the proofs of the main theorems.

§2. A global expressivist framework. An *expressivist model* is a structure $\mathcal{E} = (\mathcal{A}, \mathcal{M}, \vdash, R, \mathcal{C}, \oplus, \circledast)$ where $\mathcal{A}$ is the set of attitudes, $\mathcal{M}$ is the set of mental states, $\vdash$ is a support relation, R is a relation on mental states, $\mathcal{C}$ is the set of *coherent* mental states (so $\mathcal{C} \subseteq \mathcal{M}$), $\oplus$ is an expansion operator, and $\circledast$ is a revision operator. The remainder of this section explains these notions and spells out the properties they are required to satisfy.

Throughout the object language consists of a set of propositional letters $p, q, r, \ldots$, the boolean connectives $\neg, \wedge, \vee$, and $\supset$, and the conditional $\rightarrow$. The language L is defined by the following:

[8] To this one should add that the basic idea has for some time been employed in accounts that provide an epistemic semantics for conditionals (see e.g. Segerberg, 1998; Arló-Costa, 1999b; Rott, 2011), and is anticipated in probabilistic accounts (e.g Adams, 1975) where the probability of a conditional is the probability of the consequent *conditioned* on the antecedent (this is also the entry-point for McGee's paper).

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1. The propositional letters are sentences.
2. If φ and ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \supset \psi$, $\varphi \equiv \psi$ and $\varphi \rightarrow \psi$.

A sentence containing no instance of a conditional will be called *factual*. A conditional $\varphi \rightarrow \psi$ where both φ and ψ are factual will be called a *base* conditional. Apart from L I will also consider the reduced language L_R which is just like L except that it only allows conditional sentences containing conditionals with factual antecedents (thus L_R does not contain left-embedded conditionals).

2.1. The basic ingredients. An *attitude* is a type of cognitive entity or state, a type of functional state individuated by its functional role. There are different kinds of attitudes with different kinds of functional roles (e.g *beliefs*, *hopes*, and *desires*) and they can come in both categorical and hypothetical form. The only attitudes that will be considered here are the attitudes of *acceptance* and *rejection* of a sentence φ, denoted by Acc φ and Rej φ, respectively. The set of such attitudes is denoted by $\mathcal{A}$. An attitude of acceptance or rejection towards a factual sentence will be called a *factual* attitude, while an attitude towards a modal sentences will be called a *modal* attitude.

A *mental state* is a cognitive state supporting whole configurations of attitudes. Where m denotes a mental state and a denotes an attitude, the claim that *m supports* the attitude a will be written $m \vdash a$. The set of mental states is denoted by $\mathcal{M}$.

Mental states support attitudes with functional roles and are subject to norms (norms of functionality, and via their relation to social behavior, social norms); they can accordingly be divided into those mental states that are *coherent* (conform to the norms) and the mental states that are *incoherent* (fail to conform to the norms).[9] The set of coherent mental states is denoted by $\mathcal{C}$. Note that an important part of the work in this section is to spell out in more detail what kind of properties a mental state should have in order to count as coherent; coherence is thus not an antecedently given notion.

The two modes of mental simulation or hypothetical thought will be written $m \oplus a$ and $m \circledast a$, respectively, where $m \oplus a$ denotes the mental state that results from a hypothetical expansion with the attitude a, and $m \circledast a$ denotes the mental state that results from a hypothetical revision with the attitude a. The operations can be iterated ,for instance, $m \oplus a \circledast b$ would be the mental state of first hypothetically expanding with a, and then hypothetically revising with b. An expressivist model will said to be *restricted* when it only allows hypothetical revision of *factual* attitudes.

The accessibility relation R what mental states that are ‘reachable’ from a mental state, is assumed to be reflexive, transitive and satisfy:

$$mR(m \oplus a).$$
$$mR(m \circledast a).$$

[9] The treatment of incoherence is arguably one of the biggest weaknesses of the current model. A more plausible treatment would make a distinction between ‘incomplete’ mental states – states that fail to contain attitudes that are normatively required – and ‘incoherent’ mental states – states that require revision in the form of withdrawal of some attitudes (such as when one believes a contradiction). The present model can thus justly be held to be over-idealized in this respect (indeed not only in this respect). I will accept this as the price of technical simplicity.

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Due to the transitivity of reachability this implies that any sequence of expansion and revision operations will lead to a reachable mental state. Using the accessibility relation R one can define what it means for a mental state m to *force* an attitude a, in symbols, $m \Vdash a$:

$$m \Vdash a \text{ iff } m' \vdash a, \text{ for all } m' \text{ such that } mRm'.$$

2.2. Requirements on the boolean connectives. The boolean connectives are taken to be governed by the following requirements on acceptance and rejection; for any coherent m:

$\neg$**Acc** $m \vdash \mathrm{Acc}\,\neg\varphi$ iff $m \vdash \mathrm{Rej}\,\varphi$.
$\neg$**Rej** $m \vdash \mathrm{Rej}\,\neg\varphi$ iff $m \vdash \mathrm{Acc}\,\varphi$.
$\vee$**Acc** $m \vdash \mathrm{Acc}\,(\varphi \vee \psi)$ iff $m \oplus \mathrm{Rej}\,\varphi \vdash \mathrm{Acc}\,\psi$ and $m \oplus \mathrm{Rej}\,\psi \vdash \mathrm{Acc}\,\varphi$.
$\vee$**Rej** $m \vdash \mathrm{Rej}\,(\varphi \vee \psi)$ iff $m \vdash \mathrm{Rej}\,\varphi$ and $m \vdash \mathrm{Rej}\,\psi$.
$\wedge$**Acc** $m \vdash \mathrm{Acc}\,(\varphi \wedge \psi)$ iff $m \vdash \mathrm{Acc}\,\varphi$ and $m \vdash \mathrm{Acc}\,\psi$.
$\wedge$**Rej** $m \vdash \mathrm{Rej}\,(\varphi \wedge \psi)$ iff $m \oplus \mathrm{Acc}\,\varphi \vdash \mathrm{Rej}\,\psi$ and $m \oplus \mathrm{Acc}\,\psi \vdash \mathrm{Rej}\,\varphi$.
$\supset$**Acc** $m \vdash \mathrm{Acc}\,(\varphi \supset \psi)$ iff $m \oplus \mathrm{Acc}\,\varphi \vdash \mathrm{Acc}\,\psi$.
$\supset$**Rej** $m \vdash \mathrm{Rej}\,(\varphi \supset \psi)$ iff $m \vdash \mathrm{Acc}\,\varphi$ and $m \vdash \mathrm{Rej}\,\psi$.

Note how hypothetical expansion is involved in the acceptance conditions for disjunctions (and the material conditional $\supset$), and in the rejection conditions for conjunctions.

2.3. Requirements on the indicative conditional. The indicative conditional is taken to be governed by the following requirements; for any coherent m:

$\rightarrow$**Acc** $m \vdash \mathrm{Acc}\,(\varphi \rightarrow \psi)$ iff $m \circledast \mathrm{Acc}\,\varphi \vdash \mathrm{Acc}\,\psi$.
$\rightarrow$**Rej** $m \vdash \mathrm{Rej}\,(\varphi \rightarrow \psi)$ iff $m \circledast \mathrm{Acc}\,\varphi \vdash \mathrm{Rej}\,\psi$ and $m \circledast \mathrm{Acc}\,\varphi$ is coherent.

The acceptance clause for the conditional is simply the Ramsey Test in a formal guise, and I will not comment further on it here. The rejection clause, however, needs some justification as there is no consensus (among Ramsey Test adherents) on how to formulate the ‘negative’ Ramsey Test. See Appendix A for a discussion.

2.4. Structural requirements on expansion and revision. Hypothetical expansion, it will be assumed, is subject to the following rationality postulates:

Success $m \oplus a \vdash a$.
Monotonicity If $m \vdash b$, then $m \oplus a \vdash b$.
Redundancy If $m \vdash a$, then $m \oplus a = m$.
Permutation $m \oplus a \oplus b = m \oplus b \oplus a$.

The following are the postulates that are assumed to govern hypothetical revision:

Strong Success $m \circledast a \Vdash a$.[10]
Weak Redundancy If $m \Vdash a$, then $m = m \circledast a$.

[10] Strong Success states that once one hypothetically revises with a one cannot get rid of a. This would be implausible if $\circledast$ was interpreted as a proper revision operator, that is,

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R-Permutation $m \circledast a \circledast b = m \circledast b \circledast a$, when a and b are factual.
R-Inclusion If $m \circledast a \vdash b$, then $m \oplus a \vdash b$, when a and b are factual.
R-Vacuity If $m \oplus a$ is coherent and $m \oplus a \vdash b$, then $m \circledast a \vdash b$, when a and b are factual.

It should be noted that the above restrictions to factual attitudes are all necessary: if they are removed the system will collapse into triviality.

2.5. The classical requirements. In order to get classical logic we need two additional requirements:[11]

Explosion: If m is incoherent, then $m \Vdash$ a, for all a.
Classicality: For any sentence φ: Acc φ and Rej φ are (i) incompatible and (ii) complementary attitudes.

Where:

> Two attitudes a and b are said to be *incompatible* iff there is no coherent mental state m such that $m \vdash a$ and $m \vdash b$.
>
> Two attitudes a and b are said to be *complementary* iff for every mental state m: (i) if $m \oplus a$ is incoherent, then $m \vdash b$, and (ii) if $m \oplus b$ is incoherent, then $m \vdash a$.

Attitudes that are both incompatible and complementary have some important properties:

OBSERVATION 1. If a and $\tilde{a}$ are complementary and incompatible attitudes, and b and $\tilde{b}$ are complementary and incompatible attitudes, then for any coherent m: $m \oplus a \vdash b$ iff $m \oplus \tilde{b} \vdash \tilde{a}$.

Proof. Assume that m is coherent. Assume that $m \oplus a \vdash b$. If $m \oplus a$ is incoherent then $m \vdash \tilde{a}$ by complementarity, and so, by Monotonicity, $m \oplus \tilde{b} \vdash \tilde{a}$. So assume that $m \oplus a$ is coherent. By Monotonicity $m \oplus a \oplus \tilde{b} \vdash b$. By Success $m \oplus a \oplus \tilde{b} \vdash \tilde{b}$ and so, as b and $\tilde{b}$ are incompatible, $m \oplus a \oplus \tilde{b}$ is an incoherent mental state. By Permutation $m \oplus \tilde{b} \oplus a$ is an incoherent mental state. As a and $\tilde{a}$ are complementary, $m \oplus \tilde{b} \vdash \tilde{a}$. A similar argument shows that $m \oplus \tilde{b} \vdash \tilde{a}$ implies $m \oplus a \vdash b$. □

The classical requirements are logically substantial and will ultimately be the constraints that ensure that the logic is classical (rather than, say, intuitionist or dialethist). So, if one wishes, the classical requirements can be weakened in order to account for nonclassical logics. For instance, the classical requirements give us:

as an operator modeling what happens when one 'learns' a (rather than an operator for *hypothetical* revision). However, in hypothetical revision with a, a is to be understood as an *assumption* and assumptions are cumulative. Strong Success thus deviates from the standard AGM-framework but is a feature that one finds, for instance, in 'irrevocable' revision of Segerberg (1998).

[11] The classical requirements correspond in the present setting to what Rumfitt (2000) called 'Smilean Reductio' (after Smiley, 1996), although here formulated in a nonmonotonic setting. Indeed, Dickie (2010) argues that they are central features of the classical conception of negation.

Observation 2. The binary boolean connectives are (classically) interdefinable (satisfy the standard De Morgan equivalences).

Proof. First show that $m \oplus \text{Acc}\neg\varphi = m \oplus \text{Rej}\ \varphi$ and $m \oplus \text{Rej}\neg\varphi = m \oplus \text{Acc}\ \varphi$. By Success $m \oplus \text{Acc}\neg\varphi \vdash \text{Acc}\neg\varphi$. By the acceptance conditions for $\neg$, $m \oplus \text{Acc}\neg\varphi \vdash \text{Rej}\ \varphi$. By Redundancy and Permutation, $m \oplus \text{Acc}\neg\varphi = m \oplus \text{Rej}\ \varphi \oplus \text{Acc}\neg\varphi$. By Success and the acceptance conditions for $\neg$, $m \oplus \text{Rej}\ \varphi \vdash \text{Acc}\neg\varphi$. By Redundancy, $m \oplus \text{Rej}\ \varphi \oplus \text{Acc}\neg\varphi = m \oplus \text{Rej}\ \varphi$ and so $m \oplus \text{Acc}\neg\varphi = m \oplus \text{Rej}\ \varphi$. A similar proof shows that $m \oplus \text{Rej}\neg\varphi = m \oplus \text{Acc}\ \varphi$.

Given this it is trivial to show that $\vee$ and $\wedge$ are interdefinable as their requirements for acceptance and rejection are symmetric. What remains to be shown is that $\supset$ and $\vee$ are interdefinable. As their rejection conditions are symmetric one need only consider their acceptance conditions.

$m \vdash \text{Acc}(\varphi \supset \psi)$ iff $m \oplus \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$ iff $m \oplus \text{Rej}\neg\varphi \vdash \text{Acc}\ \psi$ iff (Observation 1) $m \oplus \text{Rej}\neg\varphi \vdash \text{Acc}\ \psi$ and $m \oplus \text{Rej}\ \psi \vdash \text{Acc}\neg\varphi$ iff $m \vdash \text{Acc}(\neg\varphi \vee \psi)$. ☐

So far no semantic relations between the sentences involved have been presupposed. However, one can use the framework to define a consequence relation. The sentences $\varphi_1, \ldots, \varphi_n$ *expressively entail* ψ, in symbols $\varphi_1, \ldots, \varphi_n \models_{\mathfrak{E}} \psi$, iff for every coherent mental state m in any expressivist model $\mathcal{E}$, if $m \vdash \text{Acc}\ \varphi_1$, ..., $m \vdash \text{Acc}\ \varphi_n$, then $m \vdash \text{Acc}\ \psi$. In the coming sections it will be shown how this 'expressivist' consequence relation can be given a model-theoretic semantics.

As already indicated hypothetical expansion and revision are analogous to their AGM-counterparts. Indeed, one can make the connection explicit. Where m is a mental state let

$$K(m) = \{\varphi : m \vdash \text{Acc}\ \varphi\}.$$

Define the operators $+$ and $*$ as follows:

$$K(m) + \varphi = K(m \oplus \text{Acc}\ \varphi).$$
$$K(m) * \varphi = K(m \circledast \text{Acc}\ \varphi).$$

$K(m)$ – the set of sentences accepted in m – will be a set of sentences closed under the (supra-classical) consequence relation $\models_{\mathfrak{E}}$, and $+$ and $*$ will be AGM-style expansion and revision operators. If we restrict attention to factual sentences the $*$-operator will satisfy all the AGM postulates (1–8) except 5 (inconsistency).

Now, the AGM postulates are often criticized (see e.g. Rott, 2004) as being too strong (in particular 7 and 8) when viewed as postulates governing belief revision. In the present framework they follow, in particular, from R-Vacuity and R-Inclusion, so these could potentially also be considered too strong. However, once again I emphasize that the present operators are supposed to govern *hypothetical* revision not 'real' revision (e.g. revision when new information comes to light), and within the context of hypothetical revision the principles seem more plausible.

§3. A semantic framework. Let W be the set of possible worlds and $\mathcal{B}$ a set-algebra on W. A *selection function* on W is a partial function f from elements of $\mathcal{B}$ to worlds such that there is a nonempty set of worlds D satisfying:

1. For every set of worlds $A \in \mathcal{B}$, $f(A)$ is defined iff $A \cap D \neq \emptyset$.
2. For every set of worlds $A \in \mathcal{B}$, such that $A \cap D \neq \emptyset$: $f(A) \in D \cap A$.

The set D will be called the *modal background* of f and will be denoted MB_f. In addition, a selection function is to satisfy:

3. For $A, B \in \mathcal{B}$: if $f(A) \in B$, then $f(A \cap B) = f(A)$.

This condition ensures that a selection function has a 'preferred' world which always will be selected if possible: the world $f(W)$. Let w_f denote the 'preferred' world of f, that is $w_f = f(W)$.

Where A is an element of $\mathcal{B}$, the *restriction of f to A*, in symbols f/A, is a selection function with modal background $\text{MB}_f \cap A$ such that for every B in its domain:

$$f/A(B) = f(A \cap B).$$

When $\text{MB}_f \cap A = \emptyset$, let $f/A = \emptyset$. It is easy to show that if f/A is nonempty, then it is a selection function according to (1–3) above.

A key target for the semantic analysis is to show that it can semantically characterize the class of expressivist models. A central contention is that the propositional content of a belief state cannot be adequately represented by its representational or factual content, that is, it cannot be adequately represented as merely a set of worlds. We need the further structure afforded by *sets of selection functions*.

With this in mind let a *proposition* be a set of selection functions. The factual or representational content of a proposition P is its set of preferred worlds:

$$F(P) = \{w_f : f \in P\}.$$

A proposition P is said to have a *purely factual content* iff for every $w \in F(P)$: if f is a selection function such that $w_f = w$, then $f \in P$. That is, a proposition P has purely factual content if it doesn't in any way differentiate between selection functions over and above their preferred worlds.

A proposition P is said to have a *fixed modal background* iff for any $f, f' \in P$: $\text{MB}_f = \text{MB}_{f'}$. When P has a fixed modal background we can let $\text{MB}_P = \text{MB}_f$ for some arbitrary $f \in P$.

A proposition P is said to *prefer its own factual content* iff for every $f \in P$ and $A \in \mathcal{B}$: if $A \cap F(P) \neq \emptyset$, then $f(A) \in F(P)$.

A proposition P is said to be *modally reflexive* if it both has a fixed modal background and prefers its own factual content.

When P is a proposition with a fixed modal background and $A \in \mathcal{B}$, let

$$P/A = \begin{cases} \{f/A : f \in P\}, \text{ if } \text{MB}_P \cap A \text{ is nonempty,} \\ \emptyset, \text{ otherwise.} \end{cases}$$

3.1. A model for the language. An *ICL-model* is a triple $(W, \mathcal{B}, V)$ where W is a set of worlds, $\mathcal{B}$ is a set-algebra, and V is a function such that $V(p) \in \mathcal{B}$, for each sentential atom p in the language.

A sentence φ will be said to be *true relative to model M and a selection function f*, and this relation is written $f \models_M \varphi$ (correspondingly, where P is a set of selection functions, let $P \models_M \varphi$ hold iff $f \models_M \varphi$ for all $f \in P$). Define:

$$|\varphi|_M = \{f : f \models_M \varphi\}.$$

$|\varphi|_M$ is the proposition expressed by the sentence φ in the model M. Correspondingly, $F(|\varphi|_M)$ will represent the factual content of φ in M (i.e. the set of worlds not excluded by φ in M). When it is clear from context reference to the model will be suppressed. To simplify notation I will often write f/φ instead of $f/F(|\varphi|)$.

One can now give the truth-conditions for the language relative to a model M and a selection function f:

$$f \models p \text{ iff } w_f \in V(p).$$
$$f \models \neg\varphi \text{ iff } f \not\models \varphi.$$
$$f \models \varphi \wedge \psi \text{ iff } f \models \varphi \text{ and } f \models \psi.$$
$$f \models \varphi \vee \psi \text{ iff } f \models \varphi \text{ or } f \models \psi.$$
$$f \models \varphi \supset \psi \text{ iff } f \not\models \varphi \text{ or } f \models \psi.$$
$$f \models \varphi \equiv \psi \text{ iff } f \models \varphi \supset \psi \text{ and } f \models \psi \supset \varphi.$$
$$f \models \varphi \to \psi \text{ iff } f/\varphi \models \psi \text{ or } f/\varphi = \emptyset.$$

Note in particular the truth conditions for the conditional. To evaluate a conditional $\varphi \to \psi$ relative to a selection function is to evaluate the consequent ψ relative to the selection function f constrained by the factual content of the antecedent φ.

3.2. Left-nested conditionals. The above semantics will not work as intended when conditionals are allowed to embed arbitrarily in the antecedent of a conditional. In fact, to get a plausible semantics for such constructs I think one needs more semantic structure than is given by selection functions alone. At this stage however, given the triviality results, it can be helpful to consider even a semantics that is not very plausible, as long as it demonstrates that adding left-nested conditionals to the language does not collapse the expressivist framework. With that goal in mind, I will define some new notions and provide a semantics that at least allows us to demonstrate nontriviality.

Define, for any selection function f and set of selection functions P:

$$f \textit{ forces } P \text{ iff } f/A \in P \text{ for all } A \in \mathcal{B} \text{ such that } A \cap \textsc{mb}_f \neq \emptyset.$$

Furthermore, define:

$$U_{f,P} = \text{ the inclusion maximal } A \in \mathcal{B} \text{ such that } f/A \text{ forces } P.$$

If it is not the case that there is a unique inclusion maximal $A \in \mathcal{B}$ such that f/A forces P, then set $U_{f,P} = \emptyset$. Define:

$$f//P = f/U_{f,P},$$

Note that, trivially, $f//P$ forces P, when $f//P \neq \emptyset$. Furthermore, when P has a purely factual content, $f//P = f/F(P)$. So the following revised truth-conditions will not change the behaviour of conditionals with factual antecedents, but will allow for a nontrivial (although, ultimately, perhaps not very plausible) semantics of left-embedded conditionals

$$f \models \varphi \to \psi \text{ iff } f//|\varphi| \models \psi \text{ or } f//|\varphi| = \emptyset.$$

Why is this an implausible semantics? One reason is that it will validate (for factual φ and ψ):

$$(\varphi \to \psi) \to \chi \equiv (\varphi \supset \psi) \to \chi.$$

When an indicative conditional appears in the antecedent of an indicative conditional it reduces to the material conditional; this is a principle we have reason to be suspicious of. Borrowing an example from McGee (2018):

(2) If Mimi moves to Aruba if she wins the lottery, she really likes the beach.

'Materialising' the antecedent we get:

(3) If Mimi moves to Aruba or doesn't win the lottery, she really likes the beach.

The two do not seem to be equivalent.

[Note that a semantics that doesn't 'materialise' left-embedded conditionals in the above fashion would need to violate the principle that if $\varphi \to \psi$ and $\psi \to \varphi$ are both logically valid, then $(\varphi \to \chi) \equiv (\psi \to \chi)$ is logically valid. For quite apart from the semantics of left-embedded conditionals we have the following validity (for factual φ and ψ): $(\varphi \supset \psi) \to (\varphi \to \psi)$, i.e. $(\neg\varphi \vee \psi) \to (\varphi \to \psi)$ (not to be confused with the nonvalidity $(\varphi \supset \psi) \supset (\varphi \to \psi)$): this follows from the fact that $((\varphi \supset \psi) \wedge \varphi)$ logically entails ψ and so given CN (see next section for the logical principles) $((\varphi \supset \psi) \wedge \varphi) \to \psi$ is valid and so, given Import-Export (again see next section), $(\varphi \supset \psi) \to (\varphi \to \psi)$. Furthermore, given CN and that $\varphi \to \psi$ logically entails $\varphi \supset \psi$ when φ and ψ are factual we would also expect $(\varphi \to \psi) \to (\varphi \supset \psi)$ to be valid (indeed it is on the present semantics). So we quite naturally get both $(\varphi \supset \psi) \to (\varphi \to \psi)$ and $(\varphi \to \psi) \to (\varphi \supset \psi)$ as logical validities. What is questionable, however, is that this should lead to the validity of the equivalence: $((\varphi \supset \psi) \to \chi) \equiv ((\varphi \to \psi) \to \chi)$. We get the latter in the current semantics.]

3.3. The logic. *Indicative conditional logic*, ICL, is the smallest Tarskian consequence relation $\models_{\mathrm{ICL}}$ that satisfies:[12]

MP $\varphi, \varphi \supset \psi \models_{\mathrm{ICL}} \psi$.
CL $\models_{\mathrm{ICL}} \tau$, for any classical tautology τ.

CN If $\varphi \models_{\mathrm{ICL}} \psi$, then $\models_{\mathrm{ICL}} \varphi \to \psi$.
CK If $\psi_1, \dots, \psi_n \models_{\mathrm{ICL}} \chi$, then $\varphi \to \psi_1, \dots, \varphi \to \psi_n \models_{\mathrm{ICL}} \varphi \to \chi$.
LLE If $\models_{\mathrm{ICL}} \varphi \equiv \psi$, then $\models_{\mathrm{ICL}} (\varphi \to \chi) \equiv (\psi \to \chi)$.

CEM $\models_{\mathrm{ICL}} \varphi \to \psi \vee \varphi \to \neg\psi$.

For factual φ and ψ (where χ can be modal):

Import-Export $\models_{\mathrm{ICL}} \varphi \to (\psi \to \chi) \equiv (\varphi \wedge \psi) \to \chi$.
rMP $\varphi, \varphi \to \psi \models_{\mathrm{ICL}} \psi$.
rCS $\varphi, \psi \models_{\mathrm{ICL}} \varphi \to \psi$.

The second half of the following result is originally due to McGee (1989) but is restated and the completeness part is also proven here:

THEOREM 1. *ICL is sound with respect to L, and sound and complete with respect L_R.*

12 So, $\models_{\mathrm{ICL}}$ is a relation between sets of sentences and sentences (taken from either L or L_R) such that (i) $\Gamma, \varphi \models_{\mathrm{ICL}} \varphi$, (ii) if $\Gamma \models_{\mathrm{ICL}} \varphi$, then $\Gamma \cup \Delta \models_{\mathrm{ICL}} \varphi$, and (iii) if $\Gamma, \varphi \models_{\mathrm{ICL}} \psi$ and $\Gamma \models_{\mathrm{ICL}} \varphi$, then $\Gamma \models_{\mathrm{ICL}} \psi$. Throughout I will use the standard notational convention that $\Gamma \cup \{\varphi\} = \Gamma, \varphi$.

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(Proofs of theorems are found in Appendix B). One can now also characterize the notion of expressivist entailment provided in Section 2.

THEOREM 2. *The expressivist consequence relation* $\models_{\mathfrak{E}}$ *on* L_R *is exactly* $\models_{ICL}$.

The following property indicates the distinctive nature of ICL:

THEOREM 3. *Any sentence* φ *in* L_R *is logically equivalent to a disjunction of logically incompatible disjuncts* $\delta_1 \vee \cdots \vee \delta_n$ *where each* δ_j *has the form*

$$\bigwedge_{1 \leq i \leq k} (\psi_i \rightarrow a_i) \wedge \bigwedge_{1 \leq i \leq m} \neg(\chi_i \rightarrow \perp).$$

Here each a is a (possibly negated) atomic sentence or an arbitrary contradiction (denoted by $\perp$*).*

That is: any sentence can be equivalently written in a disjunctive normal form where conditionals have noncomplex consequents. It is a nonstandard property for conditional logics (although a corresponding property holds, of course, for the material conditional), and it can be traced back to normality and the combined validity of Conditional Excluded Middle (CEM) and Import-Export.

Given the semantics it is easy to ascertain that the logic has some of the standard properties that we would expect on bare conditionals governed by a selection-function semantics, e.g. the failure of antecedent strengthening ($\varphi \rightarrow \psi$ does not entail $(\varphi \wedge \chi) \rightarrow \psi$) and *modus tollens* ($\varphi \rightarrow \psi$ does not entail $\neg\psi \rightarrow \neg\varphi$). In addition, due to LEM we have:

$$\neg(\varphi \rightarrow \neg\varphi) \models_{\text{ICL}} \neg(\varphi \rightarrow \psi) \equiv \varphi \rightarrow \neg\psi.$$

Given that the antecedent of a conditional is not contradictory, negation will distribute across the conditional.

The logic also supports the Import-Export principle (with factual antecedents) and, consequently, only a *restricted* version of Modus Ponens (it need not be valid when the consequent is nonfactual). These are known to be interconnected; for if we have both Import-Export and unrestricted modus ponens the conditional collapses into the material conditional (the proof is due to Gibbard, 1981: by CN $((\varphi \supset \psi) \wedge \varphi) \rightarrow \psi$; by Import-Export $(\varphi \supset \psi) \rightarrow (\varphi \rightarrow \psi)$; if $\rightarrow$ satisfied unrestricted modus ponens we would thus get $\varphi \rightarrow \psi$ from $\varphi \supset \psi$).

McGee (1985) has provided a well-known counterexample to unrestricted modus ponens:

> Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:
>
> (i) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
> (ii) A Republican will win the election.
> Yet they did not have reason to believe
> (iii) If it's not Reagan who wins, it will be Anderson. (McGee, 1985, p.462, numbering added).

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Crucially, the premise (i) has the form $\varphi \to (\psi \to \chi)$: a conditional with a conditional in the consequent (there is no suggestion that modus ponens fails for factual consequents). Now, the counterexample is by no means uncontested. Some deny its status as a counterexample by claiming that it illicitly appeals to the Import-Export principle: if people 'illicitly' interpret $\varphi \to (\psi \to \chi)$ as $(\varphi \wedge \psi) \to \chi$ then – the argument goes – the counterexample evaporates as we would not expect φ together with $(\varphi \wedge \psi) \to \chi$ to entail $\psi \to \chi$ (consider the case where we believe ψ to be false). But this explanatory strategy fails if the reason why people interpret $\varphi \to (\psi \to \chi)$ as $(\varphi \wedge \psi) \to \chi$ is that the two are in fact equivalent (and so taking them to be equivalent is not 'illicit'). Indeed, while the empirical data is not conclusive, there are good reasons to think that speakers treat them as equivalent.[13]

Others concede that McGee's counterexample is effective 'on the surface' but argue that modus ponens should not be understood as a syntactic principle, instead modus ponens should be viewed as a principle working on propositional contents (e.g. Schulz, 2018). Indeed, one can note that with a restrictor style semantics there is at least one sense in which a sentence in the consequent of a conditional can be viewed as having a different semantic content than when it stands alone. For instance, $|\psi \to \chi|$ (the proposition expressed by the stand-alone conditional) is not the same proposition as $|\psi \to \chi|/F(|\varphi|)$ (the proposition expressed by the conditional after it has been constrained by φ). So, it is argued, if we allow for some such semantic shift, the counterexample is not really a counterexample to the schema $A, A \Rightarrow B \therefore B$ where A and B are propositions, but rather to something like $A, A \Rightarrow B \therefore B'$, where B and B' are distinct propositions, a schema we would not expect to hold in any case. While this may be a plausible explanation it doesn't challenge McGee's contention that at the level of sentence use, unrestricted modus ponens is not valid.[14]

§4. Semantic interpretation of expressivist models. A *semantic interpretation* of an expressivist model $\mathcal{E}$ is a pair $(M, ||.||)$ where M is a semantic model and $||.||$ assigns propositional content to all the mental states and attitudes of the expressivist model, such that:

[13] Importantly, the literature contains no counterexample to Import-Export for the indicative conditional in its linguistic form. If it were invalid one could expect this to be demonstrated by some natural counterexample. Still, Douven & Dietz (2011) (also discussed in Douven, 2015) report that speakers probability assignments for a $\psi \to \chi$ *assuming that* φ differ from their probability assignments for $(\varphi \wedge \psi) \to \chi$, a result that would seem to undermine Import-Export. However, van Wijnbergen-Huitink, Elqayam, & Over (2014) testing Import-Export directly found strong evidence in its support and suggest that the contrary results of Douven and Dietz could be explained by pragmatic factors.

[14] Interestingly, in their recent article Khoo & Mandelkern (2019) the authors' argue that while Import-Export appears valid on the 'sentential' level (that is, at the level of English syntax) it is questionable whether it is valid on a deeper propositional level. Thus while one can make the case that modus ponens is valid on a deeper propositional level but not on the sentential, with Import-Export it seems to be the other way around. So even if modus ponens is valid on some deeper propositional level, the corresponding failure of Import-Export at the deeper propositional level would ensure that the conditional, on that level, does not – *pace* Gibbard – collapse to the material conditional. This is an interesting line of inquiry, however, it does not affect the sentential validity of Import-Export, which is the property that is crucial in the current analysis.

1. For every mental state m: $||m||$ is modally reflexive (i.e. it has a fixed modal background and prefers its own factual content).
2. For every sentence φ:
   (a) $||\text{Acc } \varphi|| = |\varphi|_M$.
   (b) $||\text{Rej } \varphi|| = |\neg\varphi|_M$.
3. For every mental state m and attitude a in the model: $m \vdash a$ iff $||m|| \subseteq ||a||$.

So the propositional content of the attitude of accepting φ is the proposition expressed by φ, and the propositional content of the attitude of rejecting φ is the proposition expressed by $\neg\varphi$. A semantic interpretation ensures that:

$$m \vdash \text{Acc } \varphi \text{ iff } ||m|| \subseteq |\varphi|_M.$$
$$m \vdash \text{Rej } \varphi \text{ iff } ||m|| \subseteq |\neg\varphi|_M.$$

So which expressivist models have a semantic interpretation?

THEOREM 4. *Every expressivist model on L_R has a semantic interpretation.*

COROLLARY 1. *Let $(M, ||.||)$ be a semanticinterpretation of $\mathcal{E}$ such that the interpretation of every mental state m is a 'closed' proposition*:

$$||m|| = \bigcap\{|\varphi|_M : ||m|| \subseteq |\varphi|_M\};$$

then:

1. $||m \oplus \text{Acc } \varphi|| = ||m|| \cap |\varphi|_M$.
2. $||m \circledast \text{Acc } \varphi|| = ||m||/F(|\varphi|_M)$.
3. $m \vdash \text{Acc}(\varphi \to \psi)$ *iff* $||m||/F(|\varphi|_M) \subseteq |\psi|_M$.

This gives a 'nice' *semantic* rendition of the Ramsey Test: an agent accepts $\varphi \to \psi$ iff the agent holds ψ true on restricting the possible worlds to the φ-worlds. It also establishes a connection to dynamic semantic frameworks (e.g. Kolodny & MacFarlane, 2010; Yalcin, 2012; Starr, 2014a, 2014b): $||m||$ can be regarded as an 'information state' and $||m||/F(|\varphi|_M)$ as an 'update' of the information state. So to 'test' a conditional is thus to test whether the consequent is true on updating with the antecedent. An important difference, however, is that in dynamic semantic frameworks the 'information state' is typically represented as a set of worlds (or world-like entities) whereas here it becomes a set of selection functions: it is this added structure that allows us to represent expressivist models semantically.

So it has been established that the semantic apparatus provides a semantic interpretation of the expressivist structures invoked in expressivist models. But are there any nontrivial expressivist models to begin with? That is: does the expressivist framework avoid the threat of triviality? Indeed it does.

An expressivist model $\mathcal{E}$ is *generated from* a semantic model M if there is a semantic interpretation $(||.||, M)$ of $\mathcal{E}$ such that for every modally reflexive proposition P there is some mental state m in $\mathcal{E}$ such that $||m|| = P$.

An expressivist model generated by a semantic model is thus an expressivist model where any modal proposition with a fixed modal background constitutes the content of some mental state. If one can prove that there are 'nontrivial' semantic models (e.g. models with more than three worlds) that can generate an expressivist model this would establish that there are 'nontrivial' expressivist models. Indeed, one can prove something stronger:

THEOREM 5. *Every semantic model (on the full language L or on the restricted language L_R) generates an expressivist model.*

§5. Discussion.

5.1. McGee: ICL and restrictor semantics for probability measures. McGee (1989) was first to characterize ICL with a restrictor semantics. But in his rich paper this was a mere side-result stemming from the real goal of the paper. McGee showed that one can define a function $*$ that for any probability measure Pr on factual sentences can be extended to an ICL-based probability measure Pr^* on L_R[15] in such a way that:

$$\mathrm{Pr}^*(\varphi \to \psi) = \mathrm{Pr}^*(\psi \mid \varphi) = \mathrm{Pr}(\varphi \wedge \psi)/\mathrm{Pr}(\varphi),$$

provided (i) that $\mathrm{Pr}(\varphi) \neq 0$, and (ii) that both φ and ψ are factual, thus (letting probabilities correspond to degrees of belief or acceptability) satisfying a restricted version of the quantitative Ramsey Test. Let us call this the *restricted probabilistic Ramsey Test* (similar to, but due to the restrictions, not identical with Adams' Thesis/Stalnaker's hypothesis). The restriction (ii) is what blocks Lewis' impossibility theorem: we can maintain the graded Ramsey Test if we restrict it to conditionals with factual consequents. If we think of the probabilistic Ramsey Test as an axiom this suggests an analysis that is 'silent' on all conditionals that contain modally complex consequents.

Now, McGee proved something stronger. He noted that from the plausible *Simple Independence* (SI) one could derive the restricted Ramsey Test.

> **(SI)** Given that φ, ψ and χ are factual sentences, that φ and χ are logically incompatible and that $\mathrm{Pr}(\varphi) \neq 0$:
>
> $$\mathrm{Pr}(\chi \wedge (\varphi \to \psi)) = \mathrm{Pr}(\chi) \times \mathrm{Pr}(\varphi \to \psi).$$

McGee then showed that if one adds the requirement that a probability measure on L_R satisfies the following property of *Generalized* Independence GI (all sentence letters are factual sentences, χ is logically incompatible with each of φ_i, all antecedents are assumed to have nonzero probability):

$$\mathrm{Pr}(\chi \wedge \varphi_1 \to \psi_1 \wedge \cdots \wedge \varphi_n \to \psi_n) = \mathrm{Pr}(\chi)\mathrm{Pr}(\varphi_1 \to \psi_1 \wedge \cdots \wedge \varphi_n \to \psi_n), \qquad \text{(GI)}$$

then any probability measure on the factual propositions has a *unique* extension to L_R. However, GI has been criticized as being too strong (Lance, 1991; Edgington, 1995) and it is subject to plausible counterexamples. Indeed, to ask for uniqueness of extensions here is perhaps to ask for too much. Lance (1991) suggests:

> it is no more likely that there will be a general formula for determining the probability of $(\varphi \to \psi) \wedge (\chi \to \theta)$ on the basis of the probabilities of the conjuncts (and their components) than that there will be a

[15] That is: Pr^* satisfies: (1) $0 \leq \mathrm{Pr}^*(\varphi) \leq 1$, (2) $\mathrm{Pr}^*(\neg\varphi) = 1 - \mathrm{Pr}^*(\varphi)$, and (3) $\mathrm{Pr}^*(\varphi \vee \psi) = \mathrm{Pr}^*(\varphi) + \mathrm{Pr}^*(\psi)$ whenever $\varphi \models_{\mathrm{ICL}} \neg\psi$. Any measure satisfying these properties will ensure that all ICL-valid sentences acquire probability 1, and only ICL-valid sentences acquire probability 1 in all measures with these properties.

> general formula for determining the probability of $\varphi \wedge \psi$ on the basis of that of φ and of ψ. (p. 275, notation changed)

Note, however, that McGee didn't show that *every* ICL-based extension of Pr to Pr* will satisfy GI, he only showed that there is a unique extension of Pr to Pr* *given* the constraint that Pr* should satisfy GI. Thus there is nothing in the ICL-based framework itself that forces us to treat GI as a valid principle.[16]

So McGee demonstrated that ICL at least caters for a restricted form of the probabilistic Ramsey Test. Is there a way of strengthening this? Due to completeness we can (for a given model) define a probability measure on the space of propositions expressed by some sentence in the language with the property $\Pr(\varphi) = p(|\varphi|)$. As we are now dealing directly with propositions we can do formal justice to the idea (echoing the discussion on modus ponens and Import-Export above) that the reason why we cannot get the conditional version of the restricted Ramsey Test is that when the conditioned sentence is a conditional, the conditional expresses a different proposition than when it stands alone.[17]

The conditional version of the restricted Ramsey Test states that for factual φ, ψ and χ:

$$\Pr(\varphi \to \psi \,|\, \chi) = \Pr(\psi \,|\, \varphi \wedge \chi).$$

Under very weak assumptions (c.f. Fitelson, 2015) this is known to give rise to triviality. However, if we allow that a conditional can express a different proposition when conditioned upon than it does stand-alone, the principle can be satisfied. Let:

$$|\varphi|_\psi = (|\psi \to \varphi| \cap |\psi|) \cup (|\varphi| \cap -|\psi|).$$

Intuitively: $|\varphi|_\psi$ is the proposition φ expresses *given* ψ (from, as it were, an 'outside' perspective).

When φ is factual $|\varphi|_\psi = |\varphi|$. This follows trivially from the fact that when φ and ψ are factual, $|\psi \to \varphi| \cap |\psi| = |\varphi| \cap |\psi|$.

Now *define* (assuming $p(|\psi|) \neq 0$):

$$\Pr(\varphi \,|\, \psi) =_{df} \frac{p(|\varphi|_\psi \cap |\psi|)}{p(|\psi|)}.$$

This is (nearly) the usual quotient definition of conditional probability, but now with the added twist that we allow that the proposition expressed by φ given ψ depends on ψ. This analysis of conditional probability is probabilistically well-behaved:

OBSERVATION 3. If $\Pr(\psi) > 0$: (i) $0 \leq \Pr(\varphi \,|\, \psi) \leq 1$, (ii) $\Pr(\neg\varphi \,|\, \psi) = 1 - \Pr(\varphi \,|\, \psi)$, and (iii) $\Pr(\varphi \vee \chi \,|\, \psi) = \Pr(\varphi \,|\, \psi) + \Pr(\chi \,|\, \psi)$, whenever $\varphi \models_{ICL} \neg\chi$.

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16 One strategy is to abandon the selection function framework adopted here and by McGee and work with a Stalnaker–Bernoulli space as in Kaufmann (2009) where probabilities are assigned to conditionals in a way that avoids some of the problems with GI. However, this strategy will not be pursued any further here.

17 The idea is old, going back to van Fraassen (1976) and has been revived a number of times, e.g. Douven (2015) (chap. 2) contains a lengthy discussion. Part of the problem lies in finding a semantic 'shiftiness' that actually delivers. The approach sketched by Khoo & Mandelkern (2019) comes, I think, close in spirit to the present approach, but their suggestion is not spelled out in detail.

(Proofs are again in the Appendix).

Moreover, it is easy to see that for factual φ and ψ:

$$\Pr(\varphi \mid \psi) = \frac{\Pr(\varphi \wedge \psi)}{\Pr(\psi)}.$$

We *wont* get:

$$\Pr(\varphi \to \psi \mid \chi) = \frac{\Pr((\varphi \to \psi) \wedge \chi)}{\Pr(\chi)}.$$

So it is a nonstandard analysis of conditional probability (that is the whole point). Given SI, we do, however, get the probabilistic Ramsey Test for arbitrarily right-nested conditionals.

OBSERVATION 4. If (i) Pr satisfies (SI), (ii) $\Pr(\theta) = 0$ implies $\Pr(\theta \to \sigma) = 1$, and (iii) $\Pr(\varphi \wedge \chi) \neq 0$, then for factual $\psi, \varphi_1, \dots, \varphi_n$ and χ:

$$\begin{aligned} \Pr(\varphi_1 \to (\varphi_2 \to \cdots (\varphi_n \to \psi) \cdots) \mid \chi) &= \Pr(\varphi_2 \to \cdots (\varphi_n \to \psi) \cdots) \mid \varphi_1 \wedge \chi) \\ &\vdots \\ &= \Pr(\psi \mid \varphi_1 \wedge \cdots \wedge \varphi_n \wedge \chi). \end{aligned}$$

Indeed, if we assume GI then we get the full unrestricted Ramsey Test, applying to consequents of arbitrary complexity.

OBSERVATION 5. If (i) Pr satisfies (GI), (ii) $\Pr(\theta) = 0$ implies $\Pr(\theta \to \sigma) = 1$, and (iii) $\Pr(\varphi \wedge \chi) \neq 0$, then for factual φ and χ (ψ can be any sentence of L_R):

$$\Pr(\varphi \to \psi \mid \chi) = \Pr(\psi \mid \varphi \wedge \chi).$$

This shows that a completely unrestricted probabilistic Ramsey Test is attainable under the assumption that the proposition expressed by a sentence depends in a structured way on the conditioning sentence. But it relies on the principle GI that we have independent reasons to be suspicious of.

However, the standard quotient analysis of conditional probability – rather than being a *definition* – is but one *analysis* of conditional probability. In line with the theses of the present paper it would seem appropriate to adopt a *suppositional* analysis of conditional probabilities. We thus seek a suppositional operator $p * A$ on the probability measure p (where A is a set of worlds), without any *a priori* commitment that such an operator can be analyzed by the standard quotient rule (although, clearly, a criterion for the success of the analysis is that it coincides with the quotient analysis in the 'normal' cases). Indeed, following Corollary 1, the analysis that corresponds to the categorical case would be:[18]

[18] Again this is equivalent to McGee's analysis where he *defines* $\Pr_\varphi(\psi)$ as $\Pr(\varphi \to \psi)$. Here $\Pr_\varphi$ is the analogue of $p * F(|\varphi|)$. Indeed McGee provided a diachronic Dutch Book argument to the effect that $p * A$ represents the probability that one should acquire upon learning A for certain; thus the analysis has one of the properties that we associate with conditioning.

$$p * A(X) =_{df} p(\{f \mid f/A \in X\} \cup \{f \mid f/A = \emptyset\}).$$
$$\Pr(\psi \mid \varphi) =_{df} p * F(|\varphi|)(|\psi|).$$

On this analysis the unrestricted probabilistic Ramsey Test follows trivially (although, of course – absent a plausible account of left-embedded conditionals – the restriction that the conditioning sentence be factual still stands) while the standard quotient analysis of conditional probability only covers the special case when the consequent is nonmodal. Under the new definition of $\Pr(\cdot \mid \cdot)$, Observations 4 and 5 could then be reinterpreted as revealing ways of partially 'saving' the quotient analysis by appeal to the shiftiness of content when conditionals occur in the consequent.

5.2. Varieties of epistemic possibility. The expressivist framework allows a distinction between two kinds of epistemic possibility, 'strong' epistemic possibility and 'weak' epistemic possibility. A sentence φ, intuitively, is *weakly possible* if one can coherently *revise* with φ. Meanwhile, φ is *strongly possible* if one can coherently *expand* with φ, that is, φ is strongly possible if one doesn't accept $\neg\varphi$.

Weak possibility can be readily defined in the existing language:

$$\Diamond_w \varphi =_{df} \neg(\varphi \rightarrow \neg\varphi).$$

Strong possibility, the kind of possibility that comes closest to being captured by the English word "Might" or "Maybe," cannot be defined in the language by existing means, and adding it forces us to extend the expressivist and the semantic framework. Here I will only briefly consider an extension of the semantic framework (for a bilateral inferentialist treatment see Incurvati & Schlöder, 2019). We extend our restricted language L_R to L'_R (which contains L_R) so that Might φ is a sentence in L'_R whenever φ is a sentence in L'_R. Might-sentences are assumed to embed freely, except in the antecedent of conditionals.[19] A point of evaluation will now be a pair X, f where X is a modally reflexive proposition (has a fixed modal background that prefers its own content), and where $f \in X$.

The truth-clauses for atomic sentences and boolean connectives will be independent of the X-parameter and will be the same as in the original semantics. For the might-modal we chose a standard clause (see, e.g. Yalcin, 2007; MacFarlane, 2014)):

$$X, f \models \text{Might } \varphi \text{ iff } \exists f' \in X : X, f' \models \varphi.$$

For the conditional we get:[20]

$$X, f \models \varphi \rightarrow \psi \text{ iff } f/\varphi \neq \emptyset \text{ or } X/\varphi, f/\varphi \models \psi.$$

[19] The reason for avoiding might-claims in the antecedent of a conditional is the same as for mostly omitting talk of left-embedded conditionals: the only way of getting a remotely plausible semantics for such constructs given a restrictor semantics for the conditional is by adding new elements to the formal framework, and this goes beyond the scope of the paper. For recent work illustrating some of the problems one must deal with, see Yalcin, 2007; Russell & Hawthorne, 2016; Mandelkern, 2019.

[20] $|\varphi|$ is now defined as the set $\{w_f \mid \text{there exists } X \text{ such that } f \in X \text{ and } X, f \models \varphi\}$. Here we can see the source of the problem of allowing a might-modal in the antecedent: its propositional content wont correspond in any interesting way to its factual content, and it is the latter that we use to constrain the modal background. To make sense of might-modals in the antecedent of a conditional we would thus need to enrich the notion of 'constraining' the modal background.

We can still represent a mental state as a modally reflexive proposition X. But now we say:

$$X \text{ accepts } \varphi \text{ iff } X, f \models \varphi \text{ for all } f \in X.$$

Note that X accepts Might φ iff X does not accept $\neg\varphi$.

This analysis also squares with other observed features of the might-modality. For instance, we have that for *no* nonempty X:

$$X \text{ accepts both } \varphi \text{ and Might } \neg\varphi.$$

This seems right: no coherent agent would accept "It is raining and it might not be raining." Furthermore, it gives us a way of linguistically expressing Restricted Or-to-If so that it holds valid in our expressivist logic:

(rOr-to-If) If X accepts $\varphi \vee \psi$, and Might $\neg\varphi$, then X accepts $\neg\varphi \to \psi$.

We also get a restricted form of SDA (Strengthening of disjunctive antecedents) for factual sentences:

(rSDA) If X accepts $(\varphi \vee \psi) \to \chi$, Might φ, and Might ψ, then X accepts $(\varphi \to \chi) \wedge (\psi \to \chi)$.

The 'weaker' epistemic possibility $\Diamond_w$ has neither of these features: a coherent agent can accept $\varphi \wedge \Diamond_w \neg\varphi$, furthermore, a coherent agent can accept both $\varphi \vee \psi$ and $\Diamond_w \neg\varphi$ without accepting $\neg\varphi \to \psi$.

A central feature of the AGM-framework is that it allows for coherent nontrivial revision with propositions that go contrary to what one believes. The present framework correspondingly allows for coherent nontrivial hypothetical revision with propositions that go contrary to what one believes: it allows an agent to explore the possibility that the agent is wrong in some respect. As conditionals are interpreted in terms of the hypothetical revision operator, the present framework thus allows that *belief-contravening* conditionals – conditionals with an antecedent that one believes false – can nontrivially be accepted or rejected. Such belief-contravening conditionals are not 'counter-factual' in the usual sense. Compare the familiar pair:

1. If Shakespeare hadn't written Hamlet someone else would have.
2. If Shakespeare didn't write Hamlet someone else did.

Most would reject (1) and accept (2). The antecedent of (1) is a standard counterfactual: it explores an alternative way that the world could have evolved. The antecedent of (2) is a belief-contravening possibility: it explores the possibility that contrary to what we believe – nay: *know*! – Shakespeare wasn't the author of Hamlet.

Of course, what one is willing to accept, or what one takes oneself to know, can depend on context. Different contexts can, for instance, come with different standards of evidence. But in hypothetically considering a possibility that one rejects one need not be taken to be seriously questioning the standards of evidence on which the rejection is based. Compare the following pair:

(4) Shakespeare wrote Hamlet, I don't doubt that, but *say* that he didn't.
(5) Shakespeare wrote Hamlet, I don't doubt that, but *maybe* he didn't. (??)

It makes perfect sense (as in (4)) to invite an audience to consider as true a possibility that they reject, and one can do so without suggesting that one is seriously challenging the falsity of the proposition. The 'might'- or 'maybe'-modality can't (compare (5)) be made to play the same function. But the conditional can. For the following makes good sense:

(6) Shakespeare wrote Hamlet, I don't doubt that, but *if* he didn't, then someone else did.

When framed in the right way one can allow oneself to make a belief-contravening conditional claim without signaling a serious challenge to the falsity of the antecedent (it helps if one puts emphasis on the 'if'). Here the antecedent of the conditional – "if he didn't..." – will function in much the same way as the suppositional injunction – "say that he didn't."

The idea that one can meaningfully accept and assert belief-contravening conditionals is controversial. It has become standard lore that the antecedent of a conditional must be possible with respect to what is mutually accepted in a context. But the claim here is that one can *use* a conditional to widen the space of possibilities allowed by context, thus what is taken to be known or believed in a context doesn't by itself put any hard border on what kind of conditionals that can be accepted or asserted in a context. Those who deny this usually appeal to the oddness of pairs of assertions like:

(7) Jim didn't win on the lottery. If Jim won on the lottery, he is happy. (??)

Clearly, it usually sounds odd or infelicitous to first assert φ and then assert "If $\neg\varphi$...." But to take this as direct evidence against the idea that one can believe and assert belief-contravening conditionals is to disregard important elements of the pragmatics of assertion. Typically, when one makes an assertion, one doesn't merely express that one is willing to accept that the assertion is true, one is in addition actively trying to constrain what it is to be considered possible in the context. There is a difference between what is tacitly taken for granted and an explicit assertion which is an attempt to take some possibilities off the table in the common ground. In the assertion of a conditional, by contrast, one is trying to open up the space of possibilities in order to allow for the antecedent to be true. In (7) there is a clash as one is first trying to conversationally exclude possibilities and then trying to open up the same possibilities. So the oddness can be explained by purely pragmatic means. Indeed, one can override the oddness of this pragmatic clash in various ways. For instance, the following does not come out nearly as odd:

(8) Jim didn't win on the lottery, but *if* he won, he is happy.

The phenomenon has a not-to-distant analogue in disjunctions. An assertion of a disjunction like

(9) Either the butler did it or the gardener did

typically would signal that the speaker holds both disjuncts open. This is witnessed by the oddity of:

(10) The butler did it. Either the butler did it or the gardener did. (??)

But this pragmatic effect can be overridden:

(11) Either the butler did it or the gardener did. But it wasn't the butler, so it was the gardener.

Several authors have argued (e.g. Veltman, 1985, 1996); Gillies, 2009; Starr, 2014a) that *as* conditionals with ‘impossible’ antecedents (impossible with respect to what is taken for granted in context) are infelicitous, much of the linguistic data that has driven the search for a semantics for indicative conditionals in the last decades can be given a nonsemantic/pragmatic explanation. Consider a standard example illustrating how indicative conditionals do not satisfy the principle of antecedent strengthening.

(12) If there is sugar in the coffee, it tastes sweet.
So: if there is sugar *and diesel* in the coffee, it tastes sweet. (??)

Someone who is in position to assert the premise need not be in a position to assert the conclusion, and this has traditionally been taken to show that antecedent strengthening is not a semantically valid mode of inference when indicative conditionals are involved. But – the argument goes – this misses something important. For one wouldn’t accept the premise of the argument if one thought that the coffee might contain diesel. So one should accept the premise only if one believes that there is no diesel in the coffee. But then the conclusion is a belief-contravening conditional and this, the presumption is, disqualifies it on its own by pragmatic effects. So, the argument has been, we don’t need a *semantic* explanation for the failure of antecedent strengthening, the failure of transitivity, the failure of modus tollens and so on: a pragmatic explanation will suffice (so one could, for instance, embrace a ‘strict’ semantics for conditionals, more on this below).

I think this argument has merits. But as noted above, I reject its starting point: conditionals with ‘impossible’ antecedents *can* be felicitously accepted and asserted. It does not, of course, necessarily follow that the apparent failure of antecedent strengthening (transitivity, modus tollens, etc.) should be given a semantic explanation. But if one – as I have done here – takes the Ramsey Test to be the norm of use that endows the conditional with the semantic content that a semantic analysis is to account for, then it is natural to give the ‘apparent’ failures of antecedent strengthening and their ilk a semantic explanation.

5.3. Modal distribution. On the given semantics for might-modals we find that the might-modality distributes over the conditional:

OBSERVATION 6. For any point X, f:

$$X, f \models \text{Might}(\varphi \to \psi) \equiv (\varphi \to \text{Might}\,\psi).$$

So X accepts $\text{Might}(\varphi \to \psi)$ iff X accepts $\varphi \to \text{Might}\,\psi$. This fits with the general pattern that the attitude one has towards a conditional corresponds to the attitude one would have to the consequent on supposing the antecedent. One who *accepts* a conditional should accept its consequent on supposing the antecedent, and vice versa (the positive qualitative Ramsey Test); one who *rejects* a conditional should reject its consequent on supposing the antecedent, and vice versa (the negative qualitative Ramsey Test); one’s *degree of belief* in a conditional should be one’s degree of belief in the consequent on supposing the antecedent, and vice versa (the probabilistic Ramsey Test); one who is *undecided* on a conditional (ignorant of its truth-value) should be undecided on its consequent on supposing the antecedent, and vice versa (the above equivalence); one who *wants* that a conditional be true should want that its be consequent to be true on supposing the antecedent, and vice versa. And so on for class of modalities that include epistemic modals and deontic operators (no one suggests that it holds for all modalities).

The existence of such a 'pattern' is, of course, a controversial issue. However, it has empirical support. Over, Douven, & Verbrugge (2013) showed that speakers tend to equate [MODALITY] $(\varphi \to \psi)$ with $\varphi \to$ [MODALITY]ψ when the modality involved is an epistemic or deontic modal. The pattern itself has often been interpreted as suggesting that when the modality occurs in the consequent of a conditional it is ambiguous between a wide-scope reading and a narrow-scope reading, where the wide-scope reading is strongly preferred (indeed this is the position taken in the cited study). However, with the present restrictor semantics the explanation (echoing Observation 6) is simpler: the wide-scope and the narrow-scope reading are logically equivalent, there is no need to postulate ambiguity. The thesis is, then, that for epistemic modals we have the equivalence:

(13) [MODALITY] $(\varphi \to \psi) \equiv (\varphi \to [\text{MODALITY}]\psi)$.

Such a thesis holds promise of demystifying the conditional nature of some key modalities. For instance, add probability modalities $P_n(\varphi)$ and $P_n(\psi\,|\,\varphi)$ to the object language (that only operate on sentences of the original L_R) where n is a term for numbers – interpreted by V. $P_n(\varphi)$ is, intuitively, to be interpreted as stating that φ has probability $V(n)$. Now give a semantic interpretation of the probability modality along the following lines (assuming the model supplies a probability measure p satisfying the restricted probabilistic Ramsey Test and a suppositional operator $*$ as given at the end of Section 5.1):[21]

$$f \models P_n(\varphi) \text{ iff } p * \text{MB}_f(|\varphi|) = V(n).$$
$$f \models P_n(\psi\,|\,\varphi) \text{ iff } p * (\text{MB}_f \cap F(|\varphi|))(|\psi|) = V(n) \text{ or } f/F(|\varphi|) = \emptyset.$$

That is, any probability claim is taken to be implicitly conditionalised on the modal background. Whenever the modal background has a positive probability our defined operators will satisfy the standard Kolmogorov axioms, and when φ and ψ are factual we get the equivalent of the quotient analysis of conditional probability:

$$\neg(\varphi \to \bot) \supset (P_n(\psi\,|\,\varphi) \equiv (P_m(\varphi \wedge \psi) \wedge P_k(\varphi)))$$

will hold whenever $V(k) > 0$ and $V(n) = V(m)/V(k)$. Importantly, we get:

OBSERVATION 7. For factual φ and arbitrary ψ:

1. $\models \neg(\varphi \to \bot) \supset (P_n(\varphi \to \psi) \equiv [\varphi \to P_n(\psi)])$.
2. $\models P_n(\psi\,|\,\varphi) \equiv [\varphi \to P_n(\psi)]$.

The probability operator not only distributes over the conditional in the same manner as negation (1 above), the standard notion of a 'conditional probability' is quite literally analysed as a *conditional* probability (2 above). And when we combine the two we get the unrestricted probabilistic Ramsey Test:

$$\neg(\varphi \to \bot) \supset (P_n(\varphi \to \psi) \equiv P_n(\psi\,|\,\varphi)).$$

[21] When φ is impossible the analysis will imply that $P_n(\psi\,|\,\varphi)$ holds for every n. I take this to be a formal way of characterizing the idea that the conditional probability is 'undefined'.

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Three seemingly (or superficially) distinct notions, *The probability of* $[\psi - given - \varphi]$, [*The probability of* ψ] *if* φ, and *The probability of* $[\,\psi - if - \varphi]$, on this analysis turn out to be equivalent (modulo that the antecedent is possible). Much as one would expect from the empirical data.

5.4. Stalnaker or Lewis? There is a related underlying semantic issue concerning the modal status of the antecedent of a conditional and its relationship to the modal status of the consequent. Any restrictor semantics will treat modalities occurring in the consequent of a conditional as being 'constrained' by the antecedent. This, however, does not have any immediate implications for the relationship between [MODALITY] $(\varphi \to \psi)$ and $\varphi \to$ [MODALITY]ψ. Two different semantic traditions give two different treatments.

On the one hand we have the Stalnaker-style semantics that take selection functions to select a single 'preferred' world. It is this feature that yields the principle CEM and the near-equivalence of $\neg(\varphi \to \psi)$ and $\varphi \to \neg\psi$ (the equivalence fails when φ is a contradiction). When a Stalnaker-style selection function is combined with a restrictor semantics (as has been done in this paper) we expect the equivalence (13) to hold for a variety modalities that are sensitive to the modal context of the antecedent.

On the other hand we have the Lewis-style semantics (as, for instance, adopted by Kratzer) whereby a selection function is taken to return a *set* of 'preferred' worlds, a semantics that rejects CEM and the near-equivalence of $\neg(\varphi \to \psi)$ and $\varphi \to \neg\psi$ (this holds for 'strict' semantic analyses as well as they can be viewed as a special case of a Lewis-style analysis). On a Lewis-style semantics, even a restrictor version, one would not expect the equivalence (13) to hold (although compare Gillies, 2018).

Given the cited empirical data on negated and modalised conditionals a Stalnaker-style restrictor semantics has an upper hand on a Lewis-style restrictor semantics. However, an important appeal of Lewis-style semantics is that it has a natural way of dealing with 'indeterminate' cases. For instance, in evaluating the conditional

(14) If the coin was flipped it landed heads.

it seems natural to treat a world in which the coin was flipped and landed heads as 'equally preferred' to a world in which the coin was tossed and instead landed tails. A Lewis-style selection function allows for this, a Stalnaker-style doesn't as a Stalnaker-style selection function always returns a unique 'preferred world'. Relative to such a Lewis-style selection function (14) comes out false, while with a Stalnaker-style analysis the conditional will come out either true or false, depending on how the selection function is chosen.

Nonetheless, the present analysis is well-prepared for such 'indeterminate' conditionals. For mental states are represented as *sets* of Stalnaker-style selection functions: such a set would presumably contain selection functions that prefer heads-worlds as well as selection functions that prefer tails-worlds; so the mental state of an agent who doesn't know if the coin was flipped or not or how it landed will not as a whole prefer one kind of world over another. So on some such selection functions (14) comes out true, on others false. Thus rather than *rejecting* (14) (as one would on a Lewis-style semantics as the consequent is not true in all the preferred worlds in which the antecedent is true) one should, on a Stalnaker-style analysis, be undecided about it.

Indeed the very feature that is to speak in favor of a Lewis-style analysis (a natural way of dealing with 'indeterminate' conditionals) can be turned against it. Given the probabilistic Ramsey Test it would be natural to have degree of belief 0.5 in (14). This is in line with a Stalnaker-style semantics, but goes contrary to a Lewis-style semantics (according to which the conditional should be rejected). Again the empirical studies (that uniformly favor the probabilistic Ramsey Test) point in favor of a Stalnaker-style semantics.

5.5. Missing-link conditionals. As with most accounts that take the Ramsey Test as given, the present analysis validates the '*centering*' axiom (rCS). Indeed, in virtue of Restricted Or-to-If it satisfies the stronger principle (for factual φ and ψ):

(rCS) If X accepts Mightφ and ψ, then X accepts $\varphi \to \psi$.

As a result, the analysis exposes itself to the objection that conditionals with antecedents and consequents that have no connection at all for an agent, will still be acceptable for that agent. For instance, I know that Shakespeare wrote Hamlet. I have no idea, however, whether or not it is right now raining in Oslo. There is no link between the two whatsoever (as far as I know). But given (rCS) I should *accept*:

(15) If it is raining in Oslo, then Shakespeare wrote Hamlet.

Yet it is hard to imagine the context in which it would be reasonable for me to *assert* it.

The proposed analysis thus introduces a general discrepancy between the acceptability and assertability of missing-link conditionals. While the nonassertability of missing-link conditionals is well established by both intuition and empirical studies (see e.g. Skovgaard-Olsen, Singmann, & Klauer, 2016), this does not in itself constitute a compelling argument against the analysis. For we have the same problem with disjunctions:

(16) Either it isn't raining in Oslo or Shakespeare wrote Hamlet.

It is unassertable, yet by most accounts I should accept it (as I know that it is true). The standard pragmatic explanation for the unassertability of (16) goes by appeal to Grice's maxim of Quantity, roughly: don't make a weaker claim (e.g. 16) when you are in a position to make a stronger (i.e. Shakespeare wrote Hamlet). This works for conditionals as well: for when I am undecided about the antecedent of a conditional (as in 15) but know that the consequent is true this expressively entails the conditional (see rCS above), and so the consequent is in this sense stronger than the conditional. In the end we need more than a sketch of a Gricean account to make the explanation plausible, but due to the close semantic relationship between disjunctions and conditionals one can expect that whatever the correct pragmatic explanation for the unassertability of 'missing-link' disjunctions, a similar explanation will exist for missing-link conditionals as well.

Where to draw the semantics-pragmatics division, however, is a highly controversial matter. So even if one can expect that a pragmatic account of the unassertability of (16) will have a close analogue for the unassertability of (15), one can reasonably require some *positive* argument for why a quite general phenomenon is to be explained by pragmatic rather than semantic means. Why not simply adjust the semantics – the truth-conditions and the acceptability conditions – so as to capture the phenomenon directly, and be done with it? In the case of conditionals this would mean that the semantics itself should encode the idea of an epistemic dependence. This, for instance, is forcefully argued for in Skovgaard-Olsen *et al.*, (2016) (see also Douven, 2015; Krzyżanowska, 2015; Skovgaard-Olsen, 2016). But there are at least two kinds of reasons that speak in favor of a pragmatic rather than semantic explanation.

Take an unassertable missing-link conditional $\varphi \to \psi$ with an antecedent that one does not reject and a consequent that one accepts. Negate the consequent, giving us: $\varphi \to \neg\psi$. The result is a conditional that one rejects, and so one accepts its negation $\neg(\varphi \to \neg\psi)$. But if negation distributes one should accept $\varphi \to \neg\neg\psi$ and so, if we have dubbel-negation elimination, one should accept $\varphi \to \psi$. So given that negation distributes and that we have double negation elimination, missing-link conditionals like (15) should be *acceptable* even when they are not *assertable*.

Of course, as the saying goes, one person's modus ponens is another's modus tollens: the unassertability of missing-link conditionals like (15) could be taken to be a counterexample to negation-distribution. There is, however, a more direct argument for giving the unassertability of missing-link conditionals a pragmatic rather than semantic explanation. It concerns how to assess other speakers' conditional claims.

Whether a conditional qualifies as a missing link conditional depends on what one knows. What may be a missing-link conditional for me need not be so for you or someone else. Indeed, whenever the antecedent and consequent of a conditional do not directly contradict each other and where the consequent is true, one can always conjure up a possible factually correct epistemic state in which the antecedent and consequent are epistemically linked. That is, for any conditional $\varphi \to \psi$ where both antecedent and consequent are logically contingent and logically independent there will be an epistemic state that supports neither $\neg\varphi$ nor ψ but that nontrivially supports $\neg\varphi \vee \psi$ and so (by restricted Or-to-If) $\varphi \to \psi$. Such epistemic states will often seem contrived (try thinking of a natural situation where one would nontrivially know that the disjunction (16) is true), adding to the apparent oddness of conditionals like (15). But this very oddity is a confounder; for an important part of the explanatory work that we want from the theoretical distinction between acceptability and assertability (and for that matter between truth and assertability) concerns the distinction between assessments of what oneself can reasonably assert as opposed to what one would accept if it were asserted by some other speaker.

Consider the following exchange:

Alex enters Bill's room, explaining that he is looking for Mary's mobile phone.
Alex: I have checked her room, it isn't there; she said that if it isn't in her room it is in yours.
Bill: Yes, she is right, the phone is right there on the table.

The exchange seems natural enough. However, the conditional that Bill apparently assents to is for him a missing link conditional; for he *knows* that Mary's mobile phone is in his room, and *he* could not have felicitously asserted a conditional with the same content (e.g. he could not assert "If Mary's mobile phone is not in her room then it is in mine"). If acceptance has as much to do with how to assess other people's assertions as it has to do with what to assert oneself, this suggests that unassertability and unacceptability come apart for conditionals. Experimental studies on missing-link conditionals to date focus on whether respondents find such conditionals 'acceptable' in a context where there is no epistemically plausible speaker for whom the conditional is not a missing-link conditional, and so miss one of the key domains where the distinction between acceptability and assertability can be expected to play an important role. If, as it seems, judgments of acceptability and assertability do come apart in such cases, we have a direct argument for treating the unassertability of missing-link conditionals as a pragmatic rather than semantic phenomenon.

The debate doesn't end there, of course. One could hold that the semantic content of a conditional depends on who asserts it, and, yes, this would yield a semantic explanation for the unassertability of a missing link conditional while accounting for exchanges like the above (e.g. the conditional in the above exchange would be true-when-uttered-by-Alex but false-when-uttered-by-Bill). I think there are independent reasons for rejecting such a solipsistic contextualist semantics of conditionals, but this is a major debate on its own. What is important for the present discussion is that it shows that encoding epistemic dependency in the semantics is not just a matter of slightly tweaking truth-conditions or acceptability conditions: the 'slight tweak' comes with a heavy semantic baggage with consequences that extends well beyond missing link conditionals. If one does not accept these consequences one has a positive reason for adopting a pragmatic explanation for the phenomenon at hand.

§6. Concluding remarks. As promised in the introduction a global expressivist analysis of the conditional has been presented. Building on the Ramsey Test it allows conditionals to embed freely in complex constructions – including arbitrarily complex left-nested conditionals – and is sensitive to the two features that drive the logic of the indicative conditional: conditionals can be belief- or knowledge contravening, and the antecedent of a conditional constrains the modal context within which the consequent is evaluated. Drawing only on conditions of acceptance and rejection and structural conditions, a consequence relation was defined, giving us the logic ICL. This is in analogy with McGee's derivation of ICL from probabilistic considerations. I take the results presented here to demonstrate that an expressivist interpretation of the conditional along the lines of the Ramsey Test need not be susceptible to the set of issues that are sometimes dubbed the Frege–Geach problem.

The semantic analysis, in turn, gives a complete characterization both of ICL and of expressivist models, at least when the language is restricted so as to omit left-nested conditionals. While left-nested conditionals have been given a semantics as well, it arguably results in a logic that is too strong, but the semantics at least fills the purpose of showing that expressivist models do not collapse into triviality when left-nested conditionals are allowed.

The primary motivation for the semantic apparatus is that it enables a semantic representation of the epistemic states of speakers in order to explain why speakers use conditionals the way they do. As shown, the semantics validates the Ramsey Test (in both graded and categorical form) in addition gives a plausible logic (e.g. validates Import-Export and the distribution of negation over conditionals). As indicated the resulting analysis is well in line with many of the major empirical findings in the literature.

As the focus has been on representing the epistemic state of speakers, one cannot – from the semantic apparatus alone – read off the conditions under which conditionals are in some objective sense true or false; for the semantic apparatus itself gives no indication of what it is in the world (other than epistemic states) that yield the kind of structures that can be represented as sets of selection functions. (The semantics in this sense allows conditionals to have 'nonrepresentational' contents.) However, as it stands, the semantic analysis itself is compatible with both contextualist, relativist and objectivist analyses of how to make sense of the idea that conditionals have objective truth values (or, in the case of relativism, make sense of the idea that they do not have any objective truth values).

§A. Appendix: The negative Ramsey Test. This appendix gives a more detailed justification for the choice of 'negative' Ramsey Test:

$\rightarrow$Rej $m \vdash \text{Rej}(\varphi \to \psi)$ iff $m \circledast \text{Acc}\ \varphi \vdash \text{Rej}\ \psi$ and $m \circledast \text{Acc}\ \varphi$ is coherent.

Equivalently:

$\rightarrow$ Rej $m \vdash \text{Acc}\neg(\varphi \to \psi)$ iff $m \circledast \text{Acc}\ \varphi \vdash \text{Acc}\neg\psi$ and $m \circledast \text{Acc}\ \varphi$ is coherent.

The first argument in favor of $\rightarrow$Rej is that it complies with linguistic intuitions and usage data, for except when $m \circledast \text{Acc}\ \varphi$ is incoherent, $\neg(\varphi \to \psi)$ becomes equivalent to $\varphi \to \neg\psi$ (see, for instance, Handley, Evans, & Thompson, 2006; Pfeifer, 2012 for empirical support for this equivalence). This is further bolstered by the fact that the near equivalence follows straightforwardly from standard principles of probability and the Probabilistic Ramsey Test (the thesis that $P(\varphi \to \psi) = P(\psi \mid \varphi)$, when $P(\varphi) \neq 0$) which itself has strong empirical support. Assuming $P(\varphi) \neq 0$:

$$\begin{aligned} P(\neg(\varphi \to \psi)) &= 1 - P(\varphi \to \psi) = 1 - P(\psi \mid \varphi) = P(\neg\psi \mid \varphi) \\ &= P(\varphi \to \neg\psi). \end{aligned}$$

A second argument is that $\rightarrow$Rej provides the rejection conditions for the conditional that ensure that accepting and rejecting a conditional can be treated as mutually incompatible and complementary attitudes: the requirements are in a weak form of *bilateral harmony*.

For incompatibility, note that no coherent mental state can satisfy the conditions for *accepting* a conditional (according to $\rightarrow$Acc) and also (at the same time) satisfy the conditions for *rejecting* a conditional (according to $\rightarrow$Rej). For assume that m is coherent and satisfies both sets of conditions. Then $m \circledast \text{Acc}\ \varphi$ is coherent (according to $\rightarrow$Rej), yet $m \circledast \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$ (according to $\rightarrow$Acc) and $m \circledast \text{Acc}\ \varphi \vdash \text{Rej}\ \psi$ (according to $\rightarrow$Rej), and so $m \circledast \text{Acc}\ \varphi$ is incoherent, contrary to assumption. Thus one is incoherent if one satisfies both the grounds for accepting the conditional (as spelled out by $\rightarrow$Acc), and at the same time the grounds for rejecting the conditional (as spelled out by $\rightarrow$Rej).

Consider the following *weak* form of complementarity.

(wComp) Two attitudes a and b are *weakly complementary* iff for every coherent m, if there is no coherent m' accessible from m such that $m' \vdash a$, then $m \vdash b$ (and vice versa).

With the given acceptance and rejection conditions for the conditional, we find that the attitude of accepting a conditional weakly complements rejecting it. For assume that m is coherent and that there is no coherent mental state m' accessible from m such that m' satisfies the acceptance conditions for the conditional $\varphi \to \psi$. This implies that $m \circledast \text{Acc}\ \varphi$ is coherent (if $m \circledast \text{Acc}\ \varphi$ is incoherent it would trivially support Acc ψ, and so we would have $m \vdash \text{Acc}(\varphi \to \psi)$). It also implies that there is no coherent extension of $m \circledast \text{Acc}\ \varphi$ that supports Acc $\varphi \to \psi$ and so, in light of Strong Success and Weak

Redundancy, there is no coherent extension of $m \circledast \mathrm{Acc}\ \varphi$ that supports $\mathrm{Acc}\ \psi$. So $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi$ is incoherent. But as $\mathrm{Acc}\ \psi$ and $\mathrm{Rej}\ \psi$ are complementary attitudes, it follows that $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Rej}\ \psi$. So in m the rejection conditions for the conditional $\rightarrow$Rej is satisfied. So if one *cannot* (by expanding or revising one's mental state) come to coherently accept $\varphi \rightarrow \psi$ then one should, on the basis of $\rightarrow$Rej, accept $\neg(\varphi \rightarrow \psi)$. An analogous argument can be given for the other direction: if there is no coherent mental state accessible from m in which one satisfies the rejection condition for the conditional (as spelled out by $\rightarrow$Rej), then one in m satisfies the acceptance conditions for the conditional (as spelled out by $\rightarrow$Acc).

Although encouraging, satisfaction of this weak form of complementarity is not a conclusive argument for establishing $\rightarrow$Rej given $\rightarrow$Acc; for it doesn't establish uniqueness. A conclusive argument would be one that demonstrated that if *expansion* of m with $\varphi \rightarrow \psi$ (as governed by the requirements $\rightarrow$Acc) leads to an incoherent mental state, then m satisfies the requirements for rejecting $\varphi \rightarrow \psi$ (as spelled out by $\rightarrow$Rej). Such a stronger form of complementarity can, by contrast, be demonstrated for the standard boolean connectives (see Cantwell, 2015). So, as it stands, one cannot exclude that other rejection requirements for the conditional would exhibit the same form of weak bilateral harmony as $\rightarrow$Rej. The final argument I offer for the present rejection requirements is thus that the alternatives proposed in the literature all lead to a collapse in one way or another.

What are the alternatives to $\rightarrow$Rej? Consider:

Alt1 $m \vdash \mathrm{Rej}(\varphi \rightarrow \psi)$ iff $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Rej}\ \psi$.

This is nearly identical to ($\rightarrow$Rej) except that one has dropped the constraint that $m \circledast \mathrm{Acc}\ \varphi$ is coherent. The problem with this alternative is that it collapses into inconsistency: every mental state is incoherent. For assume that m is coherent. As $m \circledast \mathrm{Acc}(p \wedge \neg p)$ is incoherent we have $m \circledast \mathrm{Acc}(p \wedge \neg p) \vdash \mathrm{Acc}\ q$ and $m \circledast \mathrm{Acc}(p \wedge \neg p) \vdash \mathrm{Rej}\ q$. So $m \vdash \mathrm{Acc}(p \wedge \neg p) \rightarrow q$ and $m \vdash \mathrm{Rej}(p \wedge \neg p) \rightarrow q$, i.e. one accepts both $(p \wedge \neg p) \rightarrow q$ and its negation $\neg((p \wedge \neg p) \rightarrow q)$, and so m is incoherent, contrary to assumption.

Consider instead the following alternative:

Alt2 $m \vdash \mathrm{Rej}(\varphi \rightarrow \psi)$ iff $m \vdash \mathrm{Acc}\ \varphi$ and $m \vdash \mathrm{Rej}\ \psi$.

This gives the indicative conditional the same rejection conditions as the material conditional, and quickly leads to a breakdown. For it makes $\neg(\varphi \rightarrow \psi)$ logically equivalent to $\varphi \wedge \neg\psi$, and so (given a classical logic for the boolean connectives) $\varphi \rightarrow \psi$ is logically equivalent to $\neg\varphi \vee \psi$, and so $\varphi \rightarrow \psi$ becomes logically equivalent to the material implication $\varphi \supset \psi$.

Consider instead:

Alt3 $m \vdash \mathrm{Rej}(\varphi \rightarrow \psi)$ iff $m \circledast \mathrm{Acc}\ \varphi \nvdash \mathrm{Acc}\ \psi$.

This is perhaps the most commonly proposed rejection condition for the conditional (reject a conditional if you do not accept it). In my mind it conflates *rejection* of a conditional (willingness to accept its negation) with dismissing a conditional on the grounds that one is not justified in accepting it. As such I think it lacks linguistic plausibility as it makes it impossible to be undecided about a conditional. For instance, given Alt3 one should accept one of the following conditionals:

(17) If Caesar was wearing socks when he crossed the Rubicon, they were green.

(18) It is not the case that if Caesar was wearing socks when he crossed the Rubicon, they were green.

I would suggest (contrary to Alt3) that one can be undecided about both (17) and its negation.

Linguistic implausibility aside, Alt3 leads to a kind of collapse. As is seen from the following observation:

OBSERVATION 8. If (Alt3) replaces ($\to$Rej), then for any φ and ψ and any coherent m: either $m \vdash \mathrm{Acc}(\varphi \to \psi)$ or $m \vdash \mathrm{Acc}(\varphi \to \neg\psi)$.

Proof. Assume that $m \nvdash \mathrm{Acc}(\varphi \to \psi)$. So $m \circledast \mathrm{Acc}\ \varphi \nvdash \mathrm{Acc}\ \psi$. By Strong Success and Weak Redundancy, $m \circledast \mathrm{Acc}\ \varphi \circledast \mathrm{Acc}\ \varphi \nvdash \mathrm{Acc}\ \psi$. So, by Alt3, $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\neg(\varphi \to \psi)$. By Monotonicity, $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi \vdash \mathrm{Acc}\neg(\varphi \to \psi)$. By Success $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi \vdash \mathrm{Acc}\ \psi$. By Strong Success, $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\psi \Vdash \mathrm{Acc}\ \varphi$, so, by Weak Redundancy, $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\psi = m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi \circledast \mathrm{Acc}\ \varphi$. So $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi$ and so $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi \vdash \mathrm{Acc}(\varphi \to \psi)$. As we also have $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi \vdash \mathrm{Acc}\neg(\varphi \to \psi)$, $m \circledast \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi$ is incoherent. So by Classicality $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\neg\psi$. So $m \vdash \mathrm{Acc}(\varphi \to \neg\psi)$. □

The result is a collapse as no one (as far as I know) have proposed the implausible principle that for any φ and ψ, either you should accept $\varphi \to \psi$ or you should accept $\varphi \to \neg\psi$. [One should not confuse this fallacious principle with the principle "You should accept the disjunction $\varphi \to \psi \vee \varphi \to \neg\psi$" (also known as CEM); the latter principle is valid in the present framework and flows quite naturally from the rejection condition for the conditional.]

§B. Appendix: Proof of Theorems.

Proof of Theorem 1. The consistency part of the proof is omitted. For the language L_R it is largely trivial. For the full language L note that Modus Ponens, Classicality, CN, K and LLE do not make use of any particular feature of // other than that it is a function of the proposition it takes as an argument, while Import-Export and rMP and rCS only involve factual antecedents.

The completeness part has a standard structure (it proceeds by showing how a 'canonical' ICL model can be constructed and then shows that if $\Gamma \not\models_{\mathrm{ICL}} \varphi$ we can find a point of evaluation in the canonical model where the sentences of Γ are true but φ is false). Note throughout that due to CN, K and LLE, $\to$ is a *normal* conditional modality and I will rely on this in a number of steps.

Let W_c be the set of maximal ICL-consistent sets of factual sentences. Let $V_c(p) = \{w \in W_c : p \in w\}$ and let $\mathcal{B}_c$ be the set we get by closing the set $\{V_c(p) : p \text{ atomic}\}$ under complements, finite intersections and finite unions. When $A \in \mathcal{B}_c$ let $S(A)$ denote some canonically chosen factual sentence such that $F(|S(A)|) = A$. Let M_c be the model $(W_c, \mathcal{B}_c, V_c)$.

Let Σ denote a maximal ICL-consistent set on the whole language L_R (so not only on factual sentences). Define, for any $A \in \mathcal{B}_c$ such that $S(A) \to \bot \notin \Sigma$ (Where $\bot$ is an arbitrary factual contradiction):

$$f_\Sigma(A) = \{\varphi : S(A) \to \varphi \in \Sigma \text{ and } \varphi \text{ is factual}\}.$$

Due to LLE it doesn't matter which sentence $S(A)$ picks out (so $f_\Sigma(A)$ is well-defined When $S(A) \to \bot \notin \Sigma$. Furthermore, $f_\Sigma(A)$ is an element of W_c (for due to CEM and the fact that $S(A) \to \bot \notin \Sigma$, $f_\Sigma(A)$ will be a maximal consistent set of factual sentences, i.e. an element of W_c).

Let MB_f be the range of f_Σ, the set $\{f_\Sigma(A) : A \in \mathcal{B}_c \text{ and } S(A) \to \bot \notin \Sigma\}$.

LEMMA B.1.

1. *$f_\Sigma(A)$ is defined iff $A \cap \text{MB}_{f_\Sigma} \neq \emptyset$.*
2. *If $\text{MB}_{f_\Sigma} \cap A \neq \emptyset$, then $f_\Sigma(A) \in \text{MB}_{f_\Sigma} \cap A$.*
3. *If $f_\Sigma(A) \in B$, then $f_\Sigma(A) = f_\Sigma(A \cap B)$.*

Proof. (1) The left-to-right direction is trivial given the construction. For the right-to-left direction, assume that $A \cap \text{MB}_{f_\Sigma} \neq \emptyset$. Then there is some B such that $f_\Sigma(B) \in A$. So $S(B) \to S(A) \in \Sigma$ and $S(B) \to \bot \notin \Sigma$. Assume for reductio that $S(A) \to \bot \in \Sigma$. By normality $S(A) \to (S(B) \to \bot) \in \Sigma$. By applying Import-Export twice (and normality) $S(B) \to (S(A) \to \bot) \in \Sigma$. By rMP and normality $S(B) \to \bot \in \Sigma$: a contradiction. So $S(A) \to \bot \notin \Sigma$ and so $f_\Sigma(A)$ is defined.

(2) Assume that $A \cap \text{MB}_{f_\Sigma} \neq \emptyset$. By (1) $f_\Sigma(A)$ is defined and so $S(A) \to \bot \notin \Sigma$. By CN, $S(A) \to S(A) \in \Sigma$. So $f_\Sigma(A) \in A$. Furthermore, by construction $f_\Sigma(A) \in \text{MB}_{f_\Sigma}$.

(3) Assume that $f_\Sigma(A) \in B$. So $S(A) \to S(B) \in \Sigma$. First show that $f_\Sigma(A \cap B)$ is defined. Assume that it isn't. Then $S(A \cap B) \to \bot \in \Sigma$. So $(S(A) \wedge S(B)) \to \bot \in \Sigma$. By Import-Export $S(A) \to (S(B) \to \bot) \in \Sigma$. By rMP and normality $S(A) \to \bot \in \Sigma$ and we have a contradiction (for, by assumption, $f_\Sigma(A)$ is defined).

Assume that $f_\Sigma(A) \in C$. So $S(A) \to S(C) \in \Sigma$. By rCS and normality $S(A) \to (S(B) \to S(C)) \in \Sigma$. By Import-Export, $(S(A) \wedge S(B)) \to S(C) \in \Sigma$. But $F(|S(A) \wedge S(B)|) = A \cap B$. So $S(A \cap B) \to S(C) \in \Sigma$ and so $f_\Sigma(A \cap B) \in C$. As this holds for all C, $f_\Sigma(A) = f_\Sigma(A \cap B)$. □

LEMMA B.2. *If $f_\Sigma(A)$ is defined: $f_\Sigma/A = f_{\Sigma'}$, where $\Sigma' = \{\psi : S(A) \to \psi \in \Sigma\}$.*

Proof. Take any $B \in \mathcal{B}_c$ such that $f_\Sigma/A(B)$ is defined. By definition: $f_\Sigma/A(B) = f_\Sigma(A \cap B)$. So assume that $f_\Sigma/A(B) \in C$. So $S(A \cap B) \to S(C) \in \Sigma$. Thus $(S(A) \wedge S(B)) \to S(C) \in \Sigma$. By Import-Export $S(A) \to (S(B) \to S(C)) \in \Sigma$. So $S(B) \to S(C) \in \Sigma'$. But then $f_{\Sigma'}(B) \in C$. This holds for all C so $f_\Sigma/A(B) = f_{\Sigma'}(B)$. So $f_\Sigma/A = f_{\Sigma'}$. □

LEMMA B.3. *For any maximal consistent set Σ: $f_\Sigma \models_{M_c} \varphi$ iff $\varphi \in \Sigma$.*

Proof. By induction over the length of φ.

Let $\varphi = p$. $f_\Sigma \models_{M_c} p$ iff $f_\Sigma(W_c) \in V_c(p)$ iff $S(W_c) \to p \in \Sigma$ iff (by rMP and the fact that $S(W_c)$ is a tautology) $p \in \Sigma$.

The boolean connectives are straightforward and left as an exercise.

Let $\varphi = \psi \to \chi$ (where ψ is factual). $f_\Sigma \models_{M_c} \psi \to \chi$ iff (a) $f_\Sigma/F(|\psi|) = \emptyset$ or (b) $f_\Sigma/F(|\psi|)) \models_{M_c} \chi$. The remainder of the proof is split into the two cases (a) and (b). Throughout the fact that ψ is logically equivalent to $S(F(|\psi|))$ will be used. Furthermore, let $\Sigma' = \{\theta : \psi \to \theta \in \Sigma\}$.

Case (a). Note that $f_\Sigma/F(|\psi|) = \emptyset$ iff $\psi \to \bot \in \Sigma$ iff both $\psi \to \bot \in \Sigma$ and (by normality) $\psi \to \chi \in \Sigma$.

Case (b). Assuming that $f_\Sigma/F(|\psi|) \neq \emptyset$. $f_\Sigma/F(|\psi|) \models_{M_c} \chi$ iff (by Lemma B.2) $f_{\Sigma'} \models_{M_c} \chi$ iff (by the induction hypothesis) $\chi \in \Sigma'$ iff (by construction) $\psi \to \chi \in \Sigma$. □

Continuing the proof of the completeness theorem. Assume that $\Gamma \not\models_{\mathrm{ICL}} \varphi$. One can show (using standard techniques) that $\Gamma \cup \{\neg\varphi\}$ can be extended to a maximal ICL-consistent set Σ. By Lemma B.3, $f_\Sigma \models_{M_c} \psi$ for all ψ in Γ, furthermore, $f_\Sigma \not\models_{M_c} \varphi$.

□

Proof of Theorem 2. First show that the expressivist consequence relation $\models_{\mathfrak{E}}$ contains $\models_{\mathrm{ICL}}$. The proof of the opposite direction follows from Theorem 5 which is proved below.

Take any expressivist model $\mathcal{E}$ and any coherent mental state m: one needs to show that $m \vdash$ is closed under the rules and axioms of $\models_{\mathrm{ICL}}$.

MP: Assume that $m \vdash \mathrm{Acc}\ \varphi$ and $m \vdash \mathrm{Acc}\ \varphi \supset \psi$. Then $m \oplus \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi$; by Redundancy $m \vdash \mathrm{Acc}\ \psi$.

CL: As the binary boolean connectives are classically interdefinable, it is enough to show that every instance of the following axiomatisation of classical logic is accepted: (1) $\varphi \supset (\psi \supset \varphi)$, (2) $(\varphi \supset (\psi \supset \chi)) \supset ((\varphi \to \psi) \supset (\varphi \supset \chi))$, (3) $(\neg\psi \supset \neg\varphi) \supset (\varphi \to \psi)$.

Recall throughout that by Explosion if a mental state is incoherent, then it supports every attitude.

(1) $m \oplus \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\ \psi \vdash \mathrm{Acc}\ \varphi$ by Success and Monotonicity, so $m \vdash \mathrm{Acc}\ \varphi \supset (\psi \supset \varphi)$ for all m.

(2) $m \oplus \mathrm{Acc}\ \varphi \supset (\psi \supset \chi) \oplus \mathrm{Acc}\ \varphi \supset \psi \oplus \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \varphi \supset \psi$ by Success and Monotonicity. By Success and modus ponens, $m \oplus \mathrm{Acc}\ \varphi \supset (\psi \supset \chi) \oplus \mathrm{Acc}\ \varphi \supset \psi \oplus \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi$. By Success and Monotonicity, $m \oplus \mathrm{Acc}\ \varphi \supset (\psi \supset \chi) \oplus \mathrm{Acc}\ \varphi \supset \psi \oplus \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \varphi \supset (\psi \supset \chi)$. By modus ponens $m \oplus \mathrm{Acc}\ \varphi \supset (\psi \supset \chi) \oplus \mathrm{Acc}\ \varphi \supset \psi \oplus \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi \supset \chi$. By modus ponens $m \oplus \mathrm{Acc}\ \varphi \supset (\psi \supset \chi) \oplus \mathrm{Acc}\ \varphi \supset \psi \oplus \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \chi$. So $m \vdash \mathrm{Acc}(\varphi \supset (\psi \supset \chi)) \supset ((\varphi \to \psi) \supset (\varphi \supset \chi))$.

For the last axiom, note that $\mathrm{Acc}\ \psi$ and $\mathrm{Acc}\neg\psi$ are complementary attitudes. For recall that $m \oplus \mathrm{Acc}\ \psi = m \oplus \mathrm{Rej}\neg\psi$ for any m and so, as $\mathrm{Rej}\neg\psi$ and $\mathrm{Acc}\neg\psi$ are complementary (Classicality), so are $\mathrm{Acc}\ \psi$ and $\mathrm{Acc}\neg\psi$.

(3) $m \oplus \mathrm{Acc}\neg\psi \supset \neg\varphi \oplus \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\neg\psi \vdash \mathrm{Acc}\ \varphi$ by Success and Monotonicity while $m \oplus \mathrm{Acc}\neg\psi \supset \neg\varphi \oplus \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\neg\psi \vdash \mathrm{Acc}\neg\varphi$ by Success, Monotonicity and modus ponens. So $m \oplus \mathrm{Acc}\neg\psi \supset \neg\varphi \oplus \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\neg\psi \vdash \mathrm{Rej}\ \varphi$. By Classicality, $m \oplus \mathrm{Acc}\neg\psi \supset \neg\varphi \oplus \mathrm{Acc}\ \varphi \oplus \mathrm{Acc}\neg\psi$ is incoherent. By Classicality (complementarity) $m \oplus \mathrm{Acc}\neg\psi \supset \neg\varphi \oplus \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi$. So $m \vdash \mathrm{Acc}(\neg\psi \supset \neg\varphi) \supset (\varphi \to \psi)$.

CN: Assume that for every coherent m: if $m \vdash \mathrm{Acc}\ \varphi$, then $m \vdash \mathrm{Acc}\ \psi$. By Strong Success $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \varphi$, so either $m \circledast \mathrm{Acc}\ \varphi$ is incoherent (in which case $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi$) or $m \circledast \mathrm{Acc}\ \varphi$ is coherent and so by assumption $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi$. So $m \vdash \mathrm{Acc}(\varphi \to \psi)$.

K: Assume that for every m: $m \vdash \mathrm{Acc}\chi$ whenever $m \vdash \mathrm{Acc}\ \psi_i$ (for all i: $1 \leq i \leq n$). Take any m such that $m \vdash \mathrm{Acc}(\varphi \to \psi_i)$ (for all i: $1 \leq i \leq n$). Then $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \psi_i$ (for all i: $1 \leq i \leq n$). By assumption $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\ \chi$. So $m \vdash \mathrm{Acc}(\varphi \to \chi)$.

LLE: Assume that φ and ψ are non modal and that $\models_{\mathrm{ICL}} \varphi \equiv \psi$, that is, φ and ψ are classically equivalent. From the above we know that that for every coherent m: $m \vdash \mathrm{Acc}\ \varphi$ iff $m \vdash \mathrm{Acc}\ \psi$. We need to show that $m \vdash \mathrm{Acc}(\varphi \to \chi)$ iff $m \vdash \mathrm{Acc}(\psi \to \chi)$. Assume that $m \vdash \mathrm{Acc}(\varphi \to \chi)$. So $m \circledast \mathrm{Acc}\ \varphi \vdash \mathrm{Acc}\chi$. By Strong Success $m \circledast \mathrm{Acc}\ \varphi \Vdash \mathrm{Acc}\ \varphi$, so $m \circledast \mathrm{Acc}\ \varphi \Vdash \mathrm{Acc}\ \psi$. By Weak Redundancy $m \circledast \mathrm{Acc}\ \varphi = m \circledast \mathrm{Acc}\ \varphi \circledast \mathrm{Acc}\ \psi$. By R-Permutation $m \circledast \mathrm{Acc}\ \varphi = m \circledast \mathrm{Acc}\ \psi \circledast \mathrm{Acc}\ \varphi$. As $m \circledast \mathrm{Acc}\ \psi \Vdash \mathrm{Acc}\ \varphi$, by Weak Redundancy $m \circledast \mathrm{Acc}\ \varphi = m \circledast \mathrm{Acc}\ \psi$. So $m \circledast \mathrm{Acc}(\psi \vdash \mathrm{Acc}\chi)$ and so $m \vdash \mathrm{Acc}(\psi \to \chi)$. The converse direction follows analogously.

CEM: By Success $m \oplus \text{Rej}(\varphi \to \psi) \vdash \text{Rej}(\varphi \to \psi)$. If $m \oplus \text{Rej}(\varphi \to \psi)$ is incoherent then $m \oplus \text{Rej}(\varphi \to \psi) \vdash (\text{Acc } \varphi \to \neg\psi)$. So assume that $m \oplus \text{Rej } \varphi \to \psi$ is coherent. By the rejection condition for the conditional: $m \oplus \text{Rej } \varphi \to \psi \circledast \text{Acc } \varphi \vdash \text{Rej}\psi$, and so $m \oplus \text{Rej}(\varphi \to \psi) \circledast \text{Acc}(\varphi \vdash \text{Acc}\neg\psi)$ and so $m \oplus \text{Rej}(\varphi \to \psi) \vdash \text{Acc}(\varphi \to \neg\psi)$. An analogous argument yields $m \oplus \text{Rej}(\varphi \to \neg\psi) \vdash \text{Acc}(\varphi \to \psi)$. So $m \vdash \text{Acc}(\varphi \to \psi \vee \varphi \to \neg\psi)$.

Import-Export: Assume that φ and ψ are factual; so Acc φ and Acc ψ are factual. It is enough to show that $m \circledast \text{Acc } \varphi \circledast \text{Acc } \psi = m \circledast \text{Acc}(\varphi \wedge \psi)$. By Strong Success, $m \circledast \text{Acc } \varphi \circledast \text{Acc } \psi \Vdash \text{Acc}(\varphi \wedge \psi)$. Thus, by Weak Redundancy, $m \circledast \text{Acc } \varphi \circledast \text{Acc } \psi = m \circledast \text{Acc } \varphi \circledast \text{Acc } \psi \circledast \text{Acc}(\varphi \wedge \psi)$. By R-Permutation, $m \circledast \text{Acc } \varphi \circledast \text{Acc } \psi \circledast \text{Acc}(\varphi \wedge \psi) = m \circledast \text{Acc}(\varphi \wedge \psi) \circledast \text{Acc } \varphi \circledast \text{Acc } \psi$. As $m \circledast \text{Acc}(\varphi \wedge \psi) \Vdash \text{Acc } \varphi$ and $m \circledast \text{Acc}(\varphi \wedge \psi) \Vdash \text{Acc } \psi$, it follows that $m \circledast \text{Acc}(\varphi \wedge \psi) \circledast \text{Acc } \varphi \circledast \text{Acc } \psi = m \circledast \text{Acc}(\varphi \wedge \psi)$, so $m \circledast \text{Acc } \varphi \circledast \text{Acc } \psi = m \circledast \text{Acc}(\varphi \wedge \psi)$.

[rMP] Assume that φ and ψ are syntactically factual. Take any m such that $m \vdash \text{Acc } \varphi \to \psi$ and $m \vdash \text{Acc } \varphi$. We have $m \circledast \text{Acc } \varphi \vdash \text{Acc } \psi$. By R-Inclusion $m \oplus \text{Acc } \varphi \vdash \text{Acc } \psi$. By Redundancy $m \oplus \text{Acc } \varphi = m$ so $m \vdash \text{Acc } \psi$.

[rCS] Assume that φ and ψ are syntactically factual. Take any m such that $m \vdash \text{Acc } \varphi$ and $m \vdash \text{Acc } \psi$. If $m \oplus \text{Acc } \varphi$ is incoherent then by Explosion $m \oplus \text{Acc } \varphi \vdash \text{Acc}(\varphi \to \psi)$. So assume that $m \oplus \text{Acc } \varphi$ is coherent. By R-Vacuity $m \circledast \text{Acc } \varphi \vdash \text{Acc } \psi$ and so $m \vdash \text{Acc}(\varphi \to \psi)$. □

Proof of Theorem 3. Begin by noting that any sentence φ is ICL-equivalent to a sentence φ' where all negations have been showed inward, that is where every occurrence of $\neg$ either applies to an atom or has the form $\neg(\psi \to \perp)$ (this is a standard property of conditional logics that satisfy CEM). For instance, $\neg(p \to q)$ is logically equivalent to $\neg(p \to \perp) \vee p \to \neg q$. So we will only consider sentences where $\neg$ has been showed inward. Let $\bar{\varphi}$ denote a sentence that is logically equivalent to $\neg\varphi$ but where all negations have been showed inward. Define a function* on sentences that returns a set of sets of sentences of either the form $\varphi \to a$ (where a is a possibly negated atomic sentence) or the form $\neg(\varphi \to \perp)$. As follows:

1. $(a)^* = \{\{\top \to a\}\}$, here a is an atomic sentence (possibly negated) and $\top$ is an arbitrary tautology.
2. $(\neg(\varphi \to \perp))^* = \{\{\neg(\varphi \to \perp)\}\}$.
3. $(\varphi \wedge \psi)^* = \{X \cup Y \mid X \in \varphi^* \text{ and } Y \in \psi^*\}$.
4. $(\varphi \vee \psi)^* = (\varphi \wedge \psi)^* \cup (\varphi \wedge \bar{\psi})^* \cup (\bar{\varphi} \wedge \psi)^*$.
5. $X \in (\varphi \to \psi)^*$ iff $X = \{\varphi \to \perp\}$ or there is a $Y \in \psi^*$ such that:
   $Y = \{\sigma_1 \to a_1, \ldots, \sigma_n \to a_n, \neg(\theta_1 \to \perp), \ldots, \neg(\theta_m \to \perp)\}$,
   and:
   $X = \{(\varphi \wedge \sigma_1) \to a_1, \ldots, (\varphi \wedge \sigma_n) \to a_n, \neg((\varphi \wedge \theta_1) \to \perp), \ldots, \neg((\varphi \wedge \theta_m) \to \perp)\}$.

Let $D(\varphi) = \bigvee\{\bigwedge X \mid X \in \varphi^*\}$. Each set in φ^* thus corresponds to a conjunction of its elements and multiple sets indicate that these in $D(\varphi)$ are connected through disjunction. Note that $D(\varphi)$ has the form $\delta_1 \vee \cdots \vee \delta_n$ and that each δ_j has the form:

$$\bigwedge_{1 \leq i \leq k_j} (\psi_i^j \to a_i^j) \wedge \bigwedge_{1 \leq i \leq m_j} \neg(\chi_i^j \to \perp),$$

Moreover, if φ is the formula $\sigma \to \theta$, each δ_j has the form:

$$\bigwedge_{1\leq i\leq k_j}((\sigma \wedge \psi_i^j) \to a_i^j) \wedge \bigwedge_{1\leq i\leq m_j} \neg((\sigma \wedge \chi_i^j) \to \bot).$$

That is, the antecedent σ in φ occurs everywhere as an antecedent in the subformulae of $D(\varphi)$.

From (4) and (5) the only clauses that introduce new disjunctive sets – it is evident that the disjuncts will be mutually logically exclusive. We need to prove by induction over the length of φ that $D(\varphi)$ is logically equivalent to φ. But all the cases except possibly the last are straightforward. For the last case note that $\varphi \to (\delta_1 \vee \cdots \vee \delta_n)$ is logically equivalent to $\varphi \to \delta_1 \vee \cdots \vee \varphi \to \delta_n$, and that

$$\varphi \to \left(\bigwedge_{1\leq i\leq k_j}(\psi_i^j \to a_i^j) \wedge \bigwedge_{1\leq i\leq m_j} \neg(\chi_i^j \to \bot)\right)$$

is equivalent to

$$\bigwedge_{1\leq i\leq k_j}(\varphi \to (\psi_i^j \to a_i^j)) \wedge \bigwedge_{1\leq i\leq m_j} (\varphi \to \neg(\chi_i^j \to \bot))$$

which, by Import-Export and CEM, is equivalent to:

$$(\varphi \to \bot) \vee \left(\bigwedge_{1\leq i\leq k_j}((\varphi \wedge \psi_i^j) \to a_i^j) \wedge \bigwedge_{1\leq i\leq m_j} \neg((\varphi \wedge \chi_i^j) \to \bot).\right) \qquad \square$$

Proof of Theorem 4. Take any expressivist model $\mathcal{E}$ on L_R. Let the ICL-model $M_c = (W_c, \mathcal{B}_c, V_c)$ be the canonical model of the proof of Theorem 1.

Define:

$$||\text{Acc } \varphi|| = |\varphi|_{M_c}$$
$$||\text{Rej } \varphi|| = |\neg\varphi|_{M_c}$$
$$||m|| = \{f : \text{ for every sentence } \varphi, \text{ if } m \vdash \text{Acc } \varphi, \text{ then } f \models \varphi\}.$$

One needs to show that this is a semantic interpretation of $\mathcal{E}$.

Begin by showing that $||m||$ is modally reflexive, i.e. (1) that it has a fixed modal background and (2) that it prefers its own factual content.

(1) Take any selection functions f and f' in $||m||$. If one can show that $f(F(|\varphi|))$ is defined iff $f'(F(|\varphi|))$ is defined, for all factual φ, then f and f' have the same modal background. So assume that $f(F(|\varphi|))$ is defined and for reductio that $f'(F(|\varphi|))$ is not defined. As $f(F(|\varphi|))$ is defined, $\text{MB}_f \cap F(|\varphi|)$ is nonempty. As a result $f \not\models \varphi \to \bot$. So $m \not\vdash \text{Acc } \varphi \to \bot$. So $m \circledast \text{Acc } \varphi$ is coherent. By Theorem 2, $m \circledast \text{Acc } \varphi \vdash \text{Acc}\neg\bot$ and so $m \circledast \text{Acc } \varphi \vdash \text{Rej}\bot$. So $m \vdash \text{Rej}(\varphi \to \bot)$ and so $m \vdash \text{Acc}\neg(\varphi \to \bot)$. By the construction $f' \models \neg(\varphi \to \bot)$. But as $\text{MB}_{f'} \cap F(|\varphi|)$ is empty, $f' \models \varphi \to \bot$ and we have a contradiction.

(2) Let $X = F(||m||)$. Assume that $f \in ||m||$. Assume that $A \cap F(||m||) \neq \emptyset$. There is thus some $f' \in ||m||$ such that $f'(W) \in A$. So $m \not\vdash \text{Acc}\neg S(A)$ ($m \not\vdash \text{Rej}S(A)$). Assume for reductio that $f(A) \notin F(||m||)$. There is then some B such that $f(A) \in B$ and $B \cap F(||m||) = \emptyset$. So $m \vdash \text{Acc}\neg S(B)$ but $m \not\vdash \text{Acc}(S(A) \to S(B))$. But given

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Redundancy this violates R-Vacuity. For by Redundancy $m \oplus \text{Acc}\ S(A) \vdash \text{Acc}\neg S(B)$ and so by R-Vacuity $m \circledast \text{Acc}S(A) \vdash \text{Acc}\neg S(B)$. So we have a contradiction: $f(A) \in F(||m||)$.

Note that as the set of accepted sentences in m is closed under $\models_{\text{ICL}}$ (follows from the above Theorem 2) we have (this can be shown by familiar techniques from modal logic), where $a = \text{Acc}\ \varphi$ or $a = \text{Rej}\ \varphi$:

$$m \vdash a \text{ iff } ||m|| \subseteq ||a||. \qquad \square$$

Proof of Corollary 1. Assume that mental states are represented as closed propositions (so $||m|| = \bigcap\{|\varphi| : ||m|| \subseteq |\varphi|\}$). It follows that if $||m|| \subseteq |\varphi|$ iff $||m'|| \subseteq |\varphi|$ for all φ, then $||m|| = ||m'||$.

1. $||m \oplus \text{Acc}\ \varphi|| \subseteq |\psi|$ iff $||m \oplus \text{Acc}\ \varphi|| \subseteq ||\text{Acc}\ \psi||$ iff $m \oplus \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$ iff $m \vdash \text{Acc}(\varphi \supset \psi)$ iff $||m|| \subseteq ||\text{Acc}\ \varphi \supset \psi||$ iff $||m|| \subseteq |\varphi \supset \psi|$ iff $||m|| \cap |\varphi| \subseteq |\psi|$ iff $||m|| \cap ||\text{Acc}\ \varphi|| \subseteq |\psi|$. So $||m \oplus \text{Acc}\ \varphi|| = ||m|| \cap ||\text{Acc}\ \varphi||$.

2. $||m \circledast \text{Acc}\ \varphi|| \subseteq |\psi|$ iff $||m \circledast \text{Acc}\ \varphi|| \subseteq ||\text{Acc}\ \psi||$ iff $m \circledast \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$ iff $m \vdash \text{Acc}(\varphi \to \psi)$ iff $||m|| \subseteq ||\text{Acc}\ \varphi \to \psi||$ iff $||m|| \subseteq |\varphi \to \psi|$. By showing that $||m|| \subseteq |\varphi \to \psi|$ iff $||m||/F(|\varphi|) \subseteq |\psi|$, we thus show that $||m \circledast \text{Acc}\ \varphi|| = ||m||/F(|\varphi|)$. So assume that $||m|| \subseteq |\varphi \to \psi|$. If $m/F(|\varphi|)$ is empty, then trivially it is a subset of $|\psi|$. So assume that $m/F(|\varphi|)$ is nonempty. Take any $f \in ||m||/F(|\varphi|)$. There is some $f' \in ||m||$ such that $f = f'/F(|\varphi|)$. As $f' \in |\varphi \to \psi|$, $f'/F(|\varphi|) = f \in |\psi|$. So $||m||/F(|\varphi|) \subseteq |\psi|$. For the other direction assume that $||m||/F(|\varphi|) \subseteq |\psi|$. If $||m||/F(|\varphi|)$ is empty then $||m|| \subseteq |\varphi \to \psi|$. So assume that $||m||/F(|\varphi|)$ is nonempty. Take any $f \in ||m||$. As $f/F(|\varphi|) \in |\psi|$, $f \in |\varphi \to \psi|$. So $||m|| \subseteq |\varphi \to \psi|$.

3. A direct consequence of (2) and the construction. $\square$

Proof of Theorem 5. The claim can be proven by showing how to construct a signature expressivist model for any ICL-model on either L or L_R. The construction provided equates mental states and attitudes with their propositional content (so mental states and attitudes become modal propositions). As a result, for any mental state m, $||m|| = m$, and for any attitude a, $||a|| = a$ (this is 'cheating' somewhat as logically equivalent attitudes such as $\text{Acc}\neg\varphi$ and $\text{Rej}\ \varphi$ will turn out to be identical rather than merely equivalent but this won't affect the main result).

Take any ICL-model M. Let the set of mental states $\mathcal{M}$ be the set of propositions that are modally reflexive. A mental state m is thus a modal proposition with a fixed modal background; we can denote this modal background by MB_m.

The set of incoherent mental states is defined: $\mathcal{C} = \{\emptyset\}$. Define:

$$\text{Acc}\ \varphi =_{df} |\varphi|_M.$$
$$\text{Rej}\ \varphi =_{df} |\neg\varphi|_M.$$

The set of attitudes $\mathcal{A}$ is thus the set of modal propositions that can be expressed with a sentence. Define (for $a \in \mathcal{A}$):

$$m \vdash a \text{ iff}_{df}\ m \subseteq a.$$
$$m \oplus a =_{df} m \cap a.$$
$$m \circledast a =_{df} m//a.$$

Finally, for mental states m and m':

$$mRm' \text{ iff}_{\text{df}} \text{ there is some } A \in \mathcal{B} \text{ such that } m' \subseteq m/A.$$

This ends the construction. One now needs to show that it satisfies all the requirements on expressivist models for the language L (and so for the language L_R). It is easy to see that R is reflexive and transitive and that $mR(m \circledast a)$ and $mR(m \oplus a)$.

Consider the requirements governing acceptance and rejection. The proof that the boolean connectives satisfies the stated requirements on acceptance and rejection is left as an exercise.

$\rightarrow$Acc: $m \vdash \text{Acc}\ \varphi \rightarrow \psi$ iff for all $f \in m$: $f \models \varphi \rightarrow \psi$ for all $f \in m$ iff $\text{MB}_m \cap |\varphi| = \emptyset$, or for all $f \in m$: $f /\!/ |\varphi| \models \psi$ iff (as $m \circledast \text{Acc}\ \varphi = m /\!/ |\varphi|$)$m \circledast \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$.

$\rightarrow$Rej: $m \vdash \text{Rej}\ \varphi \rightarrow \psi$ iff $f \not\models \varphi \rightarrow \psi$ for all $f \in m$ iff $m /\!/ |\varphi| \neq \emptyset$ and, for all $f \in m$: $f /\!/ |\varphi| \not\models \psi$ iff $m \circledast \text{Acc}\ \varphi$ is coherent and $m \circledast \text{Acc}\ \varphi \vdash \text{Rej}\ \psi$.

Now the structural requirements. The proof that $\oplus$ as defined satisfies the requirements is left as an exercise, as are the requirements [Classicality 1], [Classicality 2] and [Explosion].

[Strong Success] Assume that $(m \circledast \text{Acc}\ \varphi)Rm'$. By construction $m' \subseteq (m \circledast \text{Acc}\ \varphi)/A$, for some A, i.e. by construction, $m' \subseteq (m /\!/ |\varphi|)/A$. Take any $f \in m'$. There is some $f' \in m$ such that $f = (f' /\!/ |\varphi|)/A$. By definition $f' /\!/ |\varphi|$ forces $|\varphi|$. So $f \in |\varphi|$, i.e. $f \models \varphi$. So $m' \models \varphi$. So $m \circledast \text{Acc}\ \varphi \Vdash \text{Acc}\ \psi$.

[Weak Redundancy] Assume that $m \Vdash \text{Acc}\ \varphi$. So $m' \models \varphi$ for every m' such that mRm'. Take any $f \in m$. Assume for reductio that f does not force $|\varphi|$. So there is some A such that $f/A \notin |\varphi|$. But then $m/A \not\subseteq |\varphi|$, i.e. $m/A \not\models \varphi$. But $mR(m/A)$ and so we have a contradiction. So f forces $|\varphi|$; so $f /\!/ |\varphi| = f$. This holds for all $f \in m$ so $m /\!/ |\varphi| = m \circledast \text{Acc}\ \varphi = m$.

[R-Permutation] Follows trivially from the construction (as $f/A/B = f/B/A$ for any A and B).

[R-Inclusion] Assume that $m \circledast \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$ (where φ and ψ are factual). Two cases. (1) $m \cap |\varphi| = \emptyset$. Trivially: $m \cap |\varphi| \subseteq |\psi|$ so, $m \oplus \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$. (2) $m \cap |\varphi| \neq \emptyset$. Take any $f \in m \cap |\varphi|$, $f(W) = f(F(|\varphi|)) = f/F(|\varphi|)(W)$ and by assumption $f/F(|\varphi|) \in |\psi|$. So $f \in |\psi|$. So $m \cap |\varphi| \subseteq |\psi|$ and so $m \oplus \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$.

[R-Vacuity] Assume that $m \oplus \text{Acc}\ \varphi$ is coherent and $m \oplus \text{Acc}\ \varphi \vdash \text{Acc}\ \psi$, where φ and ψ are factual. So $m \cap |\varphi| \neq \emptyset$ and $m \cap |\varphi| \subseteq |\psi|$. Take any $f \in m$. As m prefers its own factual content $f(F(|\varphi|)) \in F(m)$. So there is some $f' \in m$ such that $f(F(|\varphi|)) = f'(W)$. So $f' \in m \cap |\varphi|$. As $m \cap |\varphi| \subseteq |\psi|$, we have $f' \in |\psi|$. So, as ψ is factual, $f/F(|\varphi|) \in |\psi|$. So $m/F(|\varphi|) = m \circledast \text{Acc}\ \varphi \subseteq |\psi|$. $\square$

Proof of Observation 3. (i) is trivial. (ii) By definition $p(|\neg\varphi|_\psi \cap |\psi|) = p(|\psi \wedge (\psi \rightarrow \neg\varphi)|)$. Note that $|\psi \wedge (\psi \rightarrow \neg\varphi)| = |\psi \wedge \neg(\psi \rightarrow \varphi)|$. So: $p(|\psi \wedge \psi \rightarrow \neg\varphi)|) = p(|\psi|) - p(|\psi \wedge (\psi \rightarrow \varphi)|)$. But then

$$\begin{aligned}\Pr(\neg\varphi \mid \psi) = \frac{p(|\varphi|_\psi \cap |\psi|)}{p(|\psi|)} &= \frac{p(|\psi|) - p(|\psi \wedge (\psi \rightarrow \varphi)|)}{p(|\psi|)} \\ &= 1 - \frac{p(|\varphi)|_\psi \cap |\psi|)}{p(|\psi|)} = 1 - \Pr(\varphi \mid \psi).\end{aligned}$$

(iii) Assume that φ and χ are logically incompatible. Note that $|\psi \wedge (\psi \rightarrow (\varphi \vee \chi))| = |\psi \wedge (\psi \rightarrow \varphi)| \cup |\psi \wedge (\psi \rightarrow \chi)|$. Moreover, $|\psi \wedge (\psi \rightarrow \varphi)| \cap |\psi \wedge (\psi \rightarrow \chi)| = \emptyset$. From this it follows in just a few steps that $\Pr(\varphi \vee \chi \mid \psi) = \Pr(\varphi \mid \psi) + \Pr(\chi \mid \psi)$. $\square$

Proof of Observation 4. First prove the claim for base conditionals. It is assumed that if $p(|\varphi|) = 0$, then $p(|\varphi \rightarrow \psi|) = 1$. Note that $|\varphi \rightarrow \psi|_\chi \cap |\chi| = |\chi \rightarrow (\varphi \rightarrow \psi)| \cap |\chi| =$

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(by Import-Export) $|((\chi \wedge \varphi) \to \psi) \wedge \chi|$. We find

$$\begin{aligned} p(|((\chi \wedge \varphi) \to \psi) \wedge \chi|) &= p(|(\chi \wedge \varphi) \to \psi|) - p(|((\chi \wedge \varphi) \to \psi) \wedge \neg\chi|) \\ &= p(|(\chi \wedge \varphi) \to \psi|) - p(|(\chi \wedge \varphi) \to \psi|) \times p(|\neg\chi|) \quad \text{(by SI)} \\ &= p(|(\chi \wedge \varphi) \to \psi|) - p(|(\chi \wedge \varphi) \to \psi|) \times (1 - p(|\chi|)) \\ &= p(|(\chi \wedge \varphi) \to \psi|) - p(|(\chi \wedge \varphi) \to \psi|) + p(|(\chi \wedge \varphi) \to \psi|) \times p(|\chi|) \\ &= p(|(\chi \wedge \varphi) \to \psi|) \times p(|\chi|). \end{aligned}$$

So (where $\Pr(\varphi \wedge \chi) > 0$):

$$\begin{aligned} \Pr(\varphi \to \psi \,|\, \chi) &= \frac{p(|\varphi \to \psi|_\chi \cap |\chi|)}{p(|\chi|)} = \frac{p(|(\chi \to (\varphi \to \psi)) \wedge \chi|)}{p(|\chi|)} \\ &= \frac{p(|((\chi \wedge \varphi) \to \psi) \wedge \chi|)}{p(|\chi|)} = \frac{p(|(\chi \wedge \varphi) \to \psi|) \times p(|\chi|)}{p(|\chi|)} \\ &= p(|(\chi \wedge \varphi) \to \psi|) = \frac{p(|(\chi \wedge \varphi) \to \psi|) \times p(|\chi \wedge \varphi|)}{p(|\chi \wedge \varphi|)} \\ &= \frac{p(|((\chi \wedge \varphi) \to \psi)| \cap |\chi \wedge \varphi|)}{p(|\chi \wedge \varphi|)} = \frac{p(|\psi|_{\chi \wedge \varphi} \cap |\chi \wedge \varphi|)}{p(|\chi \wedge \varphi|)} \\ &= \Pr(\psi \,|\, \varphi \wedge \chi). \end{aligned}$$

Next, note that by Import-Export an arbitrarily right-hand nested conditional $\varphi_1 \to (\varphi_2 \to \cdots (\varphi_n \to \psi) \cdots)$ is logically equivalent to a base conditional $(\varphi_1 \wedge \cdots \wedge \varphi_n) \to \psi$. Thus $|\varphi_1 \to (\varphi_2 \to \cdots (\varphi_n \to \psi) \cdots)|_\chi = |(\varphi_1 \wedge \cdots \wedge \varphi_n) \to \psi|_\chi$ and so they will get the same conditional probability $\Pr(\cdot \,|\, \chi)$. □

Proof of Observation 5. It is assumed that if $p(|\varphi|) = 0$, then $p(|\varphi \to \psi|) = 1$. So consider the following form of (GI):

$$p\left(\left|\neg\varphi \wedge \bigwedge_{1 \le i \le n} ((\varphi \wedge \psi_i) \to a_i)\right|\right) = p(|\neg\varphi|) \times p\left(\left|\bigwedge_{1 \le i \le n} ((\varphi \wedge \psi_i) \to a_i)\right|\right). \quad \text{(A)}$$

Here each a_i is an atomic sentence or a negated atomic sentence (in light of the original assumption we do not need the restriction that the antecedents have nonzero probability). From (A) we can establish:

$$p\left(\left|\neg\varphi \wedge \bigwedge_{1 \le i \le n} ((\varphi \wedge \varphi_i) \to a_i) \wedge \bigwedge_{1 \le i \le m} \neg((\varphi \wedge \chi_i) \to \bot)\right|\right) = \\ p(|\neg\varphi|) \times p\left(\left|\bigwedge_{1 \le i \le n} ((\varphi \wedge \varphi_i) \to a_i) \wedge \bigwedge_{1 \le i \le m} \neg((\varphi \wedge \chi_i) \to \bot)\right|\right). \quad \text{(B)}$$

For the probability of any conjunction containing negated terms can always be written out as the sum or difference of probabilities of conjuncts containing only unnegated terms. E.g. $p(|\neg\varphi \wedge ((\varphi \wedge \psi) \to a) \wedge \neg((\varphi \wedge \chi) \to \bot)|) = p(|\neg\varphi \wedge ((\varphi \wedge \psi) \to a)|) - p(|\neg\varphi \wedge ((\varphi \wedge \psi) \to a) \wedge ((\varphi \wedge \chi) \to \bot)|)$. So (B) can be written as a series of terms where there are no negated conditionals, in which case (GI) applies to each term.

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From (B) it follows by basic probabilistic principles (see previous observation) that:

$$p\left(\left|\varphi \wedge \bigwedge_{1\leq i\leq n}((\varphi\wedge\varphi_i)\to a_i)\wedge \bigwedge_{1\leq i\leq m}\neg((\varphi\wedge\chi_i)\to\bot)\right|\right) =$$
$$p(|\varphi|)\times p\left(\left|\bigwedge_{1\leq i\leq n}((\varphi\wedge\varphi_i)\to a_i)\wedge \bigwedge_{1\leq i\leq m}\neg((\varphi\wedge\chi_i)\to\bot)\right|\right). \quad \text{(C)}$$

Recall (Theorem 3) that the disjunctive normal form of a conditional $\varphi\to\psi$ is a disjunction $D(\varphi\to\psi)$ of mutually inconsistent disjuncts $\delta_1\vee\cdots\vee\delta_n$ where each δ_j has the form

$$\bigwedge_{1\leq i\leq k_j}((\varphi\wedge\psi_i^j)\to a_i^j)\wedge \bigwedge_{1\leq i\leq m_j}\neg((\varphi\wedge\chi_i^j)\to\bot).$$

Together with claim (C) this ensures that for any sentence $\varphi\to\psi$:

$$\begin{aligned}
&p(|\varphi\wedge(\varphi\to\psi)|)\\
&\quad = p(|\varphi\wedge D(\varphi\to\psi)| = p(|(\varphi\wedge\delta_1)\vee\cdots\vee(\varphi\wedge\delta_n)|) = \sum_{1\leq j\leq n} p(|\varphi\wedge\delta_j|)\\
&\quad = \sum_{1\leq j\leq n} p\left(\left|\varphi\wedge \bigwedge_{1\leq i\leq k_j}((\varphi\wedge\psi_i^j)\to a_i^j)\wedge \bigwedge_{1\leq i\leq m_j}\neg((\varphi\wedge\chi_i^j)\to\bot)\right|\right).\\
&\quad = p(|\varphi|)\times \sum_{1\leq j\leq n} p\left(\left|\bigwedge_{1\leq i\leq k_j}((\varphi\wedge\psi_i^j)\to a_i^j)\wedge \bigwedge_{1\leq i\leq m_j}\neg((\varphi\wedge\chi_i^j)\to\bot)\right|\right).\\
&\quad = p(|\varphi|)\times p(|\varphi\to\psi|). \qquad \text{(D)}
\end{aligned}$$

Now assume that ψ has the form $\chi\to\sigma$, where χ is factual and σ can be an arbitrary sentence of L_R. By the definition of $\Pr(\cdot\,|\,\cdot)$ it follows that (where $\Pr(\varphi\wedge\chi)>0$):

$$\begin{aligned}
\Pr(\chi\to\sigma\,|\,\varphi) &= \frac{p(|\chi\to\sigma|_\varphi\cap|\varphi|)}{p(|\varphi|)} = \frac{p(|(\varphi\to(\chi\to\sigma))\wedge\varphi|)}{p(|\varphi|)}\\
&= \frac{p(|\varphi\to(\chi\to\sigma)|)\times p(|\varphi|)}{p(|\varphi|)} \quad \text{(by (D))}\\
&= p(|\varphi\to(\chi\to\sigma)|) = p(|(\varphi\wedge\chi)\to\sigma|)\\
&= \frac{p(|(\varphi\wedge\chi)\to\sigma|)\times p(|\varphi\wedge\chi|)}{p(|\varphi\wedge\chi|)}\\
&= \frac{p(|((\varphi\wedge\chi)\to\sigma)\wedge\varphi\wedge\chi|)}{p(|\varphi\wedge\chi|)} \quad \text{(by (D))}\\
&= \frac{p(|\sigma|_{\varphi\wedge\chi}\cap|\varphi\wedge\chi|)}{p(|\varphi\wedge\chi|)}\\
&= \Pr(\sigma\,|\,\varphi\wedge\chi).
\end{aligned}$$

□

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Proof of Observation 6. Assume that $X, f \models \text{Might}\,(\varphi \rightarrow \psi)$. Then there is an $f' \in X$ such that $X, f' \models \varphi \rightarrow \psi$. The case when f'/φ is empty is trivial (recall that X has a fixed modal background). So consider the case when $X/\varphi, f'/\varphi \models \psi$. As $f/\varphi \in X/\varphi$: $X/\varphi, f/\varphi \models \text{Might}\ \psi$, so $X, f \models \varphi \rightarrow \text{Might}\ \psi$.

Assume instead that $X, f \models \varphi \rightarrow \text{Might}\,\psi$. The case when f/φ is empty is trivial. So consider the case when $X/\varphi, f/\varphi \models \text{Might}\,\psi$. So there is some $f' \in X/\varphi$ such that $X/\varphi, f' \models \psi$. But there is some $f'' \in X$ such that $f''/\varphi = f'$. So $X/\varphi, f''/\varphi \models \psi$. So $X, f'' \models \varphi \rightarrow \psi$. But then $X, f \models \text{Might}\,(\varphi \rightarrow \psi)$. □

Proof of Observation 7. Let $\bot A = \{f \mid f/A = \emptyset\}$. Note first:

$$\begin{aligned}
&\{f' \mid f'/\text{MB}_f \in |\varphi \rightarrow \psi|\} \cup \bot\text{MB}_f \\
&\quad = \{f' \mid [f'/\text{MB}_f \cap F(|\varphi|)] \in |\psi|\} \cup \bot(\text{MB}_f \cap F(|\varphi|)) \cup \bot\text{MB}_f \\
&\quad = \{f' \mid [f'/\text{MB}_f \cap F(|\varphi|)] \in |\psi|\} \cup \bot(\text{MB}_f \cap F(|\varphi|)). \qquad \text{(A)}
\end{aligned}$$

The last step holds in virtue of $\bot\text{MB}_f \subseteq \bot(\text{MB}_f \cap F(|\varphi|))$.

(1) Take any f such that $f \models \neg(\varphi \rightarrow \bot)$. $f \models P_n(\varphi \rightarrow \psi)$ iff $p * \text{MB}_f(|\varphi \rightarrow \psi|) = V(n)$ iff $p(\{f' \mid f'/\text{MB}_f \in |\varphi \rightarrow \psi|\} \cup \bot\text{MB}_f) = V(n)$ iff (by (A)) $p * (\text{MB}_f \cap F(|\varphi|))(|\psi|) = V(n)$ iff (as $\text{MB}_{f/F(|\varphi|)} = \text{MB}_f \cap F(|\varphi|)$) $f/F(|\varphi|) \models P_n(\psi)$ iff (as $f/F(|\varphi|) \neq \emptyset$) $f \models \varphi \rightarrow P_n(\psi)$. So: $\models \neg(\varphi \rightarrow \bot) \supset (P_n(\varphi \rightarrow \psi) \equiv (\varphi \rightarrow P_n(\psi)))$.

(2) $f \models P_n(\psi \mid \varphi)$ iff $p * (\text{MB}_f \cap F(|\varphi|))(|\psi|) = V(n)$ or $f/F(|\varphi|) = \emptyset$ iff $f/F(|\varphi|) \models P_n(\psi)$ or $f/F(|\varphi|) = \emptyset$ iff $f \models \varphi \rightarrow P_n(\psi)$.

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