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BOOK REVIEWS

Computational Differential Equations. By K. Eriksson, D. Estep, P. Hansbo & C. Johnson. Cambridge University Press, 1997. 538 pp. ISBN 0521 56738 6. £29.95

This unconventional book emerged from the extensive rewriting of its precursor entitled *Numerical Solution of Partial Differential Equations*, single-authored by C. Johnson. It is the second part of a trilogy consisting of an introductory book entitled *Introduction to Computational Differential Equations* where fundamental calculus and linear algebra are discussed, and an advanced book entitled *Advanced Computational Differential Equations* where applications of the methods to the physical and engineering sciences are discussed and specific numerical implementations are outlined. The goal of this book, as stated at the preface, is to provide a student with the essential theoretical and computational tools that make it possible to use differential equations in science and engineering effectively. Furthermore, the authors state, and a quick glance through the table of contents confirms, that the backbone of this book is a unified presentation of numerical solution techniques for differential equations based on Galerkin methods. Accordingly, several classes of other methods, including finite-difference, finite-volume, and spectral methods, are not discussed.

I used the word unconventional in the first paragraph for several reasons. Most important, the authors introduce the subject from a historical point of view, and motivate the reading of their book by arguing that to interface calculus and numerical computation is both useful and necessary. To the unsuspecting reader, this book may be erroneously mistaken for a tribute to Leibniz, for every chapter contains extensive discussions of, drawings and excerpts from, the work and life of the co-founder of differential and integral calculus. Parts of the book may also be mistaken for a brief history of mathematical modelling and numerical simulation based on differential equations. Indeed, the first two chapters are devoted entirely to historical retrospectives.

The book is divided into three parts: the first part is an introduction; the second part discusses ordinary differential equations; and the third part discusses partial differential equations. The authors' enthusiasm in writing this text is apparent and is clearly reflected in the manner by which they introduce new concepts; intellectual curiosity takes priority over the need to build an apparatus. An example is the ingenious introduction of the Cauchy sequence in Chapter 3 entitled a Review of Calculus. This chapter contains a summary of key concepts and theorems, often accompanied by rigorous mathematical proofs. Among other things, one learns that Leibniz conceived the idea of integral calculus on October 29, which also happens to be the birthday of this reviewer. Each subsequent chapter concisely but clearly introduces and explains a key idea or class of methods, often by example and always on a need-to-know basis.

Chapters 4 and 5 introduce selected concepts of linear algebra, polynomial and trigonometric interpolation and integration. The level of the presentation is typical of an intermediate-level text on numerical analysis. The purpose is to prepare the ground for the finite-element method, and the material is screened carefully for that purpose. Chapter 6 introduces the Galerkin projection and the finite-element method for problems in one dimension. Part I concludes with Chapter 7 which reviews the solution of systems of linear algebraic equations by direct and iterative methods.

Chapters 8–12, comprising Part II, discuss the solution of several classes of problems involving linear ordinary differential equations by finite-element methods. This includes two-point boundary value problems, and initial-value problems for a single, or a system of, first-order equations. The intentional exclusion of nonlinear equations explains the absence of the shooting and related methods, reminds us that the authors' interest lies in the Galerkin projection method, and emphasizes that this text should be regarded as an introduction to finite-element methods from the perspective of the applied mathematician.

Part III extends established concepts and procedures to two and higher dimensions. Chapter 13 is an introduction to differential and integral calculus for functions of more than one variable; Chapter 14 discusses polynomial interpolation in two dimensions, with an informative section on triangulation: Chapter 15 focuses on the finite-element solution of the Poisson equation; Chapters 16, 17, 18 and 19 concentrate, respectively, on the one-dimensional unsteady heat conduction equation, the two-dimensional wave equation, the steady two-dimensional convection-diffusion equation, and the unsteady two-dimensional convection-diffusion equation. The penultimate chapter, 20, discusses eigenvalue problems. The final chapter, 21, entitled the power of abstraction, explains how a generalized formalism for strongly elliptic equation may be used to derive useful results for equations commonly encountered in mathematical physics.

This book is not about practical numerical computation but is intended to explain the framework upon which the computation relies. It is addressed to students of, or researchers with a sound background on, applied mathematics. Frequent error analyses and proofs of convergence are reminders of this focus. The careful reader will learn the fundamentals of Galerkin and finite-element methods, and should then be in a position to use them rigorously and skilfully to solve particular problems, carrying out error and convergence analyses if so desired. A software package called *Femlab* accompanies the book and is freely available through the internet, although I was unable to locate the distribution URL or e-mail addresses anywhere in the text. The programs reportedly solve several classes of differential equations with adaptive error control. I suspect that the software package is under production and will be available upon publication of the introductory volume.

One cannot find the heart to criticize any aspect of this delightful and illuminating book which I recommend for general reading and one's continuing education. The book could be used as a text in an advanced undergraduate course or introductory graduate course in curricula of applied mathematics. It could also be used as a text in corresponding courses in a number of disciplines of applied sciences and engineering, but the students must have a strong mathematics background as well as carry a certain amount of intellectual curiosity, or they may complain about the authors' rightful demand for attention to the fundamentals.

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SHORT NOTICES

Mathematical Analysis in Engineering. By C. C. Mei. Cambridge University Press, 1997. 479 pp. ISBN 0 521 58798 0. £19.95.

This textbook was first published as a hardback edition in 1995 and reviewed in *J. Fluid Mech.* vol. 316, 1996, p. 375. This paperback edition can certainly be welcomed, as it will make the text more widely available at a lesser cost. The text is based around a lecture course on engineering mathematics for beginning graduate students at MIT. However, much of the material could well be encountered in second and third year undergraduate courses; in particular, this comment would apply to the first five chapters which cover such common topics as separation-of-variable techniques and Fourier series. Much of the later part of the text is concerned with integral transforms and complex-variable techniques. For the most part, the emphasis is on mathematical techniques for linear problems. The last two chapters on perturbation methods are particularly good as they at least indicate to students how to go beyond traditional 'exact' methods. An unusual feature here is the brief introduction to symbolic computing in this context. The main feature of this text which distinguishes it from the many texts on engineering mathematics is the wealth of worked examples. Indeed, each topic is motivated and illustrated by applications, which range widely over mechanics, fluid flow, diffusion and elasticity. Some of the illustrative examples are quite challenging and others are drawn from application areas not commonly featured in text books at this level. The text is clearly laid out and very readable. It is highly recommended.

Thinking About Ordinary Differential Equations. By R. E. O'Malley. Cambridge University Press, 1997. 259 pp. ISBN 0 521 55742 9. £14.95.

This short text on ordinary differential equations is aimed at the higher undergraduate level for an applied mathematics or engineering programme. The first four chapters cover the traditional topics of linear second-order equations, power series methods, special functions and linear first-order systems. Chapter five on stability begins with the familiar phase plane for a two-dimensional autonomous system, but then moves into higher-dimensional systems and the area of Liapunov functions. The distinguishing feature of this text is the last chapter, on singular perturbation theory, which introduces the notions of both boundary layers and multiple scales. This text is certainly suitable for an advanced lecture course, particularly as it contains a wealth of examples and solutions.

From Calculus to Chaos: an Introduction to Dynamics. By D. Acheson. Oxford University Press, 1997. 269 pp. ISBN 0 19 850077 7. £25.00.

This is a delightful introductory text to classical mechanics with a very modern flavour. Throughout the text is enriched with historical interludes, numerical solutions and the qualitative flavour of the modern dynamical systems approach. The topics covered range from the classical area of oscillations and planetary motion, to very brief but illuminating discussions of waves, diffusion, and fluid flow. Experts in these areas might well find these discussions too brief to display clearly even the basic subtleties, but they can serve as a starting point to motivate students to go further. The last three chapters introduce the reader to instability, bifurcations and chaotic motion, primarily through well-chosen examples. While this is a very enjoyable book to read, and highly recommended, it would be quite challenging to base an undergraduate subject around it, or to use it as a text in a 'traditional' mathematical methods subject within a science or engineering curriculum. But the reader is strongly encouraged to try! A very useful feature is that even although some of the examples are at quite a difficult level, lengthy solutions and hints are provided.