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LIQUIDITY REGULATION AND FINANCIAL STABILITY

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Anticipating a bailout in the event of a crisis distorts financial intermediaries' incentives in multiple dimensions. Bailout payments can, for example, lead intermediaries to issue too much short-term debt while simultaneously underinvesting in liquid assets. To correct these distortions, policymakers may choose to regulate the composition of both the assets and liabilities of intermediaries. I examine these regulations in a version of the Diamond and Dybvig [(1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3), 401–419] model with limited commitment. I demonstrate that, contrary to common wisdom, introducing a minimum liquidity requirement can increase intermediaries' susceptibility to a run by their investors.

Keywords: Bailouts, Liquidation Cost, Macroprudential Policy, Time-Inconsistency

1. INTRODUCTION

Following the global financial crisis of 2007–2009, regulating financial institutions' liquidity holdings has become the topic of both political and academic debates. To promote financial stability, the Basel Committee on Banking Supervision (BCBS) has adopted a new liquidity standard for financial intermediaries, called the liquidity coverage ratio (LCR), as part of the Basel III accords. A main concern motivating liquidity regulation is intermediaries' incentive to issue too much short-term debt and to underinvest in liquid assets, because they anticipate government support in periods of financial distress. As a result, there is widespread agreement by policymakers and regulators on the importance and utility of liquidity regulation.[1] Unfortunately, there is less consensus among academic researchers on the need for liquidity regulation. Allen and Gale (2017), in their survey of the recent literature on liquidity regulation, conclude their paper by writing "With liquidity regulation, we do not even know what to argue about."

This paper contributes to the current debate by examining the role liquidity regulation plays in mitigating the distortions caused by government bailouts. I employ a version of the Diamond and Dybvig (1983) model with the following

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three features: first, I introduce fiscal policy in the form of a public good as in Keister (2016). There is a benevolent policymaker with the ability to tax financial intermediaries' deposits and who can use the revenue from this tax to produce a public good. A second feature is the incorporation of a non-trivial portfolio choice in which intermediaries divide their resources between liquid and illiquid assets, as in Cooper and Ross (1998). Third, I study a model in the spirit of Green and Lin (2003) and Peck and Shell (2003) with no arbitrary restrictions on payment schedule. Together, these adjustments imply that the model is not only suitable for capturing distorted incentives on both sides of intermediaries' balance sheets, but can also be used to explore potential policy options (e.g. liquidity regulation, early payments restriction, and prohibition of bailouts) that are currently being proposed to prevent future crises.[2]

In the environment I study, policymakers in a financial crisis will choose to transfer resources from the public sector to the private sector. From an ex-ante point of view, these bailouts distort intermediaries' incentives in two dimensions. First, intermediaries will choose to increase the amount of consumption they give to investors demanding early withdrawal, since intermediaries do not internalize the effects of these actions on the government's ability to provide the public good when a crisis occurs. Second, intermediaries underinvest in the liquid asset, optimally ignoring the costs that liquidation imposes on the public sector during a crisis.

I consider two regulatory-policy regimes that aim to eliminate the negative incentive effects associated with bailouts. In the first regime, the policymaker places an upper bound on the payments an intermediary can make before it is able to infer whether the crisis has occurred. For some parameter values, the economy is fragile under this policy regime in the sense that there exists an equilibrium in which investors run on their intermediary depending on the realization of a sunspot variable. The allocation in this equilibrium is inefficient because intermediaries underinvest in the liquid asset from a social point of view.

I next consider a secondary regulatory regime in which the policymaker has the additional ability to place a minimum liquidity requirement on intermediaries' asset holdings. Because the policymaker now has two tools, such a regime can potentially eliminate the distortions on both sides of intermediaries' balance sheets. I show that, taking the withdrawal behavior of investors as given, this regime allows the policymaker to generate the welfare-maximizing allocation of resources. In other words, if we take as given that investors will run on the financial system in the bad sunspot state, adding liquidity regulation reliably raises welfare.

Paradoxically, however, liquidity regulation may actually make the financial system fragile. Introducing a restriction on the composition of intermediaries' assets and liabilities has two competing effects on investors' incentive to run. First, the regulation forces intermediaries to hold a more liquid portfolio, which tends to make the financial system more stable. At the same time, however, the fact that intermediaries are more liquid will lead the policymaker to loosen the

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early payments restriction and allow intermediaries to increase their early payments. This change tends to increase the incentive for investors to run. I show that the optimal policy regime in my model depends on which of these two effects dominates.

I further show that the relationship between the desirability of liquidity regulation and the configuration of liquidation costs is non-monotone. If there is no liquidation cost, in fact, the minimum liquidity requirement chosen by a policymaker is not a binding constraint on intermediaries' behavior. In such case, adding liquidity regulation clearly has no effect on the financial system. If the liquidation cost is low, intermediaries will not have much incentive to hold a large amount of the liquid asset as a buffer. In this case, a tightening of the liquidity requirement that forces intermediaries to hold a more liquid portfolio will generate a strong incentive to increase early payments, precisely because the insurance value of holding the extra liquid asset is small. By encouraging patient investors to withdraw early, adding liquidity regulation can thus lead to an increase in fragility. When the liquidation cost is large, I find that either of the two competing effects can dominate and I derive the conditions under which each policy regime is desirable. The precise cutoff points will depend on the specific features of the economy. In this sense, my model thus provides meaningful policy advice to guide policymakers' decisions to impose liquidity regulation.

Note that a time-inconsistency problem arises here. If the policymaker could commit to choices of both regulatory tools before investors choose their withdrawal strategies, then adding liquidity regulation would always be at least as effective as merely imposing the cap on early payments. In particular, the policymaker could choose to keep the early payments restriction at the same level and to optimally choose the level of the liquidity requirement. Doing so cannot decrease and would in most cases increase welfare. Without commitment, however, the timing of the two policy decisions matters. In this setting, once intermediaries are required to hold a more liquid portfolio of assets, the policymaker will optimally choose to ease up on the early payments restriction. Investors anticipate this action, which increases their incentive to withdraw early. I show that, contrary to the conventional wisdom, these equilibrium changes can in some case lead to an increase in fragility.[3]

The model in this paper is most closely related to that in Keister (2016), which studies a version of the Diamond and Dybvig (1983) model with bailouts but with a single asset and hence no portfolio choice. In that paper, the incentive distortion has only one dimension: when bailouts are permitted, intermediaries offer larger payments to investors who withdraw early than a social planner would advise. In other words, the moral hazard only affects the liabilities side of intermediaries' balance sheets. In my paper, the source of the moral hazard is the same (bailouts), but intermediaries also make a portfolio choice and, therefore, the distortion affects both sides of their balance sheets. This expanded model allows me to explore current regulatory tools that Keister (2016) cannot, especially the imposition of regulations on intermediaries' asset holdings, and the interaction between regulatory tools.

Two other recent papers argue that the optimal approach is a mix of bailouts and prudential policy tool(s). Stavrakeva (2017) builds a three-period model in the spirit of Lorenzoni (2008) where markets are endogenously incomplete. In order to replicate what a central planner would optimally do, regulators impose a minimum capital requirement as well as an instrument that limits intermediaries' liabilities during a crisis when a bailout is expected. Bianchi (2016) develops a non-linear dynamic stochastic general equilibrium model to assess the interaction between bailouts in credit markets and the build-up of risk ex ante. The optimal policy requires, in general, a mix of ex-post intervention (bailouts) and ex-ante prudential policy (taxes on debt and capital income). Keister (2016), Stavrakeva (2017), and Bianchi (2016) stress that the optimal policy mix involves bailouts combined with macro-prudential policy tool(s) correcting distortions associated with bailouts. In my model, I show that adding a liquidity requirement to the policy regime with a cap on early payments can correct the resulting distortions, but may end up making the financial system more fragile.

Other recent research shows that introducing liquidity regulation can have unintended consequences; this finding dates back to Peck and Shell (2010) who show that forcing intermediaries to hold only liquid assets can create the incentive for liquidity-based runs. Malherbe (2014) suggests that imposing liquidity requirements on financial institutions, a policy that mitigates fire-sale externalities, deters market participation, and makes the markets more prone to adverse selection. Using a network model of optimizing intermediaries featuring contagion on both sides of their balance sheets, Aldasoro and Faia (2016) find that a phased-in increase in the LCR produces undesired consequences in the dynamics of systemic risk. Instead of making the system more resilient, the regulation renders it more fragile. Gorton and Muir (2016) argue that the LCR represents a step back towards an immobile collateral system, similar to the system that was in place in the US national banking era. Based on the evidence from that period, they show that such a system contributes to scarcity of safe debt and encourages other forms of short-term debt to emerge, possibly making the system riskier.

This paper also belongs to the literature analyzing the optimal combination of different types of regulatory instruments. Perotti and Suarez (2011) suggest that an optimal regulatory design may combine Pigouvian tax and LCR instruments. The emphasis on each will depend on what is the dominant dimension of heterogeneity across intermediaries. Kashyap et al. (2014) argue that any attempt to implement the social planner's allocations using regulation will involve different regulatory tools including capital regulation, liquidity regulation, deposit insurance, loan to value limits, and dividend taxes. My analysis highlights the importance of time-inconsistency issues in this policy debate by demonstrating that imposing liquidity regulation in an environment without commitment can increase fragility.

2. THE MODEL

My model is a generalization of that in Keister (2016), which is a version of the Diamond and Dybvig (1983) model augmented to include fiscal policy and limited commitment. I add a portfolio choice as in Cooper and Ross (1998) so that I can study how bailouts distort financial intermediaries' asset holdings as well as consider policy interventions to correct this distortion.[4] I begin by describing the physical environment and the basic elements of the model and then define financial fragility in this environment.

2.1. The Environment

Time. I consider an economy with three periods indexed by $t = 0, 1, 2$.

Investors. The economy is populated by a [0, 1] continuum of ex-ante identical investors, indexed by i. I assume that each investor has preferences of the form:

$$U(c_1, c_2, g; \omega_i) = u(c_1, c_2; \omega_i) + v(g) = \frac{(c_1 + \omega_i c_2)^{1-\gamma}}{1-\gamma} + \delta \frac{g^{1-\gamma}}{1-\gamma},$$

where c_t represents private consumption at date $t = 1, 2$ and g is the level of public good provided at date 1. The parameter $\delta > 0$ measures the relative importance of the public good. The preference type of investor i, denoted by ω_i, is a binomial random variable with support $\Omega \equiv \{0, 1\}$. With probability π an investor is impatient (i.e. $\omega_i = 0$) and only values the date 1 consumption; with probability $1 - \pi$ the investor is patient and values the sum of date 1 and date 2 consumption. An investor's type ω_i (impatient or patient) is private information and is revealed to the investor at the beginning of date 1. The fraction of investors in the population who will be impatient is also π due to a law of large numbers. As in Diamond and Dybvig (1983), the coefficient of relative risk-aversion γ is assumed to be greater than one.

At date 0, each investor is endowed with one unit of all-purpose good that can be used for consumption or investment. Investors pool their resources through an intermediation technology to insure against individual liquidity risk. This technology is operated in a central location by a large number of identical and competitive financial intermediaries. After depositing her endowment, each investor chooses either to withdraw her funds at date 1 or to wait to withdraw until date 2. Those investors who contact their intermediary at date 1 arrive one at a time in the order determined by their index i. This index is private information and intermediaries only observe that an investor has arrived to withdraw.

Sunspot. As in Peck and Shell (2003) and others, I introduce an extrinsic sunspot signal on which investors can base their withdrawal decisions. The economy will be in one of two states, $s \in S \equiv \{\alpha, \beta\}$, with probabilities $\{1 - q, q\}$. Investors observe the realization of the state of nature at the beginning of date 1. The policymaker and intermediaries do not observe the sunspot state, but must infer it based on the fundamental withdrawal demand.[5]

Financial intermediaries. Each intermediary has a large number of investors and aims to maximize these investors' expected utility at all times. There are two kinds of assets, a short-term liquid asset and a long-term illiquid asset. Each asset is represented by a constant-returns-to-scale investment technology. The liquid asset is represented by a *storage* technology that allows one unit of the good placed at date t to be converted into one unit of the good at date $t + 1$, for $t = 0, 1$. The illiquid asset is represented by an *investment* technology that allows one unit of the good at date 0 to be converted into $R > 1$ units of the good at date 2. If the illiquid asset is liquidated prematurely at date 1, it yields $0 < r < 1$ units of the good for each unit invested.

Intermediaries take one unit of the good from each of their investors at date 0 and form a portfolio consisting of x units of the illiquid asset and $1 - x$ units of the liquid asset. As in Green and Lin (2003), Peck and Shell (2003), and others, there are no restrictions on the payment an intermediary can make to a withdrawing investor other the fact that it can only depend on information available to the intermediary when the payment is made.[6] At the beginning of date 1, intermediaries are initially unable to make any inference about the state of nature and choose to give the same level of consumption c_1 to each withdrawing investor with $i \leq \pi$. Note that this payment is determined after investors have made their withdrawal decisions and thereby cannot be used to influence investors' incentive to withdraw early [see Ennis and Keister (2010)]. Once π withdrawals have taken place, intermediaries will, in equilibrium, be able to infer the state of nature by observing whether or not withdrawals stop. If withdrawals continue, they realize that the sunspot state is $s = \beta$ and a run is underway, and they provide payments $c_{1\beta}$ to the remaining investors who withdraw at date 1. This payment will be chosen optimally based on the intermediaries' updated information. At date 2, each of the remaining investors will receive a common amount c_{2s} from their intermediary's remaining resources.

Following Wallace (1988, 1990), intermediaries face the sequential service constraint and hence determine the payment to each withdrawing investor based on the number of withdrawals that have been made so far. Investors cannot directly observe intermediaries' portfolio choice or the payments made to other investors, but they are able to infer the chosen values in equilibrium. As in Ennis and Keister (2009, 2010), intermediaries are unable to pre-commit to future actions, implying that the payment given to each investor is determined as a best response depending on the current situation.[7]

The policymaker. As in Keister and Narasiman (2016), the policymaker collects a fraction τ_s of intermediaries' total deposits as tax revenue immediately after π withdrawals have been made and has the ability to transform these resources into public good. For simplicity, I employ a technology for transforming units of the private good one-for-one into units of the public good. The policymaker is unable to commit to future actions and will choose tax rates and implement regulations as a best response to the situation at hand. The objective of the policymaker is to maximize welfare measured by the equal-weighted sum of investors' expected utilities ex ante:

$$\mathcal{W} = \int_0^1 E\left[U(c_1(i), c_2(i), g; \omega_i)\right]\mathrm{d}i.$$

2.2. Bailouts

The policymaker chooses a tax rate τ_s after it learns the state. When a run is underway, the policymaker will choose to collect fewer taxes, effectively "bailing out" the intermediaries at a cost of lower public consumption.[8] This response will be ex-post efficient because, during a crisis, the policymaker knows that some of the π investors who have already withdrawn were in fact patient, effectively causing a decline in the amount of remaining resources available. The policymaker can mitigate this decline by decreasing public consumption and leaving intermediaries with more funds, which they will optimally distribute among their remaining investors.

In addition, during a crisis, the policymaker is able to reallocate resources across intermediaries. In particular, the policymaker will choose to redistribute assets in such a way that each intermediary has the same ratio of liquid to illiquid assets. While this policy is ex-post efficient, it will tend to undermine intermediaries' incentive to provision for a crisis by holding more liquid assets. This reallocation can be interpreted either as resulting from a publicly directed resolution process for troubled institutions, or as a short-hand representation of other policies (such as a lender of last resort facility) that have the effect of mitigating any differences in the liquidity of intermediaries' portfolios.

2.3. Macroprudential Regulation

I consider two regulatory tools that attempt to eliminate the incentive distortions associated with bailouts.

Early payments restriction. First, the policymaker is allowed to set a cap η_1 on those payments that are made before the intermediaries and the policymaker are able to infer the state, that is

$$c_1 \leq \eta_1. \tag{1}$$

This cap can be interpreted as a restriction on intermediaries' short-term debt and, in fact, is functionally equivalent to the policy of taxing short-term debt studied in Keister (2016). Alternatively, the policy can be interpreted as a restriction on all types of payouts made by intermediaries before they and the policymaker are able to infer the state of nature. In this sense, the policy can be thought of

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representing limits on dividend payments or other measures used to preserve the intermediaries' capital position.

Liquidity requirement. The policymaker is also able to require intermediaries to hold a minimum share of the liquid asset in their portfolio. Letting η_2 denote this fraction, the liquidity requirement can be written as

$$1 - x \geq \eta_2. \tag{2}$$

In practice, liquidity regulations sometimes involve a joint restriction on an intermediary's assets and liabilities. For example, the Basel III LCR requires an intermediary to hold enough liquid assets to cover a specific fraction of its liabilities that mature over the next 30 days. For expositional purposes, it is useful to have one policy that affects only the liabilities side of the balance sheet and another that affects only the asset side, as with equations (1) and (2). The effects I identify below will also be present for more general policies that involve joint restrictions on c_1 and x.

2.4. Financial Crises and Fragility

At the beginning of date 0, each investor can choose either to withdraw at date 1 or to wait until date 2, based on her own preference type ω_i and the state s,

$$y_i : \Omega \times S \longrightarrow \{0, 1\},$$

where $y_i = 0$ corresponds to withdrawing at $t = 1$ and $y_i = 1$ corresponds to withdrawing until $t = 2$. Let y denote a profile of withdrawal strategies for all investors. An impatient investor will clearly choose to withdraw at date 1, since she receives no utility from consuming at date 2. Without loss of generality, I assume a run only potentially occurs in state β. The interesting question is then how patient investors will behave in state β. I study the following partial-run strategy profile for investors[9]:

$$\begin{aligned} &y_i(\omega_i, \alpha) = \omega_i \qquad \text{for all } i, \text{ and} \\ &y_i(\omega_i, \beta) = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \text{ for } \left\{ \begin{array}{c} i \leq \pi \\ i > \pi \end{array} \right\}. \end{aligned} \tag{3}$$

Under this specific profile, each patient investor with $i \leq \pi$ has an opportunity to withdraw early in state β before intermediaries and the policymaker learn the state. Notice that after they have inferred the state, the run halts and the remaining patient investors, those with $i > \pi$, wait to withdraw at date 2. I show below that once intermediaries and the policymaker know the state, their reaction will be such that the remaining patient investors in both states have no incentive to withdraw early and the run must stop. The following definition provides the notion of financial fragility I use in the paper.

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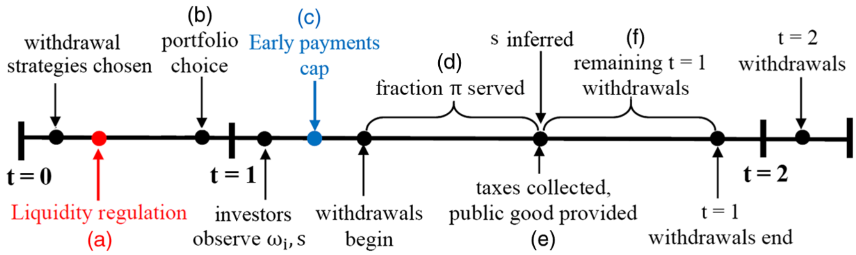

FIGURE 1. Timeline of events.

DEFINITION 1. *A financial system is said to be fragile if the strategy profile in (3) is part of an equilibrium; otherwise the financial system is said to be stable.*

A run equilibrium in this model is the strategy profile (3) for all investors, together with strategies for the policymaker and intermediaries, such that every agent is best responding to the strategies of others.

2.5. Timeline

The timing of decisions is summarized in Figure 1. I begin with investors' endowments already deposited with their intermediary, abstracting from what Peck and Shell (2003) call the "pre-deposit game." Investors are isolated from each other and make a state-contingent withdrawal plan. Intermediaries make a portfolio choice at $t = 0$ after investors choose (state-contingent) withdrawal strategies. At the beginning of date 1, each investor observes her own preference type and the state; she then follows her withdrawal strategy. Intermediaries serve their early withdrawing investors as they arrive. Once π withdrawals have taken place, intermediaries and the policymaker are able to infer the state of nature s. The policymaker chooses the state-contingent tax rates τ_s and provides the public good. Intermediaries continue to provide payments to their remaining investors based on this updated information.

Figure 1 also depicts the timeline of the regulatory decisions. In the environment I study, intermediaries make their portfolio choice at $t = 0$ and choose payments to investors at $t = 1$. In this setting, liquidity regulation is naturally implemented before the early payments restriction. In particular, the minimum liquidity requirement in my model places restrictions on intermediaries' portfolio choice at $t = 0$. Then, after intermediaries' portfolios have been set, the policymaker places a upper bound on the payments that intermediaries are able to provide to their investors at $t = 1$. The key assumption in my model is that the two regulatory parameters are set after investors make their withdrawal strategies. This timing is what allows a time-inconsistency problem to arise in the policymaker's decision problem.

3. EQUILIBRIUM WITH EARLY PAYMENTS RESTRICTION

In this section, I begin by deriving the best response of financial intermediaries and the policymaker to investors' withdrawal strategies profile (3) under the regime without liquidity regulation. In this regime, the policymaker chooses the cap on early payments to offset the distorted incentives on both sides of intermediaries' balance sheets created by bailouts. I then use the allocations generated by these best responses to derive conditions under which an economy is fragile.

3.1. The Best-Response Allocation

In the analysis that follows, I derive the best responses by working backward through the decision points labeled with letters in Figure 1.

Ex-post efficient private allocation. First, consider decision point (**f**) in Figure 1: intermediary j's decision after the state has been revealed and taxes have been collected. After learning the state of nature, each intermediary will use this information to calculate the fraction of its remaining investors who are impatient. Note that (3) implies this fraction will be zero in state α and π in state β. Since all uncertainty has been resolved, the intermediary will choose to give a common amount $c^j_{1\beta}$ to each (impatient) investor who withdraws at date 1 in state β. In addition, each of the remaining patient investors will receive a common amount c^j_{2s} from intermediary j's remaining resources when they withdraw at date 2. Intermediary j will distribute its remaining available resources to solve

$$\mathcal{V}^j_\alpha = \max_{\{c^j_{2\alpha}\}}(1-\pi)u(c^j_{2\alpha}) \text{ and } \mathcal{V}^j_\beta = \max_{\{c^j_{1\beta},c^j_{2\beta}\}}(1-\pi)\left[\pi u(c^j_{1\beta}) + (1-\pi)u(c^j_{2\beta})\right].$$

In writing the constraint set for this problem, it will be useful to recognize some features of the intermediary's overall optimization problem. First, it will never be optimal for an intermediary to liquidate any units of the illiquid asset in state α. If it were doing so, the intermediary could instead adjust its choice of x^j at decision point (b) and provide more consumption to all investors by holding more of the liquid asset and less of the illiquid asset. Similarly, the assumption $R > 1$ implies that it will never be optimal for an intermediary to hold units of the liquid asset until $t = 2$ in state β. In this case, holding less of the liquid asset and more investment would raise consumption for all investors. Intermediaries may, however, hold units of the liquid asset until $t = 2$ in state α, and they may choose to meet additional early withdrawal demand by liquidating some units of the illiquid asset in state β. Thus, I can write the intermediary j's overall resource constraints as follows:

$$\pi c^j_1 \leq 1 - x^j - \tau_\alpha, \tag{4}$$

$$(1-\pi)c^j_{2\alpha} = \underbrace{1 - x^j - \pi c^j_1 - \tau_\alpha}_{\text{resources held in storage for two dates}} + \underbrace{Rx^j}_{\text{matured investment}}, \tag{5}$$

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$$\underbrace{1 - x^j - \pi c_1^j - \tau_\beta}_{\textit{resources placed into storage}} \leq (1-\pi)\pi c_{1\beta}^j, \tag{6}$$

$$(1-\pi)^2 c_{2\beta}^j = R\Big\{x^j - \tfrac{1}{r}\underbrace{\Big[(1-\pi)\pi c_{1\beta}^j - (1 - x^j - \pi c_1^j - \tau_\beta)\Big]}_{\textit{prematurely liquidated investment}}\Big\}. \tag{7}$$

The constraint in (4) states that the consumption of the first π investors to withdraw will always come from the resources placed into storage. This constraint may or may not hold with equality at the solution because the intermediary may choose to hold excess liquidity in state α. Equation (5) says that in state α, the remaining patient investors will consume all of intermediary j's matured investment plus any resources held in storage for two dates. The constraint in (6) reflects the fact that any excess liquidity must be used for date 1 consumption in state β. This expression may or may not hold with equality at the solution because the intermediary may choose, in addition, to liquidate some investment at date 1. The last constraint, equation (7), is the standard pro rata division of remaining resources that determines the payment to the remaining patient investors at date 2.

In solving the intermediary's overall problem, it is helpful to divide the constraint set into four regions according to which constraints hold with equality/inequality as shown in Table 1. In Case A, an intermediary does not hold excess liquidity for the purpose of providing funds to investors in the event of a run and will liquidate investment to provide additional date 1 payments. If the intermediary instead chooses to hold excess liquidity and also chooses to liquidate investment to provide date 1 consumption in state β, I say the solution lies in Case B. In Case C, the additional early payments come only from the resources in storage without liquidating investment if a crisis occurs. Finally, an intermediary could choose to occupy Case D, where there is no excess liquidity and no liquidation.

Letting $\mu_1^j, \mu_{2\alpha}^j, \mu_{1\beta}^j, \mu_{2\beta}^j$ denote the multipliers associated with the resource constraints in equations (4)–(7), the intermediary's optimal choice of $(c_{1\beta}^j, c_{2\alpha}^j, c_{2\beta}^j)$ is characterized by the conditions

$$u'(c_{2\alpha}^j) = \mu_{2\alpha}^j \text{ and } u'(c_{1\beta}^j) = \tfrac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j, u'(c_{2\beta}^j) = \mu_{2\beta}^j. \tag{8}$$

State-contingent tax rates. Now consider decision point (**e**) in Figure 1: the best response of the policymaker after the state of nature has been revealed. Let σ represent the distribution of investors across intermediaries. The policymaker will choose the state-contingent tax rates τ_s to maximize

$$\int \mathcal{V}_s^j \mathrm{d}\sigma(j) + v(\tau_s).$$

The policymaker will choose τ_α to equate the marginal values of public and private $t = 1$ consumption averaged across intermediaries in the state where no bailouts occur:

$$v'(\tau_\alpha) = \int (\mu_1^j + \mu_{2\alpha}^j)\mathrm{d}\sigma(j). \tag{9}$$

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TABLE 1. Four possible cases

		State α	
		No excess liquidity	Excess liquidity
State β	Liquidation	Case A	Case B
	No liquidation	Case D	Case C

Once the policymaker bails out intermediaries during the crisis, he will choose the tax rate τ_β to again equalize the marginal values of public and private consumption, now measured in state β:

$$v'(\tau_\beta) = \int (\tfrac{R}{r}\mu^j_{2\beta} - \mu^j_{1\beta}) \mathrm{d}\sigma(j). \tag{10}$$

Distorted incentives. As described above, the policymaker will choose to reallocate resources across intermediaries during a financial crisis. The policymaker will use this reallocation policy to maximize welfare, which requires equating the consumption of the remaining investors across all intermediaries. To achieve this goal, the policymaker will leave each intermediary with the same amount of both liquid assets and investment per remaining investor. Let $\bar{c}_1 \equiv \int c^j_1 d\sigma(j)$ be the economy-wide average of the payment c^j_1 given to the fraction π of investors who withdraw early and $\bar{x} \equiv \int x^j d\sigma(j)$ be the average amount of the illiquid asset across all intermediaries and all investors. Then the resources available to each intermediary will depend on the aggregate variables $\bar{c}_1$ and $\bar{x}$ and not on the intermediary's own choice of c^j_1 and x^j. If, after this reallocation has taken place, an intermediary finds it optimal to liquidate some units of investment at date 1, its resource constraint can be written as

$$(1-\pi)[\pi c^j_{1\beta} + (1-\pi)\tfrac{r}{R}c^j_{2\beta}] = 1 - (1-r)\bar{x} - \pi\bar{c}_1 - \tau_\beta. \tag{11}$$

If the intermediary instead chooses not to liquidate its illiquid asset, its resource constraints in state β will be

$$(1-\pi)\pi c^j_{1\beta} = 1 - \bar{x} - \pi\bar{c}_1 - \tau_\beta, \tag{12}$$

$$(1-\pi)^2 c^j_{2\beta} = R\bar{x}. \tag{13}$$

The key point in these expressions is that, in all cases, the right-hand side depends only on aggregate variables and not on intermediary j's own choices. This fact distorts incentives on the both sides of intermediaries' balance sheets, as I will show below.

Early payments. At decision point (**d**) in Figure 1, as the first π withdrawals take place, the intermediary j will choose to give the same amount c^j_1 to each of these investors. This value is chosen to maximize

$$\pi u(c^j_1) + (1-q)\mathcal{V}^j_\alpha + q\mathcal{V}^j_\beta \tag{14}$$

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subject to the above resource constraints and the early payments restriction (1). The first term of (14) corresponds to the fraction π of investors who withdraw before intermediaries learn the state. The sum of the last two terms denotes the expected utility from private consumption for its remaining investors. Recall, however, that equations (11)–(13) imply that the last term is fixed from the individual intermediary's point of view. Letting μ_c^j denote the multiplier on early payments restriction (1), the first-order condition characterizing the solution to this problem is, therefore,

$$u'(c_1^j) = (1-q)(\mu_1^j + \mu_{2\alpha}^j) + \tfrac{1}{\pi}\mu_c^j. \quad \textbf{(15)}$$

This condition highlights the incentive distortion on the liabilities side of an intermediary's balance sheets. Intermediary j has no incentive to provision for the bad state β. Instead, it will balance the marginal value of early payments against the marginal value of resources placed into storage in state α only and the additional value of resources to meet the early payments restriction.

Partially corrected incentives. At decision point (**c**) in Figure 1, the policymaker will choose the cap η_1 to maximize welfare:

$$\pi u(c_1^j) + (1-q)[\mathcal{V}_\alpha^j + v(\tau_\alpha)] + q[\mathcal{V}_\beta^j + v(\tau_\beta)]. \quad \textbf{(16)}$$

The first-order condition that characterizes the policymaker's optimal choice of η_1 is given by

$$\left[u'(c_1^j) - (1-q)(\mu_1^j + \mu_{2\alpha}^j) - q(\tfrac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j)\right]\frac{\mathrm{d}c_1^j}{\mathrm{d}\eta_1} = \tfrac{1}{\pi}\mu_c^j(-1 + \tfrac{\mathrm{d}c_1^j}{\mathrm{d}\eta_1}). \quad \textbf{(17)}$$

It is clear from combining the first-order conditions (15) and (17) that $\mu_c^j > 0$ must hold at the solution, and hence by complementary slackness the early payments restriction (1) must bind. This result yields $-1 + dc_1^j/d\eta_1 = 0$, which in turn implies that the right-hand side of (17) is zero. With this finding, the upper bound of early payments η_1 will be set to make the left-hand size zero, which requires

$$u'(c_1^j) = (1-q)(\mu_1^j + \mu_{2\alpha}^j) + q(\tfrac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j). \quad \textbf{(18)}$$

This expression shows that the cap on early payments will be chosen to increase the cost of using resources to meet early withdrawals until the marginal utility of early consumption is equal to the expected marginal value of remaining resources, taking all states into account. Specifically, imposing a cap on early payments can correct the distortion on the liabilities side of intermediaries' balance sheets, much as in Keister (2016).

Still distorted portfolio choice. At decision point (**b**), intermediary j makes its portfolio choice $(1-x^j, x^j)$. How does the early payments restriction affect intermediaries' behavior regarding asset holdings? Since this regulatory tool is used after the portfolio choice has been made, intermediary j chooses the value of x^j in a way that rationally ignores the losses in state β as a run occurs. The value of

x^j will be chosen to maximize the objective in (14) subject to the resource constraints (4), (5), and (11)–(13) depending on whether intermediary j decides to liquidate the illiquid asset in state β. The solution to this problem is characterized by the condition

$$(1-q)(\mu_1^j + \mu_{2\alpha}^j) = (1-q)R\mu_{2\alpha}^j. \quad \textbf{(19)}$$

Condition (19) shows that the portfolio choice does not balance the expected marginal value of private consumption across periods, instead ignoring the losses of resources in the event of a run.[10] Therefore, the first-order condition for x^j implies that the distortion on the asset side of the intermediaries' balance sheets still appears (i.e. overinvesting in the illiquid asset from a social point of view).[11]

To sum up, the first-order conditions (8)–(10), (18), and (19), combined with the resource constraints (4), (5), and (11)–(13), define the allocation of resources that results from the best responses by intermediaries and the policymaker to the strategy profile (3) under the policy regime with a cap on early payments. Since all intermediaries face the same decision problem, I omit the index j to simplify the notation in what follows. This allocation is summarized by the vector $\mathcal{A}^I \equiv \left(x^I, c_1^I, c_{2\alpha}^I, \tau_\alpha^I, c_{1\beta}^I, c_{2\beta}^I, \tau_\beta^I\right)$ that specifies the portfolio choice, the early payments, and the private and public consumption levels in each state under this first policy regime, labeled I. The explicit derivation of the allocation is presented in Supplemental Appendix A.1.

3.2. Equilibrium Fragility and No Excess Liquidity

I now ask under what conditions the strategy profile in (3) is part of an equilibrium and, hence, the financial system is fragile. Recall that an impatient investor will always strictly prefer to withdraw early whatever payment she receives, since she values date 1 consumption only. Therefore, I only need to consider the actions of patient investors. First note that since all uncertainty has been resolved after intermediaries learn the state, the ex-post efficient allocation of remaining resources always satisfies $c_{1\beta}^I < c_{2\beta}^I$.[12] Thus, a patient investor with $i > \pi$ prefers to wait in state β.

For patient investors with $i \leq \pi$, consider separately each of the two possible sunspot states. In state α, a patient investor receives $c_{2\alpha}^I$ if she waits until date 2, but receives c_1^I if she withdraws at date 1. After some algebra (see the explicit derivation of this allocation in Supplemental Appendix A.1), it can be shown that the solution to this problem will always satisfy $c_1^I < c_{2\alpha}^I$. In other words, a patient investor will strictly prefer to wait in state α as specified in (3).

In state β, a patient investor with $i \leq \pi$ whose opportunity to withdraw arrives before intermediaries learn the state receives c_1^I if the investor joins the run and $c_{2\beta}^I$ if she leaves her deposits in the financial system. The discussion above establishes that the profile (3) emerges as an equilibrium under the regime with an early payments restriction if and only if the allocation $\mathcal{A}^I$ satisfies

$$c_1^I \geq c_{2\beta}^I.$$

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I use this condition below to generate examples of economies that are fragile and not fragile under this policy regime. Before presenting the examples, however, it is worth noting that intermediaries have no incentive to hold units of the liquid asset between dates 1 and 2 as a precaution against the liquidation cost, since an anticipated bailout still encourages intermediaries to overinvest their funds in the illiquid asset. As a result, the best-response allocation $\mathcal{A}^I$ will be never in Cases B or C of Table 1.

Note also that when fewer investors have a real need to consume early (i.e. π is small), there will be sufficient assets left and hence intermediaries with more resources will avoid liquidation in the event of a crisis. In this situation, the allocation $\mathcal{A}^I$ lies in Case D. In this case, intermediaries have sufficient remaining resources to offer a comparatively high payment to patient investors who wait until date 2, which implies that the financial system is always stable. When intermediaries realize that a crisis is occurring relatively late (i.e. π is large), all of the resources in storage will have been paid out to the first π investors already. In this situation, additional date 1 payments will come only from liquidating investment, which corresponds to Case A. In this case, the remaining resources are comparatively small, which would optimally lead intermediaries to provide smaller payments to patient investors who withdraw at date 2. Thus, the financial system tends to be fragile in such a situation. I provide the following result. Formal proofs of all propositions are given in Supplemental Appendix B unless otherwise noted.

PROPOSITION 1. *If the financial system is fragile under the regime with a cap on early payments, then $\mathcal{A}^I$ must lie in Case A of Table 1.*

3.3. Measuring Fragility

Let Φ^I denote the set of economies for which the particular strategy profile in (3) is an equilibrium under the regime with a cap on early payments. Figure 2 plots this set as a function of q and r given the other parameters $(\gamma, \pi, \delta, R) = (8, 0.15, 10^{-4}, 1.25)$. For a given value of r, the figure shows that there is a maximum value for q such that the economy is fragile. If the probability of a run were higher than this threshold, intermediaries and the policymaker would be conservative enough in their choices that patient investors would have no incentive to run.[13]

Figure 2 shows that this threshold value of q may either increase or decrease with r. To understand intuitively the non-monotone pattern in this figure, it is helpful to write $c_1^I/c_{2\beta}^I$ as $c_1^I/c_{2\beta}^I = (c_{1\beta}^I/c_{2\beta}^I) \cdot (c_1^I/c_{1\beta}^I)$. I then have the following result, which shows that the non-monotone pattern comes from the composition of two monotone effects.

PROPOSITION 2. *If the economy lies in Φ^I, then $\begin{pmatrix} c_{1\beta}^I/c_{2\beta}^I \\ c_1^I/c_{1\beta}^I \end{pmatrix}$ is strictly $\begin{pmatrix} \text{increasing} \\ \text{decreasing} \end{pmatrix}$ in r.*

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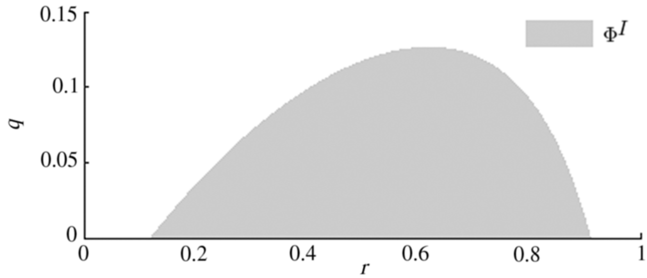

FIGURE 2. Fragile set Φ^I.

When the economy lies in Φ^I, an increase in r (i.e. a decrease in the liquidation cost $1 - r$) will lead intermediaries to decrease $c_{2\beta}$ relative to $c_{1\beta}$, because providing consumption in date 1 is now relatively less expensive. This fact encourages patient investors to withdraw early if a crisis occurs rather than leaving their funds in the financial system. At the same time, however, intermediaries decrease the spread between the payments to the early withdrawing investors in good times and in bad times ($c_1/c_{1\beta}$). In this situation, more consumption is available in state β because the liquidation cost becomes smaller. This latter effect decreases the ex-ante incentive for investors to run. The composition of these two competing effects results in the non-monotone pattern illustrated in Figure 2.

4. ADDING LIQUIDITY REGULATION

Now suppose the policymaker is given the ability to impose liquidity regulation, as discussed in Section 2.3. Does this liquidity regulation combined with the cap on early payments allow the policymaker to fully correct the distortions created by bailouts? Is adding this new tool better than the regime with an early payments restriction alone? I first analyze equilibrium allocations with such a regulation, then study the desirability of this policy regime.

4.1. The Best-Response Allocation

The steps for deriving the best responses of financial intermediaries and the policymaker to the profile of withdrawal strategies in (3) under this new regime are the same as in Section 3. The analysis begins with the portfolio choice, since the decisions on payment schemes, taxing deposits, and restricting early payments are unchanged.

Correcting portfolio choice. At decision point (**b**), intermediary j makes its portfolio choice to maximize (14), taking into account the liquidity requirement (2). Letting μ_l^j denote the multiplier on the liquidity constraint (2), the solution to this problem is characterized by the first-order condition

$$(1-q)(\mu_1^j + \mu_{2\alpha}^j) + \mu_l^j = (1-q)R\mu_{2\alpha}^j. \quad \textbf{(20)}$$

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Compared with the first-order condition for x^j in equation (19) under the regime with early payments restriction alone, this condition shows how the liquidity regulation creates the incentive for intermediaries to privately provision for a crisis.

Liquidity regulation. At the decision point (**a**), the policymaker chooses the minimum level of the liquid asset η_2 to maximize welfare (16). The first-order condition for η_2 is

$$\left[(1-q)R\mu_{2\alpha}^j + qR\mu_{2\beta}^j - (1-q)(\mu_1^j + \mu_{2\alpha}^j) - q(\tfrac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j)\right]\tfrac{\mathrm{d}x^j}{\mathrm{d}\eta_2}$$
$$= \mu_l^j(1 + \tfrac{\mathrm{d}x^j}{\mathrm{d}\eta_2}). \qquad \mathbf{(21)}$$

It is clear from combining the first-order conditions (20) and (21) that $\mu_l^j > 0$ must hold at the solution, and hence by complementary slackness the liquidity requirement (2) must bind. This result yields $1 + \mathrm{d}x^j/\mathrm{d}\eta_2^j = 0$, which in turn implies that the right-hand side of (21) is zero. With this finding, the η_2 will be set to make the left-hand side zero, which requires

$$(1-q)(\mu_1^j + \mu_{2\alpha}^j) + q(\tfrac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j) = (1-q)R\mu_{2\alpha}^j + qR\mu_{2\beta}^j. \qquad \mathbf{(22)}$$

The minimum liquidity requirement is used to offset the distortion in intermediaries' choice of asset holdings. Combined with (18), I have

$$u'(c_1^j) = (1-q)(\mu_1^j + \mu_{2\alpha}^j) + q(\tfrac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j) = (1-q)R\mu_{2\alpha}^j + qR\mu_{2\beta}^j, \qquad \mathbf{(23)}$$

which highlights that intermediary j now allocates resources to equate the marginal utility of its investors across periods, taking into account both states.

4.2. Equilibrium Fragility and Excess Liquidity

The first-order condition (23), combined with the earlier first-order conditions and resource constraints (4), (5), and (8)–(13), defines the best-response allocation $\mathcal{A}^{II}$ under the regime with both regulatory tools. As in the previous section, I provide the precise condition that determines whether the financial system is fragile by comparing the amount of consumption each patient investor with $i \leq \pi$ receives in state β. The financial system will be fragile under the regime with added liquidity regulation if and only if

$$c_1^{II} \geq c_{2\beta}^{II}.$$

Let Φ^{II} denote the set of economies for which this condition holds. In Supplemental Appendix A.2, I show that this allocation can take each of four distinct forms, again as described in Table 1. Recall that if the economy lies in Case D, intermediaries will realize that a crisis is occurring relatively early. After a small number of investors have been served, there will be sufficient remaining

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resources for intermediaries to offer a relatively high payment to patient investors, which implies that runs never occur in equilibrium. If the economy lies in Case C, intermediaries are conservative and hence hold excess liquidity as a precaution against the losses, because the probability of a crisis is sufficiently large. The above logic can be applied to this situation again and the financial system is always stable. If not, the losses created by the run are significant and, as a result, it may be attractive for patient investors to join the run. Thus, the financial system tends to be fragile under the regime with both regulatory tools when the best-response allocation lies in either Case A or Case B of Table 1. Subsequently, I present the following result.

PROPOSITION 3. *If the financial system is fragile under the regime with both regulatory tools, then $\mathcal{A}^{II}$ must lie in either Case A or Case B of Table 1.*

As this proposition indicates, the equilibrium allocation can fall in Case B under the policy regime with liquidity regulation. Recall that intermediaries will hold excess liquidity in such case. Is requiring intermediaries to hold a large buffer of the liquid asset desirable? I next analyze the effects of adding liquidity regulation to answer this question.

4.3. The Benefit of Adding Liquidity Regulation

If the economy lies in one fragile set but not the other, the optimal policy is to select the non-fragile regime. If the economy is fragile under both regimes, the policymaker chooses the regime that generates the higher expected investors' welfare level $\mathcal{W}$ conditional on the financial system being fragile. With these above expressions in hand, I can identify situations when the liquidity regulation should be implemented. The next proposition shows that adding liquidity regulation can generate higher welfare in the run equilibrium compared to the regime with early payments restriction alone.[14]

PROPOSITION 4. *Suppose the economy is in both Φ^I and Φ^{II} and $q > 0$. Then $\mathcal{W}^{II} > \mathcal{W}^I$.*

The proof of this proposition is straightforward. One can show that the competitive equilibrium allocation generated by the regime with both regulatory tools generates the welfare-maximizing allocation of resources conditional on the behavior of investors specified in (3). Suppose the policymaker and intermediaries are replaced by a single benevolent planner who aims to maximize welfare and faces all of the informational constraints described above. This planner will allocate resources efficiently conditional on investors' behavior and, therefore, the planner's best-response allocation to the strategy profile (3) is exactly $\mathcal{A}^{II}$. (See the formal proof in Supplemental Appendix B.) Since this solution is unique, the equilibrium allocation vector $\mathcal{A}^{II}$ must create strictly higher welfare than that of the equilibrium allocation $\mathcal{A}^I$.

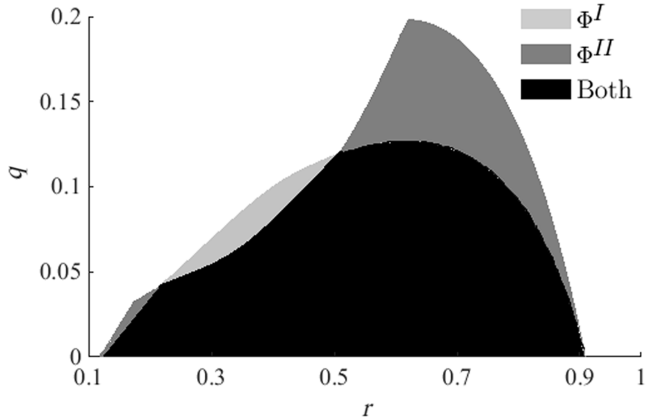

FIGURE 3. Comparing the fragile sets Φ^I and Φ^{II}.

4.4. The Cost of Adding Liquidity Regulation

As I demonstrated before, adding liquidity regulation can fully correct distorted incentives and generate the welfare-maximizing allocation of resources conditional on the behavior of investors specified in (3). In the one-asset model, Keister (2016) shows that introducing a liabilities tax on intermediaries' short-term liabilities to fully correct distortion is always desirable because it generates both a more stable financial system and higher welfare in the run equilibrium. Does this property hold in the two-asset model as well? To answer this question, I need to determine how imposing liquidity regulation would change the fragile set, Φ. In the next proposition, I show that adding liquidity regulation is, unexpectedly, undesirable in some cases because it causes the economy to become fragile.

PROPOSITION 5. *There exist economies in Φ^{II} that are not in Φ^I and vice versa.*

This proposition is portrayed graphically in Figure 3, which adds the fragile set Φ^{II} to Figure 2. Notice in particular that there are two kinks in the boundary of the set Φ^{II}, one when r is at around 0.2 and the other around 0.6. This pattern arises because the equilibrium allocation under the policy regime with both regulatory tools can be in either Case A or Case B of Table 1. Figure 3 depicts a situation in which the best-response allocation $\mathcal{A}^{II}$ in equilibrium lies in Case A for small and large values of r, but lies in Case B for intermediate values. Thus, an increase in r causes the maximum value for q in the set Φ^{II} to switch between Cases A and B, which translates into two kinks.

Figure 3 illustrates that adding liquidity regulation may actually make the financial system fragile. The intuition for this result is as follows. Introducing a restriction on the composition of intermediaries' assets and liabilities has two

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competing effects on investors' incentive to run. First, the regulation forces intermediaries to hold a more liquid portfolio, which tends to make the financial system more stable. At the same time, however, the fact that intermediaries are more liquid will lead the policymaker to loosen the early payments restriction and allow intermediaries to increase their early payments. This tends to increase the incentive for investors to run. Which of these two effects dominates in terms of financial fragility depends on parameter values.

When the liquidation cost is sufficiently high (i.e. r is small enough), intermediaries will choose to hold a fairly liquid portfolio. In this region, imposing a liquidity requirement causes an increase in the value of reserves. This increase, in turn, raises the ex-ante incentives associated with c_1 for patient investors with $i \leq \pi$ to run, which makes the economy more fragile. (Look at an economy that is in Φ^{II} but not in Φ^{I}.) As the liquidation cost decreases further (i.e. as r increases), however, regulating intermediaries' choice of asset holdings leads them to raise $c_{2\beta}$ because they now have sufficient assets to mitigate the losses of liquidating investment. As a result, in this region, adding liquidity regulation tends to make the economy more stable. (Look at the light gray region where the economy is in Φ^{I} but not in Φ^{II}.) When the liquidation cost is sufficiently small, no excess liquidity becomes the best choice for intermediaries. In this region, requiring intermediaries to hold more of the liquid asset leads them to raise c_1, which encourages patient investors to withdraw early. Thus, adding the liquidity regulation, paradoxically, becomes by itself the source of fragility.

Taken together, when the economy lies in the dark gray region in Figure 3, adding liquidity is undesirable because it introduces a run equilibrium. However, it makes the financial system more stable when the economy is in the light gray region. Finally, the economy is fragile under both policy regimes in the black region. In this case, imposing regulation on asset holdings has higher welfare as indicated in Proposition 4. (I provide precise conditions when adding liquidity regulation is desirable in Supplemental Appendix C.)

Worse scenario. The next result identifies situations in which the cost of adding liquidity regulation is clearly dominant and thus the financial system becomes more fragile for all values of r. For this result, I need to define one expression:

$$f(r) = [(1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2(r/R) + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}}]^{\gamma} - [(1-\pi)rR^{\frac{1}{\gamma}-1} + \delta^{\frac{1}{\gamma}}]^{\gamma}.$$

PROPOSITION 6. *For all* $q > 0$, *if* $0 < f(r) \leq (R^2 - 2rR + r)/(R - Rr)$ $[(1-\pi)(r/R) + \delta^{\frac{1}{\gamma}}]^{\gamma}$, *then the set* Φ^{I} *is strictly contained in* Φ^{II}.

This proposition is portrayed graphically in Figure 4. This example verifies the claim that the minimum liquidity requirement chosen by a policymaker encourages intermediaries to distribute all of their buffer of the liquid asset to investors who withdraw early, which would raise the incentive of patient investors to join the run.

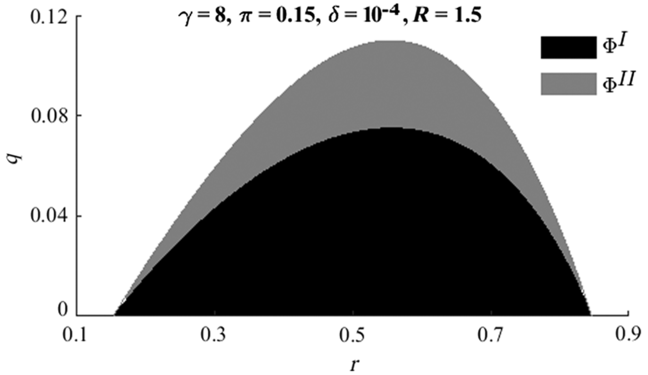

FIGURE 4. Φ^I is strictly contained in Φ^{II}.

4.5. Discussion

Note that the limited commitment assumption is crucial for the results above. If the policymaker could commit to choices of η_1 and η_2 before investors make their withdrawal decisions, then having two policy tools would always be at least as good as having only one, since the policymaker could ensure that the second requirement does not bind, for example. But without commitment, that logic does not hold. In the environment with limited commitment, the policy is chosen to allocate resources efficiently given that investors are playing the run strategy profile. In particular, when intermediaries are required to hold a more liquid asset portfolio, the policymaker will optimally choose to loosen the early payments restriction and allow intermediaries to make larger early payments. One then has to check what effect the anticipation of this choice has on the incentive for investors either to run or wait. As Figures 3 and 4 both illustrate, in some situations the net result of these changes is an increased incentive for patient investors to withdraw early and, hence, a higher degree of financial fragility.

5. CONCLUSION

In this paper, I ask when liquidity regulation should be introduced to an existing policy regime based on a cap on early payments. The benefit of adding liquidity regulation is that the distortions associated with bailouts are corrected. Consequently, such a policy regime reliably yields higher welfare than other alternative policy options conditional on investors' withdrawal behavior. The potential cost of adding liquidity regulation is that it leads the policymaker to loosen the early payments restriction due to the time-inconsistency problem. Such a change can thereby increase financial fragility by encouraging patient investors to join the run. I have shown that, in certain cases, the financial system is not fragile under the regime with early payments restriction alone, but becomes fragile when the liquidity regulation is added.

This paper offers a convenient and effective framework to evaluate the desirability of liquidity regulation, as bailouts generate multiple distortions. A key policy conclusion from my work is that liquidity regulations like the LCR in the Basel III accords must be examined carefully to ensure that they promote financial stability. One interesting extension would be to consider the role of liquidity regulation on other externalities such as contagion and fire sales.

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit http://dx.doi.org/10.1017/S1365100518000834.

NOTES

1. See, for example, Wellink (2011) and Yellen (2014).

2. This analysis applies not only to commercial banking, but also to a wide range of financial institutions, especially shadowing banking system. See Adrian and Ashcraft (2012, 2016) for a overview of shadow credit intermediation and the impact of recent reform efforts on it.

3. It is worth emphasizing that, conditional on a run by investors, welfare is always higher with liquidity regulation than without. The fact that intermediaries increase the amount they offer for early withdrawals and therefore might increase fragility does not imply they are acting against investors' interests. However, like the policymaker, intermediaries lack commitment and choose their actions as a best response to the situation at hand.

4. See also Li (2017) for an analysis of a model with a non-trivial portfolio choice together with limited commitment, but with no bailouts.

5. This approach follows Peck and Shell (2003) and others but differs from Keister (2016), where intermediaries and the policymaker are assumed to observe the sunspots state after a given number of withdrawals have been made. This assumption simplifies some expressions in what follows and allows me to focus on the effects of liquidity regulation while side-stepping some unnecessary complications.

6. Recent work following this approach includes Sultanum (2014) and Shell and Zhang (in press).

7. Note in particular that this lack of commitment implies that intermediaries cannot credibly promise to use contracts with a suspension of convertibility clause to rule out runs.

8. In Keister (2016), the policymaker collects taxes at date 0 and potentially makes bailout payments to intermediaries at date 1. Keister and Narasiman (2016) show that having the policymaker instead to collect taxes after inferring the state generates the same type of incentive distortion while removing the policymaker's ability to use the tax rate as a macroprudential policy tool. I take this latter approach because it allows me to focus more cleanly on the effects of early payments and liquidity regulation.

9. This strategy profile follows Ennis and Keister (2010), who show that run is necessarily partial in this type of model, with only some investors participating.

10. When there is no bailout and incentives are not distorted, an individual intermediary will choose x to equate the expected marginal utility at date 1 after they learn the state to the expected marginal utility at date 2 taking into account both states, that is, $(1-q)(\mu_1^j + \mu_{2\alpha}^j) + q(\frac{R}{r}\mu_{2\beta}^j - \mu_{1\beta}^j) = (1-q)R\mu_{2\alpha}^j + qR\mu_{2\beta}^j$.

11. Keister (2016) shows that this type of regulation can fully correct distortions created by bailouts in a model with a single asset and no portfolio choice. Note that the incentive distortion studied in Keister (2016) only affects the liabilities side of intermediaries' balance sheets due to the absence of portfolio choice. In my paper, the source of the moral hazard, that is, bailouts, is the same, but now

intermediaries also make a portfolio choice. Thus, the distortion now simultaneously affects both sides of their balance sheets.

12. See Supplemental Appendix A.1 for a derivation of this result.

13. See Li (2017) for a detailed analysis of how the maximum probability of a run varies with parameter values in a model with limited commitment but without bailouts.

14. It can be shown that each of these two policy regimes is unambiguously preferable to the regime with no regulation. Here, I focus on the comparison of these two policy regimes to save space.

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