Financial History Review 22.1 (2015), pp. 107–132. © European Association for Banking and Financial History e.V. 2015
doi:10.1017/S0968565015000062

Real estate transactions in ancient Israel: excavating embedded options utilizing modern finance

ELIEZER Z. PRISMAN
York University

An economic setting of ancient Jewish law is analyzed and reinterpreted in light of a modern formal financial model. Certain real estate transactions in ancient Israel, as stipulated in the Bible, involved embedded financial options that seem to have been overlooked by the commentators. This article interprets, utilizing modern financial theory, the biblical text and sheds light on a phrase used in stipulating these rules that has puzzled some commentators. The option's value and the complexity of the pricing system that would have been needed in order to capture true market prices of these assets are demonstrated.

Keywords: Jubilee, derivative securities, real estate, biblical rules

JEL classification: B11, G12, K11, N25

I

This article relates economic notions of remote past days to a branch of modern financial economic theory. Certain real estate transactions in ancient Israel, as stipulated in the Bible, involved embedded financial options that seem to have been overlooked by the commentators and the literature in general. The option's value and the complexity of the pricing system that would have been needed in order to capture true market prices of these assets are demonstrated.

The article interprets, utilizing modern financial theory, the biblical text and sheds light on a phrase used in stipulating these rules, one that has puzzled some commentators. This repositioning makes ancient Jewish law on real estate transactions more

E. Z. Prisman, Schulich School of Business, York University, 4700 Keele Street, Toronto, ON M3J 1P3, Canada; email: eprisman@yorku.ca. The author would like to thank Rabbis E. Afersemon and I. Aranon and Dr Y. Zitrin for numerous conversations and Professor Y. Rosenberg for help in finding historical information, which considerably improved the paper. As well, the careful reading and editing comments of an anonymous referee significantly enhanced the clarity of the paper. All remaining errors are my own.

accessible to professional readers. Earlier scholars who referred to other areas of Jewish law have already promoted such an approach.[1]

The rules for selling a piece of land (a field) in Israel are specified in the Hebrew Bible[2] in Leviticus chapter 25, verses 23–8 (the original Hebrew text is quoted in the footnote):[3]

23. The land shall not be sold permanently, for the land belongs to Me, for you are strangers and [temporary] residents with Me.
24. Therefore, throughout the land of your possession, you shall give redemption for the land.
25. If your brother becomes destitute and sells some of his inherited property, his redeemer who is related to him shall come forth and redeem his brother's sale.
26. And if a man does not have a redeemer, but he gains enough means to afford its redemption,
27. he shall calculate the years for which the land has been sold, and return the remainder to the man to whom he sold it, and [then] he may return to his inheritance.
28. But if he cannot afford enough to repay him, his sale shall remain in the possession of the one who has purchased it, until the Jubilee year. And then, in the Jubilee year, it shall go out and revert to his inheritance.

These rules were followed after the 12 tribes settled in Israel and when the Jubilee was in effect.[4] Land at that time was not sold in perpetuity, but rather the land

[1] Liebermann (1981) discovered the principles underlying the Coase Theorem embedded in Talmudic writings. Aumann and Maschler (1985) offered a game-theoretic generalized solution to a seemingly unsolvable Mishna passage. Koppel (1998) provides a systematic mathematical understructure to a complete Mishna tractate. More recently, Callen (2008) shows to what extent asset valuation arguments raised by Jewish law scholars can be matched with a formal modern contingent claims model.

[2] The Hebrew Bible, also referred to as the Torah or Old Testament, includes the books (English names): Genesis, Exodus, Leviticus, Numbers, Deuteronomy, Joshua, Judges, Ruth, I Samuel, II Samuel, I Kings, II Kings, I Chronicles, II Chronicles, Ezra, Nehemiah, Esther, Job, Psalms, Proverbs, Ecclesiastes, Songs of Solomon, Isaiah, Jeremiah, Lamentations, Ezekiel, Daniel, Hosea.

[3] **ויקרא פרק כה**

(כג) וְהָאָרֶץ לֹא תִמָּכֵר לִצְמִתֻת כִּי לִי הָאָרֶץ כִּי גֵרִים וְתוֹשָׁבִים אַתֶּם עִמָּדִי:
(כד) וּבְכֹל אֶרֶץ אֲחֻזַּתְכֶם גְּאֻלָּה תִּתְּנוּ לָאָרֶץ:
(כה) כִּי יָמוּךְ אָחִיךָ וּמָכַר מֵאֲחֻזָּתוֹ וּבָא גֹאֲלוֹ הַקָּרֹב אֵלָיו וְגָאַל אֵת מִמְכַּר אָחִיו:
(כו) וְאִישׁ כִּי לֹא יִהְיֶה לּוֹ גֹּאֵל וְהִשִּׂיגָה יָדוֹ וּמָצָא כְּדֵי גְאֻלָּתוֹ:
(כז) וְחִשַּׁב אֶת שְׁנֵי מִמְכָּרוֹ וְהֵשִׁיב אֶת הָעֹדֵף לָאִישׁ אֲשֶׁר מָכַר לוֹ וְשָׁב לַאֲחֻזָּתוֹ:
(כח) וְאִם לֹא מָצְאָה יָדוֹ דֵּי הָשִׁיב לוֹ וְהָיָה מִמְכָּרוֹ בְּיַד הַקֹּנֶה אֹתוֹ עַד שְׁנַת הַיֹּבֵל וְיָצָא בַּיֹּבֵל וְשָׁב לַאֲחֻזָּתוֹ:

[4] For an investigation on when and how the counting of the Jubilee was in affect see **אנציקלופדיה תלמודית כרך כב, יובל [טור קיב]**

Encyclopedia OZAAR YISRAE Shilo Publication Jerusalem, 1950.

.. אין היובל נוהג אלא בזמן שכל יושבי הארץ נמצאים עליה, שנאמר: וקראתם דרור בארץ לכל ישביה (ויקרא כה י) שבעה עשר יובלות מנו ישראל משנכנסו לארץ (אחר שבע שכבשו ושבע שחלקו) עד שיצאו (משגלו שבט ראובן גד וחצי מנשה קנ"א שנה) קודם בחורבן בית ראשון (ערכין יב ב; רמב"ם שמו"י פ"י ה"ג. ועי' ערכין שם ובהשגות הראב"ד ור"י קורקוס וכ"מ שם אם הכונה שעשו כל היובלות או שהכונה למנין בלבד...).

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

(agricultural field) was returned to its original owner in the Jubilee year (every 50 years).[5] An implication of this rule was that the distribution of land between the tribes was returned to its original state every 50 years. Thus, the selling of a piece of land was not the transfer of the land itself, but the right to work it and own its produce until the Jubilee. Accordingly, the price of the land was essentially the present value of the stream of income from the land, the crops, during those years.[6]

Leviticus chapter 25, verses 13–16, explains the calculation of the price (the original Hebrew text is quoted in the footnote):[7]

13. During this Jubilee year, you shall return, each man to his property.
14. And when you make a sale to your fellow Jew or make a purchase from the hand of your fellow Jew, you shall not wrong one another.
15. According to the number of years after the Jubilee, you shall purchase from your fellow Jew; according to the number of years of crops, he shall sell to you.
16. The more [the remaining] years, you shall increase its purchase [price], and the fewer the [remaining] years, you shall decrease its purchase [price], because he is selling you a number of crops.

The original owner of the land was given some buy-back rights, referred to as redemption (*geula* in Hebrew). The original owner could force a future owner to sell the field back to him. In modern financial terminology, this right is a 'call option'. A call option is a financial contract that gives its holder the right, not the obligation, to buy a certain asset (the underlying asset) for a certain price (called the strike price or the exercise price) on or up to a certain date (called the expiration or the

[5] There are cases in which the field is returned after the Jubilee. Such is the case if the contract specifies literally that the field is sold for a period that exceeds the Jubilee. See

תלמוד בבלי מסכת בבא מציעא דף עט עמוד א

.... דאמר רב חסדא אמר רב קטינא: מנין למוכר שדהו לששים שנה שאינה חוזרת ביובל - שנאמר +ו יקרא כ"ה+ והארץ לא תמכר לצמיתות - מי שאין שם יובל - נצמתת, יש שם יובל - אינה נצמתת, יצתה זו שאף על פי שאין שם יובל - אינה נצמתת.

and the Rambam

רמב"ם הלכות שמיטה ויובל פרק יא הלכה ב

והמוכר שדהו לס' שנה אינה יוצאה ביובל שאין חוזר ביובל אלא דבר הנמכר סתם או הנמכר לצמיתות.

אנציקלופדיה תלמודית כרך כב, יובל [טור קיב]

אין חוזר ביובל אלא דבר הנמכר סתם או הנמכר לצמיתות[604], אבל המוכר את שדהו לששים שנה - או יותר, ובלבד שיזכיר מספר שנים[605], ואפילו מוכר לאלפיים שנה, שיש בזמן הזה הרבה יובלות[606] - אינה יוצאה ביובל[607], אלא ימתין עד ששים שנה ותחזור לו[608], שנאמר: והארץ לא תמכר לצמתת[609], ולמדנו מכאן שאין היובל מוציא אלא קרקע שאם לא היתה מצות יובל היתה נצמתת, יצאה זו שאף על פי שאין שם יובל אינה נצמתת[610], שכיון שמזכירים ביניהם מספר שנים אין זה לצמיתות[611]

[6] Legally the purchaser of the field had the rights to use the field as if he/she owned it, e.g. to build on it etc. However, the value of these rights might be negligible.

[7] **ויקרא פרק כה**

(יד) וְכִי תִמְכְּרוּ מִמְכָּר לַעֲמִיתֶךָ אוֹ קָנֹה מִיַּד עֲמִיתֶךָ אַל תּוֹנוּ אִישׁ אֶת אָחִיו:
(טו) בְּמִסְפַּר שָׁנִים אַחַר הַיּוֹבֵל תִּקְנֶה מֵאֵת עֲמִיתֶךָ בְּמִסְפַּר שְׁנֵי תְבוּאֹת יִמְכָּר לָךְ:
(טז) לְפִי רֹב הַשָּׁנִים תַּרְבֶּה מִקְנָתוֹ וּלְפִי מְעֹט הַשָּׁנִים תַּמְעִיט מִקְנָתוֹ כִּי מִסְפַּר תְּבוּאֹת הוּא מֹכֵר לָךְ:

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

maturity date). The right to buy back an asset at a certain price has a monetary value as it may allow its holder to purchase the asset for less than its market value.

Thus when a landowner sells land and receives the rights to buy it back, the owner in fact is buying a call option from the buyer. Monies paid by the buyer to the owner are therefore the value of the field minus the value of the call option. The sale of land is therefore composed of two transactions: the sale of land and the purchase of the option (the option must be part of the deal), as commanded in the Bible. In modern finance, transactions that cannot be separated, such as the ones described above, are called 'structured products'.

The Bible, its commentators and other sources (such as the Mishnah[8] and the Talmud) do not seem to acknowledge the value of such an option or to address it in a discussion of the price the redeemer should pay for the field.[9] Leviticus chapter 25, verse 27, addresses the price (the option's exercise price) the redeemer should pay for the field. The Bible stipulates the calculation of the exercise price in verse 27: 'He shall calculate the number of years for which he sold the land and return the remainder (excess) to the man to whom he had sold it, and he shall return to his ancestral land.'

However, examples of these calculations in the commentaries do not acknowledge the existence of the embedded option and its effect on the price of the field. Furthermore, the Bible, while explaining the price of the field (Leviticus chapter 25, verse 17), states: 'you shall not wrong one another'. This warning, given in the middle of the stipulation about selling fields, seems out of place here. Perhaps this was what bothered Rashi[10] and prompted his explanation,[11] that the buyer and the

[8] The Mishnah (Hebrew משנה 'repetition'), redacted *c.*200 CE by Yehudah Ha-Nasi (יהודה הנשיא 'President Judah'), is the first written recording of the Oral Torah of the Jewish people, as championed by the Pharisees, and as debated between 70 and 200 CE by the group of rabbinic sages known as the *Tannaim*. It is considered the first important work of rabbinic Judaism and is a major source of rabbinic Judaism's religious texts: rabbinic commentaries on the Mishnah over the three centuries after its composition were then redacted as the Gemara (Aramaic: 'learning by tradition') or Talmud (Hebrew: תַּלְמוּד). The Talmud often ventures into other subjects and expounds broadly on the Tanakh. It is the basis for all codes of rabbinic law and is much quoted in other rabbinic literature. This explanation is taken from http://en.wikipedia.org.

[9] Knoll (2008) points out that the sages of the Talmud were aware of a relationship between different types of options (the put–call parity). In his opinion the relationship is acknowledged in the Talmud in the discussion of the redemption of houses. See also Chance (2008, pp. 7–14), whose paper about the history of derivatives attributes the first option to the patriarch Jacob: 'To start we need to go back to the Bible. In Genesis Chapter 29, believed to be about the year 1700 BC, Jacob purchased an option costing him seven years of labor that granted him the right to marry Laban's daughter Rachel … Some argue that Jacob really had forward contracts, which obligated him to the marriage but that does not matter. Jacob did derivatives, one way or the other. Around 580 BC, Thales the Milesian purchased options on olive presses and made a fortune from a bumper crop in olives. So derivatives were around before the time of Christ.'

[10] Rabbi Solomon ben Isaac (Hebrew: רבי שלמה יצחקי), better known by the acronym Rashi (Hebrew: רש"י), (22 February 1040 – 13 July 1105).

[11] **רש"י ויקרא פרק כה**
אל תונו זו אונאת ממון:

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

seller should make each other aware of the number of years until the Jubilee, so the price would be fair to both. The phrase 'one another' is also somewhat puzzling in its redundancy. Rashi therefore further stipulates the trivial fact of who is losing when the deal is calculated based on the number of years that are above or below the number of years until the Jubilee.

The effects and subtleties of the embedded option, as we shall soon see, are much more esoteric than the number of years to the Jubilee, which is public knowledge. Consequently, the determination of the price of the land requires information known only to the seller, as well as information known only to the buyer. Being aware of the intricacies of the option embedded in the deal, it would make sense that the Bible's warning 'you shall not wrong one another' should be interpreted as a warning to the buyer and the seller to be aware of option details. These details are specific to each deal, and not common knowledge. Thus both buyer and seller should make each other aware of the information known only to them. Hence the phrase 'not wrong one another' is mentioned, as each side could take advantage of the other.

Some aspects of these field transactions are discussed in Buchholz (1988) and Westbrook (1971). These articles, however, are completely silent on the embedded options. The goal of this article is to investigate the prices of land in Israel during the time these rules were in effect. The article uses modern financial theory to value these assets (land and houses). As we will see, there are a few details involved in the options embedded in such a transaction. We are not aware of any study that deals with these options and their prices.

The rest of the article is organized as follows: Section II stipulates the contract of the option and also refers to the commentary of the Torah and Talmud and specifies, to a certain extent, how conditions for these rules are arrived at. Section III develops the model used to price the embedded option, while technical issues are explained in the Appendix. Data on the prices of fields in ancient Israel are not readily available. Thus in lieu of an empirical study, Section IV numerically analyzes the value of the hidden option and its effect on the prices of fields. This section demonstrates the complexity of the pricing system needed in order to capture true market prices of these assets in this period. Conclusions and remarks are offered in Section V.

II

The sale of land in ancient Israel, at a time when the Jubilee was observed, included the provision for the owner or a relative to have the right to buy the land back

(טו) במספר שנים אחר היובל תקנה - זהו פשוטו ליישב מקרא על אופניו על האונאה בא להזהיר, כשתמכור או תקנה קרקע דע כמה שנים יש עד היובל. ולפי השנים ותבואות השדה שהיא ראויה לעשות ימכור המוכר ויקנה הקונה, שהרי סופו להחזירה לו בשנת היובל. ואם יש שנים מועטות וזה מוכרה בדמים יקרים הרי נתאנה לוקח, ואם יש שנים מרובות ואכל ממנה תבואות הרבה ולקחה בדמים מועטים הרי נתאנה מוכר, לפיכך צריך לקנותה לפי הזמן. וזהו שנאמר במספר שני תבואות ימכר לך, לפי מנין שני התבואות שתהא עומדת ביד הלוקח תמכור לו.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

(redeem) after two years (of crops) had elapsed. This is deduced from the plural use of 'years' in Leviticus chapter 25, verse 15. The buyer cannot prevent the owner (or the owner's relatives) from buying it back.[12] The period of two years, however, is contingent on none of these years being a drought year, so in fact the right to buy back is only after two rainy[13] years have elapsed since the sale.

Consequently, at the same time that the seller is selling the land, the seller is also buying a call option from the purchaser. When the land transaction takes place, the money transferred from the buyer to the seller is the price of the field less the price of the option. The buyer, in fact, is writing (selling) a call option to the owner of the land, in which a commitment is given to sell the field back to the owner for a certain price (the exercise price) during the period specified above.

The underlying asset of this option is the land's produce value until the Jubilee. This call option is of an American type (it can be exercised during a period of time and not only on one day). However, it can be exercised only after two years ('a delayed option') and up to the Jubilee (at which time the land was returned to its original owner). Since the exercise period depends on the weather, the provision requiring these two years to be rainy years suggests that this American option falls into the category of weather derivatives.

It is important to note that the land and the call option are not separable.[14] If a secondary buyer buys the field, this buyer in fact writes a call option to the original owner

[12] This rule is deduced from

רש"י מסכת ערכין דף כט עמוד ב

המוכר, בשעת היובל - בזמן שהיובל נוהג.

פחות משתי שנים אבל לאחר שתי שנים אם רוצה לפדותה פודה בעל כרחו של לוקח ונותן לו לפי מה שמכרה

[13] This is deduced from

ויקרא פרק כה פסוק טו

בְּמִסְפַּר שָׁנִים אַחַר הַיּוֹבֵל תִּקְנֶה מֵאֵת עֲמִיתֶךָ בְּמִסְפַּר שְׁנֵי תְבוּאֹת יִמְכָּר לָךְ׃

The drought year, however, should have been not only in the location of the field but all over. This is examined and deduced from

תלמוד בבלי מסכת בבא מציעא פרק ט - המקבל שדה מחבירו [דף קו עמוד א]

... מיתיבי: היתה שנת שדפון וירקון, או שביעית, או שהיו שנים כשני אליהו - אינו עולה לו מן המנין. קתני שדפון וירקון דומיא דשנים כשני אליהו, מה שני אליהו - דלא הוי תבואה כלל, אף הכא נמי - דלא הוי תבואה כלל. אבל דאיכא תבואה - סלקא ליה, ולא קאמרינן מכת מדינה היא. - אמר רב נחמן בר יצחק: שאני התם, דאמר קרא +ויקרא כה+ במספר שני תבואת ימכר לך - שנים שיש בהן תבואה בעולם. אמר ליה רב אשי לרב כהנא: אלא מעתה שביעית תעלה לו מן המנין, דהא איכא תבואה בחוצה לארץ! - אמר ליה: שביעית אפקעתא דמלכא היא. אמר ליה מר זוטרא בריה דרב מרי לרבינא: אלא מעתה שביעית לא תעלה לו מן הגירעון,...

It is also dealt with in

תלמוד בבלי מסכת ערכין דף כט עמוד ב

...מי לא תניא: אכלה שנה אחת לפני היובל - משלימין לו שנה אחרת אחר היובל!

See also the Rambam:

רמב"ם הלכות שמיטה ויובל פרק יא הלכה י

וצריך שיאכל הלוקח שתי תבואות בשתי שנים ואח"כ יגאל שנאמר שני תבואות, לפיכך אם היתה אחת משתי השנים שביעית או שנת שדפון או ירקון אינה עולה מן המנין

[14] **רמב"ם הלכות שמיטה ויובל פרק יא הלכה טו**

מכר שדהו לראשון וראשון מכר לשני ושני לשלישי אפילו מאה זה אחר זה בשנת היובל תחזור לאדון הראשון, שנאמר בשנת היובל ישוב השדה לאשר קנהו מאתו לאשר לו אחוזת הארץ.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

of the land. The original owner can force the secondary buyer to sell the field back to the original owner. The option that the first buyer wrote the owner is no longer 'alive'. The process is therefore that the first buyer sells the field to the secondary buyer and at the same time the secondary buyer (essentially) assumes the first buyer's commitment to sell the field, upon request, to the original owner. Therefore, the money that is being transferred from the buyer to the seller is the price of the field less the price of an option. This option, as we shall soon see, may have a different exercise price than the original option.

The exercise price of the option also has a few provisions. The exercise price is calculated based on the number of years until the Jubilee at the time the field was sold and the number of years until the Jubilee from the time this option is exercised. The calculation is mentioned in the Mishnah,[15] based on verse 27 in Leviticus chapter 25, and both Rambam[16] and Rashi[17] elaborate on it using an example like the following:

If the field was sold at say for 1,000 and there are ten years to the Jubilee, it means that the product of each year was valued at 100. Hence, if the option is exercised when there are three years to the Jubilee the exercise price[18] will be 300. That is, the original owner should give back the money that was paid to him initially, assuming the sale was for ten years, for the years that the field would not be with the buyer.

[15] משנה מסכת ערכין פרק ט משנה א

המוכר את שדהו בשעת היובל אינו מותר לגאול פחות משתי שנים שנאמר (ויקרא כ"ה) במספר שני תבואות ימכר לך היתה שנת שדפון וירקון או שביעית אינה עולה לו מן המנין נרה או הובירה עולה לו מן המנין רבי אלעזר אומר מכרה לו לפני ראש השנה והיא מלאה פירות הרי זה אוכל ממנה שלש תבואות לשתי שנים:

[16] Moses Maimonides (30 March 1135, Córdoba, Spain – 13 December 1204, Fostat, Egypt), was a Jewish rabbi, physician, and philosopher in Andalusia, Morocco and Egypt during the Middle Ages Maimonides's full Hebrew name was Rabbi Moshe ben Maimon (Hebrew: **משה בן מימון**) and Jewish works refer to him as the Rambam (**רמב"ם**).

[17] **רש"י מסכת ערכין דף כט עמוד ב**

המוכר, בשעת היובל בזמן שהיובל נוהג *פחות משתי שנים* אבל לאחר שתי שנים אם רוצה לפדותה פודה בעל כרחו של לוקח ונותן לו לפי מה שמכרה כדכתיב (ויקרא כה) וחשב את שני ממכרו שמחשב כמה שנים משמכרה עד היובל ומחלק הדמים לפי השנים כגון אם מכרה קודם היובל עשר שנים בעשר ליטרין נמצא שמכר פירות של כל שנה ושנה בליטרא שהרי סתם מכירה אינה אלא עד היובל הלכך אם שהתה ביד לוקח ה' שנים ואח"כ בא מוכר לגאלה מנכה לו לוקח ה' ליטרין ליטרא לכל שנה שאכלה שכך עלה חשבון כשיוצא מתחילה. *אינו עולה מן המנין* השתי שנים דהא שני תבואות כתיב שתי שנים הראויין לתבואה תשהה ביד לוקח אבל היתה שנה הראויה לתבואה ונרה ולא זרעה או הובירה שהניחה בורה שאפי' ניר לא עשה בה איהו אפסיד אנפשיה ועולה לו במנין שתי השנים.

The Rambam also explains it very similarly:

פירוש המשנה לרמב"ם מסכת ערכין פרק טמשנהא

דין מוכר שדה אחוזה כפי שנתבאר בכתוב הוא כמו שאסדיר לך, והוא, אם מכר האדם משדה אחוזתו במחיר מסויים מתחלקים אותם הדמים לפי מנין השנים שנשתיירו עד היובל, ויודע כמה ראוי לכל שנה, ולפי מנין מה שנשארה ביד הקונה מנכין לו מן הקרן, המשל בזה, אם מכר שמעון לראובן קרקע במאה דינרין, והיה הנשאר עד היובל מיום המכירה עשר שנים, ודר בה ראובן ואכלה ארבע שנים, ואחר כך רצה שמעון לפדות את שדהו, הרי זה מחזיר לראובן ששים דינר וזה הוא גרעון כסף, ועל דרך זו תדון, ומחשב עמו בעת שרוצה לפדות את שדהו על כל החדשים והימים שאכל כפי שמראה החשבון.

[18] We will come back to this calculation in view of the point of the embedded option raised by this article.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

In this example, the land is redeemed (the option was exercised) three years prior to the Jubilee and the exercise price was 300. The example assumes that the time value of money is zero (no interest is allowed to be charged by the Jewish code of law) and that the uncertainty of the value of the produce, as a function of the number of years to the Jubilee, is not an increasing function. Consequently, the present value of future crops is the same as their value at the time of sale. Furthermore, both Rambam and Rashi do not mention the option's value. For their examples to be consistent with the existence of the option, one must interpret 'If the field was sold at say 1,000' (or in Hebrew as Rashi says, כגון אם מכרה קודם היובל עשר שנים בעשר) as referring to the net price of the field (less the option value). It is also possible that the commentaries simplified the situation in order to demonstrate the main point and hence also ignored the occurrence of a sabbatical year during the ten-year period.

The fact that produce of future years has a greater risk and also a lower present value is ignored in the example of Rashi and Rambam. In fact, they treat the value of each year of produce as being deterministic and not subject to any risk at all and assume that the market price of the produce is unchanging. Under this assumption, of course, there is no value to the embedded option.

Yet, the Mishna (see footnote 19) does address the case of the field being sold to a third party at a price (per the annual product) different from that of the original transaction. In this case, the uncertainty of market prices is acknowledged, which, of course, means that the option does have a value. The exercise price can also be affected by the price of the field in a resale transaction that had taken place between the original buyer and a new buyer. If the value of the produce from the exercise time until the Jubilee, based on such a transaction, is smaller than the value of the produce based on the original price, the lower exercise price will be used.

The exercise price is therefore the minimum between the value of the produce based on the original price and the value of the produce based on a secondary transaction done from the original sale until the exercise time. The guideline,[19]

[19] **משנה מסכת ערכין פרק ט משנה ב**

מתני'. מכרה לראשון במנה ומכר ראשון לשני במאתים אינו מחשב אלא עם הראשון, שנאמר: ויקרא כ"ה [לאיש] אשר מכר לו; מכרה לראשון במאתים ומכר הראשון לשני במנה אינו מחשב אלא עם האחרון, שנאמר: ויקרא כ"ה וחשב את שני ממכרו והשיב את העודף לאיש (אשר מכר לו), לאיש אשר בתוכו.

תלמוד בבלי מסכת ערכין פרק ט - המוכר שדהו [דף כט עמוד ב]

גמ'. תנו רבנן: מכרה לראשון במנה ומכרה ראשון לשני במאתים, מנין שאינו מחשב אלא עם הראשון? ת"ל: [לאיש] אשר מכר לו; מכרה לראשון במאתים ומכר הראשון לשני במנה, מנין שאין מחשבין אלא עם השני? ת"ל: לאיש, [לאיש] אשר בתוכו, דברי רבי. רבי דוסתאי בן יהודה אומר: מכרה לו במנה והשביחה ועמדה על מאתים, מנין שאינו מחשב אלא במנה, שנאמר: והשיב את העודף, העודף שבידו; מכרה לו במאתים והכסיפה ועמדה על מנה, מנין שאין מחשבין אלא במנה? שנאמר: והשיב את העודף, העודף שבקרקע....

רמב"ם הלכות שמיטה ויובל פרק יא הלכה טז

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

as stipulated in the Mishna and the Talmud and concisely summarized by Rambam,[20] is that the original owner is always being put in an advantageous position.

The price of the option can be implicit in the prices of fields on the market. Consider a field that was sold four years prior to the Jubilee and observe its price two years after it was sold (assume these years were rainy years). Suppose that at the same time (two years prior to the Jubilee) another field is being sold on the market and assume the fields are about the same quality. The field that was sold two years prior[21] to the Jubilee is sold without the rights to buy it back, and will be returned to the original owner in the Jubilee. The other field, that was sold four years prior to the Jubilee, can be bought back. The difference between their prices is thus the price of the option.

The features of the options are such that there may be two fields of equal quality, both eligible to be redeemed but with different prices, in the market. Consider two fields that were originally sold on different dates, where the market price of the crops was different. The exercise price of these options will be different, the exercise price of the field with the lower historical price being lower than the other. The option with the lower exercise price has a higher value. Consequently, the market price (net price) of the field with the lower exercise price will be lower. This price differential can occur also between two fields that were originally sold at the same time, but one of which one was sold again later at a lower price.

Hence, the historical price at which a field was sold or rather the minimum of these prices, if it was sold a few times, affects the current price of the field. This is a hidden attribute of the field. It should be part of the field description and disclosed to potential buyers. We would like to suggest that the phrase 'you shall not wrong one another' in Leviticus chapter 25, verse 17, might refer to these hidden esoteric attributes. Perhaps this better settles Rashi's difficulties with the placement of this warning

[20] מכרה לראשון במאה דינר וראשון לשני במאתים ורצה האדון לגאול אינו מחשב אלא עם הראשון שנאמר לאיש אשר מכר לו, מכרה לראשון במאתים וראשון לשני במאה ה"ז מחשב עם האחרון, וכן אם מכר במאה והשביחה ביד הלוקח והרי היא ראויה להמכר במאתים מחשב לפי מה שמכר, ואם מכרה במאתים והכסיפה והרי היא ראויה להמכר במאה מחשב לפי מה שהיא, ולעולם מיפים כח מוכר שדה אחוזה ומריעין כח הלוקח.

[21] If the field is sold one year prior to the Jubilee the buyer, not the original owner, gets the produce of the second year. The field is then returned to the original owner, a year after the Jubilee at no cost. This is mentioned in:

תלמוד בבלי מסכת ערכין דף כט עמוד ב

מי לא תניא: אכלה שנה אחת לפני היובל משלימין לו שנה אחרת אחר היובל!

and the Rambam also says:

רמב"ם הלכות שמיטה ויובל פרק יא הלכה יב

מכרה שנה אחת לפני היובל הרי הלוקח אוכל אותה שנה שניה אחר היובל שנאמר שני תבואות.

There is also a case where the buy-back rights can be forced prior to the elapsed time of two years from the transaction time; this applies in the case of an unproductive field. The Rambam says:

רמב"ם הלכות שמיטה ויובל פרק יא הלכה יג

מכר נקעים מלאים מים או סלעים שאינם ראויין לזריעה ה"ז פודה בפחות משתי שנים שנאמר במספר שני תבואות, שדה הראוי לתבואה הוא שאינה נגאלת אלא אחר שתי שנים, ואם לא גאלה אע"פ שאינה ראויה לזריעה חוזרת לבעלים ביובל.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

in the context of selling a field. After all, these attributes are concealed from the buyers, while the number of years until the Jubilee is common knowledge.

Each time the field is sold, the buyer, as an integral part of purchasing the field, writes an option to the original owner. Thus the exercise price of the option that is always held by the original owner may be reduced. The current buyer, however, only worries about the price at which one can be forced to sell the field, i.e. about the exercise price of the option written to the original owner. The price one will be willing to pay for the field is therefore affected only by the exercise price of the written option.

The next section suggests a pricing model for the option embedded in a sale of land. Within this model the phrase (Leviticus chapter 25) 'he shall calculate the years for which the land has been sold, and return the remainder to the man to whom he sold it' is explained in a realistic way. In this interpretation the risk of future crops is not ignored but is captured by the model. The risk of future crops increases with time and their present value decreases with time.

III

There are a few features of the option embedded in the field transaction that we will relax somewhat in order to simplify matters. We start by assuming that the buy-back option can be exercised starting two years after the transaction time, regardless of whether these years are rainy or drought years.[22] This assumption obviously over-estimates[23] the value of the option.

The option, as illustrated above, gives the seller the right to purchase back the land. However, since the item sold is actually the stream of income from the field, 'according to the number of years of crops, he shall sell to you' and not the field itself, the option is to *purchase back* the remaining *stream of income* (until the Jubilee). In the terminology of option pricing the underlying asset is the *stream of income*.

Financially, it makes sense that the crops of future years have greater uncertainty relative to one of a closer year. We have already alluded to the fact that the commentaries (at least Rashi and the Rambam) in their examples of 'he shall calculate the years for which the land has been sold, and return the remainder' do not address the increasing uncertainty of future years of crops. Consequently, in their simplified examples they assign the same present value to crops of different years.

[22] The other relaxed feature is considered during the numerical calculation when the time the option becomes alive is discretized. See the explanation in the Appendix.

[23] A year in this context is qualified as a drought year, if it occurred not only in the field's location but all over the world. Hence the probability of such an occurrence is low and its effect on pricing not significant. On the other hand, incorporating this feature will require us to use pricing methods which are beyond arbitrage pricing, and necessitates some inputs about the risk tolerance of the agent in the economy.

Therefore the following framework of analysis is suggested. Assume for a moment that the field could be sold permanently; its price in that case will be the present value of the infinite sequence of the value of the crops. The crops are assumed to be a continuous stream modeled by a (deterministic) yield, which is a percentage of the value of the field.[24] The redemption option is an option to purchase the stream of crops from the time the option is exercised until the Jubilee. This way of modeling provides us with a framework that will recognize the increasing uncertainty of crops of subsequent years.

Furthermore, it also facilitates, with a slight modification, the use of the classic Black-Scholes model of option pricing. We therefore assume that if the current price of the field is S, its price in t years will be $S(t)e^{\gamma}$ where γ follows the normal distribution with an expected value of μt and a standard deviation of $\sigma\sqrt{(t)}$. The price of the field, $S(t)$, therefore is a lognormal random variable.

It is indeed the case that, in ancient times, certain fields could not be sold permanently and thus prices could not be observed.[25] However, there were fields that could be sold permanently and thus a price of a strongly correlated asset could be observed. Furthermore, as pointed out in footnote 5, there are cases where the field could be sold for a very long period. That is the case where the contract specified the number of years for which the field was sold. For example, if the contract specified that the field was sold for 5,000 years it would not be returned to the original owner in the Jubilee, but after 5,000 years. The price of a field in such a contract would be close to the value of a field that was sold permanently. We can also imagine a field owner who decides on a strategy of reselling the field for 50 years after each Jubilee. Under this strategy the value of the field would be the present value of the infinite sequences of income streams – the value of the crops.

The crops are assumed to generate a continuous stream of income, which is $div S(t)$ at time t, where div is a deterministic constant representing the crop yield. This model therefore captures the risk of crops in future years since $div S(t)$ is a random variable. It also encompasses[26] the fact that viewed from the current time, time 0, given $t_1 < t_2$ the crops at time t_2 possesses a larger volatility $\sigma\sqrt{(t_2)}$ than the volatility $\sigma\sqrt{(t_1)}$ of the crops at time t_1. The volatility in our model is thus an increasing function of time.

The analysis is done from the point of view of the time of sale, which will be denoted as 0. The time until the Jubilee will be denoted as T. Hence at a future

[24] The crops in such an analysis are the counterpart of dividends in the case of a dividend-paying stock. Dividends are usually modeled as being paid continuously by a (deterministic) dividend yield which is a percentage of the value of the stock, and we adopt this framework.

[25] Since it is not possible to conduct an empirical study of market prices of fields (due to the unavailability of these prices), we will resort to some numerical examples.

[26] It is true that we ignore the possibility of a drought year and the sabbatical year, which is not counted in the calculation of the value of the crop. However, when two rainy years passed from the time of the sale, our calculation is correct. We decided not to incorporate the rainy years' provision as otherwise the option valuation could not be done only by arbitrage arguments. Hence we settled for this approach, which can be followed without reference to utility and risk attitudes.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

time t the time to the Jubilee will be T–t. The present value of the field as of time 0, not including the crops that are obtained during the time interval $[0, t]$, is[27] $e^{-div(t)}S(0)$. The present value of the perpetual stream of the crops is of course $S(0)$.

Thus the present value of the stream of crops from time 0 to t is

$$S(0) - e^{-div(t)}S(0).$$

The present value of the crops from time 0 to $t+1$ is

$$S(0) - e^{-div(t+1)}S(0)$$

and therefore the present value of the crops from time t to $t+1$ is

$$S(0) - e^{-div(t+1)}S(0) - \left(S(0) - e^{-div(t)}S(0)\right) = S(0)\left(e^{-div(t)} - e^{-div(t+1)}\right) \quad (1)$$

It is easy to verify that

$$\left(\frac{d}{dt}\right)S(0)\left(e^{-div(t)} - e^{-div(t+1)}\right) = S(0)div\left(e^{-div(t+1)} - e^{-div(t)}\right) \quad (2)$$

is negative. Hence the present value of the crops from time t to $t+1$, as of time 0, is a decreasing function of t. Thus, we see that this model captures the time dimension.

The value of the field at time T, as of time t, not including the crops that are obtained during the time interval $[t, T]$, is $e^{-div(T-t)}S(t)$. Applying the same argument as above, the value of the crops from time t to T is

$$S(t) - S(t)e^{div(T-t)} = S(t)\left(1 - e^{-div(T-t)}\right) \quad (3)$$

which is the value received when the option is exercised at time t. This expression is a decreasing function of t and approaches zero, as one expects, when t approaches T.

If the option is exercised, say at time t, the original owner also has to pay a certain amount (the exercise price). The issue at hand now is how to interpret the phrase in Leviticus chapter 27, verse 15, 'he shall calculate the years for which the land has been sold, and return the remainder to the man to whom he sold it, and [then] he may return to his inheritance'. If one takes the simplistic approach, ignoring the uncertainty of the value of the crops, then the interpretation is as we saw above in the example of Rashi and Rambam.

Within the model presented here, if the crops were originally sold at time 0, where the field's market price was $S(0)$, then the original owner sold the crops between time t to T for $S(0)(e^{-div(t)} - e^{-div(T)})$. Therefore, we suggest that the *remainder* to be returned by the original owner is

$$S(0)\left(e^{-div(t)} - e^{-div(T)}\right) \quad (4)$$

[27] This is a standard argument by which the value of an option on a dividend paying stock is calculated.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

In our opinion the expression in equation (4) better[28] fits the biblical text of 'return the *remainder* to the man to whom he sold it' or in Hebrew

וְהֵשִׁיב אֶת **הָעֹדֶף** לָאִישׁ אֲשֶׁר מָכַר לוֹ .

By the same argument, if the field has been sold again between the original time and the redemption time when its market price was $S < S(0)$, in keeping with the advantage given to the original owner, the remainder is defined by $S(e^{-div(t)} - e^{-div(T)})$.

Based on the above model, we can revisit the example, given in the spirit of the Rambam and Rashi,[29] of a field that was sold ten years prior to the Jubilee for 1,000. If the crops of ten years were sold for 1,000, then by equation (3), $S(0)$, the price of the field which is the value of the perpetual stream of crops, satisfies $S(0)(1 - e^{-div10}) = 1{,}000$. Hence, to investigate and compare the examples of Rambam and Rashi in a manner consistent with our framework, either the price of the field or the crop yield must be assumed. If the crop yield is assumed to be 0.03 then $S(0) \approx 3{,}858.296$ and the value of the crops for year i is given by $3{,}858.296(e^{-0.03(i-1)} - e^{-0.03(i)})$. The numerical values, for years 1 to 10, are stipulated in Table 1.

The payoff from a standard call option is $Max(S(t) - K, 0)$ where K is the exercise price and $S(t)$ the price of the underlying asset at the exercising time t. In our case the maximum price the redeemer pays for the field at time t, is what the original owner received for these years. Thus the expression in equation (4) is the exercise price of this option. The market price of the crops from time t to time T, is $S(t)(1 - e^{-div(T-t)})$. Thus the payoff from the call option, when the field is redeemed, is

$$Max\left(S(t)\left(1 - e^{-div(T-t)}\right) - S(0)\left(e^{-div(t)} - e^{-div(T)}\right), 0\right) \tag{5}$$

This option can be exercised over an interval of time and not just at a particular point in time. That is, it is an American option not a European option. Moreover, in some instances (for example, when the field is sold originally), the option could not be exercised within two years after the date of sale. For this reason a numerical procedure, such as the Binomial Tree, must be used to value the option. The option described is a real option because if crop technology becomes more efficient it will affect the value of the option.

The time of the original sale, or rather the length of time to the Jubilee at that time, is known only to the original owner (the redeemer). For subsequent transactions, the price that should be used to calculate the exercise price is based on the minimum between the price of the field when it was sold originally and the price(s) of the subsequent transactions.

[28] For an alternative interpretation of the 'remainder' see the Appendix.

[29] See footnote 17 for the sources. Rashi uses a selling price of 10 and the Rambam 100, so the price of the crops for one year is 1 and 10, respectively. We scaled it up just to get more accuracy while using only three decimal digits.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

Table 1. *Numerical values of the crops, as of the transaction time, for years 1 to 10*

1	2	3	4	5	6	7	8	9	10
114.029	110.659	107.389	104.215	101.135	98.146	95.245	92.430	89.699	87.048

The current owner knows only the price of the last transaction. Equation (5) assumes that the field is redeemed from the original buyer, i.e. there was only one transaction before the redeeming time. The exercise price in fact depends on the sequence of prices of the historical transactions since the field was first sold. These prices (or the smallest price) should be kept and transferred from one buyer to another and finally to the owner at the redeeming time. If the field was sold n times after the original sale, at times $t_1, t_2, \ldots, t_n$ the exercise price, in equation (5), should have been based on the minimum price, i.e. based on

$$S_{\min} = Min[S(0), S(t_1) \ldots, S(t_n)] \tag{6}$$

The $S(0)$ in $S(0)(e^{-div(t)} - e^{-div(T)})$ in equation (5) should have been $S_{\min}$. The original owner (or the redeemer) and the owner at the redeeming time has some private information (i.e. not publicly available) necessary for calculating the correct exercise price. The Bible warning 'you shall not wrong one another' can be interpreted as a 'heavenly regulator's' instruction to record this information and to transfer it from buyer to buyer and to the redeemer so that, if and when the field were to be redeemed, the correct exercise price could be used.

It seems that Rashi tries to explain the phrase 'you shall not wrong one another' as a warning to the redeemer and the current owner to inform each other of the exact number of years from the redeeming time to the Jubilee. Rashi explains that if the price of the field is calculated based on too many years to the Jubilee, the buyer is not paying the fair price and vice versa. However, as mentioned, the number of years to the Jubilee is public information and it is hard to understand why the redeemer or the current owner would not be aware of it, in particular since they are about to execute a transaction that depends on this information.

The next section illustrates numerically the differences in the valuation using the Rambam and Rashi example and the current model.

IV

The technical issues of valuing the option with a payoff, as described in equation (5), are dealt with in the Appendix. This section illustrates the suggested model's pricing implications and compares them with the opinion of Rambam and Rashi.

The example, in the spirit of their opinion, mentioned above, is of a field that initially was sold ten years prior to the Jubilee for 1,000 shekel. Assuming that the crop yield is 0.03, then the model implications as explained above, for the price of the crops per year, are summarized in Table 1. The field cannot be redeemed during the first two years, but the option given to the original owner to redeem it later has a value. Hence, if the crop value is 1,000 shekel, the price paid by the buyer to the owner is 1,000 minus the value of the option. Assuming the field will be redeemed at the end of the sixth year, where the crop price is stochastic as specified above, the redemption price cannot exceed the implied value of the crops (as of the original

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

time of the sale) for the last four years. The implied crop value in years 7, 8, 9 and 10 (as stipulated in the last four columns of Table 1) is displayed in Table 2.

That is, if the field's market price at the redemption time (and/or at an intermediate sale time) does not imply a lower value for the crops, the exercise price of the option, which is the redemption price, will be 92.245 + 92.430 + 89.699 + 87.048 = 361.422. In contrast, Rambam and Rashi stipulate the redemption price in such a case to be (1,000/10)4 = 400.

The option's values for the different assumed parameters' values that are calculated in this section are summarized in Table 3 and discussed henceforth.

The value of this redemption option, as explained in the Appendix, depends on the assumed value of the crop yield (0.03 in our example), the interest rate in the market and the volatility of the price of the field. The price of the option, the exercising of which is at the discretion of the original owner starting two years after it was sold, when the crop yield is 0.03, interest rate is zero and the volatility is 0.25, is 154.533 (112.018 if the volatility is 0.18). Hence, the net amount to be paid by the original buyer to the owner is 1,000–154.533 = 845.467, while according to Rambam and Rashi the price paid to the original owner is 1,000.

If it is assumed that the rate of interest was 15 percent,[30] the value of the option would have been 305.905 and 282.576 for volatility of 0.25 and 0.18, respectively. Consequently, the amount to be paid by the buyer to the owner is 1,000–305.905 = 694.095 and 1,000-282.576 = 717.424 for volatility of 0.25 and 0.18, respectively.

The value of the option is of course affected also by the assumed crop yield. If the assumed crop yield rose from 0.03 to of 0.18 and assumed an interest rate of zero, the value of the option would have been 33.344 and 12.612 for volatility of 0.25 and 0.18, respectively. Consequently, the amount to be paid by the buyer to the owner is 1,000-33.344 = 966.656 and 1,000–12.612 = 987.388 for volatility of 0.25 and 0.18, respectively. For an interest rate of 15 percent these values are 103.305 and 74.751. Consequently, the amount to be paid by the buyer to the owner is 1,000–103.305 = 896.695 and 1,000–74.751 = 925.249 for volatility of 0.25 and 0.18, respectively. The change in the option's value as a result of a change in the crop yield reinforces the notion that the option is a 'real option'.

Furthermore, note that if the field had been sold originally 12 years prior to the Jubilee, and a second time ten years prior to the Jubilee, the redemption option could be exercised without delay. To illustrate the difference in the prices of the options in these two cases assume:

- that the field's market price at the second transaction implies the same value for the crops as stipulated above, and
- that this value is smaller than or equal to the value of the crops as implied by the original price of the field.

[30] Evidence regarding rates of interest in Mesopotamia suggests very high rates, up to 20% on money lent. See Hudson and Van de Mieroop (2002).

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

Table 2. *Numerical values of the crops, as of the transaction time, for years 7 to 10*

7	8	9	10
95.245	92.430	89.699	87.048

Hence, the redemption prices at the end of the sixth year, in both cases, cannot exceed 361.422. The value of the option to redeem the field, where the interest rate is zero and the redeemer does not have to wait two years to redeem, is 159.792 and 115.781 if the volatility is 0.25 and 0.18 respectively. If the interest rate is assumed to be 15 percent, these values are 305.788 and 282.704.

Obviously, the value of the option when the field was redeemed at any time would have been be greater than or equal to the value of the option when the field was redeemed after only two years. Consequently, the net amount paid by a second buyer to the first buyer, ten years prior to the Jubilee, would have been (when the interest rate is zero and the volatility is 0.25) 1,000–159.792 = 840.208. That is, smaller than the amount paid by a buyer to the original owner when the field was sold ten years prior to the Jubilee. Consequently, two fields that were of equal quality, but had different transaction histories, may have had different prices due to the embedded option (which is not public knowledge).

The actual (realized) amount to be paid at the redemption time is not known at the time of the transaction. It depends on the price of the field at the redemption time. If at that time, the implied value of the crops from that time to the Jubilee is smaller than this implied value at the time of the original sale (or any other transaction of this field in the past), then the smaller value will be the redemption cost. Valuing the option, as explained in the Appendix, is done numerically by discretization of the time and price spaces.

Based on this discretization, we can calculate possible market prices at the end of the sixth year and the implied crop values over the next four years. The redemption price will be the smaller of the implied crop values over the next four years and 361.422 (assuming a crop yield of 0.03). The discretization used for valuing the option is such that each year is divided into 40 equal parts. Over each sub-interval the price of the field can go either up or down, where the up or down percentage change is the same for all the intervals. As a result, at the end of six years there are 241 possible price realizations.

Consider the case of a field that was sold ten years prior to the Jubilee at a price of 1,000 shekel, when the volatility is 0.25, the crop yield is 0.03 and the interest rate zero. The redemption price at the end of the sixth[31] year varies between 361.422

[31] Of course it may not be optimal to redeem (exercise the option) the field at the end of the sixth year for every eventuality of the price of the field.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

Table 3. *The option value for different assumed parameter values with and without delay*

Crop value 10 years prior to the Jubilee	Crop yield	Interest rate	Volatility	Option value with 2-year delay	Option value without delay	Crop value of last 4 years given original price	Range of redemption price in year 6 average price	Rambam & Rashi's redemption price in 6 years
1000	0.03	0	0.25	154.533	159.79	361.42	0.02–361.42 **197.77**	400 = 1000/10*4
1000	0.03	0	0.18	112.01	115.781	361.42		400 = 1000/10*4
1000	0.03	0.15	0.25	305.9		361.42		400 = 1000/10*4
1000	0.03	0.15	0.18	282.57		361.42		400 = 1000/10*4
1000	0.18	0	0.25	33.34		208.81		400 = 1000/10*4
1000	0.18	0	0.18	12.612		208.81		400 = 1000/10*4
1000	0.18	0.15	0.25	103.3		208.81		400 = 1000/10*4
1000	0.18	0.15	0.18	74.75		208.81		400 = 1000/10*4
777.74	0.03	0.15	0.18		219.815	339.23	0.63–339.23 **223.64**	311.09 = 4*777.74/10

(the implied crop value of the last four years based on the original sale price) and 0.022 (the minimum implied crop value over the next four years, based on the price at the end of the sixth year). Out of the 241 possible realizations, there were 121 cases where the redemption price was below 361.422. The average redemption price was 197.777, while according to Rashi and the Rambam the redemption price is deterministic with a value of 400. Hence, the redemption price can differ significantly from the redemption price based on Rashi and Rambam. In the paradigm of Rashi and Rambam, the buyer would have paid 1,000 shekel for the crops when they were sold originally. If the value of the option is acknowledged, the price paid by the buyer to the original owner would be 1,000 minus the value of the option, i.e. 1,000-154.533 = 845.467. Assuming an interest rate of zero, as we have assumed here, the redeeming cost should be increased by the cost of the option. Hence the average redeeming cost would have been 197.777 + 154.533 = 352.310, which is still less than the redeeming cost in the paradigm of Rashi and Rambam.

Finally, a case consistent with Maimonides, but not necessarily with biblical law, is investigated. Consider a case where the interest rate is positive at 15 percent and the crop value (over the last four years), implied by the second sale (ten years prior to the Jubilee), is lower then the original sale (twelve years prior to the Jubilee). If the price of the field at the second sale was 3,000 shekel, then the implied price of the crops for the next ten years (based on the relation $3{,}000(1 - e^{-0.03 \cdot 10})$) is 777.745 and over the last four years 339.238. Hence, if the field is redeemed four years prior to the Jubilee the maximum price is 339.238. If the volatility is assumed to be 0.18 then the value of the option that can be exercised at any time is 219.815. Following the explanation given above, the average redemption price (four years prior to the Jubilee) is 223.264 and the minimum price is 0.683. Out of the 241 possible realizations in 132 cases, the redemption price was obtained by exercising the option and paying 339.238. The second buyer paid the first buyer the price of the crops minus the value of the option, i.e. 777.745–219.815 = 557.930.

V

There were a few assets in ancient Israel where embedded options were part and parcel of the deal so that the real estate transaction was in fact a 'structured product'. This article focused on land transactions, as they are more complex than others. The pricing methods employed in this article assumed that exercising the option was done at an optimal time. Since redeeming the land was considered to be a righteous deed, this assumption may not necessarily describe the behavior of the redeemer.

It is apparent from the discussion above that the prices of fields were dependent on some attributes that were not readily available. The exercise price of the option depended on the original time of the sale, as well as on the price at which the field was sold in the secondary market. Consequently, in the market there could exist two fields that were identical but their prices would be different since the exercise price of the embedded option was different. Furthermore, some of these hidden

attributes of the options were known to the current holder of the field (e.g. the price at which it was last sold) and some to the original owner (e.g. the price and time of the original sale). Both attributes affected the price of the option and hence the price of the field. It might therefore explain why in the middle of the paragraphs in which the rules of the Jubilee are stipulated the Bible states: you shall not wrong *one another*.

Even if one makes the approximation and assumes that the price of crops of each year is the same, the option value should not be ignored in calculating the redemption price. That is, the basic amount from which the price of each year is calculated is the money that was transferred from the buyer to the seller plus the value of the option. It is nearly impossible to find out how these issues and the pricing system were handled in ancient times.

Submitted: 4 September 2014
Revised version submitted: 1 February 2015
Accepted: 5 March 2015

Appendix

The alternative interpretation of the remainder

An alternative interpretation of the 'remainder' is to define it based on $S(0)(1 - e^{-div(T-t)})$ which is the crop price of the next $T-t$ years (as of the redemption time, t), but based on the field's price at the time of the original sale. The expression $S(0)(1 - e^{-div(T-t)})$ better suits the interpretation of the original owner *buying back* the next $T-t$ years of crops (and not *returning the remainder* as stated in the Bible) based on the price of the field that prevailed at the original time of sale. Thus if the field is redeemed at time t, and the original owner returned to the buyer the price of the crops paid for the last $T-t$ years, at the time of the original sale, the amount should be as in equation (4), i.e., $S(0)(e^{-div(t)} - e^{-div(T)})$.

If the alternative interpretation of the remainder is used, then the payoff from the call option, if it is exercised at time t, is not as in equation (5) but rather

$$Max\left(S(t)\left(1 - e^{-div(T-t)}\right) - S_{\min}\left(1 - e^{-div(T-t)}\right), 0\right) \tag{7}$$

If the option had been of a European type, equation (7) could be written as

$$\left(1 - e^{-div(T-t)}\right) Max(S(t) - S_{\min}, 0) \tag{8}$$

Properties of the option: relation to European options

This subsection starts by investigating some properties of the option to redeem the field and its relation to a European option. It then continues to describe the numerical valuation of the true (American) option.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

If the option described by equation (5) had been a European option, it would have been possible to price it analytically. Such analytical solutions provide lower bounds on the true value of the option. To this end, assume that the option could have been exercised only at its maturity time $v < T$, that the current time is 0 and that the Jubilee is at time T. An examination of equation (5) reveals that in this case the t that appears in the equation is a fixed number v and consequently the payoff of such a European option at its maturity is

$$Max\left(\left(1 - e^{-div(T-v)}\right)S(v) - S(0)\left(e^{-div(v)} - e^{-div(T)}\right), 0\right) \tag{9}$$

As v increases, the time to maturity of this option decreases. The value of a regular call option decreases as a result of a decrease in its time to maturity. However, in this case, as opposed to a regular option, the exercise price also decreases as the time to maturity decreases, causing an increase in the value of the option. Hence as the time to maturity decreases, even though the value of the underlying asset in this case also decreases, the value of the option may increase. Equation (9) can be written as

$$\left(1 - e^{-div(T-v)}\right)Max\left(S(v) - S(0)\frac{e^{-div(v)} - e^{-div(T)}}{1 - e^{-div(T-v)}}, 0\right) \tag{10}$$

which stipulates the payoff of $\left(1 - e^{-div(T-v)}\right)$ units of a European call option, where the underlying asset is the field *not the crops*, with an exercise price $S(0)\frac{e^{-div(v)} - e^{-div(T)}}{1 - e^{-div(T-v)}}$. Consequently, the price of this European option is obtained by applying the Black-Scholes formula.

Consider an option embedded in the transaction of selling a field at time 0 by the original owner, T years prior to the Jubilee where the crop yield is 0.03, the volatility is 0.25 and the price of the field is S(0). If the current time is V then there are T - V years to the Jubilee. If this option had been of a European type, maturing at time t where $V < t < T$ such that the crop value is minimized based on a historical transaction price of $S_{\min}$, its payoff at time t would have been

$$Max\left(\left(1 - e^{-div(T-t)}\right)S(t) - S_{\min}\left(e^{-div(t)} - e^{-div(T)}\right), 0\right) \tag{11}$$

which equals

$$\left(1 - e^{-div(T-t)}\right)Max\left(S(t) - S_{\min}\left(e^{-div(t)} - e^{-div(T)}\right)\left(1 - e^{-div(T-t)}\right)^{-1}, 0\right) \tag{12}$$

Applying the Black-Sholes formula where r is the risk-free rate and $S(V)$ the current price of the field, the value of the call is stipulated below:

$$\text{Call}(t, S_{\min}, V, T) = \left(1 - e^{-0.03(T-t)}\right)\left(S(V)N(d_1) - e^{-r(t-V)}KN(d_2)\right) \tag{13}$$

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

where

$$N(z) = \int_{-\infty}^{z} \frac{e^{\frac{-x^2}{2}}}{\sqrt{2x}}\,dx,\; K = S_{\min}\frac{\left(e^{-div(t)} - e^{-div(T)}\right)}{1 - e^{-div(t-T)}},$$

$$d_1 = \frac{\ln\left(\frac{S(V)}{K}\right) + \left(r - div + \frac{\sigma^2}{2}\right)(t - V)}{\sigma\sqrt{t - V}}$$

and $d_2 = d_1 - \sigma\sqrt{t - V}$.

Figure A1 demonstrates the value of the option as a function of the time to maturity, t, of crops sold ten years prior to the Jubilee where the rate of interest was zero, the volatility 0.18, the crop yield 0.03 and the price of the field at that time was 3,858.296.

As indicated, indeed the value of the option does not increase with its time to maturity. Rather, the value of the option, as a function of its time to maturity, possesses a maximum. The maximum value of the option is obtained by maximizing the expression in equation (13) with respect to t. The solution is $t = 3.0762$ and the value of the call at this t is 114.511.

The option that is granted to the original owner is, however, the American type. Hence, the original owner can exercise the option from two years after the sale until the Jubilee. The value of the American option is higher than the value of a European option, because it can be exercised t years prior to the Jubilee for t such that max $(V, 2) \leq t \leq T$. Thus the American option's[32] value is higher than the maximum value of the European option and indeed its value is 154.533 as calculated above.

Equations (5) and (9) are equivalent only if the option is of a European type. Furthermore, the case at hand is different from the regular call option, as both the exercise price and the underlying asset depend directly on t. For these reasons a numerical procedure is utilized to value the option.

Numerical valuation

Consider an option that was written at time 0 and could not be exercised until time h, but could be exercised at any time from time h to its maturity, time T. Assume that the risk-neutral distribution of the value of the option at time h is known. Then the value of the option at time 0 is its expected value discounted by the risk free rate.

When the value of such an option is solved with the Binomial model, time h will not necessarily coincide with one of the nodes in the tree. Let Δ be the length of a period in the Binomial Tree and $[h/\Delta]$ be the smaller integer, which is larger than h/Δ. Thus the distribution of the value of the option at time $[h/\Delta]$ can be calculated using the regular procedure of the Binomial model, taking into account the exercising

[32] The calculations of the values of the American and European options, as well as the visualization of the optimal exercising strategy, were done using the Maple library in Prisman (2000) or some slight modification of its code.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

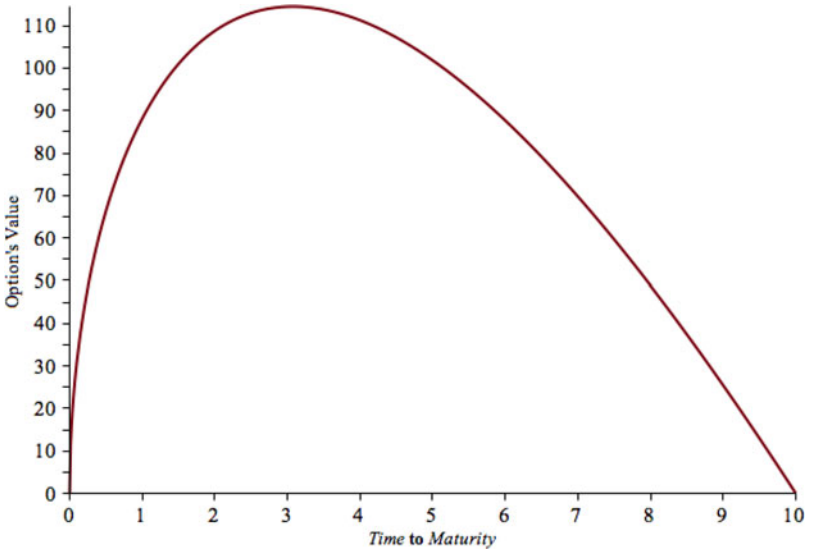

Figure A1. *The value of the European option*

provision at each node greater than $[h/\Delta]$. During the period 0 to h/Δ the option cannot be exercised since this is a condition of the option. It also cannot be exercised from time h/Δ up to (and not including) time $[h/\Delta]$ due to the discretization of the Binomial Model. Therefore the value of the option within the realm of the model, at time zero, is its discounted expected value, under the risk-neutral distribution, as of time $[h/\Delta]$. Of course the risk-neutral distribution of the option's value at time $[h/\Delta]$ is easily calculated within the Binomial model and consequently its value as of time zero is obtained. Hence, to calculate the value of the option when there is a period in which the option cannot be exercised, we value it based on the above procedure and we report as follows.

The numerical values reported in the text generated by the Binomial Tree method with 40 nodes per year was used to solve the value of the American option. The set of optimal exercising times is defined as the set of coordinates (t, y) such that t is the node (time) in the Binomial Tree and y (state of nature) is the number of up movements in the price of the underlying asset. This set is visualized for the case where the option cannot be exercised during the first two years and for a case where the option can be exercised immediately. In order to visualize the optimal exercising set in a clear way we report the case where each year was divided into four subperiods. Hence over ten years there were 40 subperiods. If the option could not be exercised during the first two years the first node at which the option could be exercised is 8.

That is, in our case both y and t can take any value from 0 to 40. The figures below demonstrate this set by placing a square at each coordinate where it is optimal to exercise the option. Figure A2 corresponds to an option that can be exercised with a delay

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

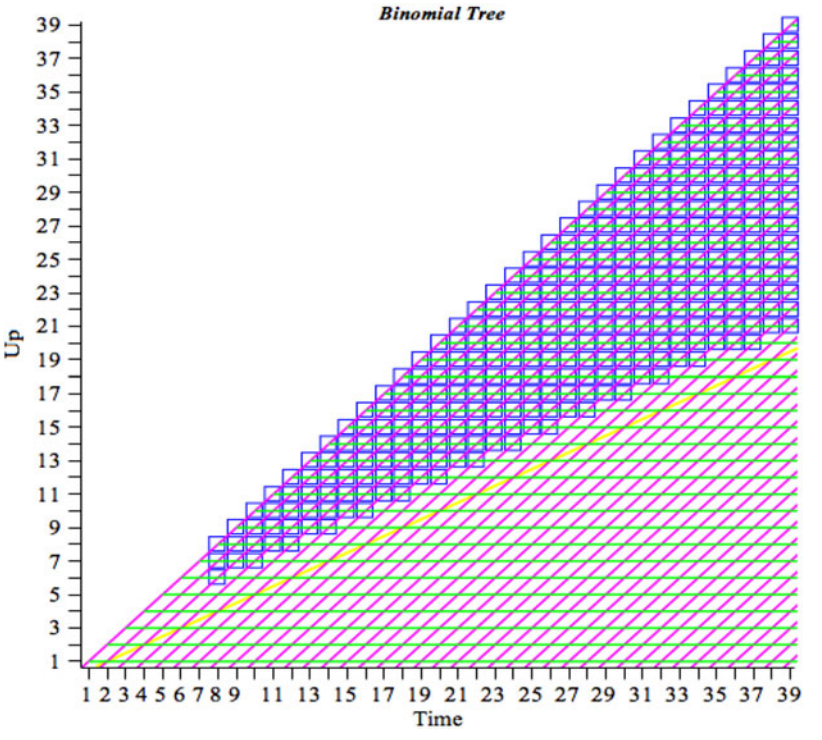

Figure A2. *The optimal exercising set with a delay of two years*

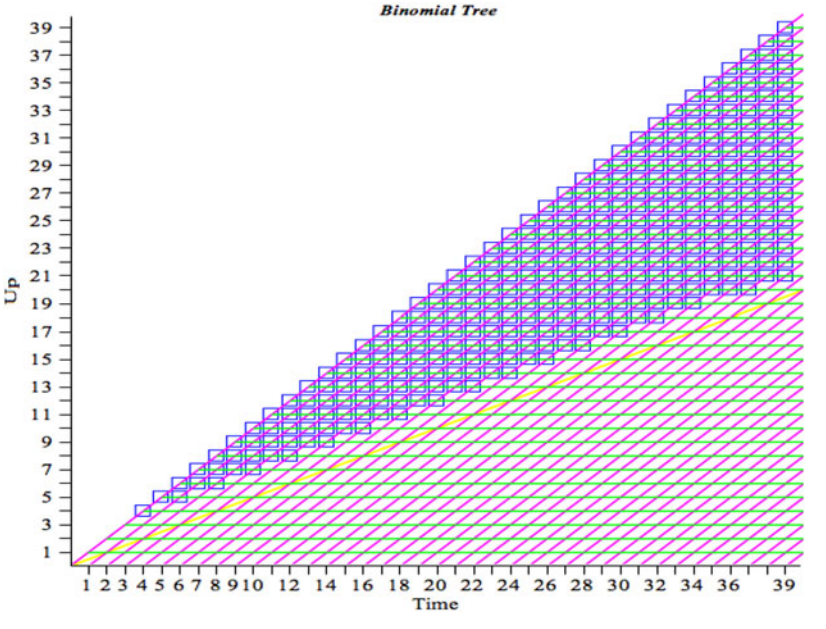

Figure A3. *The optimal exercising set without a delay*

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

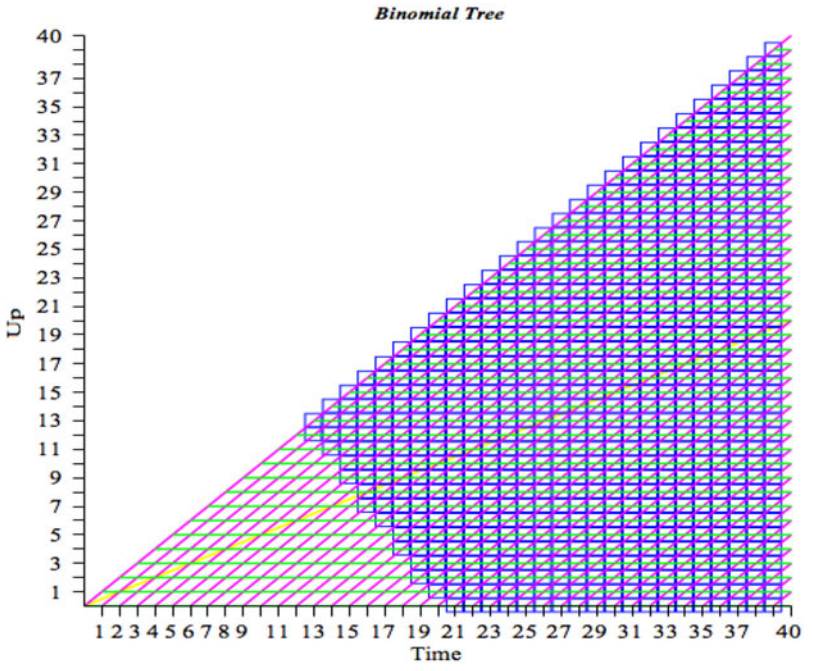

Figure A4. *The optimal exercising set without a delay for low volatility and a high interest rate*

of two years and Figure A3, an option that can be exercised without delay. Based on this discretization, the value of the first option was 153.6791119 and the second 157.9568817.

The shape of the set is of course affected by the value of the parameters. If the volatility is low (0.025), the rate of interest high (15%) and the option can be exercised immediately, based on a discretization above the value of the option is 254.843. The optimal exercising set for this case is displayed in Figure A4.

References

AUMANN, R. J. and MASCHLER, M. (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. *Journal of Economic Theory*, **36**, pp. 195–213.

BUCHHOLZ, T. G. (1988). Biblical laws and the economic growth of Ancient Israel. *Journal of Law and Religion*, **6**(2), pp. 389–427.

CALLEN, J. (2008). Differential asset valuation in the medieval post-Talmudic legal literature. *History of Political Economy*, **40**(1), pp. 183–200.

CHANCE, D. M. (2008). *Essays in Derivatives*. Hoboken, NJ: John Wiley.

HUDSON, M. and VAN DE MIEROOP, M. (eds.) (2002). *Debt and Economic Renewal in the Ancient Near East*. Potomac, MD: CDL Press.

KNOLL, M. (2008). The ancient roots of modern financial innovation: the early history of regulatory arbitrage. *Oregon Law Review*, **87**, pp. 93–116.

https://doi.org/10.1017/S0968565015000062 Published online by Cambridge University Press

KOPPEL, M. (1998). *Seder Kinnim: A Mathematical Commentary on Tractate Kinnum* (in Hebrew). Jerusalem: Aluma Publishing.
LIEBERMANN, Y. (1981). The Coase theorem in Jewish law. *Journal of Legal Studies*, **10**(2), pp. 293–303.
PRISMAN, E. Z. (2000). *Derivative Securities: An Interactive Dynamic Environment with Maple V and Matlab*. London and San Diego, CA: Academic Press.
WESTBROOK, R. (1971). Redemption of land. *Israel Law Review*, **6**(3), pp. 367–75.