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ABSTRACT M- AND ABSTRACT L-SPACES OF POLYNOMIALS ON BANACH LATTICES

QINGYING BU[1], GERARD BUSKES[1] AND YONGJIN LI[2]

[1] *Department of Mathematics, University of Mississippi, University, MS 38677, USA* (qbu@olemiss.edu; mmbuskes@olemiss.edu)
[2] *Department of Mathematics, Sun Yat-sen University, Guangzhou 510275, People's Republic of China* (stslyj@zsu.edu.cn)

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Abstract In this paper we use the norm of bounded variation to study multilinear operators and polynomials on Banach lattices. As a result, we obtain when all continuous multilinear operators and polynomials on Banach lattices are regular. We also provide new abstract M- and abstract L-spaces of multilinear operators and polynomials and generalize all the results by Grecu and Ryan, from Banach lattices with an unconditional basis to all Banach lattices.

Keywords: homogeneous polynomials; positive tensor products; AM- and AL-spaces

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1. Introduction

The theory of holomorphic functions on infinite-dimensional spaces has seen very little influence from the theory of positivity in general and Banach lattice theory in particular; indeed, most of that influence is very recent. Our interest in that influence on holomorphic functions from Banach lattice theory was ignited largely by reading the recent papers [**3**] and [**12**]. Benyamini *et al*. [**3**] showed how Banach space valued orthogonally additive polynomials on a Banach lattice can be linearized by using a concavification of the Banach lattice (see also [**16**] and, for a C^*-algebra version, [**15**]). In [**12**] Grecu and Ryan studied mulitlinear forms on Banach lattices with an unconditional basis, pointing out analogies with theorems about holomorphic functions on Banach lattices. In our paper [**5**] we provided a new unified framework to study regular multilinear operators and orthogonally additive, as well as regular, polynomials on Banach lattices and vector lattices. For its foundation we used the Fremlin tensor product of vector lattices [**10**] and Fremlin's projective tensor product of Banach lattices [**11**]. Our goal in this paper is twofold. On the one hand, we introduce new abstract M- and abstract L-spaces (AM- and AL-spaces, respectively) of multilinear operators and polynomials (Theorems 3.1 and 3.3 and Corollaries 3.2 and 3.4), therewith generalizing their classical counterparts in the theory of positive operators (see, for example, [**7**]). On the other hand, we show how the notion of bounded variation, first introduced by Buskes and van Rooij in [**6**] for bilinear maps, makes it possible to remove the condition of unconditional basis from all of the results by Grecu and Ryan in [**12**]. In the process, we obtain when all continuous multilinear operators and all continuous polynomials on Banach lattices are regular (Corollary 4.10 and Theorem 4.11). We also obtain new results on orthosymmetric multilinear operators (Theorem 4.6) and Pietsch integral operators (Proposition 5.3).

For the general theory of positive operators we refer the reader to [**2**] and [**14**]. For the wider context of infinite-dimensional holomorphy, we refer the reader to Dineen [**9**].

2. Preliminaries

For a Banach space X, let X^* denote its topological dual and B_X denote its closed unit ball. Throughout the paper, all Riesz spaces are Archimedean. For a Riesz space E, let E^+ denote its positive cone. For $x \in E^+$, a *partition* of x is a finite sequence of elements of E^+ whose sum equals x. We often denote a partition $(u_1, \dots, u_k)$ of x by just a letter u. If $u = (u_1, \dots, u_k)$ and $v = (v_1, \dots, v_m)$ are partitions of x, we call u a *refinement* of v if the set $\{1, \dots, k\}$ can be written as a disjoint union of sets $I_1, \dots, I_m$ in such a way that

$$v_i = \sum_{j \in I_i} u_j, \quad i = 1, \dots, m.$$

Any two partitions of x have a common refinement. Thus, Πx, the set of all partitions of x, in a natural way is a directed set (see [**6**]).

For Banach spaces $X, X_1, \dots, X_n, Y$, let $\mathcal{L}(X_1, \dots, X_n; Y)$ denote the space of continuous n-linear operators from $X_1 \times \cdots \times X_n$ to Y and let $\mathcal{P}(^n X; Y)$ denote the space of continuous n-homogeneous polynomials from X to Y. For an n-homogeneous polynomial $P \colon X \to Y$, let $T_P \colon X \times \cdots \times X \to Y$ denote the (unique) symmetric n-linear operator associated with P and, for a symmetric n-linear operator $T \colon X \times \cdots \times X \to Y$, let $P_T \colon X \to Y$ denote the n-homogeneous polynomial associated with T. For the basic terminology about n-linear operators and n-homogeneous polynomials we refer the reader to [**9**, §§ 1.1 and 1.2].

Let $E_1, \dots, E_n$ and F be Riesz spaces. An n-linear operator $T \colon E_1 \times \cdots \times E_n \to F$ is called *positive* if $T(x_1, \dots, x_n) \in F^+$ whenever $x_1 \in E_1^+, \dots, x_n \in E_n^+$. T is called *regular* if T is the difference of two positive n-linear operators (see [**12**,**13**]). Let $\mathcal{L}^r(E_1, \dots, E_n; F)$ denote the space of all regular n-linear operators from $E_1 \times \cdots \times E_n$ to F. If, in addition, F is Dedekind complete, then $\mathcal{L}^r(E_1, \dots, E_n; F)$ is a Dedekind complete Riesz space (see [**13**, Lemma 2.12]). If, moreover, $E_1, \dots, E_n$ and F are Banach lattices with F Dedekind complete, then $\mathcal{L}^r(E_1, \dots, E_n; F)$ is a Banach lattice with its regular n-linear operator norm (see [**5**]).

Let E and F be Riesz spaces. An n-homogeneous polynomial $P \colon E \to F$ is called *positive* if its associated symmetric n-linear operator T_P is positive. P is called *regular* if it is the difference of two positive polynomials (see [**12**,**13**]). Let $\mathcal{P}^r(^n E; F)$ denote the space of all regular n-homogeneous polynomials from E to F. If, in addition, F is Dedekind complete, then $\mathcal{P}^r(^n E; F)$ is a Dedekind complete Riesz space (see [**13**, Lemma 2.15]).

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If, moreover, E and F are Banach lattices with F Dedekind complete, then $\mathcal{P}^r({}^nE;F)$ is a Banach lattice with its regular polynomial norm (see [**5**]).

Let E be a Riesz space and let $x \in E$, $D \subseteq E$. Recall that the symbol $D \uparrow x$ (or $D \downarrow x$) means that D is directed upward (or directed downward) and $x = \sup D$ (or $x = \inf D$) holds (see, for example, [**2**, p. 15]). The lattice operations of $\mathcal{L}^r(E;F)$ are expressed in terms of directed sets (see [**2**, p. 17, Theorem 1.21]). Similar to the proofs of Proposition 2.14 and Lemma 2.16 in [**13**], we have the following two propositions in which the lattice operations of $\mathcal{L}^r(E_1,\dots,E_n;F)$ and $\mathcal{P}^r({}^nE;F)$ are expressed in terms of directed sets.

Proposition 2.1. *Let $E_1,\dots,E_n,F$ be Riesz spaces such that F is Dedekind complete and let $T,S \in \mathcal{L}^r(E_1,\dots,E_n;F)$. Then, for every $x_1 \in E_1^+,\dots,x_n \in E_n^+$,*

$$\left\{ \sum_{i_1,\dots,i_n} T(u_{1,i_1},\dots,u_{n,i_n}) \vee S(u_{1,i_1},\dots,u_{n,i_n}) \colon u_k \in \Pi x_k,\ 1 \leqslant k \leqslant n \right\} \uparrow (T \vee S)(x_1,\dots,x_n),$$

$$\left\{ \sum_{i_1,\dots,i_n} T(u_{1,i_1},\dots,u_{n,i_n}) \wedge S(u_{1,i_1},\dots,u_{n,i_n}) \colon u_k \in \Pi x_k,\ 1 \leqslant k \leqslant n \right\} \downarrow (T \wedge S)(x_1,\dots,x_n),$$

where $u_1 = (u_{1,i_1})_{i_1=1}^{m_1} \in \Pi x_1,\dots,u_n = (u_{n,i_n})_{i_n=1}^{m_n} \in \Pi x_n$.

Proposition 2.2. *Let E and F be Riesz spaces such that F is Dedekind complete and let $P,R \in \mathcal{P}^r({}^nE;F)$. Then, for every $x \in E^+$,*

$$\left\{ \sum_{i_1,\dots,i_n} T_P(v_{i_1},\dots,v_{i_n}) \vee T_R(v_{i_1},\dots,v_{i_n}) \colon (v_1,\dots,v_m) \in \Pi x \right\} \uparrow (P \vee R)(x),$$

$$\left\{ \sum_{i_1,\dots,i_n} T_P(v_{i_1},\dots,v_{i_n}) \wedge T_R(v_{i_1},\dots,v_{i_n}) \colon (v_1,\dots,v_m) \in \Pi x \right\} \downarrow (P \wedge R)(x).$$

Let E be a Riesz space and let Y be a vector space. An n-linear operator $T\colon E \times \cdots \times E \to Y$ is called *orthosymmetric* if $T(x_1,\dots,x_n) = 0$ whenever $x_1,\dots,x_n \in E$ with $x_i \perp x_j$ for some $i \neq j$, $i,j = 1,\dots,n$. It is shown in [**4**] that every positive orthosymmetric n-linear operator with values in a vector lattice is symmetric. An n-homogeneous polynomial $P\colon E \to Y$ is called *orthogonally additive* if $P(x+y) = P(x)+P(y)$ whenever $x,y \in E$ with $x \perp y$.

For Banach lattices $E,E_1,\dots,E_n$, let $E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_n$ denote the *positive n-fold projective tensor product* of $E_1,\dots,E_n$ and let $\hat{\otimes}_{n,s,|\pi|}E$ denote the *positive n-fold symmetric projective tensor product* of E. For the basic terminology about positive projective tensor products we refer the reader to [**5, 10, 11, 18**].

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3. AM- and AL-spaces of multilinear operators and polynomials

Recall that a Banach lattice E is called an *AL-space* if $\|x+y\| = \|x\| + \|y\|$ whenever $x, y \in E^+$. E is called an *AM-space* if $\|x \vee y\| = \max\{\|x\|, \|y\|\}$ whenever $x, y \in E^+$.

Theorem 3.1. *If $E, E_1, \dots, E_n$ are AL-spaces and F is a Dedekind complete AM-space, then $\mathcal{L}^r(E_1, \dots, E_n; F)$ and $\mathcal{P}^r({}^nE; F)$ are AM-spaces.*

Proof. Take $T, S \in \mathcal{L}(E_1, \dots, E_n; F)^+$, $x_1 \in E_1^+, \dots, x_n \in E_n^+$ and $y^* \in F^{*+}$ with $\|y^*\| \leqslant 1$. By Proposition 2.1,

$$
\begin{aligned}
&y^*((T \vee S)(x_1, \dots, x_n)) \\
&\quad = \lim \bigg\{ y^* \bigg(\sum_{i_1,\dots,i_n} T(u_{1,i_1}, \dots, u_{n,i_n}) \vee S(u_{1,i_1}, \dots, u_{n,i_n}) \bigg) \colon u_k \in \varPi x_k,\ 1 \leqslant k \leqslant n \bigg\} \\
&\quad \leqslant \lim \bigg\{ \sum_{i_1,\dots,i_n} \|T(u_{1,i_1}, \dots, u_{n,i_n}) \vee S(u_{1,i_1}, \dots, u_{n,i_n})\| \colon u_k \in \varPi x_k,\ 1 \leqslant k \leqslant n \bigg\} \\
&\quad = \lim \bigg\{ \sum_{i_1,\dots,i_n} \|T(u_{1,i_1}, \dots, u_{n,i_n})\| \vee \|S(u_{1,i_1}, \dots, u_{n,i_n})\| \colon u_k \in \varPi x_k,\ 1 \leqslant k \leqslant n \bigg\} \\
&\quad \leqslant \lim \bigg\{ \sum_{i_1,\dots,i_n} (\|T\| \|u_{1,i_1}\| \cdots \|u_{n,i_n}\|) \vee (\|S\| \|u_{1,i_1}\| \cdots \|u_{n,i_n}\|) \colon \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad u_k \in \varPi x_k,\ 1 \leqslant k \leqslant n \bigg\} \\
&\quad = (\|T\| \vee \|S\|) \lim \bigg\{ \sum_{i_1,\dots,i_n} \|u_{1,i_1}\| \cdots \|u_{n,i_n}\| \colon u_k \in \varPi x_k,\ 1 \leqslant k \leqslant n \bigg\} \\
&\quad = (\|T\| \vee \|S\|) \lim \bigg\{ \bigg(\sum_{i_1} \|u_{1,i_1}\| \bigg) \cdots \bigg(\sum_{i_n} \|u_{n,i_n}\| \bigg) \colon u_k \in \varPi x_k,\ 1 \leqslant k \leqslant n \bigg\} \\
&\quad = (\|T\| \vee \|S\|) \lim \bigg\{ \bigg\| \sum_{i_1} u_{1,i_1} \bigg\| \cdots \bigg\| \sum_{i_n} u_{n,i_n} \bigg\| \colon u_k \in \varPi x_k,\ 1 \leqslant k \leqslant n \bigg\} \\
&\quad = (\|T\| \vee \|S\|) \|x_1\| \cdots \|x_n\|,
\end{aligned}
$$

where $u_k = (u_{k,i_k})_{i_k=1}^{m_k} \in \varPi x_k$ for $1 \leqslant k \leqslant n$. Thus,

$$\|(T \vee S)(x_1, \dots, x_n)\| \leqslant (\|T\| \vee \|S\|) \|x_1\| \cdots \|x_n\|$$

and hence $\|T \vee S\| \leqslant \|T\| \vee \|S\|$. It follows that $\|T \vee S\| = \|T\| \vee \|S\|$, which implies that $\|T \vee S\|_r = \|T\|_r \vee \|S\|_r$. Therefore, $\mathcal{L}^r(E_1, \dots, E_n; F)$ is an AM-space. It can be shown in a similar way that $\mathcal{P}^r({}^nE; F)$ is an AM-space. □

In particular, if in Theorem 3.1 $F = \mathbb{R}$, then [**5**, Propositions 3.3 and 3.4] yields the following corollary (which was obtained by Fremlin in [**11**] for $n = 2$).

Corollary 3.2. *If $E, E_1, \dots, E_n$ are AL-spaces, then $E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_n$ and $\hat{\otimes}_{n,s,|\pi|} E$ are AL-spaces.*

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Theorem 3.3. *If $E, E_1, \dots, E_n$ are AM-spaces and F is a Dedekind complete AL-space, then $\mathcal{L}^r(E_1, \dots, E_n; F)$ and $\mathcal{P}^r({}^nE; F)$ are AL-spaces.*

Proof. Take any $T, S \in \mathcal{L}(E_1, \dots, E_n; F)^+$. For every $\varepsilon > 0$ there exist $x_1 \in B_{E_1^+}, \dots, x_n \in B_{E_n^+}$ and $y_1 \in B_{E_1^+}, \dots, y_n \in B_{E_n^+}$ such that

$$\|T\| \leqslant \|T(x_1, \dots, x_n)\| + \varepsilon/2 \quad \text{and} \quad \|S\| \leqslant \|S(y_1, \dots, y_n)\| + \varepsilon/2.$$

Then,

$$\begin{aligned}
\|T\| + \|S\| &\leqslant \|T(x_1, \dots, x_n)\| + \|S(y_1, \dots, y_n)\| + \varepsilon \\
&\leqslant \|T(x_1 \vee y_1, \dots, x_n \vee y_n)\| + \|S(x_1 \vee y_1, \dots, x_n \vee y_n)\| + \varepsilon \\
&= \|T(x_1 \vee y_1, \dots, x_n \vee y_n) + S(x_1 \vee y_1, \dots, x_n \vee y_n)\| + \varepsilon \\
&\leqslant \|T + S\| \|x_1 \vee y_1\| \cdots \|x_n \vee y_n\| + \varepsilon \\
&= \|T + S\| (\|x_1\| \vee \|y_1\|) \cdots (\|x_n\| \vee \|y_n\|) + \varepsilon \\
&\leqslant \|T + S\| + \varepsilon,
\end{aligned}$$

which implies that $\|T + S\| = \|T\| + \|S\|$, and hence that $\|T + S\|_r = \|T\|_r + \|S\|_r$. Therefore, $\mathcal{L}^r(E_1, \dots, E_n; F)$ is an AL-space. It can be shown in a similar way that $\mathcal{P}^r({}^nE; F)$ is an AL-space. □

In particular, if, in Theorem 3.3, $F = \mathbb{R}$, then [**5**, Propositions 3.3 and 3.4] yields the following corollary (which was obtained by Fremlin in [**11**] for $n = 2$).

Corollary 3.4. *If $E, E_1, \dots, E_n$ are AM-spaces, then $E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_n$ and $\hat{\otimes}_{n,s,|\pi|} E$ are AM-spaces.*

4. Bounded variation of multilinear operators and polynomials

Let E be a Banach lattice and let Y be a Banach space. For $P \in \mathcal{P}({}^nE; Y)$ the *variation* of P is defined by

$$\operatorname{Var}(P) = \sup \left\{ \left\| \sum_{i_1, \dots, i_n} \epsilon_{i_1, \dots, i_n} T_P(x_{i_1}, \dots, x_{i_n}) \right\| : \\ x_k \in E^+, \ \left\| \sum_{k=1}^m x_k \right\| \leqslant 1, \ \epsilon_{i_1, \dots, i_n} = \pm 1 \right\}.$$

Let $\mathcal{P}^{\mathrm{var}}({}^nE; Y)$ denote the space of all P in $\mathcal{P}({}^nE; Y)$ for which $\operatorname{Var}(P)$ is finite. Then $\mathcal{P}^{\mathrm{var}}({}^nE; Y)$ is a Banach space with the norm $\operatorname{Var}(\cdot)$ (see [**5**]).

Let $E_1, \dots, E_n$ be Banach lattices. Let Y be a Banach space. For $T \in \mathcal{L}(E_1, \dots, E_n; Y)$ the *variation* of T is defined by

$$\operatorname{Var}(T) = \sup \left\{ \left\| \sum_{i_1, \dots, i_n} \epsilon_{i_1, \dots, i_n} T(u_{1,i_1}, \dots, u_{n,i_n}) \right\| : \epsilon_{i_1, \dots, i_n} = \pm 1, \\ u_{k,i_k} \in E_k^+, \ \left\| \sum_{i_k=1}^{m_k} u_{k,i_k} \right\| \leqslant 1, \ 1 \leqslant k \leqslant n \right\},$$

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from which it follows that $\|T\| \leqslant \mathrm{Var}(T)$. Let $\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ denote the space of all T in $\mathcal{L}(E_1,\dots,E_n;Y)$ such that $\mathrm{Var}(T)$ is finite. If F is a Dedekind complete Banach lattice, then it is easy to see that $\mathcal{L}^r(E_1,\dots,E_n;F) \subseteq \mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;F)$ with $\mathrm{Var}(T) \leqslant \|T\|_r$ for every $T \in \mathcal{L}^r(E_1,\dots,E_n;F)$.

Buskes and Rooij [**6**] gave the above definition of $\mathrm{Var}(T)$ for $n = 2$ and showed that $\mathcal{L}^{\mathrm{var}}(E_1,E_2;Y) = \mathcal{L}(E_1\hat{\otimes}_{|\pi|}E_2;Y)$ isometrically. Similarly, we have the following proposition.

Proposition 4.1. *Let $E_1,\dots,E_n$ be Banach lattices. Let Y be a Banach space. Then for every $T \in \mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ there exists a unique $T^{\otimes}$ in $\mathcal{L}(E_1\hat{\otimes}_{|\pi|}\cdots\hat{\otimes}_{|\pi|}E_n;Y)$ such that $\mathrm{Var}(T) = \|T^{\otimes}\|$ and*

$$T(x_1,\dots,x_n) = T^{\otimes}(x_1\otimes\cdots\otimes x_n), \quad x_1 \in E_1,\dots,x_n \in E_n.$$

Moreover, $\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ is isometrically isomorphic to $\mathcal{L}(E_1\hat{\otimes}_{|\pi|}\cdots\hat{\otimes}_{|\pi|}E_n;Y)$ under the mapping $T \to T^{\otimes}$.

Lemma 4.2. *Let $E_1,\dots,E_n$ be Banach lattices, let Y be a Banach space and let T be an element in $\mathcal{L}(E_1,\dots,E_n;Y)$. Then T belongs to $\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ if and only if, for every $y^* \in Y^*$, $y^*T \in \mathcal{L}^r(E_1,\dots,E_n;\mathbb{R})$ and $\sup\{\|y^*T\|_r : y^* \in B_{Y^*}\} < \infty$. In this case,*

$$\mathrm{Var}(T) = \sup\{\|y^*T\|_r : y^* \in B_{Y^*}\}.$$

Proof. Let $T \in \mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ and let $y^* \in Y^*$. Take $x_1 \in E_1^+,\dots,x_n \in E_n^+$ and let $(u_{1,i_1})_{i_1=1}^{m_1} \in \Pi x_1,\dots,(u_{n,i_n})_{i_n=1}^{m_n} \in \Pi x_n$. Then,

$$\begin{aligned}\sum_{i_1,\dots,i_n} |y^*T(u_{1,i_1},\dots,u_{n,i_n})| &= \sum_{i_1,\dots,i_n} \epsilon_{i_1,\dots,i_n} y^*T(u_{1,i_1},\dots,u_{n,i_n}) \\ &\leqslant \|y^*\| \bigg\| \sum_{i_1,\dots,i_n} \epsilon_{i_1,\dots,i_n} T(u_{1,i_1},\dots,u_{n,i_n}) \bigg\| \\ &\leqslant \|y^*\|\|x_1\|\cdots\|x_n\|\,\mathrm{Var}(T) \\ &< \infty,\end{aligned}$$

where $\epsilon_{i_1,\dots,i_n} = \mathrm{sign}(y^*T(u_{1,i_1},\dots,u_{n,i_n}))$. Thus, by [**5**, (2.10)],

$$|y^*T|(x_1,\dots,x_n) = \sup\bigg\{\sum_{i_1,\dots,i_n} |y^*T(u_{1,i_1},\dots,u_{n,i_n})| : (u_{k,i_k})_{i_k=1}^{m_k} \in \Pi x_k, 1 \leqslant k \leqslant n\bigg\}$$

exists, and hence $y^*T \in \mathcal{L}^r(E_1,\dots,E_n;\mathbb{R})$. Moreover,

$$\begin{aligned}\|y^*T\|_r &= \||y^*T|\| \\ &= \sup\{|y^*T|(x_1,\dots,x_n) : x_1 \in B_{E_1^+},\dots,x_n \in B_{E_n^+}\} \\ &\leqslant \|y^*\|\,\mathrm{Var}(T),\end{aligned}$$

which implies that

$$\sup\{\|y^*T\|_r : y^* \in B_{Y^*}\} \leqslant \mathrm{Var}(T).$$

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On the other hand, suppose that $\sup\{\|y^*T\|_r \colon y^* \in B_{Y^*}\} < \infty$. Take $\epsilon_{i_1,\dots,i_n} = \pm 1$, $u_{k,i_k} \in E_k^+$ with $\|\sum_{i_k=1}^{m_k} u_{k,i_k}\| \leqslant 1$ for $1 \leqslant k \leqslant n$. Write $x_k = \sum_{i_k=1}^{m_k} u_{k,i_k}$ for $1 \leqslant k \leqslant n$. Then $x_k \in E_k^+$ with $\|x_k\| \leqslant 1$ for $1 \leqslant k \leqslant n$ and

$$\begin{aligned}\Big\| \sum_{i_1,\dots,i_n} &\epsilon_{i_1,\dots,i_n} T(u_{1,i_1},\dots,u_{n,i_n}) \Big\| \\ &= \sup\bigg\{ \bigg| \sum_{i_1,\dots,i_n} \epsilon_{i_1,\dots,i_n} y^*T(u_{1,i_1},\dots,u_{n,i_n}) \bigg| \colon y^* \in B_{Y^*} \bigg\} \\ &\leqslant \sup\bigg\{ \sum_{i_1,\dots,i_n} |y^*T|(u_{1,i_1},\dots,u_{n,i_n}) \colon y^* \in B_{Y^*} \bigg\} \\ &= \sup\bigg\{ |y^*T| \bigg(\sum_{i_1} u_{1,i_1},\dots,\sum_{i_n} u_{n,i_n} \bigg) \colon y^* \in B_{Y^*} \bigg\} \\ &= \sup\{|y^*T|(x_1,\dots,x_n) \colon y^* \in B_{Y^*}\} \\ &\leqslant \sup\{\|y^*T\|_r \|x_1\| \cdots \|x_n\| \colon y^* \in B_{Y^*}\} \\ &\leqslant \sup\{\|y^*T\|_r \colon y^* \in B_{Y^*}\}.\end{aligned}$$

Thus, $T \in \mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ and

$$\mathrm{Var}(T) \leqslant \sup\{\|y^*T\|_r \colon y^* \in B_{Y^*}\}.$$

□

Similarly, we have the following result for polynomials.

Lemma 4.3. *Let E be a Banach lattice, let Y be a Banach space and let P be an element in $\mathcal{P}(^nE;Y)$. Then P belongs to $\mathcal{P}^{\mathrm{var}}(^nE;Y)$ if and only if, for every $y^* \in Y^*$, $y^*P \in \mathcal{P}^r(^nE;\mathbb{R})$ and $\sup\{\|y^*P\|_r \colon y^* \in B_{Y^*}\} < \infty$. In this case,*

$$\mathrm{Var}(P) = \sup\{\|y^*P\|_r \colon y^* \in B_{Y^*}\}.$$

Combining Lemmas 4.2 and 4.3 with the polarization inequality (see, for example, [**5**]), we have the following theorem.

Theorem 4.4. *Let E be a Banach lattice, let Y be a Banach space and let $T \colon E \times \cdots \times E \to Y$ be a symmetric n-linear operator. Then $T \in \mathcal{L}^{\mathrm{var}}(E,\dots,E;Y)$ if and only if $P_T \in \mathcal{P}^{\mathrm{var}}(^nE;Y)$. In this case,*

$$\mathrm{Var}(P_T) \leqslant \mathrm{Var}(T) \leqslant \frac{n^n}{n!} \mathrm{Var}(P_T).$$

As vector spaces, $\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y) \subseteq \mathcal{L}(E_1,\dots,E_n;Y)$ with $\|T\| \leqslant \mathrm{Var}(T)$, and $\mathcal{P}^{\mathrm{var}}(^nE;Y) \subseteq \mathcal{P}(^nE;Y)$ with $\|P\| \leqslant \mathrm{Var}(P)$. The next theorem gives a sufficient condition for which $\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y) = \mathcal{L}(E_1,\dots,E_n;Y)$ and $\mathcal{P}^{\mathrm{var}}(^nE;Y) = \mathcal{P}(^nE;Y)$, which answers the implicit question asked in the last line of [**6**].

Theorem 4.5. *Let $E, E_1, \dots, E_{n-1}$ be AL-spaces, let E_n be a Banach lattice and let Y be a Banach space. The following then hold.*

(i) $\mathcal{L}^{\mathrm{var}}(E_1, \dots, E_n; Y) = \mathcal{L}(E_1, \dots, E_n; Y)$. *In this case,* $\mathrm{Var}(T) = \|T\|$ *for every* $T \in \mathcal{L}(E_1, \dots, E_n; Y)$.

(ii) $\mathcal{P}^{\mathrm{var}}({}^nE; Y) = \mathcal{P}({}^nE; Y)$. *In this case,* $\mathrm{Var}(P) = \|P\|$ *for every* $P \in \mathcal{P}({}^nE; Y)$.

Proof. Take any $T \in \mathcal{L}(E_1, \dots, E_n; Y)$. Let $\epsilon_{i_1,\dots,i_n} = \pm 1$ and $u_{k,i_k} \in E_k^+$ with $\|\sum_{i_k=1}^{m_k} u_{k,i_k}\| \leqslant 1$ for $1 \leqslant k \leqslant n$. Note that

$$\bigg\| \sum_{i_n=1}^{m_n} \epsilon_{i_1,\dots,i_n} u_{n,i_n} \bigg\| \leqslant \bigg\| \sum_{i_n=1}^{m_n} u_{n,i_n} \bigg\| \leqslant 1.$$

Then,

$$\begin{aligned}
\bigg\| \sum_{i_1,\dots,i_n} & \epsilon_{i_1,\dots,i_n} T(u_{1,i_1}, \dots, u_{n-1,i_{n-1}}, u_{n,i_n}) \bigg\| \\
&= \bigg\| \sum_{i_1,\dots,i_n} T(u_{1,i_1}, \dots, u_{n-1,i_{n-1}}, \epsilon_{i_1,\dots,i_n} u_{n,i_n}) \bigg\| \\
&= \bigg\| \sum_{i_1,\dots,i_{n-1}} T\bigg(u_{1,i_1}, \dots, u_{n-1,i_{n-1}}, \sum_{i_n} \epsilon_{i_1,\dots,i_n} u_{n,i_n}\bigg) \bigg\| \\
&\leqslant \sum_{i_1,\dots,i_{n-1}} \|T\| \|u_{1,i_1}\| \cdots \|u_{n-1,i_{n-1}}\| \bigg\| \sum_{i_n} \epsilon_{i_1,\dots,i_n} u_{n,i_n} \bigg\| \\
&\leqslant \|T\| \bigg(\sum_{i_1} \|u_{1,i_1}\| \bigg) \cdots \bigg(\sum_{i_{n-1}} \|u_{n-1,i_{n-1}}\| \bigg) \\
&= \|T\| \bigg\| \sum_{i_1} u_{1,i_1} \bigg\| \cdots \bigg\| \sum_{i_{n-1}} u_{n-1,i_{n-1}} \bigg\| \\
&\leqslant \|T\|.
\end{aligned}$$

Thus, $\mathrm{Var}(T) \leqslant \|T\|$ and hence $T \in \mathcal{L}^{\mathrm{var}}(E_1, \dots, E_n; Y)$. The proof of the polynomial statement follows similarly. □

Bu and Buskes [**5**, Theorem 6.4] show that if E is a σ-Dedekind complete Banach lattice and Y is a Banach space, then every orthogonally additive n-homogeneous polynomial $P \colon E \to Y$ is of bounded variation, that is, $P \in \mathcal{P}^{\mathrm{var}}({}^nE; Y)$. The next theorem is for the multilinear operator case, which follows from Theorem 4.4 and [**5**, Lemma 4.1, Theorem 6.4].

Theorem 4.6. *Let E be a σ-Dedekind complete Banach lattice and let Y be a Banach space. Then, for every orthosymmetric n-linear operator $T \colon E \times \cdots \times E \to Y$,*

$$\|T\| \leqslant \mathrm{Var}(T) \leqslant \frac{n^n}{n!} \|T\|.$$

Example 4.7. If E_1 is not an AL-space and $E_2 = \ell_p$ or $L_p[0,1]$ $(1 < p < \infty)$, then $\mathcal{L}^{\mathrm{var}}(E_1, E_2; \mathbb{R}) \subsetneq \mathcal{L}(E_1, E_2; \mathbb{R})$.

Proof. By Proposition 4.1 and by [**17**, p. 204, Theorem 3.2],

$$\mathcal{L}^{\mathrm{var}}(E_1, E_2; \mathbb{R}) = \mathcal{L}(E_1 \hat{\otimes}_{|\pi|} E_2; \mathbb{R}) = (E_1 \hat{\otimes}_{|\pi|} E_2)^* = \mathcal{L}^r(E_1; E_2^*).$$

Note that $\mathcal{L}(E_1, E_2; \mathbb{R}) = \mathcal{L}(E_1; E_2^*)$. Now, if we assume that $\mathcal{L}^{\mathrm{var}}(E_1, E_2; \mathbb{R}) = \mathcal{L}(E_1, E_2; \mathbb{R})$, then $\mathcal{L}^r(E_1; E_2^*) = \mathcal{L}(E_1; E_2^*)$. It follows from [**7**, Theorem 1] and [**14**, p. 169, Corollary 3.2.2] that E_1 must be an AL-space. □

As vector spaces, $\mathcal{L}^r(E_1, \dots, E_n; F) \subseteq \mathcal{L}^{\mathrm{var}}(E_1, \dots, E_n; F)$ with $\mathrm{Var}(T) \leqslant \|T\|_r$, and $\mathcal{P}^r(^nE; F) \subseteq \mathcal{P}^{\mathrm{var}}(^nE; F)$ with $\mathrm{Var}(P) \leqslant \|P\|_r$. The next theorem provides a sufficient condition for which $\mathcal{L}^r(E_1, \dots, E_n; F) = \mathcal{L}^{\mathrm{var}}(E_1, \dots, E_n; F)$ and $\mathcal{P}^r(^nE; F) = \mathcal{P}^{\mathrm{var}}(^nE; F)$.

Theorem 4.8. *Let $E, E_1, \dots, E_n$ be Banach lattices and let F be a Dedekind complete AM-space with an order unit. The following then hold.*

(i) $\mathcal{L}^r(E_1, \dots, E_n; F) = \mathcal{L}^{\mathrm{var}}(E_1, \dots, E_n; F)$. *In this case,* $\mathrm{Var}(T) = \|T\|_r$ *for every* $T \in \mathcal{L}^{\mathrm{var}}(E_1, \dots, E_n; F)$.

(ii) $\mathcal{P}^r(^nE; F) = \mathcal{P}^{\mathrm{var}}(^nE; F)$. *In this case,* $\mathrm{Var}(P) = \|P\|_r$ *for every* $P \in \mathcal{P}^{\mathrm{var}}(^nE; F)$.

Proof. Take any $T \in \mathcal{L}^{\mathrm{var}}(E_1, \dots, E_n; F)$. By Proposition 4.1 there exists a unique $T^{\otimes}$ in $\mathcal{L}(E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_n; F)$ such that $\mathrm{Var}(T) = \|T^{\otimes}\|$. It follows from [**14**, p. 48, Theorem 1.5.11] that $T^{\otimes} \in \mathcal{L}^r(E_1 \hat{\otimes}_{|\pi|} \cdots \hat{\otimes}_{|\pi|} E_n; F)$ with $\|T^{\otimes}\|_r = \|T^{\otimes}\|$. Thus, $T \in \mathcal{L}^r(E_1, \dots, E_n; F)$ with $\|T\|_r = \|T^{\otimes}\|_r$ by [**5**, Proposition 3.3]. The proof for the polynomial statement is similar. □

Example 4.9. $\mathcal{L}^r(L_2[0,1], L_2[0,1]; c_0) \subsetneq \mathcal{L}^{\mathrm{var}}(L_2[0,1], L_2[0,1]; c_0)$.

Proof. In fact, define $T \colon L_2[0,1] \times L_2[0,1] \to c_0$ by

$$T(f,g)_k = \int_0^1 f(t) g(t) \sin(2^k \pi t) \, \mathrm{d}t, \quad k \in \mathbb{N},$$

where $T(f,g)_k$ denotes the kth coordinate of $T(f,g)$ for every $k \in \mathbb{N}$. It follows from [**8**, p. 60] that T is a bounded bilinear operator with $\|T(f,g)\|_{c_0} \leqslant \|f\| \|g\|$. It is easy to see that T is also orthosymmetric, which, by Theorem 4.6, implies that $T \in \mathcal{L}^{\mathrm{var}}(L_2[0,1], L_2[0,1]; c_0)$.

Now let $A_k = \{t \in [0,1] \colon |\sin(2^k \pi t)| \geqslant 1/\sqrt{2}\}$. Then, for every $k \in \mathbb{N}$, $m(A_k) = \frac{1}{4}$ (here m denotes the Lebesgue measure on $[0,1]$) and

$$|T|(\chi_{[0,1]}, \chi_{[0,1]})_k = \int_0^1 |\sin(2^k \pi t)| \, \mathrm{d}t \geqslant \int_{A_k} |\sin(2^k \pi t)| \, \mathrm{d}t \geqslant \frac{1}{\sqrt{2}} m(A_k) = \frac{1}{4\sqrt{2}},$$

which implies that $|T|(\chi_{[0,1]}, \chi_{[0,1]}) \notin c_0$. Thus, $T \notin \mathcal{L}^r(L_2[0,1], L_2[0,1]; c_0)$. □

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It would be interesting to know when continuous n-linear operators and continuous n-homogeneous polynomials are regular. Combining Theorem 4.8 with Theorem 4.6 and [**5**, Theorem 6.4] we have the following corollary.

Corollary 4.10. *Let E be a σ-Dedekind complete Banach lattice and F a Dedekind complete AM-space with an order unit. Then the following hold:*

(i) *all continuous orthosymmetric n-linear operators $T\colon E\times\cdots\times E\to F$ are regular with $\|T\|\leqslant\|T\|_r\leqslant(n^n/n!)\|T\|$;*

(ii) *all continuous orthogonally additive n-homogeneous polynomials $P\colon E\to F$ are regular with $\|P\|\leqslant\|P\|_r\leqslant(n^n/n!)\|P\|$.*

Recall that a Banach lattice F is said to have property (P) if there exists a positive contractive projection from F^{**} to F. Every Kantorovich–Banach space has property (P) and every Banach lattice with property (P) is Dedekind complete (see [**14**, p. 47]).

Theorem 4.11. *Let $E, E_1,\dots,E_n$ be AL-spaces and let F be a Banach lattice with property (P). Then all continuous n-linear operators $T\colon E_1\times\cdots\times E_n\to F$ and all continuous n-homogeneous polynomials $P\colon E\to F$ are regular with $\|T\|=\|T\|_r$ and $\|P\|=\|P\|_r$.*

Proof. Take any $T\in\mathcal{L}(E_1,\dots,E_n;F)$. By Theorem 4.5, $T\in\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;F)$ with $\mathrm{Var}(T)=\|T\|$ and we also have, by Proposition 4.1, that there exists a unique $T^{\otimes}\in\mathcal{L}(E_1\hat{\otimes}_{|\pi|}\cdots\hat{\otimes}_{|\pi|}E_n;F)$ such that $\|T^{\otimes}\|=\mathrm{Var}(T)$. Note that $E_1\hat{\otimes}_{|\pi|}\cdots\hat{\otimes}_{|\pi|}E_n$ is an AL-space by Corollary 3.2. It follows from [**14**, p. 48, Theorem 1.5.11] that $T^{\otimes}\in\mathcal{L}^r(E_1\hat{\otimes}_{|\pi|}\cdots\hat{\otimes}_{|\pi|}E_n;F)$ with $\|T^{\otimes}\|_r=\|T^{\otimes}\|$. Thus, $T\in\mathcal{L}^r(E_1,\dots,E_n;F)$ with $\|T\|_r=\|T^{\otimes}\|_r$ by [**5**, Proposition 3.3]. The proof of the polynomial statement follows similarly. □

5. Extension of Grecu and Ryan's results

For every $T\in\mathcal{L}(E_1,\dots,E_n;Y)$, define $T_1\colon E_1\to\mathcal{L}(E_2,\dots,E_n;Y)$ by

$$T_1(x)(x_2,\dots,x_n)=T(x,x_2,\dots,x_n),\quad x\in E_1,x_2\in E_2,\dots,x_n\in E_n.$$

Then T_1 is a continuous linear operator.

Proposition 5.1. *Let $E_1,\dots,E_n$ be Banach lattices, let Y be a Banach space and let $T\in\mathcal{L}(E_1,\dots,E_n;Y)$. If $T_1\colon E_1\to\mathcal{L}^{\mathrm{var}}(E_2,\dots,E_n;Y)$ is absolutely summing, then $T\in\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$.*

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Proof. Let $\epsilon_{i_1,\dots,i_n} = \pm 1$ and let $u_{k,i_k} \in E_k^+$ with $\|\sum_{i_k=1}^{m_k} u_{k,i_k}\| \leqslant 1$ for $1 \leqslant k \leqslant n$. Then,

$$\begin{aligned}\bigg\|\sum_{i_1,\dots,i_n}\epsilon_{i_1,\dots,i_n}T(u_{1,i_1},\dots,u_{n,i_n})\bigg\| &\leqslant \sum_{i_1}\bigg\|\sum_{i_2,\dots,i_n}\epsilon_{i_1,\dots,i_n}T_1(u_{1,i_1})(u_{2,i_2},\dots,u_{n,i_n})\bigg\|\\ &\leqslant \sum_{i_1}\mathrm{Var}(T_1(u_{1,i_1}))\\ &\leqslant \pi(T_1)\sup\bigg\{\bigg\|\sum_{i_1}\lambda_{i_1}u_{1,i_1}\bigg\|\colon |\lambda_{i_1}|\leqslant 1\bigg\}\\ &\leqslant \pi(T_1)\bigg\|\sum_{i_1}|u_{1,i_1}|\bigg\|\\ &\leqslant \pi(T_1),\end{aligned}$$

where $\pi(T_1)$ is the absolutely summing operator norm of T_1. Thus, $\mathrm{Var}(T) \leqslant \pi(T_1)$ and hence $T \in \mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$. □

For every $P \in \mathcal{P}(^nE;Y)$, define $P_1\colon E \to \mathcal{P}(^{n-1}E;Y)$ by

$$P_1(x)(u) = T_P(x,u,\dots,u), \quad x,u \in E.$$

Then P_1 is a continuous linear operator (see [**12**]). Similarly to Proposition 5.1, we have the following proposition.

Proposition 5.2. *Let E be a Banach lattice, let Y be a Banach space and let $P \in \mathcal{P}(^nE;Y)$. If $P_1\colon E \to \mathcal{P}^{\mathrm{var}}(^{n-1}E;Y)$ is absolutely summing, then $P \in \mathcal{P}^{\mathrm{var}}(^nE;Y)$.*

Recall that (see [**8**, p. 165] and [**1**]) an n-linear operator $T\colon X_1\times\cdots\times X_n \to Y$ is said to be *Pietsch integral* if there exists a regular countably additive Y-valued Borel measure ν of bounded variation on the product $B_{X_1^*}\times\cdots\times B_{X_n^*}$ (with the weak *-topology) such that

$$T(x_1,\dots,x_n) = \int_{B_{X_1^*}\times\cdots\times B_{X_n^*}} x_1^*(x_1)\cdots x_n^*(x_n)\,\mathrm{d}\nu(x_1^*,\dots,x_n^*) \qquad (*)$$

for every $(x_1,\dots,x_n) \in X_1\times\cdots\times X_n$. Let $\mathcal{L}_{PI}(X_1,\dots,X_n;Y)$ denote the space of all Pietsch integral n-linear operators from $X_1\times\cdots\times X_n$ to Y endowed with the norm

$$\|T\|_{PI} = \inf|\nu|(B_{X_1^*}\times\cdots\times B_{X_n^*}),$$

where the infimum is taken over all vector measures ν satisfying the above definition.

Proposition 5.3. *Let $E_1,\dots,E_n$ be Banach lattices, let Y be a Banach space and let $T \in \mathcal{L}_{PI}(E_1,\dots,E_n;Y)$. Then $T \in \mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ and $\mathrm{Var}(T) \leqslant \|T\|_{PI}$.*

Proof. Take any $T \in \mathcal{L}_{PI}(E_1,\dots,E_n;Y)$. There exists a vector measure ν satisfying $(*)$. Let $\epsilon_{i_1,\dots,i_n} = \pm 1$ and $u_{k,i_k} \in E_k^+$ with $\|\sum_{i_k=1}^{m_k} u_{k,i_k}\| \leqslant 1$ for $1 \leqslant k \leqslant n$.

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Then,

$$\begin{aligned}\Big\|\sum_{i_1,\dots,i_n}\epsilon_{i_1,\dots,i_n}T(u_{1,i_1},\dots,u_{n,i_n})\Big\| \\ &\leqslant \int_{B_{X_1^*}\times\cdots\times B_{X_n^*}}\sum_{i_1,\dots,i_n}|x_1^*(u_{1,i_1})\cdots x_n^*(u_{n,i_n})|\,\mathrm{d}|\nu|(x_1^*,\dots,x_n^*)\\ &\leqslant \int_{B_{X_1^*}\times\cdots\times B_{X_n^*}}|x_1^*|\bigg(\sum_{i_1}u_{1,i_1}\bigg)\cdots|x_n^*|\bigg(\sum_{i_n}u_{n,i_n}\bigg)\,\mathrm{d}|\nu|(x_1^*,\dots,x_n^*)\\ &\leqslant \int_{B_{X_1^*}\times\cdots\times B_{X_n^*}}\|x_1^*\|\bigg\|\sum_{i_1}u_{1,i_1}\bigg\|\cdots\|x_n^*\|\bigg\|\sum_{i_n}u_{n,i_n}\bigg\|\,\mathrm{d}|\nu|(x_1^*,\dots,x_n^*)\\ &\leqslant |\nu|(B_{X_1^*}\times\cdots\times B_{X_n^*}),\end{aligned}$$

which implies that $T\in\mathcal{L}^{\mathrm{var}}(E_1,\dots,E_n;Y)$ and $\mathrm{Var}(T)\leqslant\|T\|_{PI}$. □

It is known that a Banach space with a 1-unconditional basis is also a Banach lattice with the order defined coordinate-wise. Now, let E_1 and E_2 be two Banach spaces, each with 1-unconditional bases. Grecu and Ryan [**12**] introduced $\mathcal{B}_\nu(E_1\times E_2)$, the space of all unconditional bilinear forms on $E_1\times E_2$, and the unconditional norm $\nu(\cdot)$ on $\mathcal{B}_\nu(E_1\times E_2)$. It is easy to see that $\mathcal{B}_\nu(E_1\times E_2)=\mathcal{L}^{\mathrm{var}}(E_1,E_2;\mathbb{R})$ and $\nu(T)=\mathrm{Var}(T)$ for every bilinear form $T\in\mathcal{B}_\nu(E_1\times E_2)=\mathcal{L}^{\mathrm{var}}(E_1,E_2;\mathbb{R})$. Thus, [**12**, Proposition 2.1] is a special case of Proposition 5.1 in this paper and [**12**, Corollary 2.2] is a special case of Proposition 5.3 in this paper.

Now let E be a Banach space with a 1-unconditional basis. Grecu and Ryan [**12**] also introduced $\mathcal{P}_\nu({}^nE)$, the space of all unconditional n-homogeneous polynomials on E, and the unconditional polynomial norm $\nu(\cdot)$ on $\mathcal{P}_\nu({}^nE)$. It is easy to see that $\mathcal{P}_\nu({}^nE)=\mathcal{P}^{\mathrm{var}}({}^nE;\mathbb{R})$ and $\nu(P)=\mathrm{Var}(P)$ for every n-homogeneous polynomial $P\in\mathcal{P}_\nu({}^nE)=\mathcal{P}^{\mathrm{var}}({}^nE;\mathbb{R})$. Thus, [**12**, Proposition 3.1] is a special case of Proposition 5.2 in this paper and [**12**, Proposition 4.2 and Proposition 4.3] are special cases of Theorem 4.8 in this paper.

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