THE REVIEW OF SYMBOLIC LOGIC
Volume 13, Number 2, June 2020

SEVENTEENTH-CENTURY SCHOLASTIC SYLLOGISTICS. BETWEEN LOGIC AND MATHEMATICS?

MIROSLAV HANKE
Institute of Philosophy of the Czech Academy of Sciences

Abstract. The seventeenth century can be viewed as an era of (closely related) innovation in the formal and natural sciences and of paradigmatic diversity in philosophy (due to the coexistence of at least the humanist, the late scholastic, and the early modern tradition). Within this environment, the present study focuses on scholastic logic and, in particular, syllogistic. In seventeenth-century scholastic logic two different approaches to logic can be identified, one represented by the Dominicans Báñez, Poinsot, and Comas del Brugar, the other represented by the Jesuits Hurtado, Arriaga, Oviedo, and Compton. These two groups of authors can be contrasted in three prominent features. First, in the role of the theory of validity, which is either a common basis for all particular theories (in this case, sentential logic and syllogistic), or a set of observations regarding a particular theory (in this case, syllogistic). Second, in the view of syllogistic, which is either an implication of a general theory of validity and a semantics of terms, or an algebra of structured objects. Third, in the role of the scholastic analysis of language in terms of *suppositio*, which either is a semantic underpinning of syllogistic, or it is replaced by a semantics of propositions.

§1. Introduction. Seventeenth-century scholastic syllogistics is ambivalent in character. The seventeenth century was an interesting period when scholastic, humanist and early modern influences were meeting and paradigm-shifting works in philosophy, mathematics and physics were being published. In contrast to that, syllogistic does not seem to have been a very exciting branch of logic per se, and even less so in the tradition-driven environment scholasticism may be suspected of sustaining. The notorious chant *Barbara*, *Celarent*, *Darii*, *Ferio*... appears to "say it all", and, in a sense, it does, as far as the practical concerns are regarded. A second, broader and more detailed look reveals diversity and innovation under the seemingly quiet surface of scholastic syllogistic. This paper aims to present a comparative analysis of two seventeenth-century systems of scholastic logic,[1] labeled $\mathfrak{B}$ and $\mathfrak{H}$, and the role syllogistic plays in them. It is important to note in advance, that $\mathfrak{B}$ and $\mathfrak{H}$ are not viewed as "different systems" because they validate different sets of theorems, in the sense in which, for instance, S4 and S5 are different systems of modal logic. Even though certain segments of $\mathfrak{B}$ and $\mathfrak{H}$ present different principles ($\mathfrak{B}$ is typically

Received: December 14, 2017.
2010 *Mathematics Subject Classification*: 01A35, 01A40, 01A45.
Key words and phrases: scholastic logic, mathematisation, second scholasticism, syllogistic reasoning, substructural logic.

[1] Even though the selection of the corpus resulted from a broader preliminary research and the authors were of considerable reputation among their peers, no complete coverage of late scholastic logic is claimed here. Ashworth (1974) and Risse (1964) and (1970) are both seminal and up to this day the most comprehensive overviews of post-medieval scholastic logic. Cf. also Broadie (1985) and Ashworth (2017).

doi:10.1017/S1755020318000357

more comprehensive and includes $\mathfrak{H}$),[2] this is not the main issue. They are viewed as different systems primarily from the point of view of their internal organisation and the roles played by the respective sub-theories. From the point of view of syllogistic, which is what the present study primarily focuses on, both theories make an attempt to validate the same inference rules, i.e., to save the complete chant "Barbara, Celarent..." However, the ways they do it, the principles they employ, and the relations of syllogistic to other sub-theories of logic are radically different. These differences will be reconstructed[3] and an attempt will be made to think along the lines of their general approach in order to show their potential.

§2. The system $\mathfrak{B}$. $\mathfrak{B}$ captures the common features of logical theories formulated by Domingo Báñez, João Poinsot, and Miguel Comas del Brugar between ca 1600 and 1660, derived from Domingo de Soto's approach of mid-sixteenth century.[4] As derived from Soto, $\mathfrak{B}$ can claim the inheritance of medieval scholasticism reaching all the way to the fourteenth century, notably to medieval treatises on *consequentiae*.[5] In other words, rather than a sixteenth- or seventeenth-century invention, $\mathfrak{B}$ is a product of continuity with medieval scholasticism. This continuity will not be explored in greater detail, but since so many parts of medieval logic were simply lost after 1550, it should not be taken for granted.[6]

The fundamental parts of $\mathfrak{B}$ are a general theory of validity, sentential logic, and syllogistic, the first being the common basis for the second and third.

2.1. Theory of validity. To stress the fact that $\mathfrak{B}$ construes logical principles as inference rules applying to inferences (*consequentia*) whose antecedents and consequents

[2] This is most striking in the case of the two theories of validity. However, that may be a practical, rather than a principal difference: $\mathfrak{H}$ is not to be considered a weaker system where some theorems of $\mathfrak{B}$ would be invalid (at least no such claim is made).

[3] Any modern representation of premodern theories is controversial, but it is hoped that the benefits will outweigh the disadvantages. This remark is especially relevant to the reconstruction of $\mathfrak{H}$, in particular its algebra of propositions.

[4] While the ultimate source of $\mathfrak{B}$ seems to have been Soto's *Summulae*, the seventeenth-century authors emphasize specific aspects of his theory and produce a more compact version of it. This difference consists in part in the changes in the genre of logical textbooks: while Soto's *Summulae* were a commentary on Peter of Spain (containing some additional material), Báñez, Poinsot, and Comas del Brugar authored standard second-scholastic "*cursus*".

[5] As opposed to at least some medieval treatises pertaining to the genre of consequences, Soto intentionally excluded principles pertaining to epistemic logic and claimed that they are part of epistemology (*posterioristice sunt*), cf. Soto (1529), 73vb.

[6] Cf. Dutilh Novaes (2008), 479–482 for a survey of general inference rules shared in the medieval scholastic community, cf. Ashworth (1974), 171–186 for the most comprehensive survey of post-medieval general inference rules and Ashworth (2016) for a general study on the transformation of logic in the post-medieval period as regards both continuity and discontinuity between the two periods. A notable change in the seventeenth-century account is the disappearance of a thorough analysis of validity. A standard fourteenth- and even early sixteenth-century treatise on consequences typically introduces several different definitions of validity together with a number of counterexamples (typically adjusting for token-based and semantically closed language, arbitrariness of signs and so forth), resulting into a highly technical formulation (for well-known examples, see Buridan (1976), 21–22 and Paul of Venice (1990), 6–11 and 79–84). This feature, to some degree present in Soto (1529), 73rb–va, did not survive to seventeenth-century texts.

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

can themselves be inferences, its principles will be represented as Gentzen-style trees applied to expressions some of which contain turnstiles, where the horizontal stroke and the turnstile will be held to have the same meaning. This notational convention is hoped to represent some interesting properties of $\mathfrak{B}$, which might be lost if a different technique of representation were employed.[7] The axioms of the general theory of validity in $\mathfrak{B}$ are three fundamental "principles of reasoning" (*fundamenta argumentationis*). First, the law of non-contradiction (*impossibile est idem simul esse et non esse*):[8]

$$\frac{\alpha, \sim \alpha}{\perp}[\text{fa1a}]$$

which, applied to validity, yields:

$$\frac{(\alpha \vdash \beta)\,, (\alpha \nvdash \beta)}{\perp}[\text{fa1a} *].$$

Second, the law of excluded middle (*quodlibet est vel non est*):

$$\frac{\top}{\alpha, \sim \alpha}[\text{fa1b}]$$

which, applied to validity, yields:

$$\frac{\top}{(\alpha \vdash \beta), (\alpha \nvdash \beta)}[\text{fa1b} *].$$

Third, the definition of validity in terms of truth-preservation, i.e., "for every valid inference, if the antecedent is true, so is the consequent" (*in omni bona consequentia, si antecedens est verum, et consequens debet esse verum*), or:

$$\frac{\alpha \vdash \beta \qquad \mathrm{Tr}(\alpha)}{\mathrm{Tr}(\beta)}[\text{fa2a}].$$

[7] These rules will be represented as schemata to be instantiated. Accordingly, some of their subformulas are, technically, variables, but this will not be emphasized in what follows. Sentences will be represented by Greek letters. Logical consequence (*consequentia*) or deductive validity (*bonitas consequentiae*) will be represented by the horizontal stroke (followed by a bracketed name of the inference rule) or by a turnstile, depending on convenience; a crossed turnstile will represent invalidity. An upper formula or a formula preceding a turnstile represents an antecedent and a lower formula or a formula following a turnstile represents a consequent. "α,β" in an antecedent corresponds to a conjunction and in a consequent to a disjunction. "$\perp$" stands for a self-contradiction and "$\top$" for a tautology. The sentential operators will include negation "$\sim$", conjunction "$\wedge$", disjunction "$\vee$", material implication "$\supset$", and strict implication "$\Rightarrow$" (*propositio conditionalis*) and the semantic operators "$\mathrm{Tr}(\alpha)$", "$\mathrm{Fa}(\alpha)$", "$\mathrm{M}(\alpha)$", and "$\mathrm{L}(\alpha)$", representing truth, falsity, possibility, and necessity. "$\alpha[\pi \mapsto \rho]$" will represent the result of replacing every occurrence of "π" by "ρ" in "α" (*mutatis mutandis* for other forms of substitution). The formal representation of the principles and proofs will use the techniques of natural deduction. However, one should not overlook the conceptual distinctions between the formal derivability of uninterpreted symbols via a syntactically definable rule (the modern approach) and a truth-preserving transformation of an interpreted sentence captured by such a rule (the scholastic approach). Last but not least, there is no genuine hierarchy of languages in these particular scholastic approaches.

[8] See Soto (1529), 73va–vb; Báñez (1559), 119–123; Poinsot (1638), 65; Comas del Brugar (1661), 459–462 (the Latin quotations for non-contradiction, excluded middle and truth-preservation in the main text are from Báñez, but the differences within the corpus are not significant).

Another way to phrase the problem is to make the modal dimension of validity explicit and emphasize, coherently with textual evidence,[9] the definition-like character of this principle (such that it is an equivalence, where truth-preservation is regarded as both a necessary and a sufficient condition of validity), by decomposing the statement that an inference is valid if and only if it is impossible for the antecedent to be true while the consequent is not into two subprinciples:

$$\frac{\alpha \vdash \beta}{\sim \mathrm{M}(\mathrm{Tr}\,(\alpha) \wedge \sim \mathrm{Tr}\,(\beta))}[\mathrm{fa2a}\ *]$$

and

$$\frac{\sim \mathrm{M}(\mathrm{Tr}\,(\alpha) \wedge \sim \mathrm{Tr}\,(\beta))}{\alpha \vdash \beta}[\mathrm{fa2a}\ **].$$

This view was sometimes restated in terms of "compatibility", where an inference is valid if and only if its antecedent is incompatible with the negation of its consequent (*oppositum consequentis repugnat antecedenti*):[10]

$$\frac{\alpha, \sim \beta \vdash \bot}{\alpha \vdash \beta}[\mathrm{fa2b}]$$

and

$$\frac{\alpha \vdash \beta}{\alpha, \sim \beta \vdash \bot}[\mathrm{fa2b}\ *].$$

These substantiate the theorems via *reductio ad absurdum* applied to inferences:[11]

$$\frac{(\alpha \nvdash \beta) \vdash \bot}{\alpha \vdash \beta}[\mathrm{red}].$$

The theorems of the general theory of validity include the value-preservation principles ("if an antecedent has the value X, so does its consequent") and their implications, the structural principles, and the so-called "paradoxes of implication" (which overlap into sentential logic, but are discussed here for their structural implications).

The value-preservation principles are, most typically:[12]

$$\frac{\alpha \vdash \beta \qquad \mathrm{Tr}(\alpha)}{\mathrm{Tr}(\beta)}[\mathrm{reg1}]$$

---

[9] Soto (1529), 73rb: "*sufficit et requiritur quod non possit antecedens esse verum sine consequenti*".

[10] Soto (1529), 73va.

[11] Possibly, this rule could take the form:

$$\frac{\top, (\alpha \nvdash \beta) \vdash \bot}{\alpha \vdash \beta}[\mathrm{red}*]$$

which would more precisely reflect the form of some proofs. But [red*] and [red] are equivalent via "weakening", which seems to be a natural implication of defining validity in terms of truth-preservation.

[12] In Báñez's version, which the formalisation attempts to follow, these are: "*Ex vero semper verum infertur*. (...) *Falsum non nisi ex falso sequitur*. (...) *In bona consequentia si antecedens est necessarium, consequens debet esse necessarium*. (...) *In bona consequentia si antecedens est possibile, consequens non potest esse impossibile*. (...) *In bona consequentia si consequens est contingens, antecedens non potest esse necessarium*. (...) *Si antecedens est contingens, consequens non potest esse impossibile.*" Báñez (1599), 121–122. Other versions only differ in details (such as not using double negation in [reg4] or omitting some of the principles).

$$\frac{\alpha \vdash \beta \qquad \mathrm{Fa}(\beta)}{\mathrm{Fa}(\alpha)}[\mathrm{reg2}]$$

$$\frac{\alpha \vdash \beta \qquad \mathrm{L}(\alpha)}{\mathrm{L}(\beta)}[\mathrm{reg3}]$$

$$\frac{\alpha \vdash \beta \qquad \mathrm{M}\,(\alpha)}{\sim(\sim \mathrm{M}\,(\beta))}[\mathrm{reg4}]$$

$$\frac{\alpha \vdash \beta \qquad \mathrm{M}\,(\beta) \qquad \mathrm{M}\,(\sim \beta)}{\sim \mathrm{L}(\alpha)}[\mathrm{reg5}]$$

$$\frac{\alpha \vdash \beta \qquad \mathrm{M}\,(\alpha) \qquad \mathrm{M}\,(\sim \alpha)}{\sim(\sim \mathrm{M}\,(\beta))}[\mathrm{reg6}].$$

[reg1] and [reg2] are construed as immediate implications of the axioms[13] and the remaining theorems are derived from [fa2a] via [red]: the assumption that a rule does not hold implies a contradiction (typically, that some valid inference is not truth-preserving or that some sentence is both true and false).[14] The same approach validates the following structural principles:[15]

$$\frac{\alpha \vdash \beta \qquad \beta \vdash \gamma}{\alpha \vdash \gamma}[\mathrm{reg7}]$$

$$\frac{\alpha \vdash \beta \qquad \beta, \gamma \vdash \bot}{\alpha, \gamma \vdash \bot}[\mathrm{reg8a}].$$

[reg8a] (whatever is incompatible with a consequent, is incompatible with its antecedent) is claimed to be equivalent to contraposibility of inference:[16]

$$\frac{\alpha \vdash \beta}{\sim \beta \vdash \sim \alpha}[\mathrm{reg8b}].$$

[13] On this particular representation, [reg1] *is* an axiom. This presentation follows the textual evidence, where what is fundamentally the same principle is listed both as an axiom and as a rule. As a result, Soto states that it “plainly follows” (*palam sequitur*) from the definition of validity (Soto, 1529, 73va–vb), for Báñez [reg1] and [reg2] are so close to the axioms that they do not require a proof (Báñez, 1599, 121), Poinsot admits that the rule is identical to the axiom (Poinsot, 1638, 66), while Comas del Brugar does not list it among the rules at all (Comas del Brugar, 1661, 460).

[14] As an example, Báñez’s proof of [reg3] is: if α entails β but [reg3] does not hold, then α is necessary, hence always true, but β is not necessary, hence can sometimes be false, in which case it can happen that α is true while β is false, which contradicts the assumption that α entails β (Báñez, 1599, 122.) While this approach to the proofs of theorems captures the approaches of Báñez, Poinsot, and Comas del Brugar, Soto relates some of the theorems to one another, namely claims that [reg3] implies [reg5] and that “$(\alpha \vdash \beta)$, M$(\alpha)\vdash$M(β)” (corresponding to [reg4]) implies “$(\alpha \vdash \beta)$, ~M$(\beta) \vdash$ ~ M(α)” (Soto, 1529, 73vb).

[15] “*Quidquid sequitur ad consequens bonae consequentiae sequitur ad eius antecedens.* (…) *Quidquid repugnat consequenti bonae consequentiae repugnat antecedenti*. (…) *Haec regula sub aliis verbis potest constitui*: *In bona consequentia ex opposito consequentis sequitur oppositum antecedentis, quae eadem ratione demonstratur.*” Báñez (1599), 122–123.

[16] A proof of this equivalence (typically introduced by saying “…which, in other words, amounts to”) is not introduced in this corpus of texts (cf. Báñez (1599), 122–123, Poinsot (1638), 67, and Comas del Brugar (1661), 461). Soto addresses [reg8a] but not [reg8b] in this context, others discuss both rules, and Soto and Comas del Brugar prove [reg8a], whereas Báñez and Poinsot

Since contraposition is also applied to validity-statements,[17] a related principle and its converse also hold:

$$\frac{(\alpha \vdash \beta) \vdash (\gamma \vdash \delta)}{(\gamma \nvdash \delta) \vdash (\alpha \nvdash \beta)}[\text{reg8b} *].$$

The authors subsumed here under the category of system $\mathfrak{B}$ discuss two readings of the paradoxes of implication “anything follows from the impossible” and “the necessary follows from anything”. On the strong reading, “anything” means literally “every” and “any” sentence:[18]

$$\frac{\sim \mathrm{M}(\alpha)}{\alpha \vdash \beta}[\text{reg9}]$$

$$\frac{\mathrm{L}(\beta)}{\alpha \vdash \beta}[\text{reg10}].$$

On the weak reading, these principles state that an impossible sentence implies and a necessary sentence is implied by sentences with all kinds of modal values (impossible,

prove [reg8b] and both proofs are designed in fundamentally the same way. Interestingly, Báñez even claims that the proofs of both principles would be the same. Let us compare the two proofs. First, to prove that “$\alpha \vdash \beta$” entails “ $\sim\beta \vdash \sim\alpha$”, Báñez and Poinsot assume the opposite, i.e., that both “$\alpha \vdash \beta$” and “ $\sim\beta \nvdash \sim\alpha$”. If “ $\sim\beta \nvdash \sim\alpha$”, then “ $\sim\beta$” and “α” are compatible (“ $\sim\beta,\alpha \nvdash \perp$”) and can hold in the same model. Then let us construct that model and assume that “α” and “ $\sim\beta$” hold. Then we would have a situation which validates “$\alpha \vdash \beta$”, “α” and “ $\sim\beta$”, contrary to the assumption that valid inferences are truth-preserving (or detachable). Which proves that [reg8b] holds. Second, to prove that if “$\alpha \vdash \beta$” and “$\beta,\gamma \vdash \perp$”, then “$\alpha,\gamma \vdash \perp$”, Soto (1529, 73vb) assumes the opposite, i.e., that “$\alpha \vdash \beta$”, “$\beta,\gamma \vdash \perp$”, and “$\alpha,\gamma \nvdash \perp$” can hold at the same time. That translates to a model where “α” and “γ” hold (since $\alpha,\gamma \nvdash \perp$). But if “$\gamma$” holds, “$\beta$” does not hold (since $\beta,\gamma \vdash \perp$). As a result, such situation would validate “$\alpha \vdash \beta$” and “α”, but not “β”, contrary to the assumption that valid inferences are truth-preserving (or detachable). Which proves that [reg8a] holds.

Both proofs are, indeed, similar. In fact, if we replace “γ” with “ $\sim\beta$” in the second proof, we get: assume that “$\alpha \vdash \beta$” and “$\beta, \sim\beta \vdash \perp$”, and “$\alpha, \sim\beta \nvdash \perp$” can hold at the same time. That translates to a model where “α” and “ $\sim\beta$” hold (since $\alpha, \sim\beta \nvdash \perp$). But if “ $\sim\beta$” holds, “β” does not hold (since $\beta, \sim\beta \vdash \perp$). As a result, such situation would validate “$\alpha \vdash \beta$” and “α”, but not “β”, contrary to the assumption that valid inferences are truth-preserving (or detachable). Which proves that “$\alpha \vdash \beta$” and “$\beta, \sim\beta \vdash \perp$” entail “$\alpha, \sim\beta \vdash \perp$”. Now, “$\alpha, \sim\beta \vdash \perp$” can be rephrased as “ $\sim\beta \vdash \sim\alpha$” (via [fa2b] and [fa2b*]). As a result, by these means one would obtain a proof of [reg8b]. And since “$\beta, \sim\beta \vdash \perp$” is a tautology, which could either be dropped from the proof or viewed as presupposed in Soto’s proof, both rules could be viewed as having the same proof (as Báñez suggests).

Also, [reg8b] reduces to [reg8a] via:

[1] $(\alpha, \sim\beta \vdash \perp) \dashv\vdash (\alpha \vdash \beta)$

[2] $(\alpha, \top \vdash \beta) \dashv\vdash (\alpha \vdash \beta)$.

Starting with the following instance of [reg8a] (via substitution):

$$\frac{\alpha \vdash \beta \qquad \beta, \sim\beta \vdash \perp}{\alpha, \sim\beta \vdash \perp}[\text{reg8a} *]$$

furthermore inferring (via [2]):

$$\frac{\alpha \vdash \beta}{\alpha, \sim\beta \vdash \perp}[\text{reg8b} **]$$

one gets, finally, [reg8b] (via [1]). While the reverse inference from [reg8b**] to [reg8a*] is possible, the generalisation resulting to [reg8a] is not obvious at this point.

[17] See §2.3 regarding the reduction to the impossible.

[18] “*Ex impossibili sequitur quodlibet*. (...) *Necessarium sequitur ex quolibet*.” Soto (1529), 73vb. Cf. also Báñez (1599), 123 and Comas del Brugar (1661), 461–462.

necessary, and contingent). The rejection of the strong reading is based on the view that truth-preservation is a necessary but not a sufficient condition of validity.

A notable instance of these is "*ex contradictione quodlibet*":

$$\frac{\alpha \quad \sim\alpha}{\beta}\text{[ecq]}.$$

Its proof is discussed in its traditional form and it has two steps.[19] [STEP 1] is the proof of "β" from "$\alpha\wedge\sim\alpha$":

$$\cfrac{\cfrac{\cfrac{\alpha\wedge\sim\alpha}{\alpha}\text{[lk1]}}{\alpha\vee\beta}\text{[la1]} \quad \cfrac{\alpha\wedge\sim\alpha}{\sim\alpha}\text{[lk1]}}{\beta}\text{[la2]}.$$

[STEP 2] deduces "$(\alpha\wedge\sim\alpha)\vdash\beta$" from the performance of [STEP 1] in this particular form, allegedly via the transitivity of inference (*a primo ad ultimum*):

$$\frac{\text{[STEP1]}}{(\alpha\wedge\sim\alpha)\vdash\beta}\text{["reg7"]}.$$

Three different solutions to this proof are discussed within $\mathfrak{B}$: operational restrictions, the redefinition of logical form targeting [STEP 1], and structural restrictions targeting [STEP 2]. Before addressing those, let us note that these paradoxes can be proved along the same lines as other theorems, hence, arguably, if this proof is not sufficient, neither are the proofs above.

The operational restriction proposed by Soto consists in distinguishing between two forms of self-contradiction, one that is "absolute" and one that is stated "for the sake of argument". To contradict a statement is either to cancel its truth (*tollere veritatem*) or to simultaneously state its opposite (*ponere contradictoriam*). The rule of disjunctive syllogism principle is assumed to rest on the first but not the second account. These remarks can be interpreted as a restriction on the disjunctive syllogism rule, or as an introduction of two forms of negation. As far as the rejection of [ecq] is concerned, the former interpretation is sufficient:

$$\frac{\alpha,\beta\nvdash\bot \quad \alpha\vee\beta \quad \sim\alpha}{\beta}\text{[la2 *]}$$

where the upper-left clause restricts the use of [la2] to consistent contexts.[20] Báñez rejects the proof in terms of an alternative view of logical form. His view of the logical structure

[19] Soto (1529), 74va: "*Ad minus ex quibuscumque duabus contradictoriis sequitur quodlibet. Probatur antecedens. Sequitur bene 'Petrus disputat et Petrus non disputat, ergo Petrus non disputat' et ex alia parte 'Petrus disputat et Petrus non disputat, ergo Petrus disputat', quia utrobique arguitur a copulativa ad eius alteram partem. Rursus 'Petrus disputat, ergo Petrus disputat vel homo est lapis', a parte disiunctive ad totam. Rursus 'Petrus disputat vel homo est lapis et Petrus non disputat* (*que erat consequens prime consequentie*)*, ergo homo est lapis', a tota disiunctiva cum destructione unius partis ad positionem alterius. Ergo de primo ad ultimum, 'Petrus disputat et Petrus non disputat, ergo homo est lapis', quia quicquid sequitur ad consequens bone consequentie sequitur ad eius antecedens, et eadem arte ex eodem antecedenti poteris inferre quodcumque consequens.*" Cf. also Báñez (1599), 125.

[20] Soto (1529), 74va: "*Hic valde est notandum quod due contradictorie possunt accipi dupliciter, uno modo absolute et sine aliqua suppositione, et sic altera est destructiva alterius, quod est dicere quod per veritatem unius tollitur veritas alterius. Alio modo accipiuntur ut concesse ab aliquo ex aliqua suppositione gratia disputationis ut videatur quid inde sequatur, et tunc certe neutra destruit alteram, quia ex hoc quod una conceditur vera, non sequitur alteram esse falsam, cum ambe concedantur vere. Igitur illa consequentia 'Petrus disputat vel homo est lapis et Petrus non disputat, ergo homo est lapis', si ille contradictorie accipiantur absolute, bona est, scilicet*

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

of [STEP 1] is:

$$\dfrac{\dfrac{\dfrac{\alpha \wedge \beta}{\alpha}[1]}{\alpha \vee \gamma}[2] \qquad \dfrac{\alpha \wedge \beta}{\beta}[3]}{\gamma}[4].$$

This, of course, fails.[21] This argument seems justified as an attempt to say that the proof of [ecq] rests essentially on the self-inconsistence of its initial assumptions. However, considering this a matter of content (rather than form) seems far-fetched, for the same type of reasoning could construe *modus tollens* as:

$$\frac{\alpha \Rightarrow \beta \qquad \gamma}{\delta}$$

and reject it as well. Similarly, one could dismantle [reg8b], which would threaten the foundations of $\mathfrak{B}$'s syllogistic.

Different editions of Soto's *Summulae* contain two structural solutions to the proof.[22] First, in the 1554 edition Soto objects that [STEP 2] is not based on the transitivity of inference, since ["reg7"] is not a straightforward form of [reg7].[23] That is correct, since

---

si accipiantur ut concesse in primo antecedenti, certe nihil valet, quia ex hoc quod negativa est vera non sequitur data suppositione quod affirmativa sit falsa, cum causa disputationis ambe sint concesse, quare non arguitur a tota disunctiva cum destructione unius partis. Nam destruere unam propositionem non est ponere eius contradictoriam, sed tollere eius veritatem, quod tali casu non sit per positionem contradictorie." See Read (1993) for analogies between this solution and relevance logic.

[21] Báñez (1599), 125–126: "*Quamvis enim prima illa consequentia, a copulativa ad partem formalis consequentia sit et etiam secunda consequentia a parte disiunctivae ad totam sit formalis, tamen in tertia consequentia, in qua argumentamur a disiunctiva cum destructione unius partis ad positionem alterius, invenitur ipsa destructio unius partis non sequi formaliter ex antecedentibus. Etenim per accidens est ad formalitatem consequentiae a copulativa ad partem, quod ipsa copulativa contineat duas contradictorias. Est etiam multo magis per accidens, quod dum argumentamur a parte disiunctivae ad totam, illa pars disiunctivae fuerit deducta ex una copulativa quae constabat ex duabus contradictoriis. Ac pro indi quando in ultima consequentia repente emergit destructio alterius partis disiunctivae, illa destructio non consecuta est formaliter per consequentiam formalem inquantum destructio est partis disiunctivae, sed materialiter contingit, quod in prima copulativa altera pars alterius fuerit destructiva. Quo circa hic discursus erit similis formae cum illo 'Petrus currit et Paulus non disputat, ergo Petrus currit'. Rursus 'Petrus currit, ergo Petrus currit vel equus est leo'.* (...) *Quod autem iste secundus discursus sit similis formae cum priore, probatur, quia prima hypothetica copulativa est similis formae cum copulativa secundi discursus, utraque enim continet duas partes eiusdem quantitatis et qualitatis. Neque in illis est differentia aliqua in proprietate logicali.* (...) *Fatemur tamen siquis concederet duas contradictorias,* (...) *conuinceretur non ex formalitate consequentiarum, sed ex materiali concessione duarum contradictoriarum in illis terminis, ubi copulativa continet duas contradictorias.*" For a detailed analysis of his position, see Ors (1998).

[22] A comparative analysis of the two texts was offered in Ors (1983).

[23] Soto (1554), 84rb: "*Tamen re vera mihi saltem non concludit. Quia ad hoc quod valeat consequentia de primo ad ultimum, necesse est quod in consequentiis intermediis* (*ut supra adnotavimus*) *id solum quod est consequens prioris consequentiae assumatur ut antecedens posterioris. Modo in illa consequentia a disiunctiva cum destructione unius partis ad positionem alterius, scilicet 'Petrus est vel homo est lapis et Petrus non est, ergo homo est lapis', assumitur illa 'Petrus non est', quae non fuerat pars consequentis praecedentis consequentie, sed fuerat consequens in alia consequentia distincta et disparata, nec illa fuerat aliquando concessa tanquam vera et tamen quando assumitur ad destruendum alteram partem disiunctivae accipitur tamquam vera.*"

[reg7] only applies to linearly concatenable proofs, whereas [STEP 1] rests on "branching together". However, this objection, even if acceptable, only solves the problem if [reg7] is the only rule enabling compound proofs, so that a proof would only be acceptable if it were a concatenation of steps enabled by single applications of elementary rules. And the concept of "being elementary" would then need to prevent the following restatement of the proof:

$$\frac{(\alpha, \sim\alpha) \vdash (\sim\alpha \wedge (\alpha \vee \beta))\,[\text{cond}] \qquad (\sim\alpha, (\alpha \vee \beta)) \vdash \beta\,[\text{la2}]}{(\alpha, \sim\alpha) \vdash \beta}[\text{reg7}]$$

where [cond] in a strong form is:

$$\frac{\alpha \vdash \beta \qquad \gamma \vdash \delta}{\alpha, \gamma \vdash \beta \wedge \delta}[\text{cond}]$$

and its weaker but for the present purposes sufficient form is:[24]

$$\frac{\alpha \vdash \beta \qquad \alpha \vdash \gamma}{\alpha \vdash \beta \wedge \gamma}[\text{cond}'].$$

[cond] would enable non-elementary rules and would reduce [STEP 1] to a form which would enable [STEP 2] via [reg7]. More generally yet, the objection brings up a question of what structural features of a proof are admissible. The crucial structural features of [STEP 1] and [STEP 2] are multiple use of a hypothesis and branching together of proof-trees. To reject the latter is to restrict the realm of admissible proofs to simple concatenations. To reject the former is to restrict admissible hypotheses, which brings us back to Soto's operational restrictions.

Second, Soto closes his 1529 analysis of [ecq] by rejecting the following principle:[25]

$$\frac{\alpha \vdash \beta \qquad \beta, \gamma \vdash \delta}{\alpha, \gamma \vdash \delta}[\text{cut} *]$$

which is an instance of "cut":[26]

$$\frac{\alpha \vdash \beta, \gamma \qquad \gamma, \delta \vdash \varepsilon}{\alpha, \delta \vdash \beta, \varepsilon}[\text{cut}].$$

When applied to the proof of [ecq], Soto rejects the following inference:[27]

$$\frac{\alpha \vdash \alpha, \beta \qquad (\alpha \vee \beta), \sim\alpha \vdash \beta}{\alpha, \sim\alpha \vdash \beta}[\text{cut} \ **].$$

[24] This could be deduced from [cond] by substitution and the idempotence of conjunction (or Gentzen's "contraction in the antecedent" (Gentzen, 1935, 192)).

[25] Soto (1529), 74va. Note that, on the present representation, [reg8a] instantiates the same principle.

[26] Gentzen (1935), 192.

[27] Soto (1529), 74va: "*Hinc sequitur quod licet aliqua consequentia sit bona et consequens cum alia premissa vel aliis premissis inferat aliud consequens, non tamen antecedens cum eadem premissa infert idem consequens* (...). *V.g. licet bene sequatur 'Petrus disputat, ergo Petrus disputat vel homo est lapis' et ex hoc consequenti cum hac premissa 'Petrus non disputat' sequatur hoc consequens 'homo est lapis', cum ille contradictorie accipiuntur absolute, tamen ex primo antecedenti, scilicet 'Petrus disputat' cum eadem premissa 'Petrus non disputat' non sequitur idem consequens, scilicet 'homo est lapis'.*"

That is a very interesting remark, since it suggests a different form of proving [ecq], one which cannot be dealt with in the abovementioned terms. Also, note that while [cut*] in its generality is rejected, [reg7], which can be viewed as yet another instance of [cut], is not.

These remarks result in a formulation of a relevance (or perhaps paraconsistent) or substructural logic, depending on the particular strategy. However, such a system combined with the principles introduced above turns out to be incoherent: one cannot let the same set of principles validate the theorems "one" to "eight" and then prevent them from validating the theorems "nine" and "ten" and "*ex contradictione quodlibet*", i.e., the paradoxes of implication, as well. Or, to put the same thing differently, once one assumes an unrestricted form of [red], some form of these paradoxes cannot be avoided.[28] In other words, $\mathfrak{B}$ has two options: to save the criticism of the paradoxes of implication and review all remaining proofs and possibly revise the principles, or to save the principles and accept the paradoxes of implication.

2.2. Sentential logic. Sentential logic in $\mathfrak{B}$ reduces to the truth-conditions of compound sentences.[29] The truth of a conjunction is defined as the truth of both subformulas[30] and the truth of a disjunction as the truth of either subformula.[31] More importantly, the truth of a conditional is regarded as equivalent to deductive validity by definition,[32] resulting

[28] A proof of that could be outlined as follows (note that the proof also requires a double negation and $\mathrm{M}(\perp) \vdash \perp$, a principle that has not been discussed):

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\alpha, \sim \alpha \nvdash \beta}{\mathrm{M}\,(\alpha \wedge \sim \alpha \wedge \sim \beta)}\,[\text{fa2a } **, \text{reg8b}]}{\mathrm{M}(\perp \wedge \sim \beta)}\,[\text{reg4, lk1}]}{\cfrac{\mathrm{M}(\perp)}{\perp}\,[\text{immediately}]}\,[\text{tautology}]}{(\alpha, \sim \alpha \nvdash \beta) \vdash \perp}\,[\text{reg7 (iterared)}].}{\alpha, \sim \alpha \vdash \beta}\,[\text{red}]$$

Now, the rules applied in this proof are all assumed to be provable via [red] in the context of $\mathfrak{B}$.

[29] Soto (1529), 74ra, 78ra, 83vb; Báñez (1599), 133–134; Poinsot (1638), 45–47; Comas del Brugar (1661), 433–434. In addition to the operators presented here, the so-called "rational" and "causal" conditionals are introduced in these texts.

[30] Cf. Soto (1529), 78ra: "*Ad veritatem copulative affirmative significantis iuxta significationem suarum partium sufficit et requiritur quamlibet eius partem principalem esse veram....*" This position was subsequently simplified to: "*...ad veritatem propositionis copulativae necesse est utramque partem esse veram et consequenter ad falsitatem sufficit unam partem esse falsam*" in Báñez (1599), 111, and similarly Poinsot (1638), 46 and Comas del Brugar (1661), 431–432. That translates into:

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta}\,[\text{tk1}]$$

$$\frac{\alpha \wedge \beta}{\alpha \quad \beta}\,[\text{tk2}].$$

[31] Cf. Soto (1529), 82vb: "*Ad veritatem disiunctive affirmative significantis iuxta significationem suarum partium sufficit et requiritur aliquam eius partem principalem esse veram....*" This, again, was simplified to: "*Ad veritatem disiunctivae sufficit unam partem esse veram...*" in Báñez (1599), 111, and similarly Poinsot (1638), 46 and Comas del Brugar (1661), 432.

[32] The most explicit formulation is Soto (1529), 73va: "*...ad veritatem condicionalis nihil aliud requiritur quam quod sit bona consequentia ...eadem est de veritate condicionalis et de consequentie bonitate disputatio...*" (Cf. also Soto (1554), 82va). For a partial statement of this correspondence, see also Báñez (1599), 112: "*Si propositio hypothetica fuerit condicionalis, nihil aliud requirit ad sui veritatem quam bonitatem consequentiae*" and Poinsot (1638), 45:

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

into:

$$\frac{\alpha \vdash \beta}{\alpha \Rightarrow \beta}[\text{ci}]$$

and

$$\frac{\alpha \Rightarrow \beta}{\alpha \vdash \beta}[\text{ce}].$$

Other principles of sentential logic are:[33]

$$\frac{\alpha \Rightarrow \beta \quad \alpha}{\beta}[\text{lc1}]$$

$$\frac{\alpha \Rightarrow \beta \quad \sim \beta}{\sim \alpha}[\text{lc2}]$$

$$\frac{\alpha \Rightarrow \beta}{\sim \alpha \vee \beta}[\text{lc3}]$$

$$\frac{\alpha \Rightarrow \beta}{\sim \beta \Rightarrow \sim \alpha}[\text{lc4}].$$

Via [ce] and [ci], these can be reformulated for the theory of validity or vice versa, thus stating, for instance, that valid inferences are detachable or that implication is transitive.

The rules governing other operators include standard introduction- and elimination-rules, de Morgan's equivalences, and the rule relating conjunctions to disjunctions:[34]

$$\frac{\alpha \wedge \beta}{\alpha}[\text{lk1}]$$

"*Ad veritatem conditionalis... sufficit bonitas consequentiae et ad falsitatem sufficit quod consequentia sit mala.*" Note that this is not a deduction theorem in the modern sense, since: a) it is a semantic principle (relating truth and validity, rather than assertibility and provability), b) it regards "strict" rather than material implication, and c) it is a matter of definition rather than a theorem to be proved.

[33] The most complete list of rules is discussed in Soto (1529), 74ra: "*A tota condicionali cum positione antecedentis ad positionem consequentis est formalis consequentia* (...) *A tota condicionali cum destructione consequentis ad destructionem antecedentis est formalis consequentia.* (...) *A condicionali ad disiunctivam compositam ex consequenti et contradictorio antecedentis consequentia est bona* (...) *A condicionali ad alteram condicionalem cuius antecedens est contradictorium consequentis prioris et consequens contradictorium antecedentis formaliter sequitur.*" Cf. Soto (1554), 83rb–va. Poinsot omits [lc4] (1638, 46) and Comas del Brugar omits [lc3] and [lc4] (1661, 433), while Báñez seems to disregard these rules entirely (1599, 133–134).

[34] The most complete list of rules is discussed in Soto (1529), 78rb and 83rb [the order of quotation follows the order in the present reconstruction, rather than the original order]): "*A copulativa affirmativa ad quamlibet eius partem est formale argumentum* (...) *A copulativa affirmativa ad disiunctivam ex eisdem partibus valere argumentum* (...) *A parte disiunctive ad totam est formale argumentum.* (...) *A tota disiunctiva cum destructione unius partis ad positionem alterius formaliter sequitur.* (...) *sequitur... copulativam affirmativam equipollere disiunctive negative ex partibus contradicentibus, itidem et disiunctiva affirmativa copulative negative.*" Cf. also Poinsot (1638), 46–47 and Comas del Brugar (1661), 433–434 (containing [lk1], [la1] and [la2]) and Báñez (1559), 133 (containing [lk1] and [la1], some other principles being discussed in the context of [ecq]).

$$\frac{\alpha \wedge \beta}{\alpha \vee \beta}[\text{lk2}]$$

$$\frac{\alpha}{\alpha \vee \beta}[\text{la1}]$$

$$\frac{\alpha \vee \beta \qquad \sim \alpha}{\beta}[\text{la2}]$$

$$\frac{\alpha \wedge \beta}{\sim (\sim \alpha \vee \sim \beta)}[\text{la3a}]$$

$$\frac{\sim \alpha \vee \sim \beta}{\sim (\alpha \wedge \beta)}[\text{la3b}].$$

On the operational solution to [ecq], some of these may be restricted to consistent contexts in the same way [la2] was in its tentative replacement by [la2*].

2.3. Syllogistic. The syllogistic of $\mathfrak{B}$ splits into two independent theories, the logic of "every S is P", "some S is P", "no S is P" and "some S is not P" (where "S" and "P" are general terms), henceforth abbreviated as, "SaP", "SiP", "SeP", "SoP", and the logic of "this S is (not) P", henceforth "s_naP" and "s_neP",[35] called "expository syllogistic". $\mathfrak{B}$ is primarily concerned with the following types of inferences:

$$\frac{\mathrm{M}x\mathrm{P} \qquad \mathrm{S}y\mathrm{M}}{X}[\text{fig1}]$$

$$\frac{\mathrm{P}x\mathrm{M} \qquad \mathrm{S}y\mathrm{M}}{X}[\text{fig2}]$$

$$\frac{\mathrm{M}x\mathrm{P} \qquad \mathrm{M}y\mathrm{S}}{X}[\text{fig3}].$$

These basic configurations of premises are called "figures" and the results of replacing "x" and "y" by "a", "i", "e", or "o" are called "modes".[36]

To say what the axioms in the syllogistic of $\mathfrak{B}$ are is problematic, because syllogistic is a traditional discipline and the conservative stance requires preserving all its parts, even if elaborating on one of them makes the other superfluous. As a result, syllogistic is organized on three different levels. First, the theory of "reduction" relates two groups of syllogisms to one another as axioms and theorems. Second, the auxiliary rules such as "nothing follows from two negative premises" are followed by correct syllogisms; these are in an auxiliary role since they are ultimately reduced to the axioms or "regulative principles": "*dici de omni*", "*dici de nullo*", and, most importantly, the formal properties of identity. Third, syllogisms are direct or indirect instances of the "regulative principles", and the indirect instances reduce to the direct ones. In terms of the axioms–inference rules–theorems view, these translate to three different systems:[37]

[35] Due to the so-called "existential import", a modern formalisation of the affirmative sentences is:
(SaP) $\exists x(\mathrm{S}(x)) \wedge \forall x(\sim \mathrm{S}(x) \vee \mathrm{P}(x))$
(SiP) $\exists x(\mathrm{S}(x) \wedge \mathrm{P}(x))$
(SoP) is a negation of (SaP) and (SeP) is a negation of (SiP).

[36] Since the figure is determined by the location of terms in the premises, the so-called "fourth figure" reduces to the first one in $\mathfrak{B}$.

[37] Rules such as "nothing follows from two negative premises" can be disregarded at this point.

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

	Axioms	**Inference rules**	**Theorems**
System 1	Perfect modes	Reductions	Imperfect modes
System 2	Regulative principles	Substitution	Perfect and imperfect modes
System 3	Regulative principles	Substitution (for the perfect modes) and reductions (for the imperfect modes)	Perfect and imperfect modes

While "2" seems to be a good representation of certain aspects of $\mathfrak{H}$, "1" and "3" are good approximations of different formulations of $\mathfrak{B}$.[38] The present reconstruction will emphasize the first reading: "reduction" will be construed as an inference relating axioms to theorems and "regulation" as part of the correctness proof. On this view, the axioms of $\mathfrak{B}$'s syllogistic are:

$$\frac{\text{MaP} \quad \text{SaM}}{\text{SaP}}[\text{sa1}]$$

$$\frac{\text{MeP} \quad \text{SaM}}{\text{SeP}}[\text{sa2}]$$

$$\frac{\text{MaP} \quad \text{SiM}}{\text{SiP}}[\text{sa3}]$$

$$\frac{\text{MeP} \quad \text{SiM}}{\text{SoP}}[\text{sa4}].$$

These correspond to the modes traditionally called "*Barbara*", "*Celarent*", "*Darii*", and "*Ferio*".

Three "regulative principles" are introduced in $\mathfrak{B}$: "*dici de omni*", "*dici de nullo*", and (for expository syllogisms) the Euclidean and transitive properties of identity. "*Dici de omni*" and "*dici de nullo*", i.e., "whatever is predicated universally about a subject-term can be predicated about everything that is subsumed under it" and "whatever is denied universally of a subject-term must be denied of everything that is subsumed under it",[39] correspond to elementary forms of instantiation:

$$\frac{\text{SaP}}{\text{s}_x\text{aP}}[\text{ddo}]$$

$$\frac{\text{SeP}}{\text{s}_x\text{eP}}[\text{ddn}].$$

These principles, assumed to capture "the nature of universal statements", are used to prove the correctness of axioms. Both are related to the theory of "*descensus*", a part of the

[38] The most explicit formulation of the first reading is Báñez (1599), 242 (saying that "*tota machina*" of the imperfect modes is reducible to the perfect ones) or Poinsot's claim that "*dici de omni*" and "*dici de nullo*" do not govern the imperfect modes (Poinsot, 1638, 55 and 58). The second reading is supported by the claim that the regulative principles govern all syllogisms either "directly" or "indirectly" (for instance, Poinsot (1638), 65).

[39] "*... quidquid universaliter dicitur de subiecto, dicitur de omni quod sub tali subiecto continetur* (...) *quidquid universaliter negatur de aliquo subiecto, negatur et de omni contento sub tali subiecto...*" Poinsot (1638), 64. Cf. Báñez (1599), 231–232 and Comas del Brugar (1661), 480–483.

semantics of *suppositio* (i.e., "standing for" an object) concerned with the way general terms can be replaced by singular terms,[40] which relates the semantic underpinning of syllogistic to a particular strategy of the logical analysis of language.[41]

Expository syllogisms are governed by the Euclidean and transitive properties of identity, in words "things which are identical to the same thing are also mutually identical" (i.e., the left-Euclidean property)[42] and "two things are different from one another if one of them is identical to a third thing, which the other thing is different from"[43] or "for two things which are mutually identical, if one of them is different from a third thing, so is the other one":[44]

$$\frac{x = z \qquad y = z}{x = y}\text{[ase1]}$$

$$\frac{x = z \qquad y \neq z}{x \neq y}\text{[ase2a]}$$

$$\frac{x = y \qquad x \neq z}{y \neq z}\text{[ase2b]}.$$

[ase2b] is equivalent to (the transitive property):

$$\frac{x = y \qquad y = z}{x = z}\text{[trans]}$$

via:

$$\frac{\alpha, \beta \vdash \gamma}{\alpha, \sim \gamma \vdash \sim \beta}$$

and its converse.[45] The exact order of the individual constants does not seem to play a crucial role here. The paradigmatic application of these principles is claimed to be third-figure syllogisms, where the subject-term in the premises is singular (the right-Euclidean property):

$$\frac{x = z \qquad x = y}{y = z}\text{[ase1 }*\text{]}.$$

[40] Báñez (1599), 68–69; Poinsot (1638), 24; Comas del Brugar (1661), 309.

[41] For an overview of the medieval versions of the theory of supposition, see Read (2015), for post-medieval developments, see Ashworth (1974).

[42] "*Quaecunque sunt eadem uni tertio sunt eadem inter se*." Báñez (1599), 266–267. Cf. Poinsot (1638), 64; Comas del Brugar (1661), 482.

[43] "*…quacunque quorum unum est idem uni tertio cui alterum est diversum sunt diversa inter se*." Báñez (1599), 267.

[44] "*…quae enim sunt eadem inter se, si unum eorum distinguitur ab aliquo tertio, etiam et aliud ab illo tertio distinguitur et separatur*." Poinsot (1638), 64. Cf. Comas del Brugar (1661), 482.

[45] The proof of this principle proposed by Buridan (1976, 87) utilizes [reg8b] (and tacitly other principles as well), and a simple proof of the converse is:

$$\cfrac{\cfrac{\sim \gamma, \alpha \vdash \sim \beta}{\sim \gamma, \alpha, \beta \vdash \bot}\text{[fa2b}*\text{]}}{\alpha, \beta \vdash \gamma}\text{[fa2b]}$$

which can be reversed. The same principle is accepted by Soto (1529, 120ra) and related to [reg8b]: "*Est enim omni syllogismo cuiuscumque figure, immo omni consequentie, commune, ut ex aliqua premissarum et contradictorio conclusionis sequatur contradictorium alterius premisse* (…) *per illam regulam*: *In omni bona consequentia ex opposito consequentis sequitur oppositum antecedentis.…*"

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

If [trans] were viewed as a primary property of identity, the Euclidean properties could be inferred from [trans], assuming that identity is symmetric. Also, since the statements discussed in this context contain general terms, the interpretation of identity is not entirely straightforward, but no closer explication is offered.

The inference rules of syllogistic or "reductions" are the "ostensive reduction" and the "reduction to the impossible".

The ostensive reduction consists in reducing a syllogism to a perfect one by transposing the premises and replacing the premises or conclusions with an equivalent or weaker statement. In $\mathfrak{B}$, the underpinning of this procedure is [reg7]. To give an elementary example, the second-figure mode *Cesare* "reduces" to [sa2] (i.e., to *Celarent*):

$$\frac{((\mathrm{PeM}, \mathrm{SaM}) \vdash (\mathrm{MeP} \wedge \mathrm{SaM}))[1] \quad (\mathrm{MeP}, \mathrm{SaM} \vdash \mathrm{SeP})[\mathrm{sa2}]}{\mathrm{PeM}, \mathrm{SaM} \vdash \mathrm{SeP}}[\mathrm{reg7}]$$

where [1] is justified by the equivalence of "PeM" and "MeP".[46] If both [1] and [sa2] are acceptable, so is *Cesare*.

The reduction to the impossible is claimed to be based on [reg8b], but the description and the examples are suggestive of a slightly different approach. The second-figure mode *Baroco* will serve as an example. Báñez starts by applying what seems to be a premise of [reg8b], namely "if *Baroco* is invalid, *Barbara* is invalid":[47]

$$\frac{\mathrm{PaM}, \mathrm{SoM} \nvdash \mathrm{SoP}}{\mathrm{MaP}, \mathrm{SaM} \nvdash \mathrm{SaP}}[\mathrm{step1}].$$

However, the actual argument goes as follows: if *Baroco* is invalid, its premises can be true and its conclusion false, hence PaM, SoM, and the negation of SoP, i.e., SaP, can hold. Then from SaP and PaM we get SaM (by [sa1], i.e., by *Barbara*). But SaM is inconsistent with SoM, hence the initial assumption implies a contradiction. As a result, either *Baroco* is valid or a contradiction is true, which is the end of the argument. The denial of [sa1] mentioned at its beginning is not mentioned again, but that could be fixed: before the application of [sa1] we already had PaM, SoM, SaP. But if SoM is true, SaM is false. Hence we have SaP, PaM, and the negation of SaM—a countermodel to [sa1], which proves [step1].

Now, the proof of *Baroco* via [step 1] and [reg8b*] can go as follows:

[46] Báñez (1599), 246–247: "*Cesare vero in nullo differt a Celarent, nisi in maiori, propterea quod cum sit secundae figurae habet medium pro praedicato. Unde facta conversione simplici emergit statim syllogismus in Celarent.*" Note that this step uses [cond] discussed as a possible underpinning for one of the proofs of [ecq].

[47] Báñez (1599), 245–246: "*Nam detur oppositum, Baroco esse malam consequentiam, sequitur quod Barbara est mala consequentia. Probo sequelam, nam si Baroco est mala consequentia, ergo possibile erit dari antecedens verum et consequens falsum. Sit igitur verum illud antecedens, omnis homo est animal, lapis non est animal, et consequens falsum, lapis non est homo. Tunc sic*: *ergo contradictoria consequentis, omnis lapis est homo, erit vera. Accipiam ergo illam cum maiore de Baroco et fient praemissae in Barbara, omnis homo est animal, omnis lapis est homo, plane infertur, ergo omnis lapis est animal, quae est contradictoria minoris de Baroco, quam tu concesseras veram esse, ergo convinceris concedere duas contradictorias aut recantare palimodiam et fateri bonum esse syllogismum de Baroco*." The accounts of Poinsot and Comas del Brugar are the same, except that they omit this opening step (Poinsot, 1638, 59; Comas del Brugar, 1661, 500).

$$\dfrac{\dfrac{(\text{PaM, SoM} \nvdash \text{SoP}) \vdash (\text{MaP,SaM} \nvdash \text{SaP})}{(\text{MaP,SaM} \vdash \text{SaP}) \vdash (\text{PaM,SoM} \vdash \text{SoP})}[\text{reg8b} *] \quad (\text{MaP, SaM} \vdash \text{SaP})\ [\text{sa1}]}{\text{PaM, SoM} \vdash \text{SoP}}[\text{detach}].$$

Furthermore, Báñez's proof works via [red] or via [reg8b]:

$$\frac{(\alpha \nvdash \beta) \vdash \bot}{\top \vdash (\alpha \vdash \beta)}[\text{reg8b}].$$

That is satisfactory, but the idea that a logical theorem is implied by a tautology or that [red] is an instance of [reg8b] does not seem to have textual support.

The chant "*Barbara*, *Celarent*, *Darii*, *Ferio*, ..." summarizes this axiomatic system by representing the type of premises and conclusions by the vocals (a,e,i,o) and the goal (B,C,D,F) and method of reduction (p,s,m,c) by the consonants.[48]

$\mathfrak{B}$ includes, in a specific sense, the proofs of correctness and of completeness for syllogistic.[49]

Correctness is proved for "perfect syllogisms" via [ddo] and [ddn] (assumed to be self-evident). The general reduction-rules reduce to [reg7] and [reg8a], and are supposed to ultimately reduce to [fa2a] (as the authors claim and try to prove), hence are truth-preserving (or, in this case, validity-preserving). Their particular applications presuppose principles governing equivalences between elementary forms of syllogistic premises and conclusions (the so-called "conversions"); let us for now assume their correctness as well.[50] All the axioms, inference rules and auxiliary principles being correct, the theorems are correct as well.

Since syllogistic is limited, its completeness can be proved mechanically by testing all the possible combinations of premises and conclusions, which would be lengthy but manageable. As opposed to modern approaches, $\mathfrak{B}$ only focuses on which combinations of *premises* have *a* syllogistic implication, rather than which combinations of premises *and conclusions* constitute a valid inference. For instance, both *Barbara* and *Barbari* are valid first-figure syllogisms, yet only the first one is introduced. Also, with the exception of the first figure, syllogisms obtained by reversing the order of terms in conclusions are disregarded. The technique of carrying out such a limited completeness proof is eliminating the "useless" (*inutilis*) combinations of premises by auxiliary principles such as "nothing follows from two negative premises".[51]

§3. The system $\mathfrak{H}$. $\mathfrak{H}$ was endorsed around mid-seventeenth century by Pedro Hurtado de Mendoza (who possibly invented it and presents its purest form), Rodrigo de Arriaga, Francisco de Oviedo, and Thomas Compton Carleton (who follow Hurtado to a significant degree). It is structurally different from $\mathfrak{B}$ even on the textual level: the classification of

[48] For example, the combination of vocals a-o-o in "*Baroco*" (together with the tacit knowledge concerning the figure) indicates that the syllogism has the form "PaM,SoM⊢SoP", "B" indicates that it reduces to *Barbara* (note the same initial letters), and "c" the reduction method (in this case, the reduction *per impossibile*). Cf. Poinsot (1638), 60–62.

[49] Consistency, which is not discussed, is implied by correctness.

[50] For instance, the above example presupposed the equivalence of "PeM" and "MeP", or so-called "e-conversion". These equivalences would require a separate debate pertaining to the logical analysis of language.

[51] "*Ex puris negativis nihil sequitur*." Báñez (1599), 239–241; Poinsot (1638), 57–58; Comas del Brugar (1661), 485–488.

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

"reasoning" (including syllogisms, enthymemes, inductions, and dilemmas) is followed by syllogistic, and finally by a general theory of inference. The last part, which plays the role of general assumptions for all sub-theories of 𝔅, only plays a secondary role of the "corollaries" of syllogistic ℌ. Also, even though "hypothetical syllogisms" (corresponding to sentential logic) are occasionally mentioned, their logic is not presented.[52]

3.1. Categorical syllogistic. ℌ's syllogistic contains all the traditional elements, but their organisation differs from ℌ in an interesting way: 1) The primary principles of ℌ are the transitive and Euclidean properties of identity ([ddo] and [ddn] reduce to them). 2) Every valid syllogism instantiates the primary principles immediately (the correctness proof is uniform for all figures). 3) Therefore, the distinction between perfect and imperfect modes is purely epistemic:[53] the transparent correctness of perfect syllogisms, rather than the reducibility of imperfect syllogisms to them, makes them into a paradigm of syllogistic validity.[54] 4) Reductions have no part in ℌ's axiomatic,[55] even though chapters on reductions are part of the textbooks in question.[56] The role of the semantic underpinning of syllogistic, played by the theories of *suppositio* in 𝔅,[57] is played by the semantics of "objects" in ℌ. The elementary statements are construed as denoting structured entities with the form x-is-"identical"-to-y, henceforth "propositions".[58] The logic of ℌ has four levels: 1) algebra of propositions, 2) logic of identity, 3) syllogistic, 4) "corollaries" (meta-syllogistic). The algebra of propositions validates the logic of identity, which in turn validates the syllogistic. The algebra of propositions is the semantics of the logic of identity and valid syllogistic inferences are validated by the logic of identity.[59] Finally, the structural and value-preserving properties of syllogisms are captured by the corollaries.

---

52 Since Soto's logic was known to Hurtado, Arriaga, and Oviedo (and referenced by them on other occasions, cf. Hurtado (1619a), 1, Arriaga (1639), 7, and Oviedo (1640), 9), the departure from 𝔅 is a matter of decision rather than ignorance.

53 The epistemic primacy of the first figure (making them the "primary principles of logic") must not be conflated with their status of implications of the formal properties of identity ("metaphysical principles"). Also, the correctness proofs in ℌ do not rest on reduction to first-figure syllogisms.

54 Hurtado (1619a), 180, 190–193.

55 Arriaga goes as far as to say that the theory of reduction is "not quite necessary and rarely useful" and backs his dismissive view by twenty years of academic experience (Arriaga, 1639, 1 and 30). Note that reductions could not be deemed as "not quite necessary" in a system where the "imperfect syllogisms" are theorems derived from the "perfect syllogisms" as axioms via reductions: imagine a modern author of an axiomatic system claiming that *modus ponens* is neither necessary nor useful. However, the justification of theorems takes a different form in ℌ, most notably in the case of Hurtado whose account is the most detailed. While Hurtado on rare occasions mentions a reducibility of a syllogistic mode to another, his justification of that particular mode never relies on that (cf. Hurtado (1619a), 186 and 187).

56 Hurtado (1619a), 180, 190–193. Hurtado (1619a), 190–194, Arriaga (1639), 30–32, Oviedo (1640), 38–40.

57 Poinsot (1638), 153–155.

58 Hurtado introduced the concept of proposition (*obiectum*) instructively by saying that "*in hac propositione 'Petrus currit' obiectum sunt tam Petrus quam cursus et utriusque unio*" (Hurtado, 1624, 693), but the distinction between "formal" and "objective" premises, conclusions, or inferences (as linguistic and extralinguistic entities, respectively) plays a major part in ℌ's philosophy of logic for all authors (cf. Arriaga (1639), 188 and Oviedo (1640), 4).

59 In the case of syllogisms only containing singular terms, syllogistic is a part of the logic of identity, i.e., level 3 immediately reduces to level 2, since syllogistic validity is a direct implication of identity being transitive and left- and right-Euclidean.

While its structure is clear as a project, $\mathfrak{H}$ remains in an outline for the most part. While it is possible to document all four levels as well as some of the connections between these levels, reconstructing $\mathfrak{H}$ is a speculative, rather than interpretative task. To some extent, the principles of the first level can be drawn directly from the original corpus, while others can be reasonably assumed. As in the case of $\mathfrak{B}$, a part of the analysis of $\mathfrak{H}$ will be asking questions not addressed in the original sources, with the motivation of showing how such a theory may work. As for the second level, there is solid textual evidence for the logic of identity, even though it must be gathered from different parts of the original corpus, and there is at least some textual evidence for its connection to the first level. There is, obviously, solid evidence for syllogistic in $\mathfrak{H}$ (not all of which will be reproduced), as well as its connections to the second level (which is trivial in case of syllogisms only containing singular terms). The fourth level will be addressed in §3.3.

The algebraic level of $\mathfrak{H}$ is the semantics of identity-statements, where "identity" is an umbrella concept for three different set-theoretical relations:[60] a) the "numerical" identity between two individuals, b) instantiation or membership between an individual and a set or general property, and c) the inclusion relation between two general properties or sets.[61] To assume that these are the same relation is to assume that their bearers are the same type of entities. In modern semantics, a uniform view could reduce these relations to inclusion by reducing individuals to unit sets. In the absence of a set-theoretical language and possibly with a nominalist ontological agenda in mind, $\mathfrak{H}$ goes the other way round and reduces general properties to as-if individuals.[62]

The core principles of the algebra of propositions in $\mathfrak{H}$ can be drawn from (and textually backed by) two observations introduced by Hurtado. First, the propositions denoted by premises are *identical* to the proposition denoted by their conclusion. As Hurtado puts it, the premises of "every M is P, this S is M, therefore this S is P" *contain* the "unity" between this S and M and between M and P, and those unities (combined) are identical to the unity between this S and P.[63] In what seems to be an extension-attempt, the

[60] In the original texts, the same term used to describe all these cases.

[61] For instance, Hurtado analyses both "every rational entity is a human being" and "Peter is a human being" in terms of identity (Hurtado, 1619a, 181). In a different context, he introduces "particular" and "universal" identity (Hurtado, 1619a, 186), which could have (but have not) been utilized to account for membership and inclusion.

[62] This applies primarily to Hurtado's view of logical universals as collections of individuals conceived "as if it were one thing" (Hurtado, 1619a, 94), and of natural kinds as "nothing but" their instances conceived in a non-discriminate way, thereby forming a totality (Hurtado, 1619a, 28). This view is based on construing the statements about "all instances", e.g., "*omnes homines*", as analogous to statements such as "All apostles are twelve". Since the term "twelve" can only apply to the totality of apostles, the universals should be viewed as totalities. However, note that Hurtado introduces the concept of a set which is a hybrid of sets in extension and in intension. The ontological justification of this view may not be shared by other proponents of $\mathfrak{H}$, but the unified approach to different forms of "identity" in syllogistic is. (On Hurtado's ontological and epistemological analysis of universals, see Heider (2014), 87–94.)

[63] "*Confirmatur hoc syllogismo 'homo est risibilis, Petrus est homo, ergo Petrus est risibilis.' Ecce Petrus est subjectum minoris et risibile praedicatum majoris, unio autem Petri cum risibili contineatur in utraque praemissa. Quod sic probatur, quia in utraque continetur unio Petri cum homine et hominis cum risibili, sed haec unio est eadem cum unione Petri et risibilis, ergo unio conclusionis continetur in utraque praemissa.*" (Hurtado, 1619a, 209–210). Hurtado's justification, in a nutshell, is based on an ontology of relations: the unity between M and P is identical to the unity between this S and M, for otherwise that would not grant the unity between this S and P, which, as the argument suggests, would undermine the formal properties of identity

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

subsequent debate splits this relation into "identity" and "inclusion" to distinguish between expository syllogisms and syllogisms containing general terms.[64] Second, a proposition and its converse, i.e., a-is-(not)-b and b-is-(not)-a, are identical.[65] As a result of these two observations, propositions can be composed and identified. As a third observation we can add that if "positive" propositions (identities) are regarded as elementary, propositions can also be negated.[66] Based on these observations, the syllogistic of $\mathfrak{H}$ will be reconstructed on the propositional level and on the sentential level, the former being primary. As a simplification, the theory will be introduced for syllogisms composed entirely of singular terms, where identity gets a straightforward "numerical" interpretation.[67]

On the propositional level, the syllogistic of $\mathfrak{H}$ is an algebra of propositions in the sense of a theory of a certain kind of algebraic structures. For a set T of terms $\{t_1, t_2, t_3, \ldots\}$ representing individuals, the syllogistic structure $\mathbb{H}$ is a set Π of propositions or ordered pairs of terms, represented by bold characters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ ..., whenever their internal structure is not relevant, and by ordered pairs such as "$<x, y>$" whenever it is, with the operations of negation "$-$" and composition "$\otimes$" defined on Π, which is closed under these operations and satisfies the subsequent principles. In accordance with the textual evidence, these principles include stronger "identities" ("$=$") and weaker "inclusions" ("$\subseteq$"),[68] but only the former plays a part in the algebra of syllogisms composed of singular terms.

The fundamental principles of this algebra are the composition principles (elaborating on Hurtado's first observation), the conversion principles (elaborating on Hurtado's second observation), the negation principle (which is a propositional underpinning of "double negation"):

[comp_p] $(<x,y>\otimes<y,z>) = <x,z>$;
[$\text{comp}_{\text{n}1}$] $(-<x,y>\otimes<y,z>) = (-<x,z>)$;
[$\text{comp}_{\text{n}2}$] $(<x,y>\otimes-<y,z>) = (-<x,z>)$, where x, y, and z are different terms.
[conv_p] $<x,y> = <y,x>$
[conv_n] $(-<x,y>) = (-<y,x>)$
[neg] $\boldsymbol{\alpha} = (-(-\boldsymbol{\alpha}))$.

A modern reconstruction could, for convenience's sake, adopt the substitution principle:

[subst] If $x=y$, then "x" and "y" are intersubstituable in all identities and inclusions.

The composition principles are fundamental in two respects. First, they ultimately translate into transitive and Euclidean properties of identity (the second level of $\mathfrak{H}$). Second, they constrain other principles by setting the composability-conditions: $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are composable

(Hurtado, 1619a, 209–210). His followers seem to have utilized this principle without subscribing to this justification.

[64] The primary context is the relation between "*obiectum conclusionis*" and "*obiectum praemissarum*", described indiscriminately as identity or inclusion by Hurtado (Hurtado, 1619a, 209–210), whereas Compton relates identity to expository syllogistic and inclusion to other branches of syllogistic (Compton, 1649, 183).

[65] Hurtado (1619a), 46: "*... negata identitate hominis cum albo, negatur identitas albi cum homine, quia est eadem.*" The context is the semantics of "*conversio*", i.e., of the relations between (e.g.,) "SeP" and "PeS". Clearly, the situation is more complicated outside the context of expository syllogisms.

[66] It is not certain whether the present authors would agree that "affirmations" are in some sense "more elementary" than "negations". Let us assume that for the sake of convenience.

[67] For examples of syllogisms composed entirely of singular terms, see Hurtado (1619a), 177 and 179.

[68] Formally, inclusions are merely transitive and reflexive, whereas identities are reflexive, transitive, and symmetric.

if at least one of them is positive (i.e., a result of even applications of negation to some elementary $<x,y>$) and if they share precisely one term. On this setting, the algebra of propositions translates into a syllogistic, in which inferences have two premises containing three different terms and sharing one of them. Via the principles of conversion and substitution, the composition principles can be rearranged to fit any syllogistic figure.

Second, the algebraic properties of composition and negation translate into the formal properties of identity and the logic of identity-statements. While the idea of such "translation" is supported by textual evidence,[69] its technical details must be constructed in a speculative manner. A possible form of an algorithm, that would save the intended results, is as follows:

Step 0. Reduce all propositions to their most elementary form via [neg].

Step 1. Decompose identities such as "$(\alpha \otimes \beta) = \gamma$" into pairs of inclusions such as "$(\alpha \otimes \beta) \subseteq \gamma$" and "$\gamma \subseteq (\alpha \otimes \beta)$".

Step 2. Replace "$\alpha \otimes \beta$" with "$|\alpha| \bullet |\beta|$" (where "$|\lambda|$" represents a sentence denoting the proposition λ) and replace "$\subseteq$" with "$\vdash$" (or, accordingly, with a horizontal stroke).

Step 3. Replace "$|\alpha| \bullet |\beta|$" in the antecedent by "$|\alpha|, |\beta|$" and in the consequent by "$|\alpha| \Rightarrow |\beta|$" or "$|\beta| \Rightarrow |\alpha|$".

Step 4. (If required) replace "$|<x,y>|$" by "$x = y$" and "$|-<x,y>|$" by "$x \neq y$".

The result of the first two steps is (e.g.,):

$$\frac{|\alpha| \bullet |\beta|}{|\gamma|}[\text{ups}]$$

$$\frac{|\gamma|}{|\alpha| \bullet |\beta|}[\text{downs}].$$

As a result of the third step, [ups] translates into:[70]

$$\frac{|\alpha| \quad |\beta|}{|\gamma|}[\text{ups}_\text{T}]$$

[69] Such a move is documented by the following passage from Hurtado: "*Tota enim necessitas illationis innititur huic principio, quae sunt eadem uni tertio, sunt idem inter se. Ex identitate autem extremorum cum medio, quae disponitur in praemissis, infertur in conclusione identitas extremorum inter se, quae si non esset eadem realiter cum identitate extremorum cum medio, inepte ex illa deduceretur.*" Hurtado (1619a), 170. In other words, the basic principle of (a fragment of) syllogistic is the Euclidean property of identity ("*quae sunt eadem… inter se*"), which, on the sentential level, means that premises and conclusions state the existence of identities between terms ("…*identitas extremorum*…"). But the fact that such inferences are not illegitimate ("…*inepte*…") is a matter of the composition principle ("…*identitas*… *si non esset eadem realiter cum identitate*…"), to which the logic of identity ultimately reduces.

[70] $[\text{ups}_T]$ can be regarded as equivalent to:

$$\frac{|\alpha|}{|\beta| \Rightarrow |\gamma|}[\text{ups}_{\text{T1}}\text{s}*].$$

and:

$$\frac{|\beta|}{|\alpha| \Rightarrow |\gamma|}[\text{ups}_{\text{T2}}*].$$

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

and [downs] translates into:[71]

$$\frac{|\gamma|}{|\alpha| \Rightarrow |\beta|}[\mathrm{downs}_{T1}]$$

and

$$\frac{|\gamma|}{|\beta| \Rightarrow |\alpha|}[\mathrm{downs}_{T2}].$$

The asymmetrical translation of "$|\alpha|\bullet|\beta|$" is a result of the difference between identities and inferences. "$(\alpha \otimes \beta) = \gamma$" simply states that γ is a product of composing α and β. However, γ can be a product of composing other propositions as well, and the information that, for instance, $(\pi\otimes\rho)=(\sigma\otimes\tau)=\upsilon$, is meaningful and possibly useful. Therefore, while composition is a straightforward procedure, a unique decomposition requires additional information regarding its components.

By this algorithm, the composition principles translate into the axioms "things which are identical to the same thing are also mutually identical" and "two things are different from one another if one of them is identical to a third thing, which the other thing is different from". These, again, are broadly conceived, since the phrasing is the same for all figures.[72] Assuming that the first-figure syllogisms are their most straightforward instance, the direct counterpart of the composition principles is the transitivity of identity and its implication (via the indiscernibility of identicals):

$$\frac{y = x \quad z = y}{z = x}[\mathrm{trans}_1]$$

$$\frac{y \neq x \quad z = y}{z \neq x}[\mathrm{trans}_2].$$

If reformulated to fit the remaining syllogistic figures, these translate into the axioms of expository syllogistic.

Other properties of identity discussed in $\mathfrak{H}$ include the principle that no two individuals (can) share a complete collection of properties;[73] depending on the exact interpretation, this translates to either:[74]

$$\frac{\alpha \wedge \sim \alpha[x \mapsto y]}{\sim (x = y)}[\mathrm{indisc}_N]$$

[71] [downs$_{T1}$] can be regarded as equivalent to:

$$\frac{|\gamma| \quad |\alpha|}{|\beta|}[\mathrm{downs}_{T1}*].$$

[downs$_{T2}$] can be regarded as equivalent to:

$$\frac{|\gamma| \quad |\beta|}{|\alpha|}[\mathrm{downs}_{T2}*].$$

[72] See, for instance, Hurtado (1619a), 177–179.

[73] Cf. Hurtado (1619a), 116; Arriaga (1639), 153; Oviedo (1640), 124; Compton (1649), 141, quoting the Porphyrian definition of "*individuum*" as "*cuius collectio proprietatum, quae in uno sunt, in alio reperiri nequeant*" or "*cuius collectio proprietatum eadem in nullo alio est*".

[74] Only Hurtado seems to have construed this principle as a general one, applying to all possible properties, while others apply it to human individuals and list seven specific properties (such as parents, name, and homeland) which form a unique collection. Furthermore, as opposed to Hurtado and Compton, Arriaga and Oviedo use the formulation "can share", emphasising the necessity of this principle; that would correspond to replacing material implication by a strict one in [indisc$_P$].

or its converse [indisc_N*], i.e., either the discernibility of non-identicals or the non-identity of discernibles.[75] By contraposition, it is possible to obtain either the indiscernibility of identicals or the identity of indiscernibles, i.e., either:

$$\frac{x = y}{(\alpha \supset \alpha[x \mapsto y]) \wedge (\alpha[x \mapsto y] \supset \alpha)}[\text{indisc}_\text{P}]$$

or its converse. If one wanted to relate these principles of the logic of identity to an algebra of propositions (another speculative task), the elementary forms of these principles correspond to the algebraic substitution principle.

Furthermore, the theory of conversion assumes that some propositions are convertible, in accordance with [conv_p] and [conv_n], and these are held to translate into:

$$\frac{x = y}{y = x}[\text{sym}].$$

By contraposition, the same holds about non-identity.[76]

While the algebra for the syllogisms only containing singular terms is relatively straightforward, its extension to traditional syllogistic is rather problematic. First, it requires introducing inclusions apart from identities; for instance, the composition principles will be inclusions rather than identities.[77] Second, the language will have to be extended to other forms of terms and propositions, corresponding to general terms and "partial" and "universal" identity,[78] and describing their formal properties, such as convertibility.[79] Third, a distinction must be drawn between a *result* of a composition and its *implications*: the former is unique, while the latter are not. For instance, if composition produces a universal

[75] Hurtado (1619a, 116) says that no pair of non-identical individuals is such that these individuals share all their properties (*nullum individuum est alteri simile in omnium proprietatum collectione*), which corresponds to [indisc_N*] in a more straightforward way. But since, despite a somewhat complicated structure, it is a "SeP" statement, the order of its terms can be reversed, which results into [indisc_N].

[76] For textual evidence, see again the "second observation" above. Note that this principle was originally formulated for non-identity by Hurtado, such that (approximately speaking) "$y \neq x \vdash x \neq y$" was validated by [conv_n]. Assuming the principle of contraposition, it is a merely technical issue whether one starts with "$y \neq x \vdash x \neq y$" or "$x = y \vdash y = x$".

Also, these principles were primarily related to "particular" rather than "numerical" identity (or, strictly speaking, non-identity) by Hurtado (1619a, 46). In the later development, symmetry of identity was introduced by Gonzáles de Santalla in the epistemology of *Dabitis* (Knebel, 2011, 545): "*... esse tamen necessariam cognitionem directam alicuius principii universalis hic et nunc applicati ad inferendam conclusionem, nempe huius*: *'Si hoc est idem cum illo, e converso illud erit idem cum hoc'*."

[77] This is based on Compton's observation and on its implications for the way the algebra of propositions translates into syllogistic. If the underlying algebraic principles were identities rather than inclusions, they would translate into untenable instances of [downs]. For instance, if the algebraic counterpart of *Darii* were an identity, it could translate into a syllogism with two SiP premises.

[78] To make things more complicated, Hurtado remarks that whether a sentence is singular, particular, or universal depends on the "number of objects" (Hurtado, 1619a, 43, 172). The meaning is not entirely clear (and he could have meant "subjects", rather than "objects"), but if he believed that there "actually" were only singular propositions and structured collections of these, such an extension might take a different form.

[79] Corresponding to the theory of conversion, universal non-identity and partial identity are convertible, while a universal identity includes the converse partial identity. Also, universal propositions include the respective particular propositions.

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

proposition, it includes both this universal proposition and whatever it includes or is identical to. That said, Hurtado makes an attempt to justify all syllogistic inferences uniformly by taking direct recourse to the logic of identity[80] and does the same with auxiliary principles such as "nothing follows from two negative premises",[81] and even the principles "*dici de omni*" and "*dici de nullo*";[82] such justifications are typically described as "*ratio a priori*". A justification of the ostensive reduction also relies on propositional identities (the argument is, again, "*a priori*").[83] Arguably, these remarks are just the beginning of a process whose outcome is not guaranteed. $\mathfrak{H}$ came up with potentially revolutionary insights, but stopped at the intuitive level.

3.2. Further observations on $\mathfrak{H}$'s algebra. Even though such question was not addressed in the original sources, one could ask what other principles hold in $\mathfrak{H}$'s algebra and what kind of an algebraic structure the syllogistic structure $\mathbb{H}$ is.

First, it seems natural to assume that composition, if defined, is commutative:

[comm] $(\alpha \otimes \beta) = (\beta \otimes \alpha)$.

If composition is defined for α and β, changing its order can only affect the order of terms in its product, which is irrelevant assuming the conversion principles.

Second, composition is associative (if both the left- and right-hand side of the following identity is defined):[84]

[assoc] $((\alpha \otimes \beta) \otimes \gamma) = (\alpha \otimes (\beta \otimes \gamma))$.

To sketch the proof: both sides are defined only if a) at most one of the propositions is negative, b) α and β share *a* term, and either α or β shares *another* term with γ (the left-hand side), and c) β and γ share *a* term and either β or γ shares *another* term with α (the right-hand side). All the conditions are satisfied only in those cases, where [assoc] holds (which can be tested mechanically).

Third:

[add] If $\alpha = \beta$ and both $(\alpha \otimes \gamma)$ and $(\beta \otimes \gamma)$ is defined, then $(\alpha \otimes \gamma) = (\beta \otimes \gamma)$

[dis] If $\alpha = \beta$ and both $(\alpha \otimes -\gamma)$ and $(\beta \otimes -\gamma)$ is defined, then $(\alpha \otimes -\gamma) = (\beta \otimes -\gamma)$.

Fourth, assuming that all compositions are defined:

[fac] $(-\alpha \otimes \beta) = (\alpha \otimes -\beta) = (-(\alpha \otimes \beta))$.

On this particular setting, the enabling condition is satisfied if α and β share a term and are positive.[85] Now, assume that α is $<x,y>$ and β is $<y,z>$ (or whatever is equivalent to

[80] For instance, the justification of a first-figure *Celarent*-syllogism "*nullus homo currit*, *omne animal rationale est homo, ergo nullum rationale currit*" is: "*Ratio a priori huius collectionis et cuiuscunque alius est, quia negatur identitas currentis et hominis et propterea negatur identitas currentis cum omni re, quae sit homo…*" (Hurtado, 1619a, 178) and the justification of a second-figure *Cesare*-syllogism "*nullus homo currit*, *omne brutum currit, ergo nullum brutum est homo*" is: "…*ratio, ob quam bene concludit, est, quia ponitur in minori identitas inter extremitatem alteram et medium, in majori autem negatur identitas eiusdem medii cum altera extremitate, unde infertur negatio utriusque extremitatis inter se.*" (Hurtado, 1619a, 186).

[81] Hurtado (1619a), 179.

[82] Hurtado (1619a), 181.

[83] Hurtado (1619a), 190: "*Ratio a priori, quia praemissae modi imperfecti vel habent aliquam modi pefecti, vel saltem illi equivalentem, vel ad quam fiat immediata conversio, habent enim obiectum realiter idem….*"

[84] Without this assumption, the following could happen: let α, β, and γ, be <a,b>, <b,c>, <a,d> respectively. Then $\alpha \bigotimes \beta$ =<a,c>, which is composible with <a,d> (the result being <c,d>). However, $\beta \bigotimes \gamma$ is undefined, since β and γ do not share a term.

[85] If both α and β are negative, $\alpha \bigotimes \beta$ will be undefined, hence $-(\alpha \bigotimes \beta)$ will also be undefined. If α is positive and β is negative, then $-\alpha$ and β will both be negative, hence $-\alpha \bigotimes \beta$ will be

these). Then $-\alpha$ will be $-$<x,y>, $-\beta$ will be $-$<y,z>, and $\alpha\otimes\beta$ will be <x,z>. In that case, $-\alpha\otimes\beta$ will be $-$<x,y>$\otimes$<y,z>, which is $-$<x,z>. Also, $\alpha\otimes-\beta$ will be <x,y>$\otimes-$<y,z>, which is $-$<x,z>. Finally, $-(\alpha\otimes\beta)$ will also be $-$<x,z>. Which is what [fac] claims (and which can be reformulated for all terms, provided that the enabling condition is satisfied).

Note that the algebraic properties of composition and negation correspond to certain algebraic properties of specific types of fractions, in particular, to the algebra of fractions that are assumed to be identical to their reciprocals, where the composition of propositions corresponds to the multiplication of the pairs of fractions, of which at least one is positive, and which enable cancellations. Another possibility is to relate the algebra of propositions to (a fragment of) the algebra of symmetric relations with the operation of relation composition defined on relations.

Finally, an extension of $\mathfrak{H}$'s algebra of propositions and, accordingly, other levels of $\mathfrak{H}$ can be outlined. The question is whether neutral and absorbing elements can be introduced in this algebra. The candidates, whose elementary[86] form is <x,x> and $-$<x,x>, will henceforth be called "triviality" and "inconsistency". Such propositions are introduced for purely speculative purposes, they are not discussed in the scholastic sources. The reason for that is the focus on syllogistic, the logic of inferences from two premises neither of which has such form. Introducing trivialities and inconsistencies generates some challenges and gives reasonable results only on some conditions.

First, the composition principles need to be loosened, for otherwise they would be undefined for trivialities and inconsistencies:

[comp$_p$*] (<x,y> $\otimes$ <y,z>) = <x,z>
[comp$_{n1}$*] ($-$<x,y> $\otimes$ <y,z>) = ($-$<x,z>)
[comp$_{n2}$*] (<x,y> $\otimes$ $-$<y,z>) = ($-$<x,z>).

The condition requiring that the terms must be different has been dropped and only the implicit requirement regarding positivity is preserved.

Second, [add] and [dis] must be replaced by:

[add*] If $\alpha=\beta$ and either ($\mathbf{a}\otimes\gamma$) or ($\beta\otimes\gamma$) is defined, then $(\alpha\otimes\gamma)=(\beta\otimes\gamma)$
[dis*] If $\alpha=\beta$ and either $(\alpha\otimes-\gamma)$ or $(\beta\otimes-\gamma)$ is defined, then $(\alpha\otimes-\gamma)=(\beta\otimes-\gamma)$.

Third, to reach interesting results, let us assume <t_m,t_n>$\in\Pi$ for every two terms t_m and t_n in T; together with the closure properties of Π, this guarantees that "triviality" and "inconsistency" can ultimately be viewed as constants. Starting with a particular triviality <x,x>, by the composition and conversion principles, we get:

(1) (<x,y> $\otimes$ <x,x>) = <x,y>
(2) (<x,y> $\otimes$ <y,y>) = <x,y>.

Since the selection of terms was arbitrary, trivialities are neutral in this limited sense. Furthermore, we get:

(3) (<x,x> $\otimes$ <x,x>) = <x,x>.

The problems start with the principle that, assuming that $\alpha\otimes\alpha$ is defined (i.e., if α is positive):

undefined. If α is negative and β is positive, then both α and $-\beta$ will be negative, hence $\alpha\otimes-\beta$ will be undefined. As a consequence, for the whole [fac] to be meaningful, α and β must share a term and be positive.

[86] The non-elementary cases are reducible to the elementary ones via [neg].

(4) $(<x,y> \otimes <x,y>) = <x,x>$

and

(5) $(<x,y> \otimes <x,y>) = <y,y>$.

That may be unexpected (unlike something along the lines of "$\boldsymbol{\alpha} \otimes \boldsymbol{\alpha} = \boldsymbol{\alpha}$"), but it is a consequence of how composition works. Also, to save both composition principles, we have to admit $<x,x>=<y,y>$. But similarly, we have:

(6) $(<x,z> \otimes <x,z>) = <x,x>$

(7) $(<x,z> \otimes <x,z>) = <z,z>$.

In which case we are forced to admit $<x,x> = <z,z>$. And with both $<x,x> = <y,y>$ and $<x,x> = <z,z>$, we have $<x,x> = <y,y> = <z,z>$. Given a sufficiently rich set of propositions in $\mathbb{H}$, this can be generalized, whence any two trivialities are identical and we can introduce "**1**" as an individual constant. The problem is that trivialities do not seem to be deliberately intersubstituable: contrast "$<x,z> \otimes <z,z>$" with "$<x,z> \otimes <y,y>$". The composition principles alone may not prohibit the latter, but they do not enable it either. However, [add*] (and, accordingly, [dis*]) can be generalized to fix the problem:

[add**] For a set of propositions $\Sigma = \{\boldsymbol{\sigma_1}, \sigma_2, \sigma_3, \ldots, \sigma_{\mathbf{n}}\}$ such that $\sigma_{\mathbf{i}} = \sigma_{\mathbf{j}}$ for every i,j from the interval [1,n], if $(\sigma_1 \otimes \gamma)$ or $(\sigma_2 \otimes \gamma)\ldots$ or $(\sigma_{\mathbf{n}} \otimes \gamma)$ is defined, then $(\sigma_1 \otimes \gamma) = (\sigma_2 \otimes \gamma) = \cdots = (\sigma_{\mathbf{n}} \otimes \gamma)$.

As a result, first, there is always *a* triviality whose composition with *a* member of Π is enabled by the composition principles in their present form.[87] Second, by [add**], if a proposition is composable with *a* triviality, it is composable with *any* triviality. To summarize, we can introduce the following properties of **1**:

[neutr$_1$] $\boldsymbol{\alpha} \otimes \boldsymbol{\alpha} = \mathbf{1}$

[neutr$_2$] $\mathbf{1} \otimes \mathbf{1} = \mathbf{1}$

[neutr$_3$] $\boldsymbol{\alpha} \otimes \mathbf{1} = \boldsymbol{\alpha}$.

As for $-\mathbf{1}$, the basic observations are:

[abs$_1$] $(\boldsymbol{\alpha} \otimes (-\boldsymbol{\alpha})) = -\mathbf{1}$

[abs$_2$] $-\mathbf{1} \otimes -\mathbf{1}$ is undefined (since inconsistencies are negative).

[abs$_1$] relates inconsistency to negation in an obvious way. Now, as before, we have:

(8) $(<x,y> \otimes -<x,y>) = (-<x,x>)$

(9) $(<x,y> \otimes -<x,y>) = (-<y,y>)$

(10) $(<x,z> \otimes -<x,z>) = (-<x,x>)$

(11) $(<x,z> \otimes -<x,z>) = (-<z,z>)$

resulting in $-<x,x> = -<y,y> = -<z,z>$, and ultimately into construing "$-\mathbf{1}$" as a constant.

Finally, if $\boldsymbol{\alpha} \otimes -\mathbf{1}$ is defined:

[abs$_3$] $\boldsymbol{\alpha} \otimes -\mathbf{1} = (-\boldsymbol{\alpha})$.

That may be surprising, but the product of $\boldsymbol{\alpha} \otimes -\mathbf{1}$ cannot be $-\mathbf{1}$, as that would violate the composition principles. As a result, $-\mathbf{1}$ is not absorbing.

Note that the principles governing triviality and inconsistency can be paired so that they instantiate the equivalence between "$(\boldsymbol{\alpha} \otimes \boldsymbol{\beta}) = \boldsymbol{\gamma}$" and "$(\boldsymbol{\alpha} \otimes (-\boldsymbol{\gamma})) = (-\boldsymbol{\beta})$". In particular: [neutr$_1$] is paired with [abs$_3$] and [neutr$_3$] is paired with [abs$_1$].

[87] By the same token, the substitution principle does not require revision.

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

On the level of the logic of identity, [neutr$_1$] translates into:

$$\frac{|\alpha|\,|\alpha|}{x = x}[\text{ne1}]$$

which, by idempotence, implies:

$$\frac{|\alpha|}{x = x}[\text{ne1}*].$$

Since the choice of "α" was arbitrary, this is equivalent to the reflexivity of identity:[88]

$$\frac{\emptyset}{x = x}[\text{refl}].$$

Similarly, we can show that:

$$\frac{x \neq x}{\perp}[\text{refl}*].$$

As already said, these principles do not translate into syllogistic, since they cover the "wrong" type of propositions and expressions.

3.3. Theory of validity and sentential logic. The general theory of validity of $\mathfrak{H}$ includes principles which must be extracted from different parts of the corpus. The most general ones include [lnc] and [lem].[89] The general theory of validity consists of [reg7], [reg8b], and [reg1].[90] This part of $\mathfrak{H}$ is not axiomatized, which, again, makes sense if the general theory of validity is merely an "a posteriori" reflection on syllogistic.[91]

Furthermore, the classification of argumentation mentions the so-called "dilemma", which (depending on the formulation)[92] corresponds to Hilbert's excluded middle[93] or (when generalized) to an instance of disjunction-elimination:[94]

$$\frac{\alpha \vdash \beta \qquad \sim \alpha \vdash \beta}{\beta}\,[\text{lem}_\text{h}]$$

$$\frac{\alpha \vee \beta \quad\; \alpha \vdash \gamma \quad\; \beta \vdash \gamma}{\gamma}\,[\text{la2i}].$$

88 A corresponding metaphysical principle introduced by Hurtado is that nothing is distinct from itself but only from something else: "*...distinctio est ad aliud, nihil enim distinguitur a seipso, sed ab alio...*" Hurtado (1619b), 397.

89 Cf. Hurtado (1619b), 327: "*Haec principia sunt quodlibet est vel non est, item impossibile est idem esse et non esse.*" Cf. Arriaga (1639), 938–939.

90 Hurtado (1619a), 196: "*Ex opposito consequentis infertur oppositum antecedentis*. (...) *Quando aliquid infertur ex consequenti, inferur etiam ex antecedenti*" and Hurtado (1624), 44: "*ex vero antecedenti non sequitur consequens falsum*". Arriaga says: "*ex vero numquam sequi falsum*" (1639, 39) and Oviedo says: "*ex vero numquam posse falsum inferri*" (1640, 36), but neither of them discusses [reg7] and [reg8b].

91 Arriaga and Oviedo even make an attempt to justify [reg1] by the logic of identity (Arriaga, 1639, 29 and Oviedo, 1640, 36).

92 Hurtado (1619a), 169: "*Dilemma est argumentum cornutum, sic vocatur, quia complectitur duas partes, quarum altera concessa necessario deprehenditur respondens, ut siquis vult convincere servum inobedientiae, sic argumentatur 'aut iuisti, quo ego tibi praecepi, aut non, si non iuisti, ergo mentiris, si iuisti, ergo vidisti Ioannem, qui ibi erat, ergo etiam mentiris', quia igitur respondens altera parte necessario petitur, vocatur argumentum cornutum, id est bicorne, aut bimembre.*"

93 Hilbert (1923), 153.

94 Gentzen (1935), 186.

The theory of equivalence describing expressions containing different logical operators as equivalent, since they say "the same thing", ipso facto validates the following principle as its most trivial instance:

$$\frac{\alpha}{\alpha}\,[\text{idem}]$$

and, explicitly, double negation:[95]

$$\frac{\sim\sim\alpha}{\alpha}[\text{dn}_{\text{U}}]$$

$$\frac{\alpha}{\sim\sim\alpha}[\text{dn}_{\text{D}}].$$

Finally, the texts contain virtually no sentential logic with the exception of [ci] establishing the correspondence between true conditionals and valid inferences[96] and double negation.[97] The texts only address the truth-conditions of compound sentences (in a standard way).[98]

§4. Concluding remarks. The situation in seventeenth- and eighteenth-century logic has repeatedly been presented in terms of the relations between logic and mathematics: non-scholastic authors tend to claim that logic is "in fact" a branch of mathematics, or at least should be adapted to match mathematics in its rigour.[99] To mention only the most obvious and famous instances: Leibniz formulated a set-theoretical view of the extensional aspects of syllogistic and an algebra of intensions and designed a diagrammatic representation for both.[100] The use of diagrams became a promising branch of logic, as witnessed by Weise-Lange's diagrams[101] or later by Euler.[102] Bernoulli contributed to a metatheory of Port-Royal logic (or a similar system),[103] theorized about a "parallelism" (that is, a morphism of a kind) between syllogistic and elementary algebra,[104] and about "the use of logic" in natural science, where he contrasted the natural-language reasoning via concatenated deductions to the mathematical approach of translating a physical problem into a set of mathematical equations and addressing the former by solving the latter.[105]

[95] This principle is an elementary problem of the theory of "equipollence", which analyses equivalent combinations of operators (typically negations, quantifiers, and modal operators); see, for instance Hurtado (1619a), 45 and 53 (*duae negationes affirmant*).

[96] See Hurtado (1619a), 44 ([*p*]*ropositio conditionalis* (...) *non solum est enunciativa, sed etiam illativa... nota illationis significat consequens necessario sequi ex antecedenti*), Arriaga (1639), 23, Compton (1649), 16 ([*c*]*onditionalis ea est, cuius partes uniuntur particula 'si' illative sumpta, denotando scilicet unum conditionate inferri ex alio*).

[97] One exception seems to be Arriaga's note on Morgan-equivalences: "...*contradictoria propositionis copulativae affirmativae, v.g. 'Petrus et Paulus currunt', est disiunctiva negativa, v.g. 'Petrus vel Paulus non currunt'*." Arriaga (1639), 24.

[98] "*Propositio hypothetica copulativa est vera, quando utraque categorica est vera, falsa vero, si vel utraque sit falsa, vel aliqua ex illis* (...) *Propositio disiunctiva non requirit, ut vera sit, veritatem utriusque, quia non affirmatur utrumque copulative, sed sufficit unius veritas*..." Arriaga (1639), 23. Cf. also Hurtado (1619a), 44–45, Compton (1649), 16.

[99] See Capozzi & Roncaglia (2008) and Mugnai (2010) for an overview.

[100] Leibniz (1903) and (1999).

[101] Weise & Lange (1712), 177–179.

[102] Euler (1770), 99–131.

[103] Bernoulli (1744a).

[104] Bernoulli (1685).

[105] Bernoulli (1744b).

https://doi.org/10.1017/S1755020318000357 Published online by Cambridge University Press

𝔅 and 𝔥 represent two trajectories in post-medieval scholastic logic, nearly contemporary to these early modern advancements. 𝔅 seems to offer a strong theory of validity and sentential logic, including interesting insights which may result into a non-classical or substructural logic. The syllogistic is viewed merely as *a* branch of logic based on a general theory of validity and a specific view of instantiation. While, at least from the modern point of view, the first feature of syllogistic seems to be an advantage, the second seems to be a drawback: since the theory of instantiation is closely tied to a specific analysis of language, it leaves little room for improvement.[106] The problem with this analysis, as becomes more and more obvious in the course of the seventeenth century, is the tendency to analyse scientific statements as "SaP"-statements. Such a traditional "Aristotelian" approach seems tenable as long as the paradigm of scientific explanation is "the planets are near and what is near does not twinkle; therefore, the planets do not twinkle". Once physics becomes "quantitative", such analysis becomes obsolete, for the "SaP"-form does not seem to capture the logical structure of even the elementary equations (not to mention geometrical proofs with their specifics), and the development of mathematics makes things ever worse. 𝔥, on the other hand, has no general theory of validity (only an "a posteriori" one), which seems to be a drawback, especially as syllogistic becomes the central part of logic. However, the syllogistic of 𝔥 is ultimately an algebraic theory describing the formal properties of certain operations defined for structured objects. This emphasis has three advantages. First, it motivates an algebraic approach to logic by focusing on the formal properties of certain operations. Second, the semantics of propositions is not directly related to the "suppositional" analysis of sentential meaning. Third, there are some hints of a set-theoretical analysis of properties. The emancipation from the suppositional view of scientific statements makes the possible extension to the semantics of equations more natural. Clearly, such an extension requires a rigorous set-theoretical language, which will not be available for almost three centuries yet.

The need for a set-theoretical language is, in fact, shared by the late scholastic and early modern mathematization attempts, should they move past the analogies between (syllogistic) reasoning and the operations of elementary algebra.[107] However, this observation needs to be qualified. The situation of logic in the seventeenth century is similar to the situation of mathematics at that time. In mathematics, the seventeenth century is an era of two linguistic revolutions, as both analytic geometry and calculus were invented (not to mention other developments), the gist of both revolutions being the translation of geometrical insights into an algebraic language.[108] The seventeenth-century situation of set-theoretical language is similar and to say that sets and functions were as yet to be discovered overlooks the advancements already available. By the end of the seventeenth century, both sets and functions will have had their geometry figured out, the former by the technique of representing sets (both in extension and in intension) by diagrams, the latter by graphical representation of change as a correlation between physical parameters available since Oresme and the reception of the Merton school.[109] A proper formulation of this position seems to be "between geometry and algebra".

[106] Note that while these theories justify the rules of instantiation, they are not strictly speaking theories of quantification, but specific analyses of language for which "SxP" is the elementary form of sentences including (most saliently) relational ones.

[107] Note that 𝔥 is algebraic in a more general sense: it addresses the formal properties of certain operations rather than simply using them.

[108] Kvasz (2008) and (2013).

[109] Oresme (2010) and Blaise of Parma (1486).

§5. Acknowledgments. This publication is an output of the project "Between Renaissance and Baroque: Philosophy and Knowledge in the Czech Lands within the Wider European Context", registration number 14-37038G, supported by The Czech Science Foundation (GAČR) and realized at the Institute of Philosophy of the Czech Academy of Sciences. I would like to thank Ansten Mørch Klev for valuable comments on the first draft of this paper and to all referees for their helpful criticism.

BIBLIOGRAPHY

Ashworth, E. J. (1974). *Language and Logic in the Post-Medieval Period*. Dordrecht: Reidel.

Ashworth, E. J. (2016). The post-medieval period. In Dutilh Novaes, C. and Read, S., editors. *The Cambridge Companion to Medieval Logic* (first edition). Cambridge: Cambridge University Press, pp. 166–191.

Ashworth, E. J. (2017). Buridan and his successors on "ex impossibili sequitur quodlibet". In Grellard, C., editor. *Miroir de l'amitié. Mélanges offerts à Joël Biard* (first edition). Paris: Vrin, pp. 243–252.

Arriaga, R. de (1639). *Cursus philosophicus*. Paris: Jacques Quesnel.

Báñez, D. (1599). *Institutiones minoris dialecticae quas summulas vocant.* Salamanca: Andreas Renaut.

Bernoulli, J. (1685). *Paralellismus ratiocinii logici et algebraici.* Basil: Johan Conrad a Mechel.

Bernoulli, J. (1744a). Centum positionum philosophicarum cento. In Bernoulli, J., editor. *Opera* (*tom. 1*). Geneva: Cramer, pp. 175–191.

Bernoulli, J. (1744b). Methodus ratiocinandi sive usus logicae in praeclaro aliquo phaenomeno physico enodando. In Bernoulli, J., editor. *Opera* (*tom. 1*). Geneva: Cramer, pp. 251–276.

Blaise of Parma. (1486). *Questiones super tractatu De latitudinibus formarum*. Padua: Mathaeus Cerdonis.

Broadie, A. (1985). *The Circle of John Mair. Logic and Logicians in Pre-Reformation Scotland*. Oxford: Oxford University Press.

Buridan, J. (1976). *Tractatus de consequentiis*. Louvain: Publications univertsitaires.

Capozzi, M. & Roncaglia, G. (2008). Logic and philosophy of logic from humanism to Kant. In Haaparanta, L., editor. *The Development of Modern Logic* (first edition). Oxford: Oxford University Press, pp. 78–158.

Comas del Brugar, M. (1661). *Quaestiones minoris dialecticae*. Barcelona: Antonius Lacavalleria.

Compton Carleton, T. (1649). *Philosophia universa.* Antwerpen: Jacob Meursius.

Dutilh Novaes, C. (2008). Logic in the 14th century after Ockham. In Gabay, D. M. and Woods, J., editors. *Handbook of the History of Logic* (first edition), Vol. 2, Mediaeval and Renaissance Logic. Amsterdam: Elsevier, pp. 433–504.

Euler, L. (1770). *Lettres à une princesse d'Allemagne sur divers sujets de physique et de Philosophie* (*tom. 1*). Mietau, Leipzig: Steidel.

Gentzen, G. (1935). Untersuchungen über das logische Schließen I. *Mathematische Zeitschrift*, **39**, 176–210.

Heider, D. (2014). *Universals in Second Scholasticism. A comparative study with focus on the theories of Francisco Suárez S. J.* (*1548–1617*)*, João Poinsot O. P.* (*1589–1644*) *and Bartolomeo Mastri da Meldola O. F. M. Conv.* (*1602–1673*)*/Bonaventura Belluto O. F. M. Conv.* (*1600–1676*). Amsterdam: John Benjamins Publishing Company.

Hilbert, D. (1923). Die logischen Grundlagen der Mathematik. *Mathematische Annalen*, **88**(1–2), 151–165.
Hurtado de Mendoza, P. (1619a). *Disputationes in universam philosophiam a Summulis ad Metaphysicam pars prior*. Mainz: typis et sumptibus Ioannis Albini.
Hurtado de Mendoza, P. (1619b). *Disputationes in universam philosophiam a Summulis ad Metaphysicam pars posterior*. Mainz: typis et sumptibus Ioannis Albini.
Hurtado de Mendoza, P. (1624). *Universa philosophia*. Lyon: Louis Prost.
Knebel, S. K. (2011). *Suarezismus, Erkenntnistheoretisches aus dem Nachlass des Jesuitengenerals Tirso González de Santalla* (*1624–1705*). *Abhandlung und Edition*. Amsterdam: B. R. Grüner.
Kvasz, L. (2008). *Patterns of Change*. Basel: Birkhäuser.
Kvasz, L. (2013). *Zrod vedy ako lingvistická udalost: Galileo, Descartes a Newton ako tvorcovia jazyka fyziky* [*The Scientific Revolution as a Linguistic Event. Galileo, Descartes, and Newton as Creators of the Language of Physics*]. Praha: Filosofia.
Leibniz, G. W. (1903). De formae logicae comprobatione per linearum ductus. In Couturat, L., editor. *Opuscules et fragments inédits de Leibniz. Extraits des manuscrits de la Bibliothèque royale de Hanovre* (first edition). Paris: Presses Universitaires de France, pp. 292–321.
Leibniz, G. W. (1999). *Generales inquisitiones de analysi notionum et veritatum.* In Schepers, H., Schneider, M., Biller, G., Franke, U., and Kliege-Biller, H., editors. *Sämtliche Schriften und Briefe. Sechste Reihe, Philosophische Schriften, Vierter Band* (first edition). Berlin: Akademie Verlag, pp. 739–788.
Oresme, N. (2010). *Questiones super Geometriam Euclidis*. Stuttgart: Steiner.
d'Ors, A. (1998). Ex impossibili quodlibet sequitur (Domingo Báñez). *Medioevo*, **24**, 177–217.
d'Ors, A. (1983). Las Summulae de Domingo de Soto. *Anuario Filosófico*, **16**(1), 209–217.
Mugnai, M. (2010). Logic and mathematics in the seventeenth century. *History and Philosophy of Logic*, **31**(4), 297–314.
Oviedo, F. de (1640). *Cursus philosophicus*. Lyon: Pierre Prost.
Paul of Venice, (1990). *Logica Magna, Part II, Fascicule 4. Secunda Pars, Capitula de Conditionali et de Rationali.* Oxford: Oxford University Press.
Poinsot, J. (1638). *Cursus philosophici Thomistici pars prima*. Cologne: Constantini Mϋnich Bibliopolae.
Read, S. (1993). Formal and material consequence, disjunctive syllogism and gamma. In Jacobi, K., editor. *Argumentationstheorie: Scholastische Forschungen zu den logischen und semantischen Regeln korrekten Folgerns* (first edition). Leiden: Brill, pp. 233–259.
Read, S. (2015). Medieval theories: Properties of terms. In Zalta, E. N., editor. *The Stanford Encyclopedia of Philosophy* (*Spring 2015 Edition*). Available at: https://plato.stanford.edu/archives/spr2015/entries/medieval-terms/.
Risse, W. (1964/1970). *Die Logik der Neuzeit*, 2 vols. Stuttgart: Frommann.
Soto, D. De (1529). *Summule*. Burgis: in officina Joannis Junte.
Soto, D. De (1554). *Summule*. Salamanca: Andreas à Portonariis.
Weise, C. & Lange, J. C. (1712). *Nucleus logicae Weisianae*. Giessen: Henning Müller.

DEPARTMENT FOR THE STUDY OF ANCIENT AND MEDIEVAL THOUGHT
INSTITUTE OF PHILOSOPHY OF THE CZECH ACADEMY OF SCIENCES
JILSKÁ1, 110 00 PRAGUE 1
CZECH REPUBLIC
E-mail: hanke@flu.cas.cz